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A possible explanation of the light curve of comet Encke

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Summary. We have constructed a theoretical model for the brightness of comet Encke as a function of heliocentric distance, R, using conservation of solar energy, and assuming that C_2 molecules are evaporated at the same rate as water molecules. An equation relating the total number of C_2 molecules in the cometary coma, and the total visual magnitude, m_V, was used to produce a graph of m_V versus $\log R$, as a function of surface albedo, A.

For comparison with the theory, we have compiled all brightness observations of comet Encke, made during this century by reliable and persistent observers. The final magnitudes adopted for the coma were those of Beyer, obtained during the apparition of 1937, 1947, 1951, 1960 and 1970. Besides these visual observations, photoelectric measurements by Mianes were used to normalize the brightness curve. For the nuclear magnitudes, measurements by Van Biesbroeck and Roemer were selected, which behaved as R^{-2}.

The final derived values were: for the nuclear radius, $r_N = 0.80 \pm 0.10$ km; for the Bond albedo, $A_{\rm bond} = 0.77 \pm 0.02$; for the ratio of C_2 molecules to H_2O molecules, $\alpha = (8.2 \pm 2.2) \times 10^{-3}$. In order to fit the observations to the theory, the solution for a fast rotating nucleus had to be adopted. A rotating nucleus is expected, since several recent comets have shown to be rotating. The value for the number of H_2O molecules evaporated per second, measured in 1970 by Bertaux *et al.* as $(3.1 \pm 0.9) \times 10^{27}$ molecule s^{-1}, lies completely within the error of our theoretical prediction of $(4.8 \pm 2.6) \times 10^{27}$ molecule s^{-1}, providing a check on our model. The standard deviation of one visual measurement of the total magnitude with respect to the theoretical curve, is ± 0.15 mag.

Introduction

Comet Encke is a periodic comet having the shortest known period (3.3 yr) and the largest number of observed apparitions (51). Considering these facts, it is surprising that its light curve has not been studied in detail to derive information about the comet's physical structure. Many visual and photographic observations of it have been published in the literature, extending into the last century, and thus can be combined to give a good idea of its brightness behaviour around its orbit.

In this work we have collected the relevant brightness observations, and constructed a model for the vaporization of the cometary nucleus which fits the observations over their total range. We will first describe the model and then the observations.

The vaporization model

DETERMINATION OF THE SURFACE TEMPERATURE OF THE NUCLEUS

We will assume that the nucleus of the comet is spherical and composed mainly of water snow with other volatiles in smaller proportions. These volatiles are evaporated at the same rate as water, as proposed by Delsemme & Miller (1971) and Delsemme & Rud (1973). Several models are available to describe the vaporization of such a snowball (Delsemme & Miller 1971; Lebofsky 1975). All of them start with the energy conservation equation for a spherical body illuminated by the Sun:

$$\frac{S}{R^2}(1-A)\cos\theta\cos\phi=(1-A_{\mathrm{IR}})\sigma T_{\mathrm{N}}^4+L\dot{E}+Q \tag{1}$$

where S = solar constant in erg cm^{-2} s^{-1} (at 1 AU); R = distance of the comet to the Sun; A = surface albedo; θ = local hour angle; ϕ = latitude of the area considered; A_{IR} = albedo of the surface in the infrared; σ = Stefan–Boltzmann constant; T_{N} = temperature of the nuclear surface at the point (θ, ϕ); L = latent heat of sublimation of the snow; $\dot{E}$ = evaporation rate of the snow (in g cm^{-2} s^{-1}); and Q = surface heat flow of the nucleus.

The term on the left hand side of the equation, represents the energy absorbed from the Sun. The first term on the right is the energy radiated back into space. The second term on the right, is the energy carried away by the molecules, and the last term, Q, is the energy conducted by the nuclear material towards the interior.

Several things are to be noted in equation (1). First, we assume that the obliquity of the nucleus is zero. This simplifies the equation considerably. However, equation (1) can be solved for the fast rotating nucleus or for a non-rotating nucleus. An indication of the influence of the obliquity can be obtained by comparing both solutions, since the non-rotating solution corresponds to obliquity equal to 90°, with the axis pointing to the Sun. We will see that the influence is not negligible. Second, note that the evaluation of the term Q is difficult. Very little is known about the thermal properties of the cometary nucleus. In order to proceed, we will assume that this energy is much smaller than the other two. This assumption can be tested by comparison with the observations. Our results will show the negligible influence of this parameter. Third, note that A is the surface albedo, not the Bond albedo. Equation (1) can be integrated over the cometary nucleus to give a mean solution with effective temperatures, in which case A is the Bond albedo. However, we have preferred to solve equation (1) exactly. In this case it is necessary to know the surface reflectivity as a function of angle of incidence of the light, $A(i)$. Fortunately this information is available. Cook, Flanklin & Palluconi (1973) have presented this data for several types of water snows.

For $\dot{E}$ we have used the value given by several authors (Delsemme & Miller 1971; Lebofsky 1975).

$$\dot{E}=P(\mu/2\pi PR_{\mathrm{G}}T_{\mathrm{N}})^{1/2} \tag{2}$$

where P = vapour pressure in dyn cm^{-2}; μ = molecular weight; and R_{G} = gas constant.

As for the vapour pressure of ice at low temperatures, we have used the result of Jancso, Pupezin & Van Hook (1970). They give an updated value for this quantity which had not been measured since 1915, and indicate that the usual handbook values contain between them, a number of contradictions and even make some outright errors. The formula given by Jancso *et al.* is

$$\log P(\text{Torr}) = -2481.60/T + 3.5721988 \log T - 3.097203 \times 10^{-3}\, T - 1.7649 \times 10^{-7}\, T^2 + 1.901973. \quad (3)$$

The solution of equation (1) can now be easily achieved if T is considered to be the independent variable instead of R. We can then solve for the quantity

$$\frac{R^2}{(1-A)\cos\theta\cos\phi} = \frac{\pi R^2}{[1-A(\phi)]\cos\phi} \text{ (fast rotating nucleus),} \quad (4)$$

$$= \frac{R^2}{[1-A(\phi)]\cos\phi} \text{ (non-rotating nucleus).} \quad (5)$$

Now by iteration, and with $A(\phi)$ given by a graph similar to Fig. 2 or 3 of Cook *et al.* (1973), it is possible to find a ϕ which satisfies equations (4) or (5) for a given R. The result is a graph of T_N versus ϕ, as a function of R, giving the temperature distribution over the nuclear surface. Several of these curves are presented in Fig. 1.

We have selected this more complicated procedure of taking into account the reflectivity of the surface, in an attempt to obtain the correct evaporation rate over the nucleus. The necessity of this procedure becomes apparent when we plot the vapour pressure P, as a

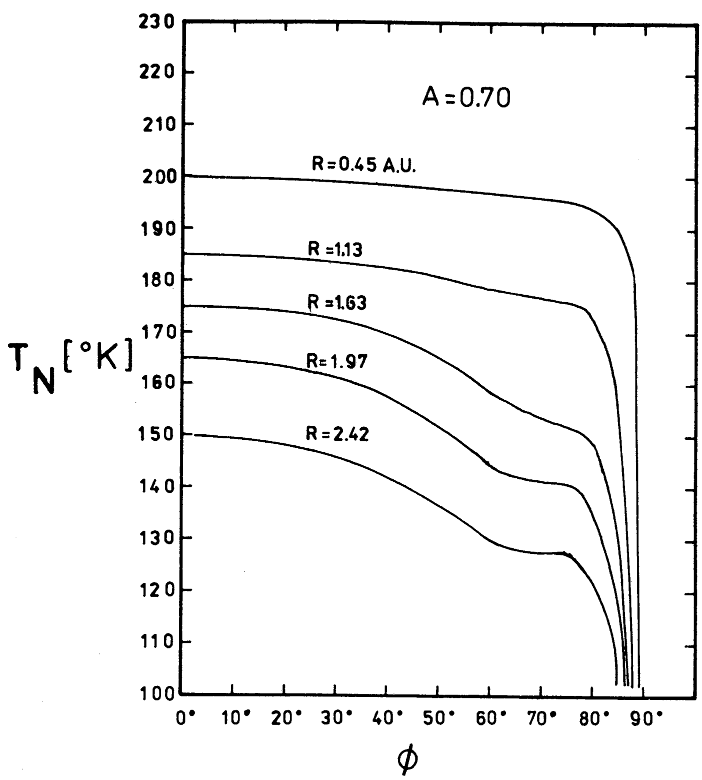

Figure 1. Temperature of the nuclear surface as a function of the latitude, for a fast rotating nucleus, for several values of the heliocentric distance, R. At large R, the temperature varies widely over the nucleus. At small R, it is almost independent of ϕ. This is due to the cooling of the sub-solar region by the evaporating molecules. This solution assumes an obliquity of zero for the nucleus.

function of T. This function is very steep, implying that a small error in the surface temperature of the nucleus produces a large error in the number of evaporated molecules (confirmation, equation 3). Of the several curves for $A(i)$ given by Cook *et al.* (1973) we have selected the one that corresponds to type III snow. However the evaporation rate is not so sensitive to the reflectivity at large ϕ since the temperature is lower, the evaporating surface is smaller (due to the high latitudes) and consequently the contribution to the total evaporation becomes less at large ϕ. So we could have selected their type I or II snow without changing the results by much.

Delsemme & Miller (1971) have pointed out that the latent heat of evaporation, L, is a function of T. For water ice, however, this change is small. We will present evidence that shows that the vaporization does not allow the temperature to change abruptly. In fact, in the range of $R = 0.4$–1.5 AU, the observed range for comet Encke, T goes from $T = 199$ to 165 K, respectively. In view of such a small change of T, and thus of L, we have adopted $L = \text{constant} = 11\,600\,\text{cal mol}^{-1}$. The albedo in the infrared, A_{IR}, is a more problematic parameter. Ney (1974) has estimated values of $A_{IR} = 0.18 \pm 0.2$ and 0.25 ± 0.1 for comets Kohoutek (1973f) and Bennett (1970II), respectively, while O'Dell (1971) gives 0.3 ± 0.15 for the same comet Bennett. Delsemme (1973) has obtained a value of $A_{IR} = 0.2$ for comet Tako–Sato–Kosaka. An unweighted mean of these values gives $A_{IR} = 0.23$, which we have adopted.

Fig. 1 presents T versus ϕ for a fast rotating nucleus, for several values of R. It confirms our suspicions: at large heliocentric distances the temperature varies widely over the nuclear latitude, and this variation introduces a large range in the vapour pressure. From this

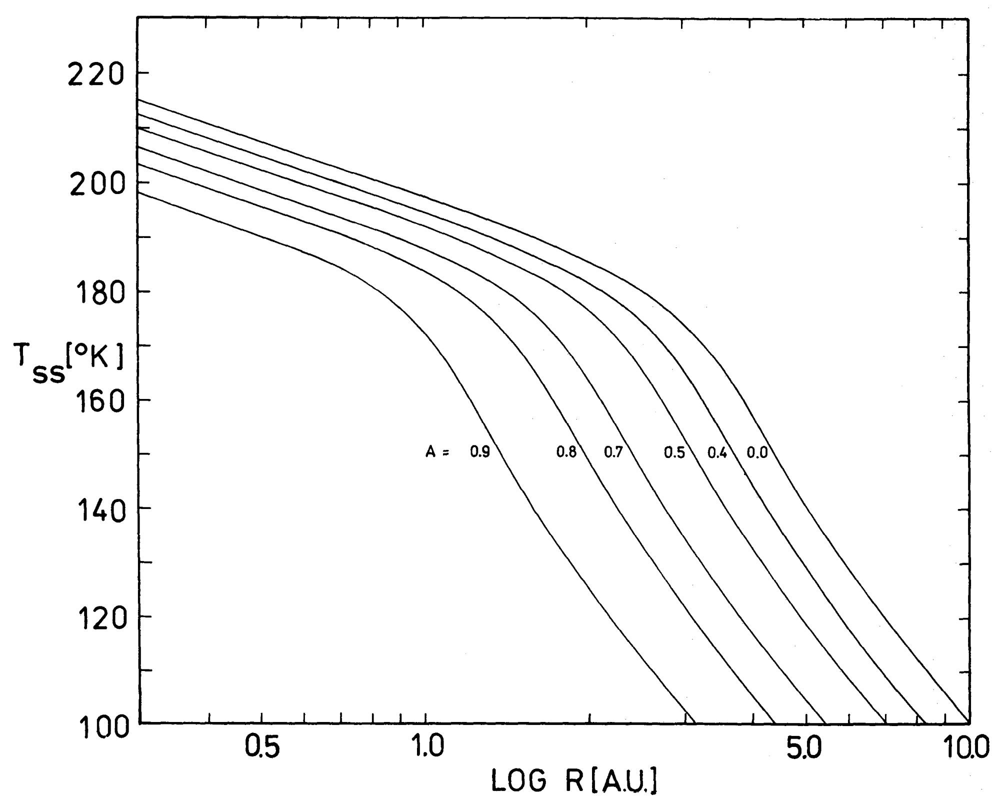

Figure 2. The subsolar temperature of the nucleus; T_{SS}, as a function of heliocentric distance, R. The change in the derivative observed between 1.0 and 2.0 AU is due to the passage from a radiating regime, to an evaporating regime. This solution corresponds to the fast rotating nucleus and if R is multiplied by $\pi^{1/2}$, it corresponds to the non-rotating nucleus. Thus the non-rotating nucleus gets hotter than the fast rotating one, for the same heliocentric distance.

figure we see the difficulty of defining a mean or effective temperature. All that can be defined is a sub-solar temperature, T_{ss}. Note also that at small heliocentric distances, the temperature levels-off as a function of ϕ. This is, of course, due to the effect of the evaporating molecules cooling the sub-solar region.

Fig. 2 shows the change in the sub-solar temperature as a function of R, for several surface albedos. The change in the derivative observed for these curves, between 1.0 and 2.0 AU, is due to the passage from a radiative regime at large heliocentric distances to an evaporating regime at small heliocentric distances. From equations (4) and (5) it can be seen that if T_{ss} is reached at a distance R_{FR} for the fast rotating nucleus, then the same temperature is achieved for the slow rotating nucleus at a distance $R_{SR} = \sqrt{\pi}\, R_{FR}$, that is, farther from the Sun.

DETERMINATION OF THE MAGNITUDE

From the theory given in the next section, we will obtain the total number of C_2 molecules in the cometary coma. We need an equation relating this quantity to the total visual magnitude. This equation has been derived by many authors, without complete agreement. Recently a precise result has been given by A'Hearn & Cowan (1975), which takes into account the distribution of molecules in the upper and lower vibrational levels. They found

$$\log N_{tot}(C_2) = 12.93 + \log L(\Delta v = +1) + 2 \log R \tag{6}$$

where N_{tot} = total number of molecules of C_2

L = luminosity in the $C_2 (\Delta v = +1)$ band, in erg s^{-1}.

Before we transform this equation into magnitudes, note that the V-filter of the UBV system measures the C_2 bands $\Delta v = -2$, -1 and 0. This is clear from a comparison of the transmission of the V-filter as given by Allen (1963) and the spectrum of comet Kohoutek reported by A'Hearn (1975). As a result of this, the above formula requires modification. If the spectral range of observation is increased, as in the case of the V-filter, the measured value of L will increase. However, this must not increase the value for the total number of molecules, N_{tot}. Consequently, the constant in the equation must decrease by an amount equivalent to the increase in $\log L$. The change can be made easily. A'Hearn & Cowan (1975) took a downward transition probability of $P(\Delta v = +1) = 0.21$. We have to replace this value for the new one, which should take into account the appropriately weighted bands transmitted by the V-filter. The probability of transition for the C_2 sequence can be obtained from the relative emission band fluxes found by A'Hearn (1975). Since these are molecular properties they should not change with the comet or with solar distance. The new value for P is

$$\int P V_\lambda \, d\lambda = \bar{P} = 0.54 \tag{7}$$

where V_λ is the transmission of the V-filter as given by Allen (1963). The replacement of this new value for the old one gives

$$\log N_{tot}(C_2) = 12.52 + \log L_V + 2 \log R. \tag{8}$$

The luminosity L_V can be transformed into observed flux F_V using

$$L_V = 4\pi\Delta^2 F_V \tag{9}$$

where Δ = distance from the comet to the Earth in cm. The flux is now converted into magnitudes with the help of an equation given by Allen (1963)

$$m_V = -2.5 \log F_V - 13.74. \tag{10}$$

The final result is

$$m_V = -2.5 \log N_{\mathrm{tot}}(\mathrm{C_2})/R^2 + 86.18, \tag{11}$$

where we have set $\Delta = 1$, since we are using absolute magnitudes for the comet. The theoretical values of the V-magnitude thus obtained, will be compared directly with the visual measurements, since these have been normalized to photoelectric V-magnitudes obtained by Mianes (1958).

The magnitudes given in equation (11) are due exclusively to the C_2 molecules, and not to dust. Dust has not been taken into account in the total magnitude because no continuum spectrum for this comet has ever been observed (Swings 1948; Swings, Fehrenbach & Woszczyk 1957; Malaise 1961).

DETERMINATION OF THE TOTAL NUMBER OF MOLECULES EVAPORATED

The total number of molecules evaporated is now given by:

$$\begin{aligned} N_{\mathrm{evap}}\ [\text{number s}^{-1}] &= \dot{E} N_{\mathrm{A}}/\mu \\ &= C_1 P N_{\mathrm{A}}/\sqrt{\mu T} \end{aligned} \tag{12}$$

where $C_1 = 0.05831$; P = vapour pressure in Torr; N_{A} = Avogadro's number.

Equation (12) has to be integrated over the whole surface. For the case of the fast rotating nucleus, we find with r_{N} = nucleus radius

$$N_{\mathrm{evap}}^{\mathrm{FR}} = \frac{C_1 N_{\mathrm{A}} 4\pi r_{\mathrm{N}}^2}{\sqrt{\mu}} \int_{\phi=0}^{\pi/2} \frac{P[T(\phi)]}{\sqrt{T(\phi)}} \cos\phi \, d\phi. \tag{13}$$

A similar equation can be obtained for the case of the non-rotating nucleus. Next we will assume that the C_2 molecules are evaporated at a rate proportional to the rate of H_2O molecules. This has been proposed by Delsemme & Rudd (1973) and we will confirm their result. Thus

$$N(\mathrm{C_2}) = \alpha\, N(\mathrm{H_2O}). \tag{14}$$

The total number of C_2 molecules in the cometary atmosphere is now

$$N_{\mathrm{tot}}^{\mathrm{FR}}(\mathrm{C_2}) = \alpha\, N_{\mathrm{evap}}^{\mathrm{FR}} \tau \tag{15}$$

with τ = lifetime of the C_2 molecules. The lifetime can be determined if we know the scale-length for the decay of C_2 in the cometary coma, $L(R)$, and the ejection velocity of these molecules. It has been pointed out by Swings (1948) that the size of the coma of comet Encke varies inversely with R^2. This result has been confirmed in quantitative form by Delsemme & Moreau (1973) for the case of comet Bennett. For the ejection velocity of the molecules we will assume that thermal velocities apply. Thus

$$\tau = L_0 R^2 \sqrt{\frac{\pi\mu}{8 R_G \bar{T}}} \tag{16}$$

where $L_0 = L(R = 1\ \mathrm{AU})$ and $\bar{T}$ = mean temperature of ejection. For L_0 we have adopted a value of 7.3×10^4 km, which is near to the mean value given by Kumar & Southall (1976) for several comets. It can be shown that the error is very small if we set $\bar{T} = T_{\mathrm{ss}}$. Then from equations (11), (13), (15) and (16) we get

$$m_{V\text{-obs}} = m_{V\text{-the}} - 2.5 \log \alpha\, r_{\mathrm{N}}^2, \tag{17}$$

$$m_{V\text{-the}} = C - 2.5 \log \frac{L_0 I}{\sqrt{\bar{T}}}, \tag{18}$$

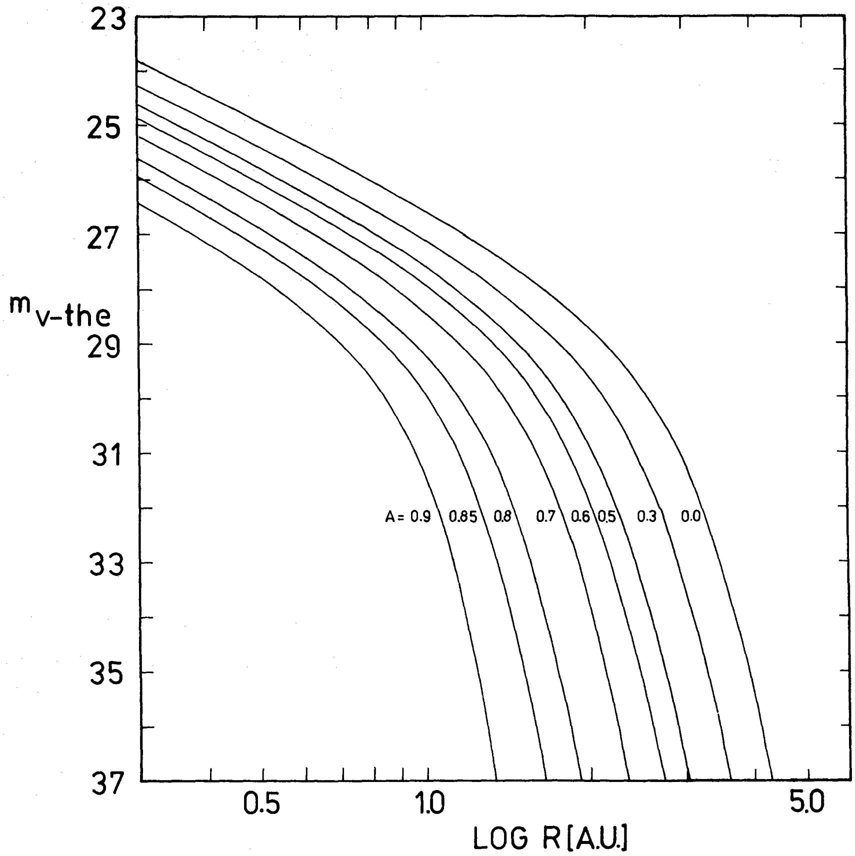

Figure 3. Theoretical magnitudes in the visual, as a function of R, for the fast rotating nucleus, for several values of the surface albedo A. At large R, all curves become very steep, with the derivative increasing with R and A. This graph shows the futility of trying to fit the magnitudes with a formula that is linear in $\log R$.

where $m_{V\text{-obs}}$ = observed visual magnitude; $m_{V\text{-the}}$ = theoretical visual magnitude; I = value of the integral in equation (13); and $C = 39.37$. Theoretical magnitudes are therefore obtained from equation (18). The resulting curve, as a function of R, can be directly compared with the observations, but is displaced downward by $-2.5 \log \alpha r_{\mathrm{N}}^2$, as seen from equation (17).

Fig. 3 shows theoretical visual magnitudes as a function of R, for different values of the parameter A. It can be seen that all the cases show curvature resulting from the transition of the radiation regime to the evaporation regime. At large heliocentric distances, all curves become very steep, with the derivative increasing with R and A. For comets that behave in this fashion, Fig. 3 shows the futility of fitting the magnitudes with a formula that is linear in the m_V versus $\log R$ graph.

DETERMINATION OF A, A_{Bond}, r_{N} AND α

The theoretical magnitude curves can be compared with the observations to derive a value of the albedo A. The fit is rather sensitive and is obtained by vertically displacing the $m_{V\text{-the}}$ versus $\log R$ curves with respect to the $m_{V\text{-obs}}$ versus $\log R$ curve. In this fashion not only a value for A is obtained, but also a value for $(m_{V\text{-the}} - m_{V\text{-obs}})$. The value of A so obtained is the surface albedo which must not be confused with the Bond albedo, A_{Bond}. Cook *et al.* (1973) have related both quantities by appropriately integrating the surface reflectivity over all emergent directions. In their notation A is the diffuse reflectivity

The integration depends on the reflectivity curve adopted for the snow. Type I snow corresponds to new snow. Type II snow corresponds to wind packed snow. Type III snow corresponds to rain crust, settling snow and surface hoar. The reflectivity curves of types I and III are very similar. Type III has been adopted in our work. The factors by which the albedo A has to be multiplied in order to get the Bond albedo are, respectively, 0.95, 0.89, and 0.93.

The value of A found will also depend on the solution adopted, that for a fast rotating nucleus, or that for a non-rotating nucleus. In the case of comet Encke, only the fast rotating nucleus fitted the observations, so this will be the case adopted.

Having obtained A_{Bond} in the above manner, we can get a value for $A_{\text{Bond}}\pi r_{\text{N}}^2$ by using a formula given by Delsemme & Rud (1973) and valid for the magnitude of the nucleus when no coma is observed

$$\log A_{\text{Bond}}\pi r_{\text{N}}^2 = \log \pi + 0.4\,\Delta m + 2\log(r\Delta) + \log[q/\phi(\alpha)]$$
$$\Delta m = m_{\text{sun}} - m_{V\text{-obs}}(\text{nucleus}), \tag{19}$$

where q = phase integral and $\phi(\alpha)$ = phase function.

Note that the third term on the right is zero, since we are using absolute magnitudes. We will also set the fourth term equal to zero since Sekanina (1976) has shown that the nucleus of comet Encke does not show any magnitude variation with phase angle. It is then simple to obtain r_{N} since

$$r_{\text{N}} \equiv \sqrt{\frac{10^{\log_{10} A_{\text{Bond}}\pi r_{\text{N}}^2}}{\pi A_{\text{Bond}}}}. \tag{21}$$

The value of the parameter α is also simply obtained with the help of equation (17), once r_{N} is known.

The light curve

Many observations of the magnitude of comet Encke are available from this and the last century. In order to avoid any heterogeneity of the data, we have considered only observations from this century obtained by persistent and experienced observers. The three most continuous observers of this comet have been G. Van Biesbroeck, E. Roemer and M. Beyer. The reliability of our light curve is strengthened by a remarkable series of photoelectric observations carried out by Mianes (1958). When all the available information is combined, the light curve can be defined in the range of 0.4–4.1 AU. No observations are available after perihelion.

The procedure for obtaining the light curve will be considered in detail elsewhere (Ferrín & Guzman 1980), but in brief it involves the following steps. We started by reducing the observations of Mianes (1958) carried out in the V-system and with diaphragms of different sizes. These are therefore partial magnitudes which correspond to the 1957 apparition. They can be converted to total magnitudes (when the diaphragm is much larger than the comet) by a procedure similar to the one followed by Svoren & Tremko (1975). The total visual magnitudes so obtained have been plotted in Fig. 6 (crosses). Next we compiled all the observations by Van Biesbroeck (1924–62) published in the *Astronomical Journal* from the beginning of this century. When the observations were reduced it was apparent that there was a large dispersion in them amounting to more than one magnitude. There was no convincing way of superimposing the different apparitions of this comet, even if corrections due to the instrument were applied. Moreover, in the range of the photoelectric observations

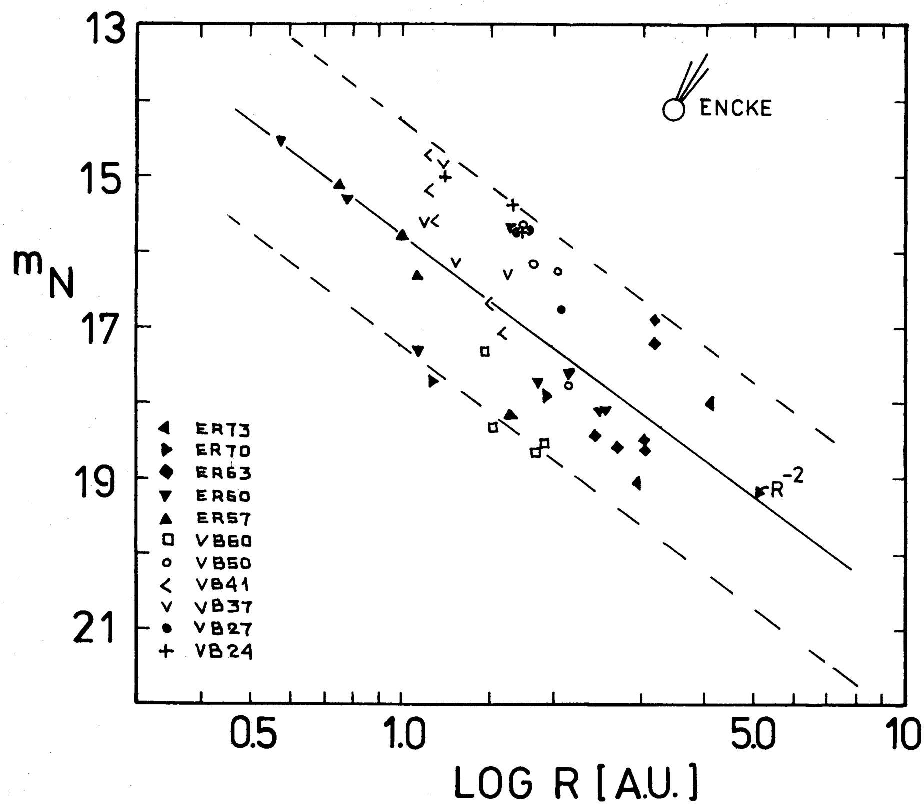

Figure 4. Nuclear photographic magnitudes for comet Encke, as a function of log R. These are observations by Van Biesbroeck and Roemer, selected for their R^{-2} behaviour. The R^{-2} behaviour (solid line) indicates that we are observing a solid object, with no atmosphere. The least-squares solution gives a slope of 1.996 and an absolute nuclear magnitude in the blue of 15.75. The large dispersion of these measurements indicates their difficulty.

of Mianes, the mean magnitude and the derivative of the two curves were different. It was, then clear that the photographic observations of Van Biesbroeck were taken mostly with blue plates (Roemer 1965) and thus isolated the CN-bands and not the C_2-bands. These observations were therefore not used. However, the m versus log R plot permitted the selection of the nucleus magnitudes, usually denoted by m_2, for which no coma is observed. These observations are plotted in Fig. 4. Observations by Roemer published in the *Astronomical Journal* were reduced in the same manner. These referred mostly to the nucleus and are presented also in Fig. 4. This figure shows a large dispersion, but a least-squares fit of this data gives a solution very near to an R^{-2} behaviour, implying that we are observing a solid object with no atmosphere. This was the criterion for selecting the nuclear magnitudes. Keep in mind that these are blue magnitudes.

The only source of visual magnitudes for comet Encke is the observations of Beyer, carried out during the apparitions of 1937, 1947, 1951, 1960 and 1970 (Beyer 1938, 1950, 1955, 1962, 1972). It was apparent that the observations of Mianes for 1957 and those of Beyer for 1951 and 1960 agreed to within 0.09 mag. The derivative of the m versus log R diagram was also the same for the two sets of data, photoelectric and visual, even for different apparitions. However, the apparitions of 1937, 1947 and 1970 had to be displaced vertically by small amounts in order to agree with those of 1951, 1957 and 1960 (see Fig. 5). This vertical displacement may be interpreted as resulting from the ageing of the comet, or as resulting from the use of different instruments by the observer. We will not try to interprete this difference. What is important to notice here is that visual and reliable photoelectric

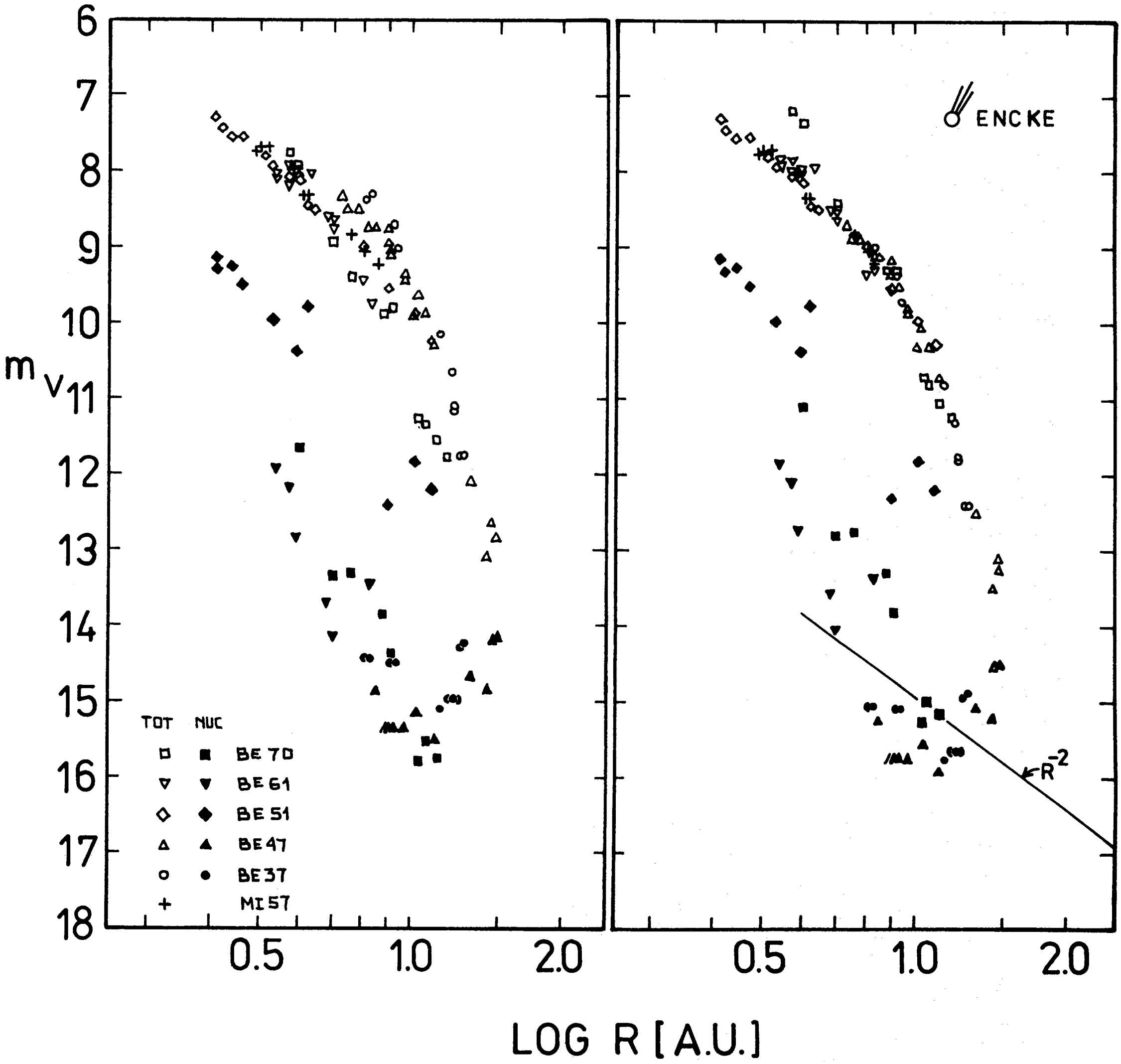

Figure 5. Measurements of the total and nuclear magnitude of comet Encke made by Beyer, for the apparitions of 1937, 1947, 1951, 1960 and 1970. The left side shows these measurements with no vertical correction. On the right side vertical displacements of +0.65, +0.39, 0.0, −0.09 and −0.55, respectively have been applied to the above apparitions. Note the improvement in the light curve. Crosses indicate photoelectric *V*-filter observations of this comet carried out by Mianes (1958), for the 1957 apparition. The agreement between the two sets of data is excellent.

observations for the apparitions of 1951, 1957 and 1960 agree to within 0.09 mag in absolute value as well as in the derivative. Thus we decided to normalize all the observations to the 1951 apparition. Fig. 5 shows all the observations by Beyer before and after this reduction has been made. For the apparitions of 1937, 1947, 1951, 1960 and 1970, the vertical corrections applied in magnitudes were +0.65, +0.39, 0.0, −0.09 and −0.55. The observational curve due to Beyer is remarkable in the small dispersion of the observations. Notice also the curvature of the light curve.

Beyer has also observed nuclear magnitudes. These have been plotted with solid dots in Fig. 5. From a comparison of Figs 4 and 5, it becomes clear that most of the 'nuclear' magnitudes by this observer refer to the inner part of the coma, and not to the nucleus. The real nucleus begins to appear at 15th magnitude and large distances, and is defined by the R^{-2} law. Fig. 6, with our final adopted magnitudes, makes this point clearer. Between $R = 0.6$ and $R = 1.7$ AU, the nuclear magnitudes are still contaminated by the coma. Only

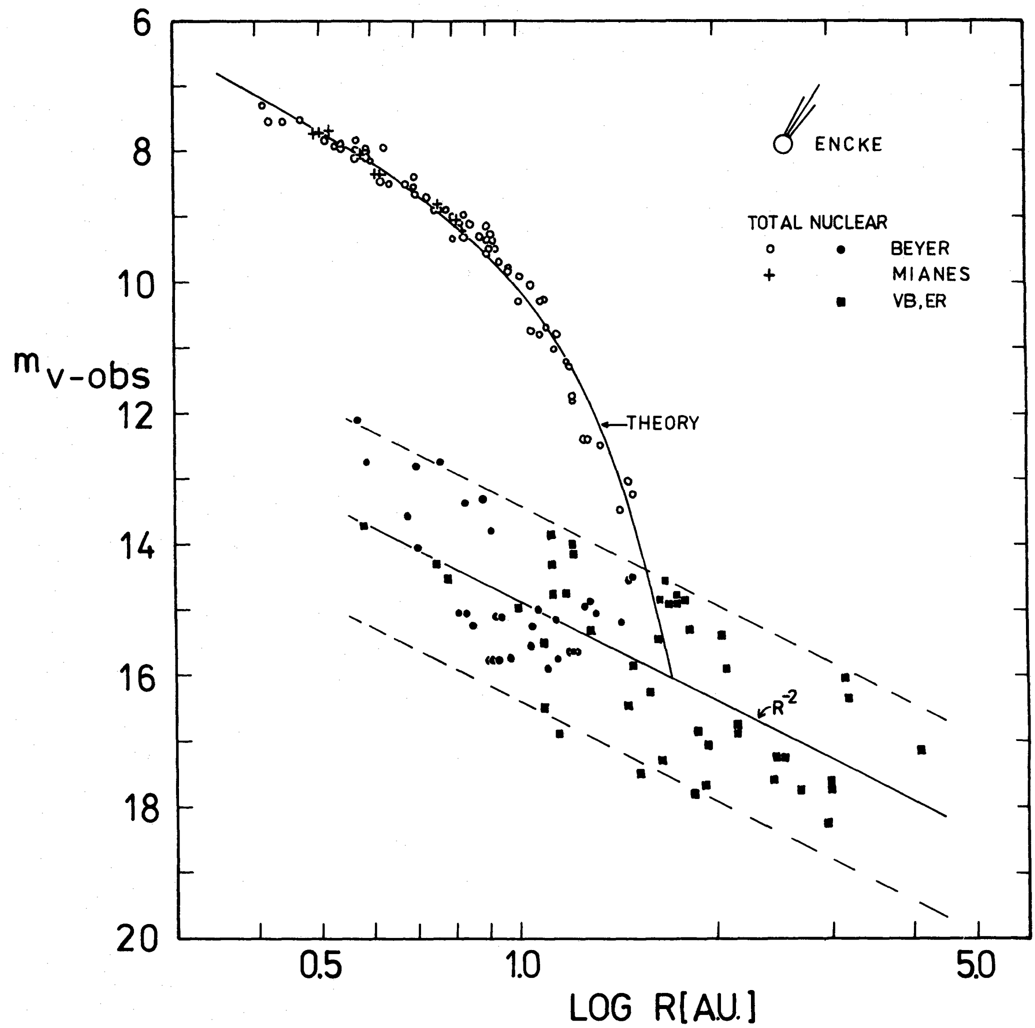

Figure 6. Comparison between the final light curve, and the theoretical light curve. The agreement is excellent. The standard deviation of one visual measurement from the theoretical curve is ± 0.15 mag. The better fitting curve corresponds to a fast rotating nucleus of radius 0.80 km, and Bond albedo 0.77.

at $R = 1.7$ AU is the nucleus observed without a coma. Note that to observe the nucleus at $R = 0.6$ AU is really very difficult, since the nuclear magnitude is approximately 14, while the magnitude of the coma is about 8, a factor of 250 in intensity! Notice however that some of the nuclear magnitudes by Beyer, those below 14.9 mag, do seem to refer to the nucleus.

For the total magnitudes, the final adopted curve is shown in Fig. 6. We mentioned before, that the nuclear magnitudes of Fig. 4 were blue magnitudes. If we want to use the nuclear magnitudes to derive the size of the nucleus, we need to transform them to visual magnitudes. This could be easily accomplished if a reflectivity curve of the nucleus were available. However, this information does not exist. In view of this we will use the reflectivity curve of the continuum, several of which are available. For example Babu (1976) gives the spectral distribution of the continuum of comet Kohoutek 1973f, and discusses the continuum of other comets. He concludes that the energy curve of this comet deviates only slightly from that of the Sun. Since this seems to be the rule, we will adopt a G2-V spectrum for the continuum.

Using the information provided by Allen (1963) for the solar continuum and the sensitivity curve of the 103-0 plate (Anonymous 1973) it is then possible to calculate that a blue sensivite plate would show a nucleus 0.85 mag fainter than a V-sensitive plate. For comparison the mean $B-V$ colour index for Saturn's rings given by Cook *et al.* (1973) is 0.84. So the above value is not unreasonable. According to this result all nuclear magnitudes of Fig. 4 have been moved upward by 0.85 mag and replotted in Fig. 6. An additional indication that this amount is approximately correct is given by the fact that some of the faintest visual magnitudes found by Beyer shown in Fig. 5, then coincide with the nuclear magnitudes. This again testifies to the excellent quality of the measurements of this extraordinary observer. The least-squares fit to the nuclear observations gives $m_{N\text{-Blue}}$ $(\Delta = 1.0) = 15.75$. When the above correction is applied we get m_{N-V} $(\Delta = 1.0) = 14.9$.

Results

Fig. 6 presents our theoretical curve for the magnitude. The agreement between theory and observations is excellent. In fact the standard deviation of one measurement from the theoretical curve over the range of the total magnitudes, from 7.3 to 13.5, is only ± 0.15 mag. This again testifies to the excellence of Beyer as an observer.

No systematic trend of deviation is apparent. The nuclear magnitudes show a dispersion of ± 1.5 mag, indicating the difficulty of these measurements at low light levels. We next consider the values of each parameter that enter into this model.

Comparing Figs 3 and 6, a value of $A = 0.81 \pm 0.02$ has been obtained. Since this corresponds more closely to type I snow, a value of $A_{\text{Bond}} = 0.77 \pm 0.02$ is found. By comparison $A_{\text{Bond}} = 0.63$ for Saturn's rings (Cook *et al.* 1973). Our result is in agreement with the fact that no dust or continuum has been observed for this comet (Swings 1948; Swings *et al.* 1957; Malaise 1961). Any contamination by dust in Saturn's rings does not have any mechanism of removal, but the expanding gases of the cometary coma may have carried away small dust particles, leaving a continuum-free comet. Delsemme & Rud (1973) found a Bond albedo of 0.66 ± 0.13 for comet Bennett, a comet that had a very strong continuum. Our value should thus be larger than theirs, as it is. Finally note that the value found, $A_{\text{Bond}} = 0.77$, equals $(1 - A_{\text{IR}})$. From physical considerations these two values should not be very different. Our result is thus quite reasonable.

Using this information and equation (21), we obtain $r_{\text{N}} = 0.80 \pm 0.10$ km, a value in agreement with that suggested by O'Dell (1976) using an albedo of 0.96. Our more complex theory has been able to reduce the albedo to more reasonable levels. For comparison Delsemme & Rud (1973) found values of $r_{\text{N}} = 2.20$ and $r_{\text{N}} = 3.76$ km for comets Tago–Sato–Kosaka and Bennett, respectively. Since these are much brighter comets than Encke, our value makes some sense.

Using the above r_{N} and $(m_{V\text{-the}} - m_{V\text{-obs}}) = 19.30 \pm 0.05$, equation (17) provides us with a value for $\alpha = (8.2 \pm 2.2) \times 10^{-3}$. Delsemme (1975) gives a value of $\alpha = 1.2 \times 10^{-3}$ obtained using information provided by Arpigny (1965). When we looked into the original reference it turned out that Arpigny had used the same observation by Beyer and an equation similar to our equation (18). Since our calculation takes into account the distribution of C_2 molecules over the upper and lower levels and a more precise treatment of the cometary coma, we consider our result more precise than his.

A check of our theory is fortunately possible. The number of H_2O molecules in the coma of comet Encke has recently been derived by Bertaux, Blamont & Festou (1973). Using *OGO-5* satellite observations taken in 1970 April, they measured a production rate of $(3.1 \pm 0.9) \times 10^{27}$ molecule s^{-1}, when the comet was at 0.715 AU from the Sun. When we calculate this quantity from our theory, we obtain $(8.1 \pm 2.6) \times 10^{27}$ molecule s^{-1}, at the same distance from the Sun, this value refering to the observations of 1951. Beyer's measurements indicate that in 1970 the comet was 0.55 mag fainter, or a factor of 1.66. Then our result for 1970 is $(4.8 \pm 2.6) \times 10^{27}$ molecule s^{-1}. The value of Bertaux *et al.* (1973) lies completely within our error.

Finally, note that our results indicate a fast rotating nucleus. There was no solution for the non-rotating nucleus that could fit the observations. This is hardly suprising. Whipple (1950) had predicted such a rotating nucleus for comet Encke, from a consideration of the change in the time of passage through perihelion for different apparitions. Also direct evidence of rotation comes from the non-gravitational parameters derived by Marsden (1972) has given in interpretation of them in terms of rotating nucleus. More recently Larson & Minton (1972) made photographic observations of comet Bennett 1970II, that revealed a rotating coma. By considering the changing pattern of spiral-shaped jets, they found a period of 1.4–1.5 day. A much shorter period of only 4.621 ± 0.004 hr was found by Whipple (1978) in the case of comet Donati, and of 5.17 ± 0.01 hr by Fay & Wisniewski (1978) for comet D'Arrest. Rotation of cometary nuclei seems to be a rather common occurrence, and seems to be the case for comet Encke.

Consideration of the nucleus of a comet rotating with a finite period, can best be made in the manner shown by Dobrovol'skij & Markovich (1972). They have considered a comet rotating with a period of 6 hr, not very different from the values quoted above. They found a lag of the maximum surface temperature compared to the maximum of solar radiation, of 1/16 of the period. It is unlikely that such a small lag would effect our results by much. A more complicated model to take into account this effect and an obliquity different from zero, is actually being developed.

Conclusions

(1) We have devised an alternative method to the one by Delsemme & Rud (1973) which allows the determination of the nuclear radius of comet Encke, given the light curve. The method also gives the nuclear albedo and the ratio of C_2 molecules to H_2O molecules, α. We found values of $A_{\text{Bond}} = 0.77 \pm 0.02$, $r_N = 0.80 \pm 0.10$ km, and $\alpha = (8.2 \pm 2.2) \times 10^{-3}$. The theoretical value for the total number of H_2O molecules evaporated per second contains within its error, the value found observationally by Bertaux *et al.* (1973), providing a nice check of our theory.

(2) Our calculations support the idea (Delsemme & Miller 1971; Delsemme & Rud 1973) that water controls vaporization of cometary molecules for this comet.

(3) We find a scale-length of C_2 that varies inversely with the square of the heliocentric distance, a conclusion first reached by Delsemme & Moreau (1973) for the case of comet Bennett.

(4) The theoretical brightness curves for a non-rotating nucleus are shifted toward larger values with respect to the fast rotating nucleus. We have been able to find a fit to the observations only if the nucleus is fast rotating.

(5) Our model used visual observations of Encke made by Beyer during several apparitions. If proper corrections are applied, these observations are very reliable. In fact in the final adopted solution, the observations had a standard deviation of ± 0.15 mag with respect to the theoretical curve. We believe that this is evidence of the excellency of Beyer as an observer of visual magnitudes.

(6) The final solution supports the idea of a dust-free nucleus, with very little conduction of energy toward the interior, with an obliquity near to zero degrees, evaporating C_2 molecules at a rate proportional to the evaporation rate of H_2O molecules, and with thermal velocities of expansion for the C_2 molecules.

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