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A Possible Interpretation of the New Event in the Cosmic Ray Experiment. II

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In a previous paper we have analysed a new event observed by Niu and others in cosmic ray experiments, and have been able to present a reasonable interpretation of it on the basis of the new (modified) Nagoya model for urbaryons. Anticipating the accumulation of such new events, we shall give a further study in this paper, especially about the details of weak decay processes which were not fully developed in the previous work.

§ 1. Introduction

We begin with a brief summary of a previous work (referred to as I).[1] A new strongly interacting particle has been observed in the nuclear jet shower.[2] Its decay life time has been reasonably understood using the usual theory of weak interactions, in a manner reminiscent of the history of the discovery of the strange particles. But it has been found impossible to incorporate the new particle in the strangeness scheme, so we have assumed it to be a particle possessing a second type of strangeness. Thus we have ascribed the existence of this fourth quantum number (after baryon number, charge and the usual strangeness of hadron physics) to that of a new entity, which we have regarded as the fourth urbaryon p' in the new Nagoya model advocated by the Sakata school.[3]

In this paper we shall proceed to some detailed study extending the earlier work. As only a single event has so far been observed, it might be premature to pursue the theory further at this stage. But the study could afford some help in the experimental detection of such new particles. Our aim is to clarify the weak decay modes, branching ratio and decay probability of these new particles, none of which was fully examined in I.

§ 2. The weak interaction

According to the new Nagoya model and to the extended ace-quark assignment for urbaryons (p, n, λ, ζ),[4] we shall take the baryon and the meson to be the composite system of three urbaryons and of an urbaryon and an anti-urbaryon, respectively, and assume the framework of $U(3)\times U(1)$ symmetry. Then the low lying states of baryons (B) and mesons (M) with $n_\zeta \leq 1$ will be expressed as

$$B\sim(N_8+\varDelta_{10})+(X_{3^*}+Y_6)$$

and

$$M\sim P_8+V_8+(Z_3+Z_{\bar{3}}),$$

where X, Y and Z are the 3*-plet, hexaplet of Π_F and the 3-plet Π_M respectively shown in Table I of paper I.

Now we shall consider the weak interaction. The weak current of urbaryons is ascribed to the leptonic current:

$$j_\alpha=(\bar{e}\nu_e)_\alpha+(\bar{\mu}\nu_\mu)_\alpha\,, \tag{2·1}$$

where $(\bar{a}b)_\alpha=(\bar{a}\gamma_\alpha(1+\gamma_5)b)$, and is given by

$$\begin{aligned} J_\alpha\equiv\langle j_\alpha\rangle_B&=(\bar{n}p)_\alpha\cos\theta+(\bar{\lambda}p)_\alpha\sin\theta\\ &\quad-(\bar{n}\zeta)_\alpha\sin\theta+(\bar{\lambda}\zeta)_\alpha\cos\theta \\ &\equiv\bar{b}\Gamma_\alpha\widetilde{W}b \quad \text{and} \quad \Gamma_\alpha=\gamma_\alpha(1+\gamma_5), \end{aligned} \tag{2·2}$$

where b denotes the urbaryon as the 4-component spinor:

$$b\equiv\begin{pmatrix} p\\ n\\ \lambda\\ \zeta \end{pmatrix},$$

and W means the current generating operator:

$$\widetilde{W}=W+W',$$

$$W\equiv\begin{pmatrix} 0 & \cos\theta & \sin\theta & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{for } \varDelta n_p=+1$$

and

$$W'\equiv\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & -\sin\theta & \cos\theta & 0 \end{pmatrix} \quad \text{for } \varDelta n_\zeta=+1\,.$$

$\widetilde{W}$ has the following property:

$$\{\widetilde{W},\widetilde{W}^+\}=1\,,$$

$$[\widetilde{W},\widetilde{W}^+]=2Q-N_B\,.$$

Then we obtain the CP-invariant semi-leptonic and non-leptonic weak interaction at the urbaryon level as follows:

$$H_w^{\text{S.L}}=\frac{G}{\sqrt{2}}J_\alpha^+ j_\alpha+\text{h.c.}$$

and

$$H_w^{\text{N.L}}=\frac{G}{\sqrt{2}}J_\alpha^+ J_\alpha, \qquad (2\cdot 3)$$

where G is the usual Fermi constant.

Now we shall estimate the decay probability of the new particles for various channels. We want to obtain the effective weak interaction in terms of the hadron starting from Eq. (2·3). But unfortunately we have no reliable method for deriving an explicit form of interaction, especially in the Lorentz space. Accordingly we shall proceed empirically. In fact there are found some phenomenological regularities in the semi-leptonic and non-leptonic decays of hadrons. The coupling strength of the effective Hamiltonians of these decays falls in the restricted value $\sim 10^{-7}$ (in the units $\hbar=c=m_\pi=1$), especially when we take the vector or axial-vector type of source function for fermions, and the derivative type for mesons.[5] Hence we shall accept the above regularity of the Hamiltonian as a standard in the following analysis. Of course the values thus estimated might not be so reliable, but if the experimental data to be accumulated in the future largely deviate from our estimation, the analysis will be still useful to obtain the method of modification more concretely.

We shall assume Π_F and Π_M to be of spin 1/2 and 0 respectively, in accordance with the lowest lying level of usual hadrons and take the following effective Hamiltonians, for each decay process:

A) Semi-leptonic decays

i) $\Pi_F\to B_8+l+\nu_l$ is described by the effective Hamiltonian

$$H=\frac{G_0}{\sqrt{2}}\{\bar{\psi}_B\gamma_\alpha(F_V+F_A\gamma_5)\psi_{\Pi_F}\}\{\bar{\psi}_l\gamma_\alpha(1+\gamma_5)\psi_{\nu_l}\}+\text{h.c.} \qquad (2\cdot 4)$$

This is the usual Fermi interaction and so the most convincing form of Hamiltonian. The decay has the possibility of giving us very interesting information because the decay probability is large and the effect of the (time-like) form factor may be remarkable, due to the large Q-value, while our calculation is performed without the form factor.

ii) $\Pi_F\to B_{10}+l+\nu_l$ is the three-body decay which finally reduces to the four-body decay, the Hamiltonian of which has no correspondence in the usual strange particle decay. To obtain the qualitative estimate, we shall calculate the decay rate from the following Hamiltonian,

$$H=\frac{G_0}{\sqrt{2}}\{\bar{\psi}_{B\alpha}(F_V+F_A\gamma_5)\psi_{\Pi_F}\}\{\bar{\psi}_l\gamma_\alpha(1+\gamma_5)\psi_{\nu_l}\}+\text{h.c.} \qquad (2\cdot 5)$$

The four-body decay can occur not only through the mediation of decuplet baryons but also by direct interaction. We have also calculated the decay rate due to the latter, but the result is not significantly different.

iii) $\Pi_M \to l+\nu_l$ is the process similar to those of π_{l_2} and K_{l_2}, and the Hamiltonian is given by

$$H=i\frac{G_0}{\sqrt{2}}\bar{\psi}_l\gamma_\alpha(1+\gamma_5)\psi_{\nu_l}\partial_\alpha\phi_{\Pi_M}+\text{h.c.} \tag{2·6}$$

iv) $\Pi_M \to P_8+l+\nu_l$ and $\to V_8+l+\nu_l$ are the processes analogous to K_{l_3} and K_{l_4}, respectively. The latter gives a four-body decay which is also derived from the direct interaction. The Hamiltonians assumed are, for the former process

$$H=\frac{G_0}{\sqrt{2}}(f_1P_\alpha{}^{\Pi_M}+f_2P_\alpha{}^P)\phi_P\phi_{\Pi_M}\{\bar{\psi}_l\gamma_\alpha(1+\gamma_5)\psi_{\nu_l}\}+\text{h.c.}, \tag{2·7}$$

where P^{Π_M} and P^P denote the 4-momentum of Π_M and P_8 respectively, and for the latter process

$$H=\frac{G_0}{\sqrt{2}}\phi_{V\alpha}\cdot\phi_{\Pi_M}\{\bar{\psi}_l\gamma_\alpha(1+\gamma_5)\psi_{\nu_l}\}+\text{h.c.} \tag{2·8}$$

(B) Non-leptonic decays

i) $\Pi_F \to B_8+P_8$ corresponds to the non-leptonic decay of usual hyperons and its Hamiltonian is taken as

$$H=i\frac{G_0}{\sqrt{2}}\bar{\psi}_B\gamma_\alpha(F_V+F_A\gamma_5)\psi_{\Pi_F}\partial_\alpha\phi_P+\text{h.c.}, \tag{2·9}$$

ii) $\Pi_F \to B_{10}+P_8$ and $\to B_8+V_8$ give the three-body decays for which there is no analogy in the usual hadron decays. We assume the following Hamiltonians,

$$H=i\frac{G_0}{\sqrt{2}}\bar{\psi}_{B\alpha}(F_V+F_A\gamma_5)\psi_{\Pi_F}\partial_\alpha\phi_P+\text{h.c.} \tag{2·10}$$

and

$$H=i\frac{G_0}{\sqrt{2}}\bar{\psi}_B\gamma_\alpha(F_V+F_A\gamma_5)\psi_{\Pi_F}\phi_{V\alpha}+\text{h.c.}, \tag{2·11}$$

respectively.

iii) $\Pi_M \to P_8+P_8$ is an analogue of $K\to 2\pi$ decay and the Hamiltonian is taken as

$$H=i\frac{G_0}{\sqrt{2}}(\phi_P\partial_\alpha\phi_\Pi-\phi_\Pi\partial_\alpha\phi_P)\partial_\alpha\phi_P+\text{h.c.} \tag{2·12}$$

iv) $\Pi_M \to V_8+P_8$ is a new process, the Hamiltonian of which is assumed to be

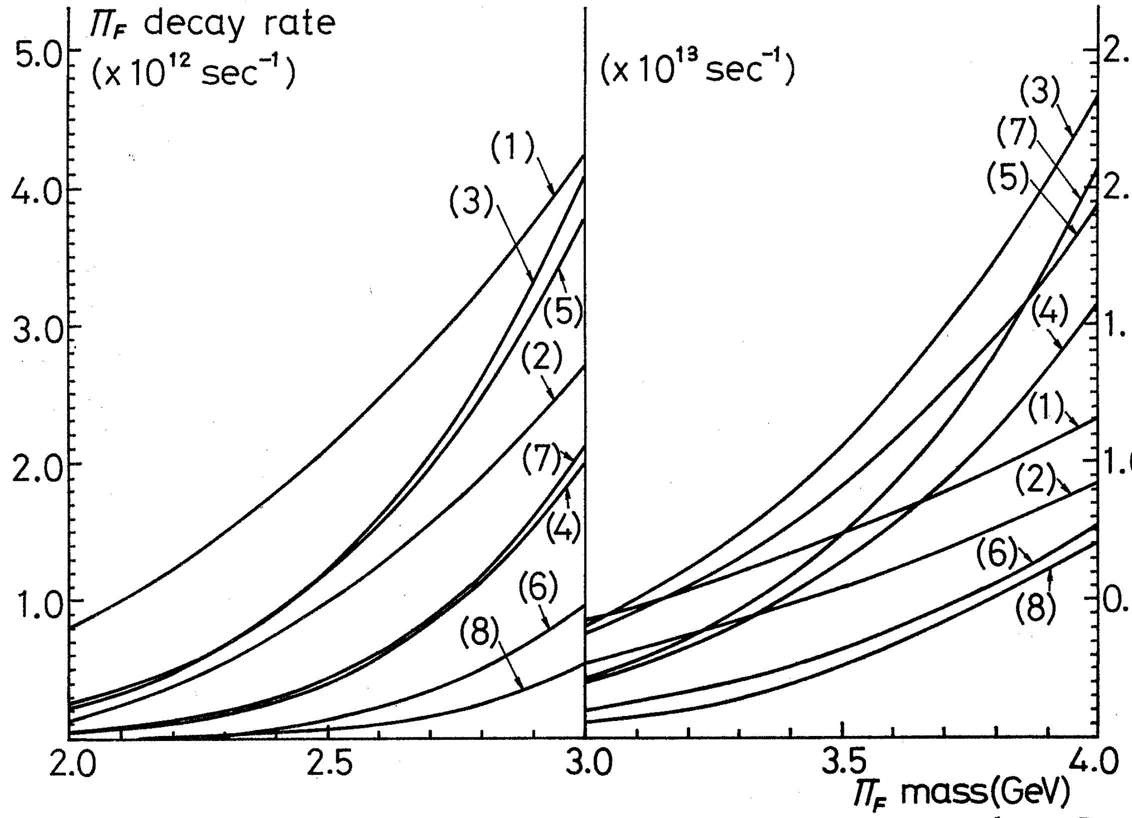

Fig. 1. The rates for the Π_F decays; $\Pi_F \to B_8+P_8$, $\Pi_F \to B_8+l\nu_l$, $\Pi_F \to B_{10}+P_8$ and $\Pi_F \to B_{10}+l\nu_l$. (1) $\Pi_F \to N\pi$, (2) $\Pi_F \to \Xi\eta$, (3) $\Pi_F \to Ne\nu_e$, (4) $\Pi_F \to \Xi\mu\nu_\mu$, (5) $\Pi_F \to \Delta\pi$, (6) $\Pi_F \to \Omega\eta$, (7) $\Pi_F \to \Delta e\nu_e$, (8) $\Pi_F \to \Omega\mu\nu_\mu$.

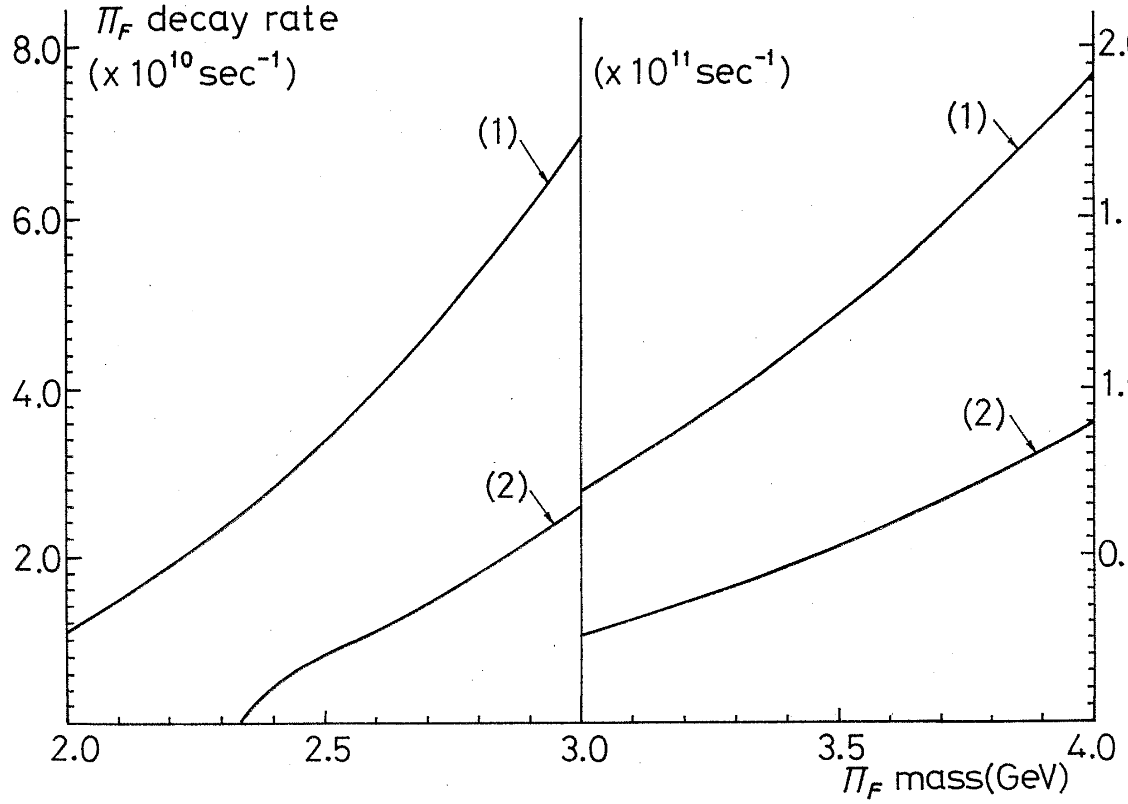

Fig. 2. The rates for the Π_F decays; $\Pi_F \to B_8+V_8$. (1) $\Pi_F \to N\rho$, (2) $\Pi_F \to \Xi\phi$.

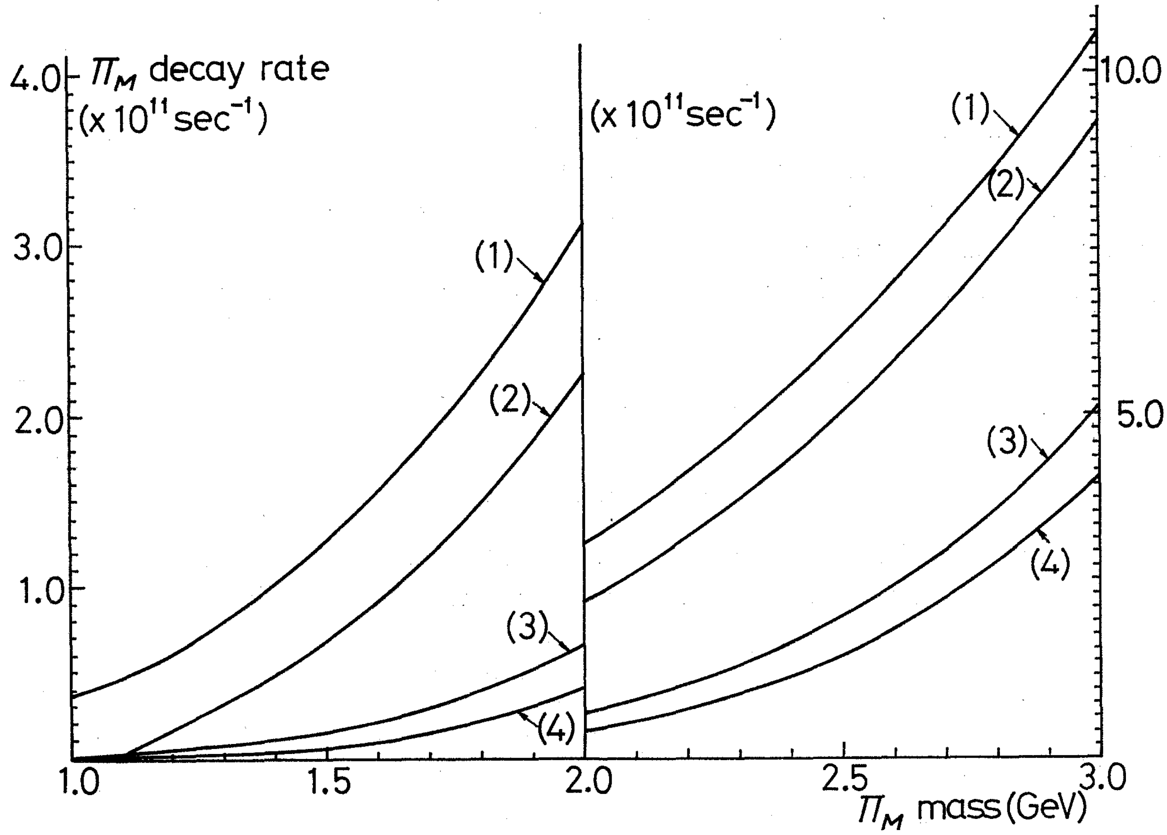

Fig. 3. The rates for the Π_M decays; $\Pi_M \to P_8 + P_8$ and $\Pi_M \to P_8 + l\nu_l$. (1) $\Pi_M \to \pi\pi$, (2) $\Pi_M \to \eta\eta$, (3) $\Pi_M \to \pi e \nu_e$, (4) $\Pi_M \to \eta \mu \nu_\mu$.

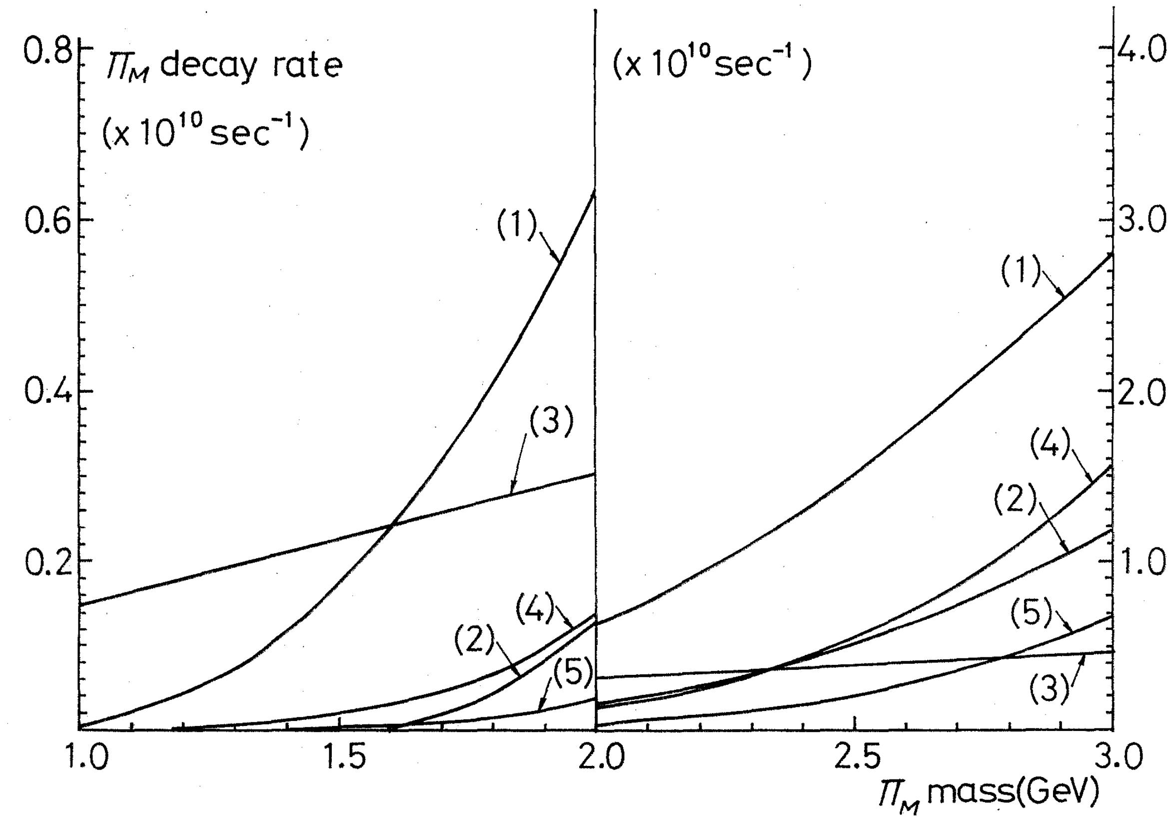

Fig. 4. The rates for the Π_M decays; $\Pi_M \to P_8 + V_8$, $\Pi_M \to l\nu_l$ anp $\Pi_M \to V_8 + l\nu_l$.
(1) $\Pi_M \to \pi\rho$, (2) $\Pi_M \to \eta\phi$, (3) $\Pi_M \to \mu\nu_\mu$, (4) $\Pi_M \to \rho e \nu_e$, (5) $\Pi_M \to \phi\mu\nu_\mu$.

$$H=\frac{G_0}{\sqrt{2}}(\phi_P\partial_\alpha\phi_\Pi-\phi_\Pi\partial_\alpha\phi_P)\phi_{V\alpha}. \tag{2·13}$$

This gives the three-body decay, which also originates from the direct interaction. We also estimate it separately.

Explicit expressions for the decay rates calculated by these Hamiltonians are presented in the Appendix.

Numerical results are given in Figs. 1∼4 for the variable mass of the Π-particle, where we put $G_0=2.27\times10^{-7}$ in units $\hbar=c=m_\pi=1$.

The Π_F mass varies from 2.0GeV to 4.0 GeV and the Π_M mass varies from 1.0 GeV to 3.0 GeV. The values of F_V, F_A, f_1 and f_2 are all taken to be 1.

§ 3. Discussion

We have calculated the rates of the various decay processes of the new particles Π_F and Π_M, some of which are shown in Fig. 1∼Fig. 4. There we

Table I. Angle factors of decay processes $\Pi_F\to B_8+P_8,\ B_{10}+P_8$.

angle factor	final states for the decay of					
	$\Pi_{F\,3^*}^{\,+}(0,0)$		$\Pi_{F\,3^*}^{\,+}(\frac{1}{2},-1)$		$\Pi_{F\,3^*}^{\,0}(\frac{1}{2},-1)$	
$\cos^2\theta$	$p\overline{K^0}$ $\Lambda\ \pi^+$ $\Sigma^+\pi^0$ $\Sigma^+\eta^0$ $\Sigma^0\pi^+$ Ξ^0K^+	$\Delta^{++}K^-$ $\Delta^+\overline{K^0}$ $\Sigma^{*+}\pi^0$ $\Sigma^{*+}\eta^0$ $\Sigma^{*0}\pi^+$ $\Xi^{*0}K^+$	$\Sigma^+\overline{K^0}$ $\Xi^0\ \pi^+$	$\Sigma^{*+}\overline{K^0}$ $\Xi^{*0}\pi^+$	$\Lambda\overline{K^0}$ Σ^+K^- $\Sigma^0\overline{K^0}$ $\Xi^0\pi^0$ $\Xi^0\eta^0$ $\Xi^-\pi^+$	Ω^-K^+ $\Sigma^{*0}\overline{K^0}$ $\Sigma^{*+}K^-$ $\Xi^{*0}\pi^0$ $\Xi^{*0}\eta^0$ $\Xi^{*-}\pi^+$
$\sin\theta\cos\theta$	$p\pi^0$ $p\eta^0$ $n\pi^+$ ΛK^+ Σ^+K^0 Σ^0K^+	$\Delta^{++}\pi^-$ $\Delta^+\pi^0$ $\Delta^+\eta^0$ $\Delta^0\pi^+$ $\Sigma^{*+}K^0$ $\Sigma^{*0}K^+$	$p\overline{K^0}$ $\Lambda\pi^+$ $\Sigma^+\pi^0$ $\Sigma^+\eta^0$ $\Sigma^0\pi^+$ Ξ^0K^+	$\Delta^{++}K^-$ $\Delta^+\overline{K^0}$ $\Sigma^{*+}\pi^0$ $\Sigma^{*+}\eta^0$ $\Sigma^{*0}\pi^+$ $\Xi^{*0}K^+$	pK^- $n\overline{K^0}$ $\Lambda\ \pi^0$ $\Lambda\ \eta^0$ $\Sigma^+\pi^-$ $\Sigma^0\pi^0$ $\Sigma^0\eta^0$ $\Sigma^-\pi^+$ Ξ^0K^0 Ξ^-K^+	Δ^+K^- $\Delta^0\overline{K^0}$ $\Sigma^{*0}\pi^0$ $\Sigma^{*0}\eta^0$ $\Sigma^{*+}\pi^-$ $\Sigma^{*-}\pi^+$ $\Xi^{*0}K^0$ $\Xi^{*-}K^+$
$\sin^2\theta$	pK^0 nK^+	Δ^+K^0 Δ^0K^+	$p\pi^0$ $p\eta^0$ $n\pi^+$ Σ^0K^+ ΛK^+ Σ^+K^0	$\Delta^{++}\pi^-$ $\Delta^+\pi^0$ $\Delta^+\eta^0$ $\Sigma^{*0}K^+$ $\Delta^0\pi^+$ $\Sigma^{*+}K^0$	$p\pi^-$ $n\pi^0$ $n\eta^0$ Σ^-K^+ ΛK^0 Σ^0K^0	$\Delta^+\pi^-$ $\Delta^0\pi^0$ $\Delta^0\eta^0$ $\Delta^-\pi^+$ $\Sigma^{*0}K^0$ $\Sigma^{*-}K^+$

angle factor	final state for					
	$\Pi_{F_6}{}^{++}(1,0)$		$\Pi_{F_6}{}^{+}(1,0)$		$\Pi_{F_6}{}^{0}(1,0)$	
$\cos^2\theta$	$\Sigma^+\pi^+$	$\Delta^{++}\overline{K^0}$ $\Sigma^{*+}\pi^+$	$p\overline{K^0}$ $\Lambda\pi^+$ $\Sigma^+\pi^0$ $\Sigma^+\eta^0$ $\Sigma^0\pi^+$ Ξ^0K^+	$\Delta^{++}K^-$ $\Delta^+\overline{K^0}$ $\Sigma^{*0}\pi^+$ $\Sigma^{*+}\pi^0$ $\Sigma^{*+}\eta^0$ $\Xi^{*0}K^+$	pK^- $n\overline{K^0}$ $\Lambda\pi^0$ $\Lambda\eta^0$ $\Sigma^+\pi^-$ $\Sigma^0\pi^0$ $\Sigma^0\eta^0$ $\Sigma^-\pi^+$ Ξ^0K^0 Ξ^-K^+	Δ^+K^- $\Delta^0\overline{K^0}$ $\Sigma^{*0}\pi^0$ $\Sigma^{*0}\eta^0$ $\Sigma^{*+}\pi^-$ $\Sigma^{*-}\pi^+$ $\Xi^{*0}K^0$ $\Xi^{*-}K^+$
$\cos\theta\sin\theta$	$p\pi^+$ Σ^+K^+	$\Delta^{++}\pi^0$ $\Delta^{++}\eta^0$ $\Delta^+\pi^+$ $\Sigma^{*+}K^+$	$p\pi^0$ $p\eta^0$ $n\pi^+$ ΛK^+ Σ^+K^0 Σ^0K^+	$\Delta^{++}\pi^-$ $\Delta^+\pi^0$ $\Delta^+\eta^0$ $\Delta^0\pi^+$ $\Sigma^{*+}K^0$ $\Sigma^{*0}K^+$	$p\pi^-$ $n\pi^0$ $n\eta^0$ ΛK^0 Σ^0K^0 Σ^-K^+	$\Delta^+\pi^-$ $\Delta^0\pi^0$ $\Delta^0\eta^0$ $\Delta^-\pi^+$ $\Sigma^{*0}K^0$ $\Sigma^{*-}K^+$
$\sin^2\theta$	pK^+	$\Delta^{++}K^0$ Δ^+K^+	pK^0 nK^+	Δ^+K^0 Δ^0K^+	nK^0	Δ^0K^0

angle factor	final state for					
	$\Pi_{F_6}{}^{+}(\frac{1}{2},-1)$		$\Pi_{F_6}{}^{0}(\frac{1}{2},-1)$		$\Pi_{F_6}{}^{0}(0,-2)$	
$\cos^2\theta$	$\Sigma^+\overline{K^0}$ $\Xi^0\pi^+$	$\Sigma^{*+}\overline{K^0}$ $\Xi^{*0}\pi^+$	$\Lambda\overline{K^0}$ Σ^+K^- $\Sigma^0\overline{K^0}$ $\Xi^0\pi^0$ $\Xi^0\eta^0$ $\Xi^-\pi^+$	Ω^-K^+ $\Sigma^{*+}K^-$ $\Sigma^{*0}\overline{K^0}$ $\Xi^{*0}\pi^0$ $\Xi^{*0}\eta^0$ $\Xi^{*-}\pi^+$	$\Xi^0\overline{K^0}$	$\Xi^{*0}\overline{K^0}$
$\cos\theta\sin\theta$	$p\overline{K^0}$ $\Lambda\pi^+$ $\Sigma^+\pi^0$ $\Sigma^+\eta^0$ $\Sigma^0\pi^+$ Ξ^0K^+	$\Delta^{++}K^-$ $\Delta^+\overline{K^0}$ $\Sigma^{*+}\pi^0$ $\Sigma^{*+}\eta^0$ $\Sigma^{*0}\pi^+$ $\Xi^{*0}K^+$	pK^- $n\overline{K^0}$ $\Lambda\pi^0$ $\Lambda\eta^0$ $\Sigma^+\pi^-$ $\Sigma^0\pi^0$ $\Sigma^0\eta^0$ $\Sigma^-\pi^+$ Ξ^0K^0 Ξ^-K^+	Δ^+K^- $\Delta^0\overline{K^0}$ $\Sigma^{*0}\pi^0$ $\Sigma^{*0}\eta^0$ $\Sigma^{*+}\pi^-$ $\Sigma^{*-}\pi^+$ $\Xi^{*0}K^0$ $\Xi^{*-}K^+$	$\Lambda\overline{K^0}$ Σ^+K^- $\Sigma^0\overline{K^0}$ $\Xi^0\pi^0$ $\Xi^0\eta^0$ $\Xi^-\pi^+$	Ω^-K^+ $\Sigma^{*0}\overline{K^0}$ $\Sigma^{*+}K^-$ $\Xi^{*0}\pi^0$ $\Xi^{*0}\eta^0$ $\Xi^{*-}\pi^+$

Table 1. (continued)

angle factor	final state for					
	$\Pi_{F_6}{}^{+}(\frac{1}{2},-1)$		$\Pi_{F_6}{}^{0}(\frac{1}{2},-1)$		$\Pi_{F_6}{}^{0}(0,-2)$	
$\sin^2\theta$	$p\pi^0$	$\Delta^{++}\pi^-$	$p\pi^-$	$\Delta^+\pi^-$	pK^-	Δ^+K^-
	$p\eta^0$	$\Delta^+\pi^0$	$n\pi^0$	$\Delta^0\pi^0$	$n\overline{K^0}$	$\Delta^0\overline{K^0}$
	$n\pi^+$	$\Delta^+\eta^0$	$n\eta^0$	$\Delta^0\eta^0$	$\Lambda\pi^0$	$\Sigma^{*0}\pi^0$
	ΛK^+	$\Delta^0\pi^+$	ΛK^0	$\Delta^-\pi^+$	$\Lambda\eta^0$	$\Sigma^{*0}\eta^0$
	Σ^+K^0	$\Sigma^{*+}K^0$	Σ^0K^0	$\Sigma^{*0}K^0$	$\Sigma^+\pi^-$	$\Sigma^{*+}\pi^-$
	Σ^0K^+	$\Sigma^{*0}K^+$	Σ^-K^+	$\Sigma^{*-}K^+$	$\Sigma^0\pi^0$	$\Sigma^{*-}\pi^+$
					$\Sigma^0\eta^0$	$\Xi^{*0}K^0$
					$\Sigma^-\pi^+$	$\Xi^{*-}K^+$
					Ξ^0K^0	
					Ξ^-K^+	

$$\Pi_F \to B_8 + \bar{l} + \nu_l$$

angle factor	final B_8 for				
	$\Pi_{F_{3^*}}^{+}(0,0)$	$\Pi_{F_{3^*}}^{+}(\frac{1}{2},-1)$	$\Pi_{F_{3^*}}^{0}(\frac{1}{2},-1)$	$\Pi_{F_6}{}^{++}(1,0)$	$\Pi_{F_6}{}^{+}(1,0)$
$\cos\theta$	Λ	Ξ^0	Ξ^-	Σ^+	Σ^0
$\sin\theta$	n	Λ, Σ^0	Σ^-	p	n

angle factor	final B_8 for			
	$\Pi_{F_6}{}^{0}(1,0)$	$\Pi_{F_6}{}^{+}(\frac{1}{2},-1)$	$\Pi_{F_6}{}^{0}(\frac{1}{2},-1)$	$\Pi_{F_6}{}^{0}(0,-2)$
$\cos\theta$	Σ^-	Ξ^0	Ξ^-	
$\sin\theta$		Λ, Σ^0	Σ^-	Ξ^-

$$\Pi_F \to B_{10} + \bar{l} + \nu_l$$

angle factor	final B_{10} for					
	$\Pi_{F_6}{}^{++}(1,0)$	$\Pi_{F_6}{}^{+}(1,0)$	$\Pi_{F_6}{}^{0}(1,0)$	$\Pi_{F_6}{}^{+}(\frac{1}{2},-1)$	$\Pi_{F_6}{}^{0}(\frac{1}{2},-1)$	$\Pi_{F_6}{}^{0}(0,-2)$
$\cos\theta$	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^-
$\sin\theta$	Δ^+	Δ^0	Δ^-	Σ^{*0}	Σ^{*-}	Ξ^{*-}

$\Pi_M \to P_8 + P_8 + P_8$

angle factor	final state for the decay of		
	$\Pi_M^+(\frac{1}{2},0)$	$\Pi_M^0(\frac{1}{2},0)$	$\Pi_M^+(0,1)$
$\cos^2\theta$	$\pi^+\pi^+K^-$ $\pi^+\pi^0\overline{K^0}$ $\pi^+\eta^0\overline{K^0}$ $K^+\overline{K^0}\,\overline{K^0}$	$\pi^+\pi^0K^-$ $\pi^+\eta^0K^-$ $\pi^+\pi^-\overline{K^0}$ $\pi^0\pi^0\overline{K^0}$ $\pi^0\eta^0\overline{K^0}$ $K^+K^-\overline{K^0}$ $K^0\overline{K^0}\,\overline{K^0}$ $\eta^0\eta^0\overline{K^0}$	$\pi^+K^0\overline{K^0}$ $\pi^+K^+K^-$ $\pi^0K^+\overline{K^0}$ $\eta^0K^+\overline{K^0}$
$\cos\theta\sin\theta$	$\pi^+\pi^+\pi^-$ $\pi^+\pi^0\pi^0$ $\pi^+\pi^0\eta^0$ $\pi^+\eta^0\eta^0$ $\pi^+K^+K^-$ $\pi^0K^+\overline{K^0}$ $\eta^0K^+\overline{K^0}$	$\pi^+\pi^-\pi^0$ $\pi^+\pi^-\eta^0$ $\pi^0\pi^0\pi^0$ $\pi^0\pi^0\eta^0$ $\pi^0\eta^0\eta^0$ $\eta^0\eta^0\eta^0$ $\pi^0K^0\overline{K^0}$ $\eta^0K^0\overline{K^0}$ $\pi^0K^+K^-$ $\eta^0K^+K^-$	$\pi^0\pi^0K^+$ $\pi^0\eta^0K^+$ $\eta^0\eta^0K^+$ $\pi^+\pi^0K^0$ $\pi^+\eta^0K^0$ $\pi^+\pi^-K^+$ $K^+K^+K^-$
$\sin^2\theta$	$\pi^+\pi^-K^+$ $\pi^+\pi^0K^0$ $\pi^+\eta^0K^0$ $K^+K^0\overline{K^0}$	$\pi^+\pi^-K^0$ $\pi^0\pi^0K^0$ $\eta^0\eta^0K^0$ $K^+K^-K^0$ $K^0K^0\overline{K^0}$ $\pi^0\eta^0K^0$ $\pi^0\eta^-K^+$ $\pi^-K^+\eta^0$	$\pi^+K^0K^0$ $\pi^0K^+K^0$ $\eta^0K^+K^0$ $\pi^-K^+K^+$

$\Pi_M \to P_8 + P_8$

angle factor	final state for the decay of		
	$\Pi_M^+(\frac{1}{2},0)$	$\Pi_M^0(\frac{1}{2},0)$	$\Pi_M^+(0,1)$
$\cos^2\theta$	$\pi^+\overline{K^0}$	π^+K^- $\pi^0\overline{K^0}$ $\eta^0\overline{K^0}$	$K^+\overline{K^0}$
$\cos\theta\sin\theta$	$\pi^+\pi^0$ $\pi^+\eta^0$	$\pi^+\pi^-$ $\pi^0\pi^0$ $\pi^0\eta^0$ K^+K^- $\eta^0\eta^0$	π^0K^+ η^0K^+
$\sin^2\theta$	π^+K^0	π^-K^+ π^0K^0 η^0K^0	K^0K^+

$\Pi_M \to P_8 + \bar{l} + \nu_l$

angle fator	final P_8 for the decay of		
	$\Pi_M{}^+(\frac{1}{2}, 0)$	$\Pi_M{}^0(\frac{1}{2}, 0)$	$\Pi_M{}^+(0, 1)$
$\cos\theta$	$\overline{K^0}$	K^-	η^0
$\sin\theta$	π^0 η^0	π^-	K^0

$\Pi_M \to P_8 + P_8 + \bar{l} + \nu_l$

angle factor	final P_8P_8 for the decay of		
	$\Pi_M{}^+(\frac{1}{2}, 0)$	$\Pi_M{}^0(\frac{1}{2}, 0)$	$\Pi_M{}^+(0, 1)$
$\cos\theta$	π^+K^- $\pi^0\overline{K^0}$ $\eta^0\overline{K^0}$	π^0K^- $\pi^-\overline{K^0}$ η^0K^-	K^+K^- $K^0\overline{K^0}$ $\eta^0\eta^0$
$\sin\theta$	$\pi^+\pi^-$ $\pi^0\pi^0$ $\pi^0\eta^0$ $K^0\overline{K^0}$ $\eta^0\eta^0$	$\pi^-\eta^0$ K^-K^0 $\pi^-\pi^0$	π^0K^0 π^-K^+ η^0K^0

have shown the decay rates for both the largest and smallest values of Q-value for each decay process of the same type, and the rate of $\Pi_F \to \Lambda\pi$ or that of $\Pi_F \to \Sigma\pi$, etc., will lie between the line (1) and the line (2) of Fig. 1. The rates of $\Pi_F \to B_8P_8P_8$ (direct decay) and $\Pi_M \to P_8P_8P_8$ (direct one) are not illustrated in the figures because of the existence of the ambiguous factors {a, b in the Hamiltonian of $\Pi_F \to B_8P_8P_8$ and $a_i (i=1, \cdots, 6)$ in the Hamiltonian of $\Pi_M \to P_8P_8P_8$ in the Appendix [B) (1) (v), (2) (iv)]}. If we put all these factors (a, b, a_i) equal to 1, the rates of both $\Pi_F \to B_8P_8P_8$ and $\Pi_M \to P_8P_8P_8$ become larger by one order than that of $\Pi_F \to B_8V_8$ and $\Pi_M \to P_8V_8$ respectively.

It should be noted that these figures 1∼4 must be multiplied by the factor due to the semi-leptonic angle θ in accordance with the new Nagoya model. These angle factors for each process are presented in Table I.

Now we comment on a few points about Fig. 1∼ Fig. 4.

(i) For the lower range of Π_F mass (almost about $\lesssim 2.5$ GeV) the non-leptonic two-body decay ($\Pi_F \to B_8P_8$) is the main mode, but the rate of the β-decay ($\Pi_F \to B_8 l \nu_l$) increases very rapidly with the Π_F mass and for the higher range (about $\gtrsim 3.5$ GeV) the latter overwhelms the former and becomes the main mode.

(ii) In Π_M decays, the non-leptonic two-body decay ($\Pi_M \to P_8P_8$) is always dominant.

Finally we want to add a comment on the form factor. As the Q-value of the decay products is larger than in the case of usual strange particle decay, the effect of the form factor might be remarkable if it exists. A simple calculation[6] gives a reduction factor of one order or more; this will be reported in the future.

Appendix

We shall present the effective Hamiltonians and the rates of various decays of Π_F- and Π_M-particle. We assume here Π_F and Π_M to be of spin $\frac{1}{2}$ and 0 respectively. The notation of B_8, B_{10}, P_8, V_8, l and ν_l means octet baryon, decuplet baryon, pseudoscalar meson, vector meson, electron or muon and neutrino respectively. M_Π, m_B, m_P, m_V and m_l represent the mass of Π_F or Π_M, baryon, ps meson, vector meson and of lepton respectively.

A) Semi-leptonic decays

(1) β-decays of Π_F-particle

(i) $\Pi_F \to B_8 l \nu_l$

$$H=\frac{G_0}{\sqrt{2}}\bar{\psi}_B\gamma_\alpha(F_V+F_A\gamma_5)\psi_{\Pi_F}\bar{\psi}_{\nu_l}\gamma_\alpha(1+\gamma_5)\psi_l+\text{h.c.},$$

$$\Gamma=\frac{G_0^2 M_\Pi}{12\pi^3}\int_b^a dx\sqrt{x^2-2Mx+N^2}\,f(x),$$

$$x=M-E_B\,,$$

$$a=\frac{m_l^2}{2M_\Pi},\qquad b=M-m_B\,,$$

$$M=\frac{M_\Pi^2+m_B^2}{2M_\Pi},$$

$$N=\frac{M_\Pi^2-m_B^2}{2M_\Pi},$$

$$f(x)=\left(1-\frac{a}{x}\right)^2\Big[|F_V|^2\Big\{\frac{a}{x}(N^2-Mx-x^2)-(2x^2+(3m_B-M)x-N^2)\Big\}$$
$$+|F_A|^2\Big\{\frac{a}{x}(N^2-Mx-x^2)-(2x^2-(3m_B+M)x-N^2)\Big\}\Big].$$

(ii) $\Pi_F \to B_{10} l \nu_l$

$$H=\frac{G_0}{\sqrt{2}}\bar{\psi}_{B\alpha}(F_V+F_A\gamma_5)\psi_{\Pi_F}\bar{\psi}_{\nu_l}\gamma_\alpha(1+\gamma_5)\psi_l+\text{h.c.},$$

$$\Gamma=\frac{G_0^2}{72\pi^3 M_\Pi m_B^2}\int_0^a dx\frac{x\sqrt{M_\Pi^2x^2-2M_\Pi(L-m_l^2)x+(L^2-M_\Pi^2m_l^2)}}{M_\Pi-2x}F(x),$$

$$x=E_\nu\,,$$

$$a=\frac{M_\Pi^2-(m_B+m_l)^2}{2M_\Pi}, \qquad L=\frac{M_\Pi^2-m_B^2+m_l^2}{2},$$

$$M=M_\Pi^4+2m_B^4+m_l^4-3M_\Pi^2m_B^2-3m_B^2m_l^2+2M_\Pi^2m_l^2,$$

$$N=M_\Pi(2L-m_B^2),$$

$$f=\frac{2\{M_\Pi x^2-(L+M_\Pi^2)x+LM_\Pi\}}{M_\Pi(M_\Pi-2x)},$$

$$g=\frac{(M_\Pi^2+m_l^2)x^2-2M_\Pi Lx+L^2}{M_\Pi(M_\Pi-2x)},$$

$$F(x)=(|F_V|^2+|F_A|^2)\{6MM_\Pi-6(2M_\Pi N+M)x+12Nx^2-3(2M_\Pi N+M)f +4N(f^2-g)+12(N+M_\Pi^3)fx-12M_\Pi^2fx^2-8M_\Pi^2(f^2-g)x\} +6m_B(|F_V|^2-|F_A|^2)\{M-2Nx-Nf+2M_\Pi^2fx\}.$$

(2) Two-body leptonic decay of meson ($\Pi_M\to l\nu_l$)

$$H=i\frac{G_0}{\sqrt{2}}\overline{\psi}_l\gamma_\alpha(1+\gamma_5)\psi_{\nu_l}\partial_\alpha\phi_{\Pi_M}+\text{h.c.},$$

$$\Gamma=\frac{G_0^2}{8\pi}M_\Pi^2m_l^2\left(1-\frac{m_l^2}{M_\Pi^2}\right)^2.$$

(3) β-decays of Π_M-particle

(i) $\Pi_M\to P_8 l\nu_l$

$$H=\frac{G_0}{\sqrt{2}}(f_1P_\alpha^{\Pi_M}+f_2P_\alpha^{P})\phi_P\phi_{\Pi_M}\overline{\psi}_{\nu_l}\gamma_\alpha(1+\gamma_5)\psi_l+\text{h.c.},$$

$$\Gamma=\frac{G_0^2}{16\pi^3M_\Pi}\int_b^a dxF(x),$$

$$x=E_P,$$

$$a=m_P,$$

$$b=\frac{M_\Pi^2+m_P^2-m_l^2}{2M_\Pi},$$

$$F(x)=\Big\{|f_1|^2M_\Pi^2\Big[(M_\Pi-m_P)E_l^2-\frac{2}{3}E_l^3-\frac{1}{2}(M_\Pi^2-2M_\Pi x+m_P^2-m_l^2)E_l\Big] +\frac{1}{2}|f_2|^2m_l^2(M_\Pi^2-2M_\Pi x+m_P^2-m_l^2) +(f_1^*f_2+f_1f_2^*)m_l^2M_\Pi\Big[-\frac{E_l^2}{2}+(M_\Pi-x)E_l\Big]\Big\}\Bigg|_{E_l^{\min}}^{E_l^{\max}},$$

$$[E_l]_{\min}^{\max}=\frac{1}{2\alpha}\{(\alpha+m_l^2)(M_\Pi-x)\pm\sqrt{(x^2-m_P^2)(\alpha^2+2\alpha m_l^2+m_l^4-4\alpha m_l^2)}\},$$

$\alpha = M_{\Pi}^2 - 2M_{\Pi}x + m_P^2.$

(ii) $\Pi_M \to V_8 l \nu_l$

$$H = i\frac{G_0}{\sqrt{2}} \phi_{V\alpha} \phi_{\Pi_M} \bar{\psi}_{\nu_l} \gamma_\alpha (1+\gamma_5) \psi_l + \text{h.c.},$$

$$\Gamma = \frac{G_0^2}{16\pi^3 M_{\Pi}} \int_0^a dx \left[\left\{ -Q + M_{\Pi}x + 2\frac{Q(K - M_{\Pi}x)}{m_V^2} \right\} (\alpha - \beta) + \frac{1}{2} \left\{ M_{\Pi} - 2\frac{M_{\Pi}(K - M_{\Pi}x)}{m_V^2} \right\} (\alpha^2 - \beta^2) \right],$$

$x = E_\nu ,$

$$a = \frac{M_{\Pi}^2 - (m_V + m_l)^2}{2M_{\Pi}} ,$$

$$Q = \frac{M_{\Pi}^2 - m_V^2 + m_l^2}{2} ,$$

$$K = \frac{M_{\Pi}^2 - m_V^2 - m_l^2}{2} ,$$

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \frac{-(3M_{\Pi}^2 x - 2M_{\Pi}x^2 - 2M_{\Pi}Q + m_l^2 x - m_V^2 x) \pm \sqrt{\gamma}}{2(M_{\Pi}^2 - 2M_{\Pi}x)} ,$$

$$\gamma = 4x^2 M_{\Pi}^2 \left(x - \frac{M_{\Pi}^2 - (m_V - m_l)^2}{2M_{\Pi}} \right) \left(x - \frac{M_{\Pi}^2 - (m_V + m_l)^2}{2M_{\Pi}} \right).$$

B) Non-leptonic decays

(1) Non-leptonic decays of Π_F-particle

(i) $\Pi_F \to B_8 + P_8$

$$H = i\frac{G_0}{\sqrt{2}} \bar{\psi}_B \gamma_\alpha (F_V + F_A \gamma_5) \psi_{\Pi_F} \partial_\alpha \phi_P + \text{h.c.},$$

$$\Gamma = \frac{G_0^2}{8\pi} \frac{M + m_B}{M_{\Pi}} N (M_{\Pi} - m_B)^2 (|F_V|^2 + K^2 |F_A|^2),$$

$$K = \frac{(M_{\Pi} + m_B) N}{(M_{\Pi} - m_B)(M + m_B)} ,$$

$$M = \frac{M_{\Pi}^2 + m_B^2 - m_P^2}{2M_{\Pi}} ,$$

$$N = (M^2 - m_B^2)^{1/2}.$$

(ii) $\Pi_F \to B_8 + V_8$

$$H = \frac{G_0}{\sqrt{2}} \bar{\psi}_B \gamma_\alpha (F_V + F_A \gamma_5) \psi_{\Pi_F} \phi_{V\alpha} ,$$

$$\Gamma=\frac{G_0^2}{8\pi M_\Pi^3}K\{(|F_V|^2+|F_A|^2)L-2(|F_V|^2-|F_A|^2)m_B M_\Pi\},$$

$$K=\{M_\Pi^4+m_B^4+m_V^4-2(M_\Pi^2 m_B^2+m_B^2 m_V^2+M_\Pi^2 m_V^2)\}^{1/2},$$

$$L=M_\Pi^2+m_B^2-m_V^2.$$

(iii) $\Pi_F\to B_{10}+P_8$

$$H=\frac{G_0}{\sqrt{2}}\bar{\psi}_{B\alpha}(F_V+F_A\gamma_5)\psi_{\Pi_F}\partial_\alpha\phi_P\,,$$

$$\Gamma=\frac{G_0^2}{192\pi m_B^2 M_\Pi^3}K\{(|F_V|^2+|F_A|^2)L+2(|F_V|^2-|F_A|^2)m_B M_\Pi\},$$

$$K=\{M_\Pi^4+m_B^4+m_P^4-2(M_\Pi^2 m_B^2+m_B^2 m_P^2+M_\Pi^2 m_P^2)\}^{3/2},$$

$$L=M_\Pi^2+m_B^2-m_P^2.$$

(iv) $\Pi_F\to B_{10}+V_8$

$$H=\frac{G_0}{\sqrt{2}}\bar{\psi}_{B\alpha}(F_V+F_A\gamma_5)\psi_{\Pi_F}\phi_{V\alpha}\,,$$

$$\Gamma=\frac{G_0^2}{192\pi m_V^2 m_B^2 M_\Pi^3}K\{(|F_V|^2+|F_A|^2)L+2(|F_V|^2-|F_A|^2)m_B M_\Pi\},$$

$$K=\{M_\Pi^4+m_B^4+m_V^4-2(M_\Pi^2 m_B^2+m_B^2 m_V^2+M_\Pi^2 m_V^2)\}^{3/2},$$

$$L=M_\Pi^2+m_B^2-m_V^2.$$

(v) $\Pi_F\to B_8+P_8+P_8$

$$H=\frac{G_0}{\sqrt{2}}\bar{\psi}_B\gamma_\alpha(F_V+F_A\gamma_5)\psi_{\Pi_F}(a\partial_\alpha\phi_{P_1}\phi_{P_2}+b\phi_{P_1}\partial_\alpha\phi_{P_2})+\text{h.c.},$$

$$\begin{aligned}\Gamma=\frac{G_0^2}{128\pi^3}\int_{a_0}^{b_0}dx\frac{\sqrt{D}}{K}\Big[&(|F_V|^2+|F_A|^2)\Big\{\frac{L}{K}(a^2(M_\Pi^2-m_B^2+m_{P_2}^2)\\&+b^2m_{P_2}^2-abM_\Pi^2-2(a+b)^2M_\Pi x)-2MM_\Pi+2x(b^2(M_\Pi^2-m_B^2+m_{P_1}^2)\\&+a^2m_{P_1}^2-2ab(M_\Pi^2+m_{P_2}^2))+abM_\Pi\Big(4x^2+\frac{L^2}{K^2}+\frac{D}{K^2}\Big)\Big\}\\&+(|F_V|^2-|F_A|^2)\Big\{2Mm_B+abM_\Pi m_B\Big(\frac{L}{K}+4x\Big)\Big\}\Big],\end{aligned}$$

$$x=E_{P_2},$$

$$a_0=m_{P_2}\,,\qquad b_0=\frac{M_\Pi^2+m_{P_2}^2-(m_{P_1}+m_B)^2}{2M_\Pi}\,,$$

$$M=a^2m_{P_1}^2+b^2m_{P_2}^2+ab(m_B^2-M_\Pi^2-m_{P_1}^2-m_{P_2}^2),$$

$$L=2M_\Pi x^2+(m_B^2-m_{P_1}^2-m_{P_2}^2-3M_\Pi^2)x+M_\Pi(M_\Pi^2-m_B^2+m_{P_1}^2+m_{P_2}^2),$$

$$K = M_\Pi{}^2 + m_{P_2}^2 - 2M_\Pi x\,,$$

$$D = 4M_\Pi{}^2 x^4 + 4M_\Pi(m_{P_1}^2 + m_B{}^2 - M_\Pi{}^2 - m_{P_2}^2)x^3$$
$$+ (M_\Pi{}^4 + m_B{}^4 + m_{P_1}^2 + m_{P_2}^2 - 2M_\Pi{}^2 m_B{}^2 - 2M_\Pi{}^2 m_{P_1}^2 - 2M_\Pi{}^2 m_{P_2}^2$$
$$- 2M_B{}^2 m_{P_1}^2 - 2m_B{}^2 m_{P_2}^2 - 2m_{P_1}^2 m_{P_2}^2)x^2 + 4m_\Pi{}^2 m_{P_2}^2(M_\Pi{}^2 + m_{P_2}^2$$
$$- m_B{}^2 - m_{P_1}^2)x - 2m_{P_2}^2(M_\Pi{}^4 + m_B{}^4 + m_{P_1}^4 + m_{P_2}^4 - 2M_\Pi{}^2 m_B{}^2$$
$$- 2M_\Pi{}^2 m_{P_1}^2 + 2M_\Pi{}^2 m_{P_2}^2 - 2m_B{}^2 m_{P_1}^2 - 2m_B{}^2 m_{P_2}^2 - 2m_{P_1}^2 m_{P_2}^2)\,.$$

(2) Non-leptonic decays of Π_M-particle

(i) $\Pi_M \to P_8 + P_8$

$$H = \frac{G_0}{\sqrt{2}}(a\phi_{\Pi_M}\partial_\alpha\phi_{P_1}\partial_\alpha\phi_{P_2} + b\partial_\alpha\phi_{\Pi_M}\partial_\alpha\phi_{P_1}\phi_{P_2} + c\partial_\alpha\phi_{\Pi_M}\phi_{P_1}\partial_\alpha\phi_{P_2}) + \text{h.c.},$$

$$\Gamma = \frac{G_0{}^2}{64\pi M_\Pi}\left\{\left(\frac{M_\Pi{}^2 + m_{P_1}^2 - m_{P_2}^2}{2M_\Pi}\right) - m_{P_1}^2\right\}^{1/2}\{-a(M_\Pi{}^2 - m_{P_1}^2 - m_{P_2}^2)$$
$$+ b(M_\Pi{}^2 + m_{P_1}^2 - m_{P_2}^2) + c(M_\Pi{}^2 - m_{P_1}^2 + m_{P_2}^2)\}^2.$$

(ii) $\Pi_M \to P_8 + V_8$

$$H = \frac{G_0}{\sqrt{2}}(a\phi_{\Pi_M}\phi_{V\alpha}\partial_\alpha\phi_P + b\partial_\alpha\phi_{\Pi_M}\phi_{V\alpha}\phi_P),$$

$$\Gamma = \frac{G_0{}^2}{16\pi M_\Pi{}^2}(Q^2 - m_P{}^2)^{1/2}F,$$

$$F = -(a^2 m_P{}^2 + b^2 M_\Pi{}^2 - 2abM_\Pi Q) + \frac{1}{m_V{}^2}\{a(Q\sqrt{Q^2 + m_V{}^2 - m_P{}^2} + Q^2 - m_P{}^2)$$
$$- bM_\Pi\sqrt{Q^2 + m_V{}^2 - m_P{}^2}\}^2,$$

$$Q = \frac{M_\Pi{}^2 - m_V{}^2 + m_P{}^2}{2M_\Pi}\,.$$

(iii) $\Pi_M \to V_8 + V_8$

$$H = \frac{G_0}{\sqrt{2}}\phi_{\Pi_M}\phi_{V_1\alpha}\phi_{V_2\alpha},$$

$$\Gamma = \frac{G_0{}^2}{16\pi M_\Pi{}^2}(Q^2 - m_{V_2}^2)^{1/2}F\,,$$

$$Q = \frac{M_\Pi{}^2 - m_{V_1}^2 + m_{V_2}^2}{2M_\Pi}\,,$$

$$F = 2 + \frac{(Q\sqrt{Q^2 + m_{V_1}^2 - m_{V_2}^2} + Q^2 - m_{V_2}^2)^2}{m_{V_1}^2 m_{V_2}^2}\,.$$

(iv) $\Pi_M \to P_8 + P_8 + P_8$

$$H=\frac{G_0}{\sqrt{2}}\{a_1\partial_\alpha\phi_{\Pi_M}\partial_\alpha\phi_{P_1}\phi_{P_2}\phi_{P_3}+a_2\partial_\alpha\phi_{\Pi_M}\phi_{P_1}\partial_\alpha\phi_{P_2}\phi_{P_3}+a_3\partial_\alpha\phi_{\Pi_M}\phi_{P_1}\phi_{P_2}\partial_\alpha\phi_{P_3}$$
$$+a_4\phi_{\Pi_M}\partial_\alpha\phi_{P_1}\partial_\alpha\phi_{P_2}\phi_{P_3}+a_5\phi_{\Pi_M}\partial_\alpha\phi_1\phi_{P_2}\partial_\alpha\phi_{P_3}+a_6\phi_{\Pi_M}\phi_{P_1}\partial_\alpha\phi_{P_2}\partial_\alpha\phi_{P_3}\},$$

If we put all $a_i=1$:

$$\Gamma=\frac{G_0^{\ 2}}{256\pi^3 M_\Pi}M^2\int_a^b\frac{\sqrt{4M_\Pi^{\ 2}E_{P_1}^4+(E_{P_1}^2-m_{P_1}^2)(K-4LM_\Pi E_{P_1})}}{L+M-4M_\Pi E_{P_1}}dE_{P_1},$$

$$a=m_{P_1},\qquad b=\frac{M_\Pi^{\ 2}-2m_{P_2}m_{P_3}}{2M_\Pi},$$

$$K=M_\Pi^{\ 4}+m_{P_1}^4+m_{P_2}^4+m_{P_3}^4+2M_\Pi^{\ 2}m_{P_1}^2-2M_\Pi^{\ 2}m_{P_2}^2-2M_\Pi^{\ 2}m_{P_3}^2$$
$$-2m_{P_1}^2m_{P_2}^2-2m_{P_2}^2m_{P_3}^2-2m_{P_1}^2m_{P_3}^2,$$

$$L=M_\Pi^{\ 2}+m_{P_1}^2-m_{P_2}^2-m_{P_3}^2,$$

$$M=M_\Pi^{\ 2}+m_{P_1}^2+m_{P_2}^2+m_{P_3}^2.$$

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