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A Possible Symmetry in Sakata's Model for Bosons-Baryons System

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In this paper we study a possible symmetry in Sakata's model for the strongly interacting particles. In the limiting case in which the basic particles, proton, p, neutron, n and Λ-particle, Λ, have an equal mass, our theory holds the invariance under the exchange of p and Λ or n and Λ in addition to the usual charge independence and the conservation of electrical and hyperonic charge.

From our theory the following are obtained: (a) iso-singlet π_0'-meson state, which is a pseudo-scalar, exists, (b) the spin of Ξ-particle may be $(3/2)^+$ and (c) several resonating states in K- and π-nucleon scattering are anticipated to exist.

§ 1. Introduction

Through the analysis of the various particles existing in nature and mutual interactions among them, we have obtained the useful concepts of family[1] and universality[2] of the interactions to clarify the complicated situation of the particle physics. For the Boson- and baryon-families which have a kind of universal interaction, e. g., the strong interaction, the well-known rule of Nakano, Nishijima and Gell-Mann[3] is valid. The complete understanding of the more fundamental origin of this rule is far from us at present, but a possible way of its realistic grasp has been proposed by Sakata.[4] Although many objections will be brought against this theory, we shall in this paper follow the idea of Sakata for its prospective insight on the present situation of the theory of elementary particles.

Now, following this idea we assume proton, p, neutron, n and Λ-particle, Λ to be the basic particles which compose other baryons and Bosons in Fermi-Yang's sense[5]. The strong interaction is characterized by the following selection rule,

$$\Delta N_p = \Delta N_n = \Delta N_\Lambda = 0, \tag{A}$$

where ΔN_i means the change of i-th particle's number.

According to the relation (A) and from the similarity of the nature of these three particles (mass, spin, etc.) and of their role in the strong interaction, we are tempted to regard the three particles as standing on an equal footing. In fact when the mass difference is neglected and the electromagnetic interaction is switched off, we may not find any difference among these three basic particles. Thus we can reasonably expect that a certain symmetry is also realized in their mutual interactions. This view has once been stressed by one of the authors (S. O.)[6] and later, independently, by Yamaguchi.

In this paper we have investigated a possible framework of such a theory. We shall give the mathematical construction of the theory in § 2 and some physical results derived from this theory in § 3.

§ 2. Mathematical construction

We propose, here, a framework which explicitly assures the equivalence of the three basic particles, p, n and Λ, in the limiting case in which they are of equal mass. This means that our theory guarantees the invariance under the exchange of Λ and p or Λ and n in addition to the usual charge independence and the conservation of hyperonic charges. Our statement on the nature of particle state, e. g., spin and parity but except mass (or energy level), still holds with the finite mass difference between Λ and nucleon, when the mass of Λ is adiabatically increased from its original value (equal to the nucleon mass) to the actual one.

Now, denote the basic particles by the generic symbol χ^{κ} ($\chi^{\kappa}=\delta_1{}^{\kappa}$, for p, etc.). Then the above mentioned symmetry is expressed by the invariance under transformations of the 3-dimensional unitary group $U(3)$:

$$\chi^{\kappa\prime}=A_{\kappa}^{\kappa\prime}\chi^{\kappa}\,;\;\bar{A}_{\kappa}^{\kappa\prime}=A_{\kappa\prime}^{\kappa}, \tag{1}$$

where the matrix $(A_{\kappa\prime}{}^{\kappa})$ is the inverse of $(A_{\kappa}{}^{\kappa\prime})$ and $\bar{A}_{\kappa}{}^{\kappa\prime}$ denote the complex conjugates of $A_{\kappa}{}^{\kappa\prime}$. An infinitesimal transformation has the form

$$\chi^{\kappa\prime}=(\delta_{\kappa}^{\kappa\prime}+i\epsilon X_{\kappa}^{\kappa\prime})\chi^{\kappa}\,;\;i^2=-1, \tag{2}$$

where $(X_{\kappa}{}^{\kappa\prime})$ is an Hermitian matrix and can be expressed linearly in terms of nine independent matrices X_{ij} $(i, j=1, 2, 3)$: e. g.,

$$(X_{ij})_{\kappa}^{\kappa\prime}=\tfrac{1}{2}\delta_{i\kappa\prime}\delta_{j\kappa}(1-i)+\tfrac{1}{2}\delta_{i\kappa}\delta_{j\kappa\prime}(1+i). \tag{3}$$

They satisfy the commutation relations

$$[X_{ij},\,X_{kl}]=X_{ij}X_{kl}-X_{kl}X_{ij}=i(\delta_{ik}X_{[jl]}-\delta_{lj}X_{[ki]}-\delta_{jk}X_{(il)}+\delta_{li}X_{(kj)}), \tag{4}$$

where ( ) and [ ] for indices denote the ordinary processes of symmetrization and alternation respectively.

In a continuous representation of $U(3)$ of degree n, X_{ij} are represented by $n\times n$ matrices M_{ij} which satisfy the same commutation laws as X_{ij}. For the sake of physical understanding it is convenient to introduce the following quantities :

$$I_1=M_{(12)},\;\; I_2=M_{[12]},\;\; I_3=\tfrac{1}{2}(M_{11}-M_{22}),$$

$$I_+=I_1+iI_2,\;\; I_-=I_1-iI_2,\;\; \boldsymbol{I}^2=\sum_i (I_i)^2, \tag{5}$$

$$N_B=\sum_i M_{ii},\quad Q=M_{11},\quad S=-M_{33},$$
$$N_{ij}=M_{(ij)}+iM_{[ij]};\ (ij=13,\ 23,\ 31,\ 32). \tag{6}$$

There are two other quantities which are important from the standpoint of representation, i. e.,

$$M=\sum_{ij}(M_{ij})^2,$$
$$M'=\sum_{ijk}(M_{ij}\{M_{jk},\ M_{ik}\}+M_{ij}\{M_{ik},\ M_{kj}\}+M_{ij}\{M_{kj},\ M_{ki}\}$$
$$-M_{ij}\{M_{jk},\ M_{ki}\})\ ;\ \{A,\ B\}=AB+BA. \tag{7}$$

Their commutation relations are given in the Appendix.

Three quantities N_B, M and M' are commutable with any M_{ij} and their eigenvalues specify each irreducible representation. On the other hand, since N_B, M, M', Q, S, $\boldsymbol{I}^2$ and I_3 are commutable with each other, there will be a basis in terms of which all these matrices are of the diagonal form. In fact, from the configuration $(p,\ n,\ \Lambda)$ we are informed that I_i, Q, S and N_B are isospin, charge number, strangeness quantum number and baryon number, respectively. If v is a simultaneous eigenstate of Q, S and I_3, so is $N_{ij}v$, and the corresponding eigenvalues change as in Table I. M and M' are the new quantum numbers which are characteristic to our theory. Their eigenvalues can be easily calculated in the following manner.

Table I

	ΔQ	ΔI_3	ΔS
N_{13}	$+1$	$+\frac{1}{2}$	$+1$
N_{23}	0	$-\frac{1}{2}$	$+1$
N_{31}	-1	$-\frac{1}{2}$	-1
N_{32}	0	$+\frac{1}{2}$	-1

We consider an irreducible representation of $U(3)$. Let s_0 be the maximum eigenvalue of S, and denote by v_0 the simultaneous eigenstate of Q, S, N_B and I_3 which corresponds to the maximum I_3 among those eigenstates with $S=s_0$. Then we have $I_+v_0=N_{13}v_0=N_{23}v_0=0$. It follows from this that the values of M and M' for v_0 are given by

$$M=q_0^2+s_0^2+(n_B-q_0+s_0)^2+2(q_0+s_0),$$
$$M'=4[q_0^3+s_0^3+(n_B-q_0+s_0)^3+3(q_0^2-s_0^2)-(q_0+s_0)+4l_0], \tag{8}$$

where n_B, q_0 and l_0 are the eigenvalues of N_B, Q and I_3 corresponding to the v_0, respectively. Since M and M' have the same values for states of an irreducible representation, (8) is the desired result.

Physical meanings of M and M' are not so obvious. In addition, we shall be confronted with some difficulty when dealing with these quantities. So, it seems better to use s_0 and l_0 rather than the values of M and M'. That is, a set of n_B, s_0 and l_0 can specify an irreducible representation of $U(3)$, which is of the degree $\frac{1}{2}(2l_0+1)[\frac{1}{2}(n_B+3s_0)+l_0+2][\frac{1}{2}n_B+3s_0)-l_0+1]$ and is composed of $(2l_0+1)\cdot[\frac{1}{2}(n_B+3s_0)-l_0+1]$ irreducible representations of isospin. Moreover, it is also easy to obtain the law of decomposing a product of representations. The details of this approach will be published elsewhere.

We next consider the $(m+n)$-body system of m baryons and n anti-baryons, and denote its Salpeter-Bethe amplitude by

$$\bar{\chi}_{\kappa_1}\cdots\bar{\chi}_{\kappa_n}\chi^{\lambda_1}\cdots\chi^{\lambda_m}$$
$$=\langle B|T(\bar{\chi}_{\kappa_1}(x_1)\cdots\bar{\chi}_{\kappa_n}(x_n)\chi^{\lambda_1}(y_1)\cdots\chi^{\lambda_m}(y_m))|\Omega\rangle, \tag{9}$$

where $|B\rangle$ is an eigenstate of the total Hamiltonian and $|\Omega\rangle$ the true vacuum. It is to be noted that the index of a baryon χ^κ is written as a superscript and that of an anti-baryon $\bar{\chi}_\kappa$ as a subscript. Such a notation is useful because an anti-baryon behaves like a covariant vector under the transformation (1) :

$$\bar{\chi}_{\kappa'}=A_\kappa^{\kappa'}\bar{\chi}_\kappa=A_{\kappa'}^{\kappa}\bar{\chi}_\kappa. \tag{10}$$

The amplitude (9) is a mixed tensor $T_{\kappa_1\cdots\kappa_n}^{\lambda_1\cdots\lambda_m}$ of contravariant valence m and covariant valence n. So the decomposition of the system into its irreducible constituents is reduced to that of the corresponding tensor space. For this purpose, we have only to decompose the tensor space according to Young's diagram with respect to the upper and lower indices separately, and then to apply the "contraction operation" for an upper and a lower index or for several such pairs of indices. The latter process is similar to the trace operation in the case of the orthogonal group. In what follows we take the cases $m=n=1$ and $m=2$, $n=1$ for illustration.

(i) Two-body system of a baryon and an anti-baryon

The corresponding tensor $T_\kappa{}^\lambda$ can be decomposed into two irreducible constituents, by means of contraction operation, thus

$$T_\kappa{}^\lambda=\underset{1}{T}_\kappa{}^\lambda+\underset{2}{T}_\kappa{}^\lambda;\quad \underset{2}{T}_\kappa{}^\lambda\overset{\text{def}}{=}\delta_\kappa{}^\lambda T_\alpha{}^\alpha/3,\quad \underset{1}{T}_\kappa{}^\lambda\overset{\text{def}}{=}T_\kappa{}^\lambda-\underset{2}{T}_\kappa{}^\lambda.$$

$\underset{2}{T}_\kappa{}^\lambda$ has the only one independent component and the corresponding Salpeter-Bethe amplitude is $\chi^\kappa\bar{\chi}_\kappa$. $\underset{1}{T}_\kappa{}^\lambda$ is characterized by $\underset{1}{T}_\alpha{}^\alpha=0$ and has eight independent components. We present the basis vectors in Table II explicitly together with the quantum numbers. Each basis vector belonging to an irreducible representation must have the same eigenvalues of energy (=mass), spin and parity.

(ii) Three-body system of two baryons and an anti-baryon

We first decompose a tensor $T_\kappa{}^{\lambda\mu}$ with respect to the upper indices as follows :

$$T_\kappa^{\lambda\mu}=T_\kappa^{(\lambda\mu)}+T_\kappa^{[\lambda\mu]}.$$

Next by applying the contraction operation we have

$$\underset{}{T}_{\varkappa}^{[\lambda\mu]}=\underset{1}{T}_{\varkappa}^{\lambda\mu}+\underset{2}{T}_{\varkappa}^{\lambda\mu}; \quad \underset{1}{T}_{\varkappa}^{\lambda\mu}\overset{\text{def}}{=}\tfrac{1}{2}(\delta_{\varkappa}^{\lambda}T_{\alpha}^{[\alpha\mu]}-\delta_{\varkappa}^{\mu}T_{\alpha}^{[\alpha\lambda]}), \quad \underset{2}{T}_{\varkappa}^{\lambda\mu}\overset{\text{def}}{=}T_{\varkappa}^{[\lambda\mu]}-\underset{1}{T}_{\varkappa}^{\lambda\mu},$$

$$T_{\varkappa}^{(\lambda\mu)}=\underset{3}{T}_{\varkappa}^{\lambda\mu}+\underset{4}{T}_{\varkappa}^{\lambda\mu}; \quad \underset{3}{T}_{\varkappa}^{\lambda\mu}\overset{\text{def}}{=}\tfrac{1}{4}(\delta_{\varkappa}^{\lambda}T_{\alpha}^{(\alpha\mu)}+\delta_{\varkappa}^{\mu}T_{\alpha}^{(\alpha\lambda)}), \quad \underset{4}{T}_{\varkappa}^{\lambda\mu}\overset{\text{def}}{=}T_{\varkappa}^{(\lambda\mu)}-\underset{3}{T}_{\varkappa}^{\lambda\mu}.$$

We thus obtain four irreducible constituents. Each of $\underset{1}{T}_{\kappa}{}^{\lambda\mu}$ and $\underset{3}{T}_{\kappa}{}^{\lambda\mu}$ has three independent components and the corresponding representation is equivalent to the

Table II

Class I $M=6$, $M=' 8$		Note
$S=-1$, $I=1/2$	$-(\Lambda\bar{n})$ $(\Lambda\bar{p})$	$\bar{K}^0$ $\bar{K}^-$
$S=0$, $I=0$	$(p\bar{p}+n\bar{n}-2\Lambda\bar{\Lambda})/\sqrt{6}$	$\pi^{0'}$
$S=0$, $I=1$	$(p\bar{n})$ $(p\bar{p}-n\bar{n})/\sqrt{2}$ $(n\bar{p})$	π^+ π^0 π^-
$S=1$, $I=1/2$	$(p\bar{\Lambda})$ $(n\bar{\Lambda})$	K^+ K^0

Class II $M=0$, $M'=0$		Note
$S=0$, $I=0$	$(p\bar{p}+n\bar{n}+\Lambda\bar{\Lambda})/\sqrt{3}$	$\pi^{0''}$

Table III $[A, B]=A(x)B(y)-B(x)A(y)$, $(A, B)=A(x)B(y)+B(x)A(y)$

Class I $M=3$, $M'=20$		Note
$S=-1$, $I=0$	$\{\bar{p}[\mathrm{p}, \Lambda]+\bar{n}[n, \Lambda]\}/2$	$K^-+p\rightarrow$
$S=0$, $I=1/2$	$\{\bar{\Lambda}[\Lambda, p]-\bar{n}[p, n]\}/2$ $\{\bar{\Lambda}[\Lambda, n]+\bar{p}[p, n]\}/2$	

Class III $M=3$, $M'=20$		Note
$S=-1$, $I=0$	$\{2\bar{\Lambda}\Lambda\Lambda+\bar{p}(p, \Lambda)+\bar{n}(n, \Lambda\}/2\sqrt{2}$	$K^-+p\rightarrow$
$S=0$, $I=1/2$	$\{\bar{\Lambda}(\Lambda, p)+2\bar{p}pp+\bar{n}(pn)\}/2\sqrt{2}$ $\{\bar{\Lambda}(\Lambda, n)+2\bar{n}nn+\bar{p}(pn)\}/2\sqrt{2}$	

Class II $M=7$, $M'=4$		Note
$S=-1$, $I=1$	$\bar{n}[p, \Lambda]/\sqrt{2}$ $\{\bar{p}[p, \Lambda]-\bar{n}[n, \Lambda]\}/2$ $\bar{p}[n, \Lambda]/\sqrt{2}$	Σ^+ Σ^0 Σ^-
$S=0$, $I=1/2$	$\{\bar{\Lambda}[\Lambda, p]+\bar{n}[p, n]\}/\sqrt{2}$ $\{\bar{\Lambda}[\Lambda, n]-\bar{p}[p, n]\}/\sqrt{2}$	
$S=1$, $I=0$	$\bar{\Lambda}[n, p]/\sqrt{2}$	$K^++n\rightarrow$

	Class IV $M=11$, $M'=76$	Note
$S=-2$, $I=1/2$	$-\bar{n}\Lambda\Lambda$ $\bar{p}\Lambda\Lambda$	Ξ^0 Ξ^-
$S=-1$, $I=0$	$\{2\bar{\Lambda}\Lambda\Lambda-\bar{p}(p, \Lambda)-\bar{n}(n, \Lambda)\}/2\sqrt{2}$	$K^-+p\to$
$S=-1$, $I=1$	$-\bar{n}(p, \Lambda)/\sqrt{2}$ $\{\bar{p}(p, \Lambda)-\bar{n}(n\Lambda)\}/2$ $\bar{p}(n, \Lambda)/\sqrt{2}$	$K^-+p\to$ $K^-+n\to$
$S=0$, $I=1/2$	$\{2\bar{p}pp+\bar{n}(p, n)-3\bar{\Lambda}(\Lambda, p)\}/2\sqrt{6}$ $\{2\bar{n}nn+\bar{p}(p, n)-3\bar{\Lambda}(\Lambda, n)\}/2\sqrt{6}$	$\pi^++n\to$ $\pi^-+p\to$
$S=0$, $I=3/2$	$-\bar{n}pp$ $\{\bar{p}pp-\bar{n}(n, p)\}/\sqrt{3}$ $\{\bar{p}(n, \mathrm{p})-\bar{n}nn\}/\sqrt{3}$ $\bar{p}nn$	$(I=3/2,\ J=3/2)$ resonance state in π-N scattering
$S=1$, $I=1$	$\bar{\Lambda}pp$ $\bar{\Lambda}(p, n)/\sqrt{2}$ $\bar{\Lambda}nn$	$K^++p\to$ $K^++n\to$

original one (1). $\underset{2}{T}{}_\kappa^{\lambda\mu}$ and $\underset{4}{T}{}_\kappa^{\lambda\mu}$ are characterized by

$$\underset{2}{T}{}_\kappa^{(\lambda\mu)}=\underset{2}{T}{}_\alpha^{\alpha\mu}=0 \quad \text{and} \quad \underset{4}{T}{}_\kappa^{[\lambda\mu]}=\underset{4}{T}{}_\alpha^{\alpha\mu}=0,$$

so they give representations of degree six and fifteen respectively. The basis vectors of each representation are given in Table III.

§ 3. Application of the theory

Before referring to the results obtained from our theory, we should like to describe explicitly our equation of motion and the interaction Hamiltonian. The free field equation is the Dirac equation with spin 1/2:

$$(i\gamma_\mu\partial_\mu+\kappa)\chi=0, \quad \chi=\begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}. \tag{11}$$

For the interaction which acts so as to compose other particles we take the four field interaction. Then the following expression is a sole one,*

$$H'=\lambda(\bar{\chi}O\chi)^2, \tag{12}$$

where O is the usual Dirac matrix. We do not enter the problem how to construct the composite particle from (11) and (12), and our approach here is quite phenomenological.

* Apparently the more general expression (B) in the preceding letter[6] can be reduced to (12) by using Fierz's formula for the ordering exchange of particle.

Now our theory has so far assumed the complete equivalence between Λ and nucleon N while the real case has the asymmetry due to the existing mass difference between them, so we should note the modification of the theory. If the true situation is attained by adiabatically increasing the mass of Λ from its original value (equal to the nucleon mass) to the actual one, then the change of Hamiltonian is expressed by the addition of such a term as

$$H'' = \Delta\kappa(\bar{\Lambda}\Lambda). \tag{13}$$

By including the term (13) the complete symmetry of our theory is broken and M and M' are no more good quantum numbers. But we notice that the parity (P), spin (J), iso-spin (I) and the strangeness quantum number (S) of each state still do not change with the inclusion of (13), namely,

$$\Delta P = \Delta J = \Delta I = \Delta S = 0\,;\ \Delta M \neq 0\,;\ \Delta M' \neq 0. \tag{14}$$

Thus we obtain the following conclusions:

(α) In the limiting case of equal mass for p, n and Λ, the corresponding particle states of an irreducible representation must have the same nature; equal mass (or energy) level, same parity and equal spin.

(β) When the finite mass difference is taken into account as (13), M and M' may not be good quantum numbers. Irreducible representation with different values of M and M' may become to mix, and the mass (or energy) level of each state will change. But the original value of spin, parity, iso-spin and strangeness of each state must still be preserved and the irreducible classes with different spin and parity will not mix with each other.

A possible reasoning for that the actual case is attained by the inclusion of such a term as (13) is as follows. We know another family—the lepton family the situation of which is very similar to our case. Within the energy region now available for us μ-meson and electron behave with close resemblance in the electromagnetic interaction as well as in the weak interaction in spite of their large mass splitting. Accordingly the origin of their mass difference, if it exists, must be confined in a far smaller region than the one where the usual interactions play a dominant role. We should like to think that the mass splitting between Λ and nucleon arises from a similar cause.

Now we shall enter into the concrete problem.

(i) Two-body system of baryon and anti-baryon.

From Table II it is informed that two neutral particle states π_0' and π_0'' are anticipated to exist in addition to the well established seven Bose particles $(\pi^+\pi^0\pi^-,\ K^+K^0,\ K^-\widetilde{K}^0)$.

π_0' belongs to the same class as that of the other seven particles and must be a pseudo-scalar particle with isotopic spin 0*. In the limiting case of equal

* A possible role of π_0' in decay process has been studied by Sawada and Yonezawa[7]. We thank them for informing us of their results.

mass for Λ and N, the mass of π_0' is equal to that of usual π-meson (in this case the mass of K is also equal to that of π-meson).

π_0'' alone forms the other irreducible base and its nature is not known to us except its isotopic spin$=0$. But we should note that if we take the strong Fermi interaction (12) to be of well-known $S-T+P$ or $V-A$ or $S-A+P$ (invariant for the exchange of ordering) type, the sign of potential in Fermi-Yang's sense between baryon and anti-baryon is opposite for class I and for class II. If this is the case, π_0'' may not be a bound state.

K-meson must be pseudo-scalar. This means that when we make up the following Yukawa type of interaction

$$\Lambda ON\cdot K,$$

the Dirac matrix O must be γ_5 or $\gamma_5\gamma_\mu$.

(ii) Three-body system of two baryons and one anti-baryon (see Table III).

Twenty-seven states appear in this case which are classified into four irreducible representations. Now we shall present a remark for each class separately.

Class IV: In this class we have the $I=1/2$ $S=-2$ state the bound level of which is Ξ-particle. There is also the $I=3/2$ $S=0$ state which corresponds to the $\pi+N$ system with $I=3/2$ in the lowest configuration of the usual theory. The $\pi+N$ system may be the free scattering state or the well-known $I=3/2$ resonant state. However, we may reasonably take the resonant state as that corresponding to the Ξ-particle state, because the scattering state (continuous spectrum) will not go into the bound (discrete spectrum) state by adiabatically changing the mass. Thus the nature of Ξ particle will be same as that of the $I=3/2$ $J=3/2$ resonant state in π-N scattering, that is, the spin of Ξ is $J=(3/2)^+$.

Our reasoning here is rather phenomenological and not logically strict. For instance, if there exist other discrete (but metastable) levels which are yet unobserved and the correspondence is such as that indicated by the arrows in Fig. 1,

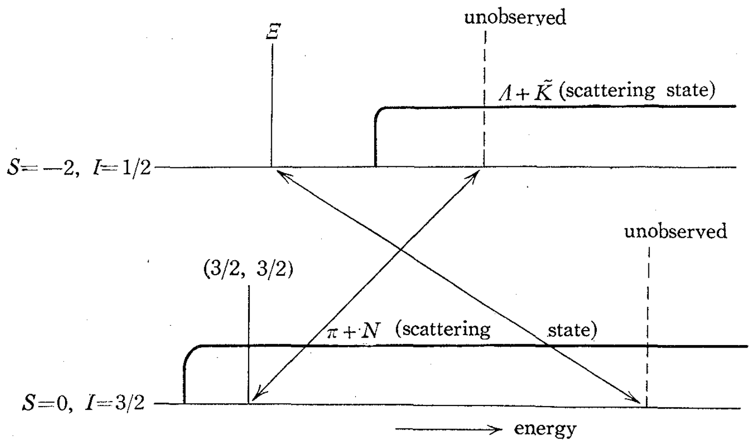

Fig. 1

then our conclusion will fail. But our original intention rests on the conjecture that the symmetry will not be drastically destroyed in the actual case and the closeness of mass level of Ξ and the $(I=3/2,\ J=3/2)$ resonant state induces us to accept the correspondence between them. And in the experiment we have not any other $I=3/2$, $S=0$ resonance-like state in the energy interval of $\sim$500 Mev from the mass of Ξ.

There are nine other corresponding states in class IV, whose spin is $J=3/2$. All of them are thought to be unstable states. But we may expect that some of them will be realized as the resonating states with $J=3/2$ in K-nucleon and π-nucleon scattering in not so high energy region (say < 1 Bev). Some possible channels leading to these states are presented in "note" of Table III.

Class II: There is the $(I=1\ S=-1)$ state which we take as Σ-particle. Although Σ-like state appears in Class IV, we regard it rather as the excited state of Σ, because the ground level of Class IV is of spin $J=3/2$ as stated above and Σ is known to be of spin 1/2. Now the other states of Class II corresponding to Σ have spin $J=1/2$ and the experimental check for this will be found in $K^+ + n$ $(I=0\ S=+1)$ scattering.

Class I and Class III: Both classes have the same quantum numbers as that of the one-body configuration. If they possess the spin, parity and other nature in common with the one-body configuration, these states will mix with p, n and Λ state correspondingly. The situation is also the same for $(I=1/2,\ S=0)$ state of Class (II). Of course the statement here is nothing beyond the speculation. Some of the states might be realized as the resonating states.

In this paper we have proposed a theory in which the systematical side of Sakata's theory is stressed, while the problem of dynamics such as a composition of the particles is left untouched. We hope, however, that if our theory is qualitatively supported by future experiments, then it will give some of clues to attack the dynamical side of composite model.

In conclusion we should like to express our deep gratitude to Profs. S. Sakata and K. Sakuma for their keen interest in this work. We should also like to thank Prof. Y. Yamaguchi at CERN who has sent us very stimulating information about his work in which the similar course to ours is developed. One of the authors (S. O.) thanks the collegues of Sakuma laboratory for their helpful discussions.

Appendix

$$[I_2,\ I_3]=iI_1,\ [I_3,\ I_1]=iI_2,\ [I_1,\ I_2]=iI_3,$$

$$[Q,\ I_1]=iI_2,\ [Q,\ I_2]=-iI_1,\ [Q,\ I_3]=0,$$

$$[S,\ I_1]=[S,\ I_2]=[S,\ I_3]=[S,\ Q]=0,$$

$$[Q,\ \boldsymbol{I}^2]=[S,\ \boldsymbol{I}^2]=[I_3,\ \boldsymbol{I}^2]=0,$$

$$[I_+, I_3]=-I_+,\ [I_-, I_3]=I_-,\ [I_+, I_-]=2I_3,$$
$$[I_+, Q]=-I_+,\ [I_-, Q]=I_-,\ [I_+, S]=[I_-, S]=0,$$
$$[N_{13}, I_1]=-\tfrac{1}{2}N_{23},\ [N_{23}, I_1]=-\tfrac{1}{2}N_{13},\ [N_{31}, I_1]=\tfrac{1}{2}N_{32},\ [N_{32}, I_1]=\tfrac{1}{2}N_{31},$$
$$[N_{13}, I_2]=-\frac{i}{2}N_{23},\ [N_{23}, I_2]=\frac{i}{2}N_{13},\ [N_{31}, I_2]=-\frac{i}{2}N_{32},\ [N_{32}, I_2]=\frac{i}{2}\mathrm{N}_{31},$$
$$[N_{13}, I_3]=-\tfrac{1}{2}N_{13},\ [N_{23}, I_3]=\tfrac{1}{2}N_{23},\ [N_{31}, I_3]=\tfrac{1}{2}N_{31},\ [N_{32}, I_3]=-\tfrac{1}{2}N_{32},$$
$$[N_{13}, I_+]=0,\ [N_{23}, I_+]=-N_{13},\ [N_{31}, I_+]=N_{32},\ [N_{32}, I_+]=0,$$
$$[N_{13}, I_-]=-N_{23},\ [N_{23}, I_-]=0,\ [N_{31}, I_-]=0,\ [N_{32}, I_-]=N_{31},$$
$$[N_{13}, Q]=-N_{13},\ [N_{23}, Q]=0,\ [N_{31}, Q]=N_{31},\ [N_{32}, Q]=0,$$
$$[N_{13}, S]=-N_{13},\ [N_{23}, S]=-N_{23},\ [N_{31}, S]=N_{31},\ [N_{32}, S]=N_{32},$$
$$[N_{13}, N_{31}]=Q+S,\ [N_{13}, N_{32}]=I_+,\ [N_{13}, N_{23}]=0,$$
$$[N_{23}, N_{32}]=Q+S-2I_3,\ [N_{23}, N_{31}]=I_-,\ [N_{31}, N_{32}]=0.$$

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