A Practical Aspect of Over-Rooftop Multiple-Building Forward Diffraction from a Low Source

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***Abstract*—A practical aspect of over-rooftop multiple-building forward diffraction from a low source is presented, including two formulations and an introduction of their application. In particular, the low-loss formulation for multiple diffraction exposes a factor that can account for strong over-rooftop radio propagation. Both low- and high-loss formulations take the advantages of the uniform geometrical theory of diffraction and physical optics (PO) and have the major advantage of significantly shortening the computing time over existing formulations. They behave well, particularly in and near the transition zone, and are written in explicit forms additionally for soft boundary that corresponds to vertical polarization transmission and reception in the horizontal plane. The application in the vertical plane for the total received signal strength prediction is introduced using the formulations for hard boundary corresponding to vertical polarization transmission and reception.**

***Index Terms*—Microcellular mobile radio, multiple diffraction, radio propagation.**

I. Introduction

MORE and more system planners have realized the importance of the over-rooftop multiple-building forward diffraction from a low transmitter, as shown in Figs. 1 and 2, which apply to microcellular mobile radio communications. It is appropriate to consider the diffraction rays 1 and 2 for the total signal reception even in the environments of the transmitter and receiver much lower than the surrounding buildings. Recently, Xia presented an analytical propagation model [1] capable for the base-station antenna near and below the average rooftop level. To account for the local scattering from obstacles surrounding the base station, the model adds a factor of two to the free-space term in the received power calculation. In order to represent the composite effects of the diffraction paths from the rooftop to the street and other scattered paths, the model also multiplies the local diffraction term by a factor of two as done by Walfisch and Bertoni [2].

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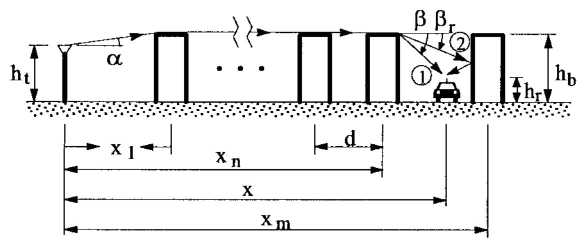

Fig. 1. Geometry showing transmitter and receiver below rooftops of several rows of buildings for microcellular mobile radio communications.

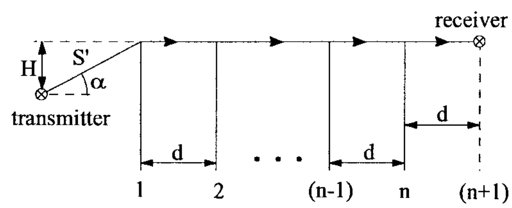

Fig. 2. Geometry representing transmitter below and near and the field point (receiver) at the average rooftop level of n rows of buildings.

The simplified factor for multiple forward diffraction past rows of buildings is based on the studies by Xia and Bertoni [3] and is extracted from the infinite series. Because of this, it is less competent in and near the transition zone. The present work provides a practical aspect of over-rooftop multiple-building forward diffraction from a low source. In particular, it derives a low-loss formulation for the multiple diffraction, which exposes a factor that can account for strong over-rooftop propagation as indicated in [4, p. 1347].

It is inappropriate to only use the uniform geometrical theory of diffraction (UTD) [5] for this multiple diffraction, because the multiple diffraction edges modeling the buildings are in the transition zone of the preceding edges as seen in Fig. 2. The physical optics (PO) approximation is accurate in and near the transition zone but involves multiple-dimension integration because of the multiple diffraction. There is a major difficulty in computing the multiple-dimension integrals. Using the summations of finite terms satisfying a recursion relation, Saunders and Bonar [6] and Juan–Llácer and Cardona [7] gave attenuation functions capable for transmitter below and at the average rooftop level. For both cylindrical and plane waves, Xia and Bertoni [3] presented infinite series rapidly convergent for the transmitter at and not very far from the average rooftop level.

In this paper, Section II presents two formulations for multiple-building forward diffraction, which are combinations of UTD and PO approaches. Both formulations are written in explicit forms involving single one-dimensional integration of the transition function [5]. They behave well particularly in and near the transition zone and additionally apply to soft boundary, which corresponds to vertical polarization transmission and reception in the horizontal plane. An engineering application of the formulations is introduced in Section III. The entire work is concluded in Section IV.

II. Formulations

Consider the electric field at the reference point $n+1$ (receiver) as seen in Fig. 2. The rows of knife edges modeling the buildings are numbered from 1 to n. Let $Q_{n+1} = \mathcal{F}D$ be a ratio of the field at position $n+1$ and the free-space field of the same path length from the source, where D is a UTD diffraction coefficient for a field at position $n+1$ diffracted by edge 1 and $\mathcal{F}$ is a hybrid function from PO and UTD. The elevation angle α is derived from

$$\sin\alpha = \frac{H}{s'} \tag{1}$$

where H is the relative height of the rooftop to the transmitter and s' is the distance from the base station antenna to edge 1. Specifically, two formulations for Q_{n+1}, which account for the multiple diffraction effects, are determined in two steps. First, by using available closed-form expressions $Q_{n+1}|_{\alpha=0}$ that are the PO-based functions of Q_{n+1} at $\alpha = 0$ [2], [3], [8], two equations for the hybrid function $\mathcal{F}$ are derived as $\mathcal{F} = Q_{n+1}|_{\alpha=0}/D|_{\alpha=0}$. Second, by making a large number of computations and comparisons with existing formulations of Q_{n+1} [3], [6], [7], the behavior of $Q_{n+1} = \mathcal{F}D$ for $\alpha \neq 0$ is examined and the applicable regions of the two formulations for Q_{n+1} are determined. Let k be the wave number, λ be the wavelength, d be the average spacing of rows of buildings, and $g = \sin\alpha\sqrt{d/\lambda}$ be a grouping parameter. The two formulations are presented in the following.

A. Low-Loss Formulation

For $0 \le g\sqrt{n} \le 0.3$, the approximation of Q_{n+1} determined by comparisons with the results of [2], [3], [6], and [7] is given as

$$Q_{n+1} = \frac{1}{\sqrt{3n+1}} \frac{D^p_{s,h}}{D^p_{s,h}\big|_{\alpha=0}} \tag{2}$$

$$D^p_{s,h} = \frac{-\exp(-j\pi/4)}{2\sqrt{2\pi k}}\left[\frac{F(X)}{-\sin(\alpha/2)} \mp \frac{1}{-\cos(\alpha/2)}\right] \tag{3}$$

$$\sqrt{X} = \sqrt{2kL}|\sin(\alpha/2)| \tag{4}$$

$$L = nd \tag{5}$$

$$F(X) = \sqrt{\pi X}\exp(j\pi/4 + jX) - 2j\sqrt{X}\exp(jX)\int_0^{\sqrt{X}}\exp(-j\tau^2)\,d\tau \tag{6}$$

$$D^p_{s,h}\Big|_{\alpha=0} = \frac{-\exp(-j\pi/4)}{2\sqrt{2\pi k}} \cdot \left[-\sqrt{2\pi kL}\cdot\exp(j\pi/4) \mp (-1)\right] \tag{7}$$

where subscripts "s" and "h" of UTD diffraction coefficients $D^p_{s,h}$ for plane wave incidence denote soft and hard boundaries, respectively. And they take signs "−" and "+" on the right-hand side of (3) and (7). The transition function $F(X)$ can be approximated as $F(X) \approx \sqrt{\pi X}\exp(j\pi/4 + jX)$ for $X < 10^{-3}$ and $F(X) \approx 1$ for $X > 10$.

At $g = 0$, (2) becomes $Q_{n+1} = 1/\sqrt{3n+1}$ by Walfisch and Bertoni [2], that approximates $\Gamma(n+1/2)/(\sqrt{\pi}n!)$ obtained by Lee [8] for both hard and soft boundaries on the condition of $s' = \infty$. This condition may not be very restrictive and may be extended to $s' > d$. In fact, Xia and Bertoni [3] obtained the same expression for plane wave incidence. Due to $kL \gg 1$ and $\cos(\alpha/2) > \sqrt{2}/2$, the second term on the right-hand side of (3) and (7) is small compared with the first term. For this and similar reasons, the two formulations for Q_{n+1} are dependent on and less sensitive to the type of boundaries. Since d ranges about from 30 to 100 m, $kL \gg 1$ is satisfied at UHF (300 MHz–3 GHz) and microwave frequencies used in mobile communications.

For $g\sqrt{n} > 0.3$, the approach to Q_{n+1} derived from comparisons with the results of [3] and [8] is formulated as

$$Q_{n+1} = \frac{1}{n+1}\frac{D_{s,h}}{D_{s,h}|_{\alpha=0}} \tag{8}$$

$$L = \frac{s'nd}{s'+nd} \tag{9}$$

where UTD diffraction coefficients $D_{s,h}$ apply to both spherical and cylindrical-wave incidence, since the distance parameters L are the same for both here. It is easy to calculate (8) expressed as $Q_{n+1} \approx 2/(n+1) \times D_{s,h}/\sqrt{L}$ for $X > 10$ leading to $F(X) \approx 1$. Because it is obtained on the condition of $s'\cos\alpha \sim d$, (8) may become incorrect when $s'\cos\alpha$ diverges far from d.

Numerical comparisons of (2) and (8) of $|Q_{n+1}|$ with existing formulations [3], [6], [7] are presented in Fig. 3, at $d/\lambda = 200$ for $-g$ from 0 to -1.414. At $g = 0$, it is seen that $1/\sqrt{3n+1}$ of (2) indeed approaches $\Gamma(n+1/2)/(\sqrt{\pi}n!)$, which all other formulations take. It is seen that (8) behaves fairly for g getting large.

In the region of $0 < g\sqrt{n} \le 0.3$, formulations by Saunders and Bonar [6] and by Juan–Llácer and Cardona [7] give nearly same numerical results. The Xia and Bertoni formulation by [3, eq. (17)] for plane wave generates almost same numerical results as formulations of [6] and [7].

In the region of $g\sqrt{n} > 0.3$, results of Xia and Bertoni formulation for cylindrical wave are computed from [3, eq. (18)]. Equation (8) of the present formulation is computed at $s' = d$. It was found that the formulations of [6] and [7] produce lower values, as well as Xia and Bertoni formulation for plane wave [3]. They may underestimate the multiple diffraction and the presentations for them are omitted.

For $\alpha \approx 2\sin(\alpha/2)$ small and $g\sqrt{n} \approx \alpha\sqrt{L/\lambda}$ small, we approximate (6) by

$$F(X) \approx \left[\sqrt{\pi X} - 2X\exp(j\pi/4)\right]\exp(j\pi/4 + jX) \tag{10}$$

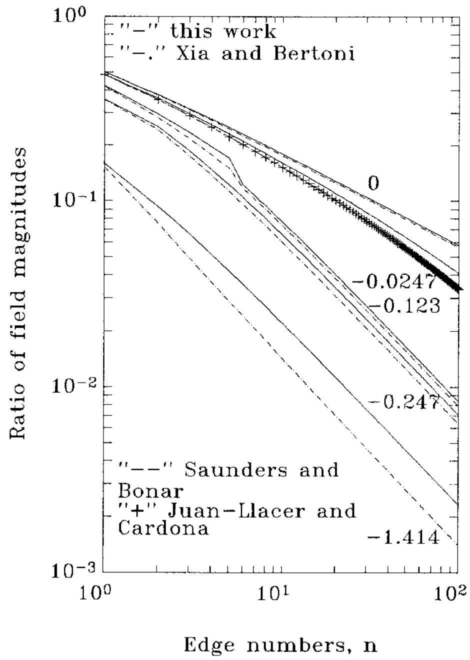

Fig. 3. Comparison of low-loss formulation with existing formulations at $d/\lambda = 200$ grouped by $-g$; ——— this works for hard boundary; —· Xia and Bertoni; - - Saunders and Bonar; + Juan–Llácer and Cardona.

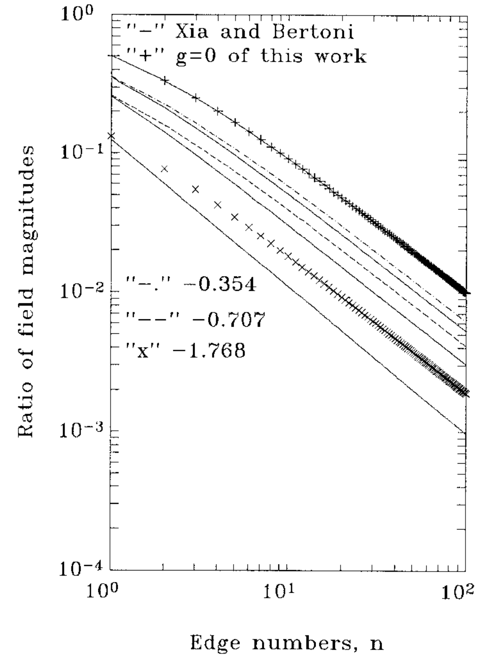

Fig. 4. Comparison of high-loss formulation for hard boundary with Xia and Bertoni formulation at $d/\lambda = 200$ grouped by $-g$ from 0 to -1.768, ——— Xia and Bertoni.

where $X \approx n\pi g^2$ is small [5]. One derives from (3)

$$D^p_{s,h}/\sqrt{L} \approx 1/2 - g\sqrt{n}\,\exp(j\pi/4). \tag{11}$$

Consequently, (2) becomes

$$Q_{n+1} \sim \frac{1}{\sqrt{3n+1}}\left[1 - 2g\sqrt{n}\,\exp(j\pi/4)\right]. \tag{12}$$

Similarly, (8) can be approximated as

$$Q_{n+1} \sim \frac{1}{n+1}\left[1 - 2g\sqrt{\frac{n}{n+1}}\,\exp(j\pi/4)\right] \tag{13}$$

for $X \approx \pi g^2 n/(n+1)$ small. Let $R \approx nd$ be the range from base station to a mobile terminal and the local row of buildings just prior to the mobile number n. The total received power calculated by (12) and (13) can have a range dependence of $n/(3R^{4+\delta\gamma})$ for $n > 3$ and of $1/R^{4+\Delta\gamma}$ as presented by Xia [1], respectively, where $0 \le \delta\gamma < 1$ and $0 \le \Delta\gamma < 1$ are small positive numbers. The loss reduction due to (12) may be evaluated by $10\log_{10}(n/3)$ for $n > 3$ and is about $10\log_{10} 2 \approx 3$ dB at $n = 6$. Hence, (2) and (8) are termed low-loss formulation. The discontinuity from (2) to (8) may be estimated by $\Delta Q_{n+1} \approx 1/\sqrt{3n+1} - 1/(n+1) - 0.6\exp(j\pi/4)[1/\sqrt{3n+1} - 1/(n+1)^{3/2}]$, that is the difference between (12) and (13) calculated at $g\sqrt{n} \approx 0.3$. The discontinuity can be seen for the curves of $g = 0.123$ and $g = 0.247$ in Fig. 3. At $g\sqrt{n} \sim 0$, it is clear that $\Delta Q_{n+1} \approx 1/\sqrt{3n+1} - 1/(n+1) \ge 0$.

B. High-Loss Formulation

Equation (8) can be used alone without the restriction of g and this is termed as high-loss formulation. Numerical comparisons of high-loss formulation at $s' = d$ with Xia and Bertoni formulation for cylindrical wave [3] are presented in Fig. 4. At $g = 0$, both formulations become $1/(n+1)$, that Lee [8] derived for both hard and soft boundaries. For g large getting edge 2 to lie outside the transition zone, (8) appears to slightly differ from Xia and Bertoni formulation.

III. Application in the Vertical Plane

This section introduces the application of the two formulations in the vertical plane to microcellular radio propagation prediction, as seen in Fig. 1. The electric field of ray 1 is determined by the multiplication of $|Q_n|$ and a single diffraction field by the local row of edges for spherical-wave incidence from the base station. Taking

$$s' = x_1/\cos\alpha + x_n - x_1 \tag{14}$$

for both amplitude factor and distance parameter $\phi' = \pi/2$ and $\phi = 3\pi/2 + \beta$, the single diffracted field can be derived from standard UTD [5]. Similarly, the reflection of diffracted fields of ray 2 can be determined [9]. To simplify the received signal strength prediction, a loss reduction $10\log_{10} 2 \approx 3$ dB may account for the multipath effects [2], [9]. In other words, reducing the path loss of ray 1 by 3 dB, one can predict the total path loss defined by the ratio of radiated to received power for isotropic antennas.

An example of predictions compared with measurements is presented in Fig. 5 and Table I, where the average error $\Delta_{\rm av}$, for difference between predicted and measured results, and the root mean square error $\Delta_{\rm rms}$ are listed. The present work, using low-loss formulation, provides better prediction in the particular case of two rows of buildings between transmitter and receiver for vertical polarization transmission and reception. The prediction of Xia model results from

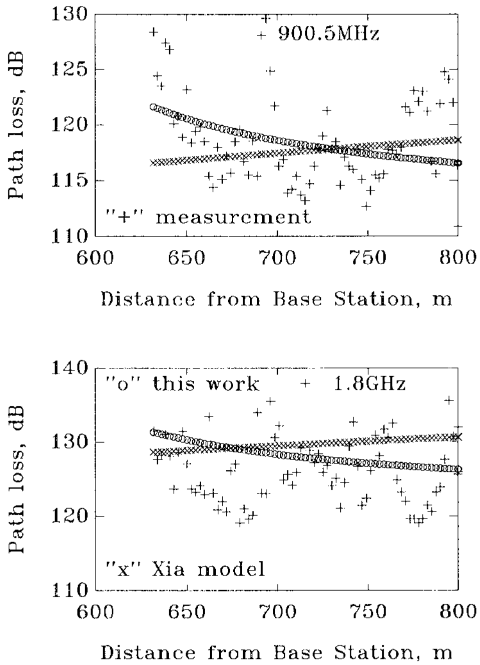

Fig. 5. Comparison of predictions with measurements for two rows of buildings between base station and mobile station; $h_t = 13.3$, $h_b = 19$, $h_r = 1.5$, $x_1 = 460$, $d = 140$, $x_n = 600$, and $x_m = 800$ m.

TABLE I
AVERAGE ERROR Δ_{av} AND ROOT MEAN SQUARE ERROR Δ_{rms} FOR COMPARISON OF PREDICTIONS WITH MEASUREMENTS AT 0.9005/1.8 GHz

items	Δ_{av} (dB)	Δ_{rms} (dB)
present work	−0.271/2.00	3.86/5.06
Xia model	−1.04/3.54	4.33/5.79

[1, eq. (29)]. Due to the complexity of urban microcellular mobile radio environments, efficient propagation prediction tools are necessary and important for the development of wireless communication systems. The present low and high-loss formulations would be efficient for site-specific propagation predictions and for the characterization of mobile radio propagation environments.

IV. CONCLUSION

A practical aspect of over-rooftop multiple-building forward diffraction from a low source has been presented, including two formulations and an introduction of their application. In particular, the low-loss formulation for multiple diffraction exposes a factor that can account for strong over-rooftop propagation. Both low- and high-loss formulations take the advantages of UTD and PO methods and have the major advantage of significantly shortening the computing time over existing formulations [3], [6], [7]. They behave well, particularly in and near the transition zone, and are written in explicit forms additionally for soft boundary. The application in the vertical plane for the total received signal strength prediction is introduced, using the formulations for hard boundary corresponding to vertical polarization transmission and reception.

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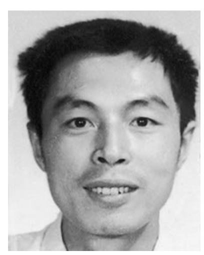

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