#### A PRACTITIONER'S GUIDE TO STOCHASTIC FRONTIER ANALYSIS USING STATA

A Practitioner's Guide to Stochastic Frontier Analysis Using Stata provides practitioners in academia and industry with a step-by-step guide on how to conduct efficiency analysis using the stochastic frontier approach. The authors explain in detail how to estimate production, cost, and profit efficiency and introduce the basic theory of each model in an accessible way, using empirical examples that demonstrate the interpretation and application of models. This book also provides computer code, allowing users to apply the models in their own work, and incorporates the most recent stochastic frontier models developed in academic literature. Such recent developments include models of heteroscedasticity and exogenous determinants of inefficiency, scaling models, panel models with time-varying inefficiency. Immensely helpful to applied researchers, this book bridges the chasm between theory and practice, expanding the range of applications in which production frontier analysis may be implemented.

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# A Practitioner's Guide to Stochastic Frontier Analysis Using Stata

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To Damayanti Ghosh

SUBAL C. KUMBHAKAR

To Yi-Yi Chen

HUNG-JEN WANG

To Maria, Joan, and Victor

ALAN P. HORNCASTLE

#### Contents

PART I GENERAL INFORMATION

1	Intr	oducti	on	3
	1.1	What	This Book Is About	3
	1.2	Who	Should Read This Book?	4
	1.3	The S	tructure of This Book	5
2	Pro	ductior	n, Distance, Cost, and Profit Functions	9
	2.1	Introduction		
	2.2	2.2 The Production Function and Technical Efficiency		
		2.2.1	Input-Oriented and Output-Oriented Technical Inefficiency	12
		2.2.2	Non-Neutral Technical Inefficiency	15
	2.3	Statistics from Production Functions		15
		2.3.1	Homogeneity and Returns to Scale	16
		2.3.2	Substitutability	17
		2.3.3	Separabilitiy	17
		2.3.4	Technical Change	18
	2.4	Transformation of Production Functions		19
	2.5	2.5 Functional Forms of Production Functions		20
		2.5.1	The Cobb-Douglas (CD) Production Function	20
		2.5.2	The Generalized Production Function (GPF)	22
		2.5.3	The Transcendental Production Function	23
		2.5.4	The Translog Production Function	24
	2.6	Multiple Output Production Technology (Distance Functions)		25
		2.6.1	Distance Functions	27
		2.6.2	The Translog Input Distance Function	30
		2.6.3	The Translog Output Distance Function	30
	2.7	The T	ransformation Function Formulation	31
		2.7.1	The Transformation Function with Inefficiency	31
	2.8	2.8 Allocative Inefficiency		
		2.8.1	Cost Minimization and Allocative Inefficiency	38
		2.8.2	Profit Maximization and Allocative Inefficiency	40

viii		Contents		
	2.9	The Indirect Production Function	41 41	
		2.7.1 Hoteling	11	
PAR	T II	SINGLE EQUATION APPROACH: PRODUCTION, COST, AND PROFIT		
3	Esti	mation of Technical Efficiency in Production Frontier Models Using	47	
	3.1	Introduction	47 47	
	3.2	Output-Oriented Technical Efficiency	48	
	3.3	Estimation Methods: Distribution-Free Approaches	49	
		3.3.1 Corrected OLS (COLS)	50	
		3.3.2 Corrected Mean Absolute Deviation (CMAD)	53	
		3.3.3 Thick Frontier Approach	54	
	3.4	Estimation Methods: Maximum Likelihood Estimators	55	
		3.4.1 A Skewness Test on OLS Residuals	56	
		3.4.2 Parametric Distributional Assumptions	59	
		3.4.4 Truncated Normal Distribution	59 73	
		3.4.5 Truncated Distribution with the Scaling Property	85	
		3.4.6 The Exponential Distribution	90	
	3.5	Input-Oriented Technical Inefficiency	95	
	3.6	Endogeneity and Input and Output Distance Functions	97	
4	Esti	mation of Technical Efficiency in Cost Frontier Models Using		
	Cro	ss-Sectional Data	100	
	4.1	Introduction	100	
	4.2	4.2.1 Price Homogeneity	101	
		4.2.2 Monotonicity and Concavity	105	
	4.3	Estimation Methods: Distribution-Free Approaches	108	
		4.3.1 Corrected OLS	109	
		4.3.2 Cases with No or Little Variation in Input Prices	110	
		4.3.3 Thick Frontier Approach	110	
		4.3.4 Quantile-Regression-Based TFA	113	
	4.4	Estimation Methods: Maximum Likelihood Estimators	115	
		4.4.1 Skewness Test on OLS Residuals	116	
		4.4.2 The Half-Normal Distribution	117	
	15	4.4.5 The Truncated Normal, Scaling, and Exponential Models	120	
	ч.5	4.5.1 Quasi-Fixed Inputs	122	
		4.5.2 Estimation Methods	125	
5	Estimation of Technical Efficiency in Profit Frontier Models Using			
	Cross-Sectional Data			
	5.1	Introduction	128	
	5.2	Output-Oriented Technical Inefficiency	130	
	5.3	Estimation Methods: Distribution-Free Approaches	134	
	5.4	Estimation Methods: Maximum Likelihood Estimators	136	

		Contents	ix
	5.5	Input-Oriented Technical Inefficiency	143
	5.6	Estimation Methods: Distribution-Free Approaches	145
	5.7	Estimation Methods: Maximum Likelihood Estimators	145
PAI	RT III	SYSTEM MODELS WITH CROSS-SECTIONAL DATA	
6	Esti	mation of Technical Efficiency in Cost Frontier Models Using System	
	Mod	lels with Cross-Sectional Data	149
	6.1	Introduction	149
	6.2	Single Output, Input-Oriented Technical Inefficiency	149
	6.3	Estimation Methods: Distribution-Free Approach	152
	6.4	Estimation Methods: Maximum Likelihood Estimators	156
		6.4.1 Heteroscedasticity, Marginal Effects, Efficiency Index, and	
		Confidence Intervals	168
	6.5	Multiple Outputs, Input-Oriented Technical Inefficiency	169
	6.6	Estimation Methods	171
	6.7	Multiple Outputs, Output-Oriented Technical Inefficiency	171
7	Estimation of Technical Efficiency in Profit Frontier Models Using System		
	Мос	lels with Cross-Sectional Data	173
	7.1	Introduction	173
	7.2	Single Output, Output-Oriented Technical Inefficiency	173
	7.3	Estimation Methods: Distribution-Free Approaches	176
	7.4	Estimation Methods: System of Share Equations, Maximum	
		Likelihood Estimators	181
	7.5	Estimation Methods: Imposing Homogeneity Assumptions, Maximum	
		Likelihood Estimators	189
	7.6	Single Output, Input-Oriented Technical Inefficiency	195
	7.7	Multiple Output Technology	196
		7.7.1 Output-Oriented Technical Inefficiency	196
		7.7.2 Estimation Methods	198
PAI	RT IV	THE PRIMAL APPROACH	
8	Esti	mation of Technical and Allocative Efficiency in Cost Frontier Models Using	
	Syst	em Models with Cross-Sectional Data: A Primal System Approach	203
	8.1	Introduction	203
	8.2	Cost System Approach with Both Technical and Allocative Inefficiency	204
	8.3	The Primal System Approach with Technical and Allocative Inefficiency	208
	8.4	Estimation Methods When Algebraic Formula Can Be Derived	210
		8.4.1 The Cobb-Douglas Production Function	210
		8.4.2 The Generalized Production Function	223
	8.5	Estimation Methods When Algebraic Formula Cannot Be Derived	224
		8.5.1 Translog Production Function	224
9	Esti	mation of Technical and Allocative Efficiency in Profit Frontier Models	
	Using System Models with Cross-Sectional Data: A Primal System Approach		
	9.1	Introduction	230
	9.2	The Profit Function Approach	231

х	Contents		
	9.3 The Primal Approach of Profit Maximization with Both Technical and		
	Allocative Inefficiency	231	
	9.4 Estimation Methods: Maximum Likelihood Estimators	233	
	9.4.1 Technical and Allocative Inefficiency Effect on Profit	236	
PAR	RT V SINGLE EQUATION APPROACH WITH PANEL DATA		
10	Estimation of Technical Efficiency in Single Equation Panel Models	241	
	10.1 Introduction	241	
	10.2 Time-Invariant Technical Inefficiency (Distribution-Free)	2.42	
	Models 10.2.1 The Fixed Effects Model (Schmidt and Sickles [1084])	243	
	10.2.2 The Pandom Effects Model	243	
	10.3 Time-Varving Technical Inefficiency Models	240	
	10.3.1 Time-Varying Technical Inefficiency Models Using Distribution-Free	200	
	Approaches	251	
	10.3.2 Time-Varying Inefficiency Models with Deterministic and		
	Stochastic Components	254	
	10.4 Models That Separate Firm Heterogeneity from Inefficiency	262	
	10.5 Models That Separate Persistent and Time-Varying Inefficiency	270	
	10.5.1 The Fixed-Effects Model	271	
	10.5.2 The Random-Effects Model	271	
	10.6 Models That Separate Firm Effects, Persistent Inefficiency and	274	
	Time-varying Inefficiency	274	
11	Productivity and Profitability Decomposition	279	
	11.1 Introduction	279	
	11.2 Productivity, Technical Efficiency, and Profitability	280	
	11.3 Productivity and Profitability Decomposition	285	
	11.3.1 Iotal Factor Productivity Decomposition: The Production Function	206	
	11.3.2 Productivity Decomposition: The Cost Function Approach	200	
	11.3.3 Multiple Outputs	300	
		200	
PAF	RT VI LOOKING AHEAD	211	
12	Looking Anead	311 211	
	12.1 Latent Class Models	311	
	12.3 Selectivity in SF Models	313	
	12.5 Selectivity in or models	515	
	Environmental Efficiency	313	
	12.5 Two-Tier SF Models	314	
	12.6 SF Models with Copula Functions (To Introduce Correlation		
	between the Noise and Inefficiency Terms)	314	
	12.7 Nonparametric and Semiparametric SF Models	314	
	12.8 Testing Distribution Assumptions	315	

$\sim$		
(.0	nte	nts
$-\omega$	1111	1115

xi

APPENDIX			
A	Deriving the Likelihood Functions of Single Equation Frontier Models		
В	Deriving the Efficiency Estimates		
С	Der	riving Confidence Intervals	326
D	Bootstrapping Standard Errors of Marginal Effects on Inefficiency		328
Ε	Software and Estimation Commands		
	E.1	Download and Install the User-Written Programs	331
	E.2	Download the Empirical Data and the Do-Files	331
	E.3	Cross-Sectional Models and Basic Utilities	331
		E.3.1 sfmodel	331
		E.3.2 sf_init	334
		E.3.3 sf_srch	335
		E.3.4 sf_transform	336
		E.3.5 sf_predict	336
		E.3.6 sf_mixtable	338
	E.4	System Models	338
		E.4.1 sfsystem	338
		E.4.2 showini	339
		E.4.3 sfsysem_profitshares	340
	E.5	Panel Data Models	342
		E.5.1 sfpan	342
		E.5.2 sf_fixeff	344
	E.6	Primal Models	
		E.6.1 sfprim	345
		E.6.2 sf_cst_compare	347
		E.6.3 sf_pft_compare	348
Bib	Bibliography		
Ind	Index		

Preface

This book deals with the estimation of productive efficiency using an econometric approach, which is popularly known as *stochastic frontier analysis*. The terminology relates to the fact that we are interested in the estimation of frontiers that envelop the data while maintaining the traditional econometric assumption of the presence of a random statistical noise. The frontiers we estimate are consistent with neoclassical microeconomic theory. Because, in reality, producers are not always efficient, the efficiency analysis can be viewed as an extension of the neoclassical theory. In this sense, the approach we consider in this book is based on sound neoclassical production theory and not purely an *ad hoc* empirical exercise.

Our primary goal in writing this book was to extend the everyday application of these tools beyond the expert practitioner or academic by making it relatively easy for the reader to carry out the complex computations necessary to both estimate and interpret these models. Our secondary goal was to ensure that the latest theoretical models can be implemented by practitioners, as many applications are limited by the software currently available.

As such, we aim at providing the reader with sufficient tools to apply many of the developed models to real data. In order to do this we have created a series of programs written for use in Stata, and they can be downloaded from the following website: https://sites.google.com/site/sfbook2014/. These commands are not part of the official Stata package, but instead are commands that we wrote ourselves in the form of Stata ado-files.

Thus, this book does not represent a comprehensive research monograph covering all areas of stochastic frontier models. Our focus is mostly on those models for which we have provided Stata codes and, as such, our list of references is limited to this purpose.

For a purely theoretical underpinning of stochastic frontier analysis the reader should consider first reading the book by Kumbhakar and Lovell (2000), *Stochastic Frontier Analysis* (Cambridge University Press). However, this is not essential as this book is intended to provide stand-alone reference materials for the reader to gain *both* a basic understanding of the theoretical underpinnings *and* a practical understanding of estimating production, profit, and cost efficiency.

As such, each chapter includes a theoretical introduction of the stochastic frontier model followed by worked examples of applying the theory to real data (examples include dairy farming, electricity generation, and airlines). These empirical examples are interwoven with the theory such that the reader can immediately apply the theory covered in the text. In order to follow these empirical examples, and thus to get the most benefit from this book, the

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xiv

Preface

reader must have Stata installed along with the programs provided with this book. Instructions on installation of the programs and explanations on the command syntax are provided in Appendix E, along with information on how to download the datasets and the empirical examples.

This book incorporates some of the most recent stochastic frontier models developed in the academic literature. Such recent developments include models of heteroscedasticity and exogenous determinants of inefficiency (Wang [2002]); scaling models (Wang and Schmidt [2002]); panel models with time-varying inefficiency (Kumbhakar [1990]); growth models (Kumbhakar and Wang [2005]); and the panel models of Greene (2005a), Wang and Ho (2010), Kumbhakar et al. (2014), and Chen et al. (2014). Other developments using semiand nonparametric approaches are not included in this book.

We wish to express our gratitude to Knox Lovell, Peter Schmidt, Robin Sickles, Bill Greene, Leopold Simar, Mike Tsionas, Subhash Ray, and many others whose work and ideas have influenced our thinking in a major way. David Drukker of StataCorp was kind enough to provide comments on some chapters. We are thankful to him for this. We also thank Scott Parris, our ex-editor, and Karen Maloney, the current Senior Editor at Cambridge University Press, for their constant support. The excellent research assistance provided by Chun-Yen Wu is also gratefully acknowledged. We would also like to thank Oxera for its support to Alan. Last, but not least, we thank our family members, especially our wives (Damayanti Ghosh, Yi-Yi Chen, and Maria Horncastle), for their constant support and encouragement in finishing this project, which took several years.

Subal C. Kumbhakar, Hung-Jen Wang, and Alan P. Horncastle