A Precise Calculation of Power System Frequency and Phasor

Jun-Zhe Yang and Chih-Wen Liu

Abstract—A series of precise digital algorithms based on Discrete Fourier Transforms (DFT) to calculate the frequency and phasor in real-time are proposed. These algorithms that we called the Smart Discrete Fourier Transforms (SDFT) family not only keep all of the advantages of DFT but also smartly take frequency deviation, and harmonics into consideration. These make the SDFT family more accurate than the other methods. Besides, SDFT family is recursive and very easy to implement, so it is very suitable for use in real-time. We provide the simulation results compared with conventional DFT method and second-order Prony method to validate the claimed benefits of SDFT.

Index Terms—Discrete Fourier Transforms (DFT), Frequency estimation, phasor measurement.

I. INTRODUCTION

FREQUENCY and phasor are the most important quantities in power system operation because they can reflect the whole power system situation. Frequency can show the dynamic energy balance between load and generating power, while phasor can constitute the state of system. So frequency and phasor are regarded as indices for the operating power systems in practice.

However, utilities have difficulty in calculating those quantities precisely. There are many devices, such as power electronic equipment and arc furnaces, etc. generating lots of harmonics and noise in modem power systems. It is therefore essential for utilities to seek and develop a reliable method that can measure frequency and phasor in presence of harmonics and noise.

With the advent of the microprocessor, more and more microprocessor-based equipments have been extensively used in power systems. Using such equipment is known to provide accurate, fast responding, economic, and flexible solutions to measurement problems [1]. Therefore, all we have to do is to find the best algorithm and implement it. There have been many digital algorithms applied to calculating frequency or phasor during recent years, for example Modified Zero Crossing Technique [2], Level Crossing Technique [3], Least Squares Error Technique [4]–[6], Newton method [7], Kalman Filter [8], [9], Prony Method [10], and Discrete Fourier Transform (DFT) [11], etc. For real-time use, most of the aforementioned methods have trade-off between accuracy and speed [12]. Unlike other methods, a series of precise digital algorithms, namely Smart Discrete Fourier Transform (SDFT) family, are presented and tries to meet the real-time use. SDFT family has the advantages that it can obtain exact solution in the presence of harmonics and frequency deviation from nominal frequency.

The organization of this paper is as follows: We describe basic principle of SDFT in section II. DFT, Prony method and SDFT are tested by four examples in section III. Finally, we give a conclusion in section IV.

II. THE PROPOSED DIGITAL ALGORITHM

This section presents the algorithm of the basic SDFT that estimates the frequency and phasor from a voltage/current signal. Consider a sinusoidal input signal of frequency $\omega = 2\pi f$ as follows:

$$x(t) = X\,\cos(\omega t + \phi) \tag{1}$$

where

X: the amplitude of the voltage/current signal,
ϕ: the phase angle of the voltage/current signal

Suppose that $x(t)$ is sampled with a sampling rate $(60^* N)$ Hz waveform to produce the sample set $\{x(k)\}$

$$x(k) = X\,\cos\left(\omega\,\frac{k}{60N} + \phi\right) \qquad k = 0,\,1,\,2,\,\cdots,\,N-1. \tag{2}$$

The signal $x(t)$ is conventionally represented by a phasor (a complex number) $\overline{x}$

$$\overline{x} = X e^{j\phi} = X\,\cos\phi + jX\,\sin\phi. \tag{3}$$

Then $x(t)$ can be expressed as

$$x(t) = \frac{\overline{x}e^{j\omega t} + \overline{x}^* e^{-j\omega t}}{2} \tag{4}$$

where $*$ denotes complex conjugate.

Moreover, the fundamental frequency (60Hz) component of DFT of $\{x(k)\}$ is given by

$$\hat{x}_r = \frac{2}{N}\sum_{k=0}^{N-1} x(k+r)e^{-j\frac{2\pi k}{N}}. \tag{5}$$

Combing Eq. (4) and Eq. (5) and taking frequency deviation $(\omega = 2\pi(60 + \Delta f))$ into consideration, at last, we obtain:

$$\hat{x}_r = \frac{\overline{x}}{N}\,\frac{\sin\dfrac{N\theta_1}{2}}{\sin\dfrac{\theta_1}{2}}\,e^{j\frac{\pi}{60N}(\Delta f(2r+N-1)+120r)}$$

$$+ \frac{\overline{x}^*}{N}\,\frac{\sin\dfrac{N\theta_2}{2}}{\sin\dfrac{\theta_2}{2}}\,e^{-j\frac{\pi}{60N}(\Delta f(2r+N-1)+120(r+N-1))} \tag{6}$$

where

$$\theta_1 = \frac{2\pi\Delta f}{60N}, \quad \text{and} \quad \theta_2 = \frac{2\pi\left(2 + \dfrac{\Delta f}{60}\right)}{N}.$$

If we define A_r and B_r as

$$A_r = \frac{\overline{x}}{N} \frac{\sin \dfrac{N\theta_1}{2}}{\sin \dfrac{\theta_1}{2}} e^{j\frac{\pi}{60N}(\Delta f(2r+N-1)+120r)} \qquad (7)$$

$$B_r = \frac{\overline{x}^*}{N} \frac{\sin \dfrac{N\theta_2}{2}}{\sin \dfrac{\theta_2}{2}} e^{-j\frac{\pi}{60N}(\Delta f(2r+N-1)+120(r+N-1))}. \qquad (8)$$

Then Eq. (6) can be expressed as

$$\hat{x}_r = A_r + B_r. \qquad (9)$$

Actually, the first half development of the algorithm of SDFT is the same as the conventional DFT method. So the SDFT can keep all advantages of DFT such as recursive computing manner. In the conventional DFT, it assumes that the frequency deviation is small enough to be ignored, and $\hat{x}_r \approx A_r$. Therefore,

$$\phi_r = \arctan(imag(\hat{x}_r)/real(\hat{x}_r)) \qquad (10)$$

$$f = 60 + \frac{\phi_r - \phi_{r-1}}{2\pi} \times 60N. \qquad (11)$$

Conventional DFT methods incur error in estimating frequency and phasor when frequency deviates from nominal frequency (60Hz). However, in the SDFT we take B_r into consideration. So we define

$$a = e^{j\left(\frac{\pi}{60N}(2\Delta f + 120)\right)}. \qquad (12)$$

And from Eq. (7) and Eq. (8), we will find the following relations

$$A_{r+1} = A_r * a \qquad (13)$$
$$B_{r+1} = B_r * a^{-1}. \qquad (14)$$

Then

$$\hat{x}_{r+1} = A_{r+1} + B_{r+1} = A_r * a + B_r * a^{-1} \qquad (15)$$

$$\hat{x}_{r+2} = A_{r+2} + B_{r+2} = A_{r+1} * a + B_{r+1} * a^{-1}$$
$$= A_r * a^2 + B_r * a^{-2}. \qquad (16)$$

There are three unknown variables in Eq. (9), Eq. (15) and Eq. (16), and after some algebraic manipulations we obtain:

$$\hat{x}_{r+1} * a^2 - (\hat{x}_r + \hat{x}_{r+2}) * a + \hat{x}_{r+1} = 0. \qquad (17)$$

Solve Eq. (17) to obtain

$$a = \frac{(\hat{x}_r + \hat{x}_{r+2}) \pm \sqrt{(\hat{x}_r + \hat{x}_{r+2})^2 - 4\hat{x}_{r+1}^2}}{2\hat{x}_{r+1}}.$$

Then from the definition of "a" in Eq. (12), we can get the exact solution of the frequency

$$f = 60 + \Delta f = \cos^{-1}(\text{Re}(a)) * \frac{60N}{2\pi}. \qquad (18)$$

Moreover, we can estimate phasor after getting exact "f" by the following equations:

$$A_r = \frac{\hat{x}_{r+1} * a - \hat{x}_r}{a^2 - 1} \qquad (19)$$

$$X = abs(A_r) * \frac{N * \sin\left(\dfrac{\pi\Delta f}{60N}\right)}{\sin\left(\dfrac{\pi\Delta f}{60}\right)} \qquad (20)$$

$$\phi = angle(A_r) - \frac{\pi}{60N} \times (\Delta f \times (N-1)). \qquad (21)$$

It is observed that SDFT can provide exact frequency and phasor using $\hat{x}_r$, $\hat{x}_{r+1}$ and $\hat{x}_{r+2}$ in the presence of frequency deviation.

Next, we take harmonics into consideration. Assume a sinusoidal signal of frequency $\omega = 2\pi f$ with m^{th} harmonic given by:

$$x(t) = X_1 \cos(\omega t + \phi_1) + X_2 \cos(m\omega t + \phi_2) \qquad (22)$$

where
 X_1, X_2: the amplitude,
 ϕ_1, ϕ_2: the phase angle.
Following similar steps developed previously, we can get:

$$\hat{x}_r = \frac{\overline{x}}{N} e^{j\frac{2\pi}{N}(1+\frac{\Delta f}{60})r} \sum_{k=0}^{N-1} e^{j2\pi\frac{\Delta f}{60N}k}$$
$$+ \frac{\overline{x}^*}{N} e^{-j\frac{2\pi}{N}(1+\frac{\Delta f}{60})r} \sum_{k=0}^{N-1}$$
$$\times e^{-j2\pi\frac{(2+\frac{\Delta f}{60})}{N}k}$$
$$+ \frac{\overline{x}}{N} e^{j\frac{2\pi}{N}(1+\frac{\Delta f}{60})mr} \sum_{k=0}^{N-1}$$
$$\times e^{j2\pi\frac{(m-1+\frac{m\Delta f}{60})}{N}k}$$
$$+ \frac{\overline{x}^*}{N} e^{-j\frac{2\pi}{N}(1+\frac{\Delta f}{60})mr} \sum_{k=0}^{N-1}$$
$$\times e^{-j2\pi\frac{(-1-m+\frac{m\Delta f}{60})}{N}k}. \qquad (23)$$

Then

$$\hat{x}_r = \frac{\overline{x}_1}{N} \frac{\sin \dfrac{N\theta_1}{2}}{\sin \dfrac{\theta_1}{2}} e^{j(\pi/60N)(\Delta f(2r+N-1)+120r)}$$
$$+ \frac{\overline{x}_1^*}{N} \frac{\sin \dfrac{N\theta_2}{2}}{\sin \dfrac{\theta_2}{2}} e^{-j(\pi/60N)(\Delta f(2r+N-1)+120(r+N-1))}$$
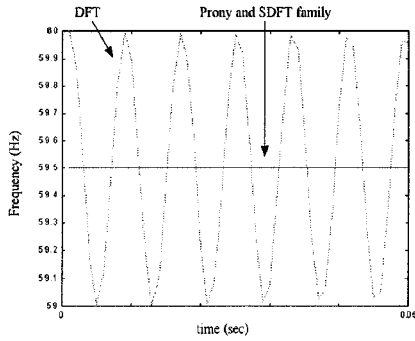$$+ \frac{\overline{x}_2}{N} \frac{\sin \dfrac{N\theta_3}{2}}{\sin \dfrac{\theta_3}{2}}$$

Fig.1a

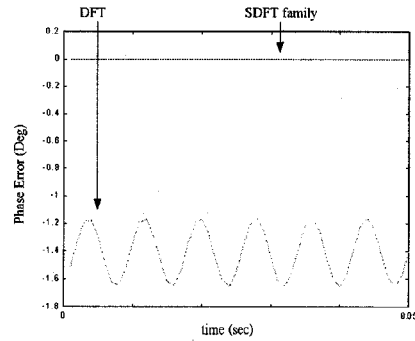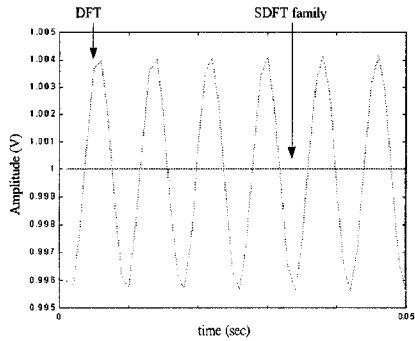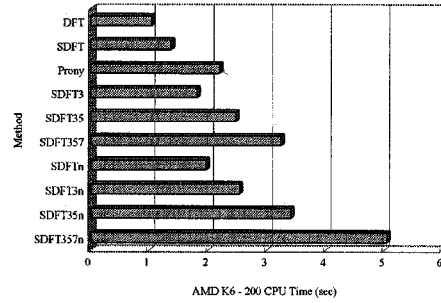

Fig.1b

Fig.1c.

Fig.1d

Fig. 1.   Test signal: $v(t) = \cos(\omega t)$; simulated frequency 59.5 Hz; sampling frequency 960 Hz.

$$\times\, e^{j(\pi/60N)(m\Delta f(2r+N-1)+60(2mr+mN-m-N+1))}$$

$$+\,\frac{\overline{x}_2^*}{N}\,\frac{\sin\dfrac{N\theta_4}{2}}{\sin\dfrac{\theta_4}{2}}$$

$$\times\, e^{-j(\pi/60N)(m\Delta f(2r+N-1)+60(2mr+mN-m+N-1))} \tag{24}$$

$$\hat{x}_r = A_r + B_r + C_r + D_r \tag{25}$$

$$\begin{aligned}\hat{x}_{r+1} &= A_{r+1} + B_{r+1} + C_{r+1} + D_{r+1}\\ &= A_r * a + B_r * a^{-1} + C_r * a^m + D_r * a^{-m}\end{aligned} \tag{26}$$

where

$$C_r = \frac{\overline{x}_2}{N}\,\frac{\sin\dfrac{N\theta_3}{2}}{\sin\dfrac{\theta_3}{2}}$$

$$\times\, e^{-j(\pi/60N)(m\Delta f(2r+N-1)+60(2mr+mN-m-N+1))}$$

$$D_r = \frac{\overline{x}_2^*}{N}\,\frac{\sin\dfrac{N\theta_4}{2}}{\sin\dfrac{\theta_4}{2}}$$

$$\times\, e^{-j(\pi/60N)(\Delta f(2mr+N-1)+60(2mr+mN-m+N-1))}$$

$$\theta_3 = \frac{2\pi\left(m-1+\dfrac{m\Delta f}{60}\right)}{N},$$

and

$$\theta_4 = \frac{2\pi\left(-m-1+\dfrac{m\Delta f}{60}\right)}{N}.$$

There are five unknown variables in Eq. (25), hence, we need five equations to solve this problem. So using $\hat{x}_r$, $\hat{x}_{r+1}$, $\hat{x}_{r+2}$, $\hat{x}_{r+3}$ and $\hat{x}_{r+4}$, we can get exact frequency and phasor in the presence of one harmonic. We use SDFT, to denote that 3rd harmonic has been taken into consideration. Of course, any other integral order harmonic can be taken into consideration too, for example: SDFT$_{35}$ and SDFT$_{357}$ take 3rd, 5th harmonic and 3rd, 5th, 7th harmonic into consideration respectively. Similarly, nonintegral harmonics also can be developed. We use SDFT$_n$ to denote that nonintegral harmonics has been taken into consideration.

III. SIMULATION RESULTS

Most of the frequency estimation methods are concerned with the performances of the four cases: frequency deviation, frequency variation, harmonics and noise. Hence, we used these four cases to compare the performance of these methods. Simulation results presented in this section were all simulated from
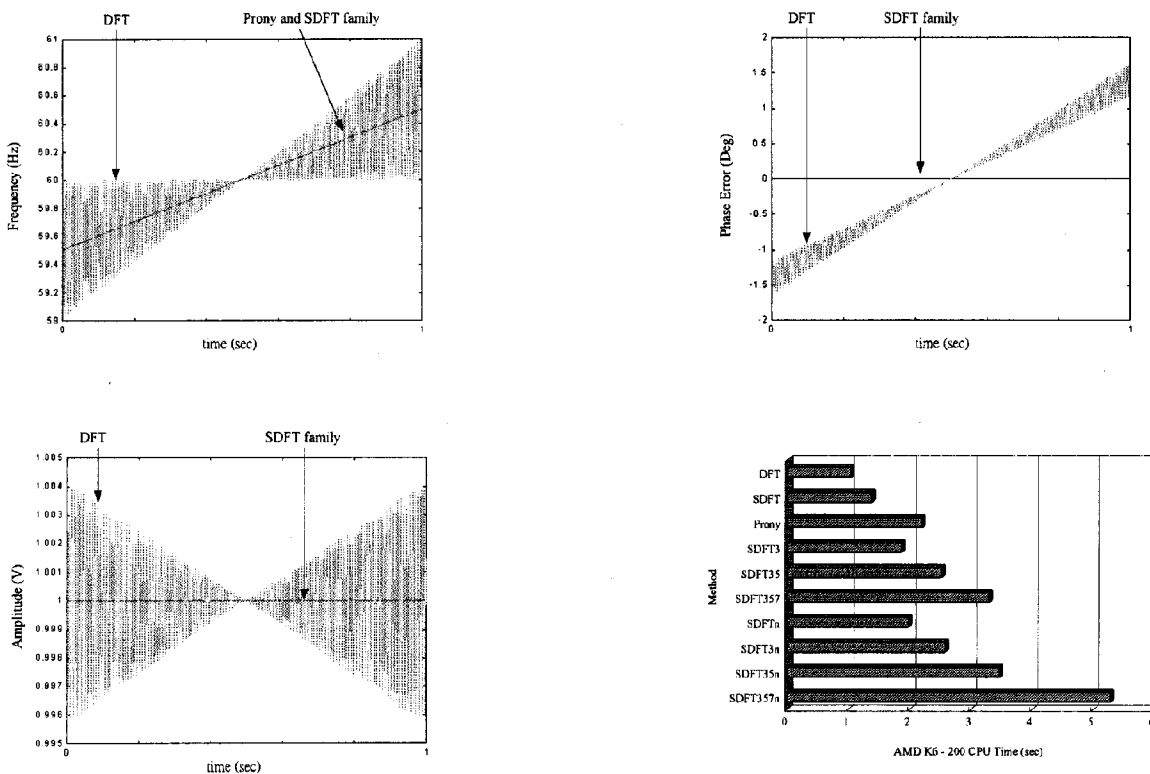
Fig. 2.   Test signal: $v(t) = \cos(\omega t)$; simulated frequency variation form 59.5 Hz to 60.5 Hz during 1 second; sampling frequency 960 Hz.
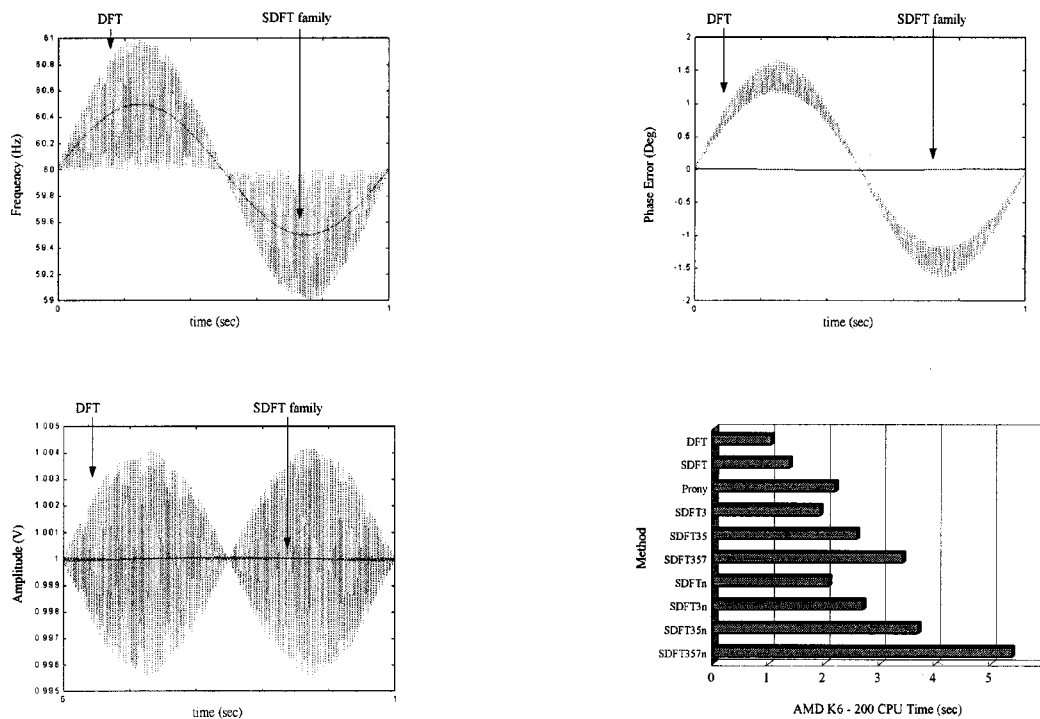

Fig. 3.   Test signal: $v(t) = \cos(\omega t)$; simulated frequency variation like sin wave during 1 second; sampling frequency 960 Hz.

Matlab and shown for a fair comparison to DFT method and Prony method.

In Fig. 1(a), we showed that SDFT family and the Prony method could obtain an exact frequency calculation under frequency deviation in a pure sinusoidal waveform. We also show the performance of conventional DFT method. It is clear that conventional DFT method gives the wrong answer in frequency and phasor. By comparison of computation speed, Fig. 1(d) shows the AMD K6-200 CPU time of each method. There are 960 data per second computed by each method.
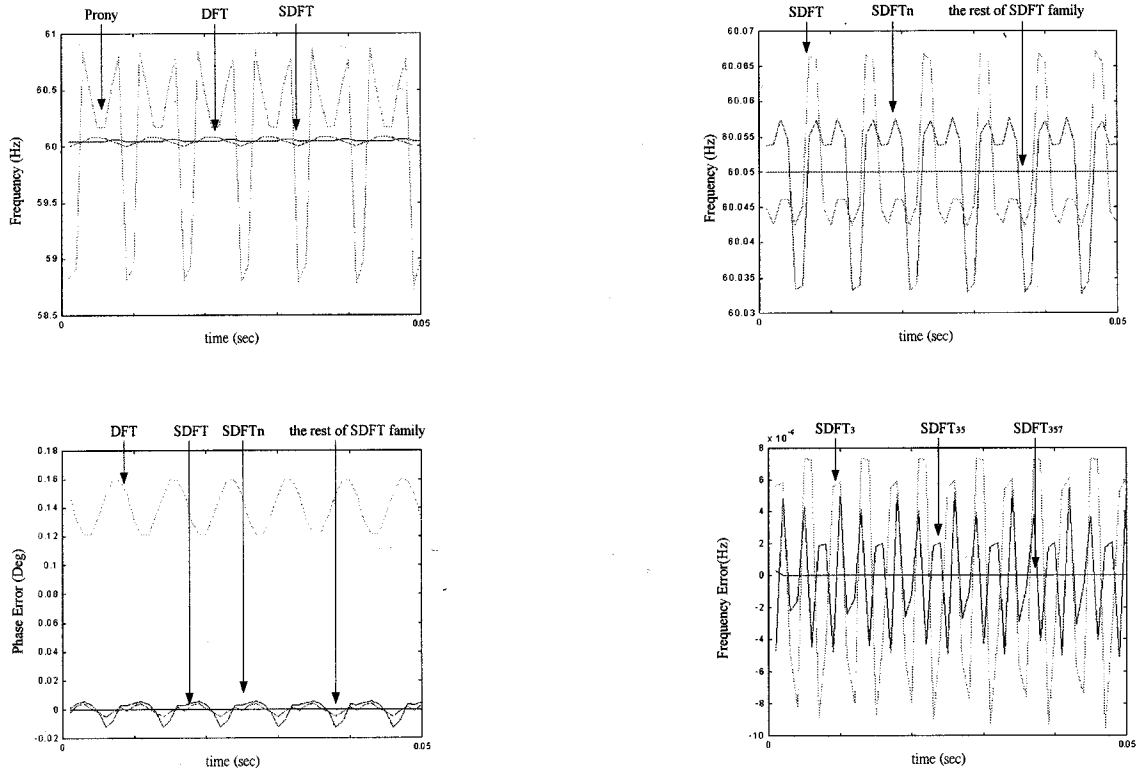
Fig. 4.   Test signal: $v(t) = \cos(\omega t) + 0.05\,\cos(3\omega t) + 0.02\,\cos(5\omega t) + O.O1\,\cos(7\omega t)$; simulated frequency 60.05 Hz; sampling frequency 960 Hz.
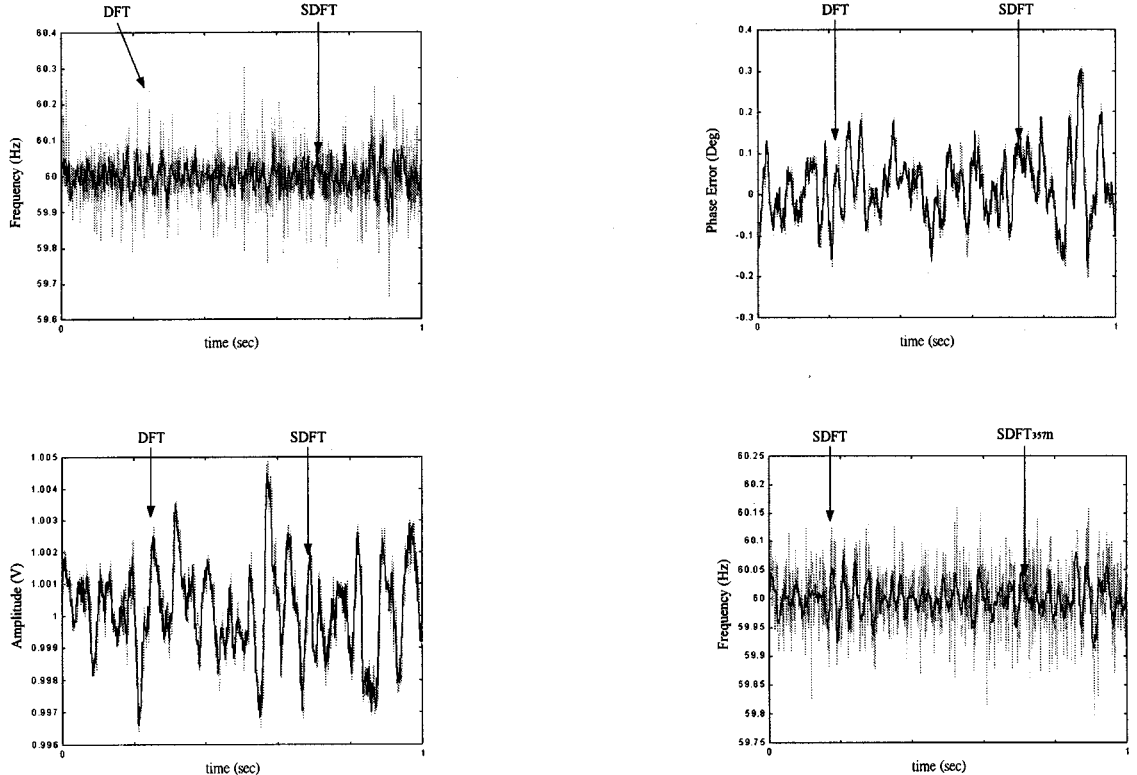

Fig. 5.   Test signal: $v(t) = \cos(\omega t)$; simulated frequency 60 Hz; sampling frequency 960 Hz; with 1% noise.

In Fig. 2 and Fig. 3, we put the emphasis on the frequency variation. The frequency of test signal in Fig. 2 is varied linearly form 59.5 Hz to 60.5 Hz during 1 second. Another test in Fig. 3 is to make the frequency change like a sinusoid: $f = 60 + \sin(2\pi t)$. It is observed that the errors of conventional DFT method are larger than SDFT family in terms of frequency and phasor.

We designed the test frequency to be 60.05Hz and added 3rd, 5th and 7th harmonics into test signal in Fig. 4. In Fig. 4a, we find that Prony method is very sensitive to harmonic. Although Prony method can change its window size, it still can't have better performance. SDFT has better performance than DFT, and the rest of SDFT family, expect SDFT_n, is better than SDFT. SDFT_{357}, SDFT_{35n} and SDFT_{357n} get the exact frequency and phasor in this case.

We know that if a method can be used in real world, it must take noise into consideration. The frequency of test signal in Fig. 5 is 60Hz, and we add one percent of white noise into signal. In Fig. 5, we didn't show the performance of Prony Method, since the performance of Prony method depends on window size in this test. It is shown that SDFT is better than DFT and SDFT_{357n} is better than SDFT.

IV. CONCLUSION

In this paper we introduce the SDFT family and demonstrate their performance. SDFT family both keeps the advantages of DFT and also deals with the difficulty of frequency deviation errors, while taking harmonics, noise into consideration. These aspects make SDFT family accurate, harmonic-resisting methods. If only rough answer is required, DFT is still a good choice, but if precise answer is essential, we recommend SDFT family. We believe that SDFT family has a great potential to replace conventional DFT method for power system frequency and phasor calculation in the future, if SDFT family can be speeded up by the advanced computing architecture.

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