A Probabilistic Approach to Modeling Two-Dimensional Pointing

TOVI GROSSMAN and RAVIN BALAKRISHNAN
University of Toronto

We investigate and model two-dimensional pointing where the target distance and size vary as does the angle of movement. We first study the spread of hits in a rapid approximate pointing task at varied distances and movement angles. Consistent with the literature, our results show that the spread of hits along the movement direction deviate more than the spread of hits in the direction perpendicular to movement, and both spreads increase with distance. Based on the distribution of this spread of hits, we propose and validate a new probabilistic model that describes two-dimensional pointing. Unlike previous models, our model accounts for more variables of two-dimensional pointing and can be generalized to any target shape, size, orientation, location, and dimension. In contrast to previous work, which suggests that target height has minimal impact on performance when it is larger than the width, our results show that, even when height is greater than width, it can significantly impact movement time.

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General Terms: Experimentation, Human Factors, Measurement

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1. INTRODUCTION

One of the most basic interaction requirements in user interfaces is pointing to targets, such as menus, buttons, and text. Fitts' law [Fitts 1954; MacKenzie 1992] is a model commonly used in HCI for modeling such pointing behavior. It predicts the time MT taken to select a target based on its width W and distance (or amplitude) A from the cursor according to the equation

$$MT = a + b\log_2\left(\frac{A}{W} + 1\right), \tag{1}$$

where a and b are empirically determined constants. The logarithmic term is the index of difficulty (ID) of the task. Over the years, numerous studies have been

Authors' address: Department of Computer Science, University of Toronto, 10 King's College Road, Toronto, Ontario, Canada M5S 3G4; email: {tovi,ravin}@dgp.toronto.edu.

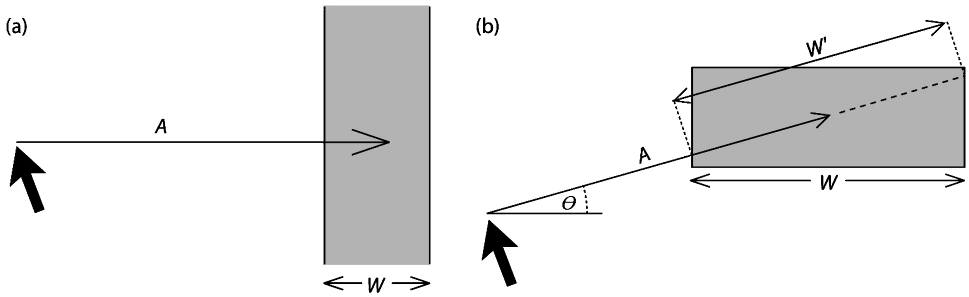

Fig. 1. (a) One-dimensional pointing where the width W is colinear to the movement direction. (b) Two-dimensional pointing where the apparent width W' may vary due to the direction of movement θ.

conducted that validate this model for one-dimensional pointing tasks (see the review by MacKenzie [1992]). This model can also be used to direct the design of new interfaces to optimize their layouts for users (e.g., Zhai et al. [2000]). Fitts' model, in its original form, is inherently one-dimensional with the target width W the only movement constraint, colinear with the direction of movement (Figure 1(a)). Most targets in current interfaces, however, are typically two or even three-dimensional, where the assumptions of the one-dimensional model may not hold. Some recent research [MacKenzie and Buxton 1992; Accot and Zhai 2003] has attempted to extend Fitts' law to two dimensions. However, none of the work to date has adequately considered relevant factors in two-dimensional pointing beyond the width and height of rectangular targets such as the effect of target orientation relative to the original cursor position.

In this article, we study and model pointing in two dimensions at targets whose size varies both in width and height and where the user's movement angle and angle of approach relative to the target are also varied. We begin by reviewing previous work on pointing and Fitts' law models in one and two dimensions. We then identify various factors—target dimensions, movement amplitude, movement angle, angle of approach, and interactions between them—that could affect pointing performance in two dimensions. Based on these factors, we propose a new approach to modeling pointing behavior which makes use of the spread of hits when pointing at targets. We then present the results of two controlled experiments: the first investigates the spread of hits in pointing actions, and the second studies the effects of the factors identified and evaluates the proposed model. We conclude by discussing the implications for interface design.

Our main contribution to the pointing literature is the probabilistic model which we develop (Section 3.2) and validate (Section 6). Unlike previous models, ours can adequately compensate for both varying movement angles and varying target orientations. We also discuss how our model could be used to predict movement times when pointing at nonrectangular targets (Section 5) which cannot be done accurately with any of the previous models.

To determine parameter values for our proposed model, we conduct an experiment which studies the spread of hits when pointing at targets in two dimensions (Section 4). This is our other major contribution to the pointing literature as we analyze how the physical movement angle and distance traveled affect the spread of hits in the directions parallel and perpendicular to the line of movement. This builds on previous research which has studied how movement time affects the spread of hits.

2. BACKGROUND

When pointing at targets in two-dimensional space, various factors must be considered beyond the target width and amplitude constraints of the one-dimensional Fitts' model. First, a two-dimensional pointing task is bivariate, constrained by both the target's width and height. Second, the location of the target relative to the cursor position is a two-dimensional vector.

MacKenzie and Buxton [1992] performed the first study on bivariate pointing in the HCI literature, examining several formulas for the index of difficulty for a rectangular target. They found two which highly correlated with their experimental data. Their first formulation, $ID_{W'}$(Equation (2)), considers W to be the apparent width (W') of the target based on the movement angle (θ) (Figure 1(b)). Their second formulation, $ID_{\min}$ (Equation (3)) only considers the minimum dimension. These formulas are expressed as follows:

$$ID_{W'} = \log_2\left(\frac{A}{W'} + 1\right), \tag{2}$$

$$ID_{\min} = \log_2\left(\frac{A}{\min(W, H)} + 1\right), \tag{3}$$

where W and H are the width and height of the target. The $ID_{\min}$ had the highest correlation with their experimental data. Since their work, the $ID_{\min}$ has been used in follow-up work [Ware and Balakrishnan 1994; Ware and Lowther 1997; Murata 1999], and was also independently proposed by Hoffman and Sheikh [1994].

Accot and Zhai [2003] identify various problems with the $ID_{W'}$ and $ID_{\min}$ formulations. They present the following desirable properties for a bivariate pointing model:

—*Scale independence*. MT should remain unchanged if A, W, and H are all multiplied by a constant.
—*Limit tasks*. When either W or H tends to infinity, the model should regress toward the one-dimensional Fitts' law.
—*Dominance effect*. The ID value should be dominated by the smaller of W or H with less impact from the other.
—*H and W duality*. The model should contain and be similarly affected by both H and W.
—*Continuity*. The effects of H and W on MT should be continuous not stepwise or segmented.

Based on their results, an additional property can be established:

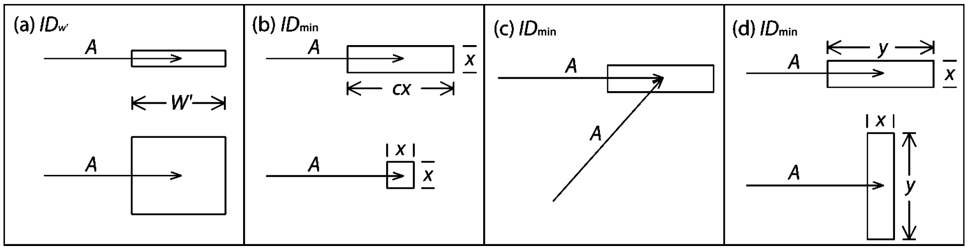

Fig. 2. Limitations of $ID_{w'}$ and $ID_{\min}$ models: all target pairs in this figure are considered identical by the respective *ID* formulations. (a) No difference in $ID_{w'}$ for targets of different heights. (b) Width has no effect on $ID_{\min}$ if greater than height, $c > 1$. (c) Movement angle has no effect on $ID_{\min}$. (d) Interchanging width and height has no effect on $ID_{\min}$.

—*W-asymmetric*. If $W > H$, then interchanging their values should increase the movement time.

The fundamental problem with the $ID_{W'}$ model (Equation (2)) is that it completely ignores the constraints imposed by the dimension of the target perpendicular to the line of movement also referred to as the directional constraint [Accot and Zhai 2003]. As a result, it does not possess the limit tasks property as increasing the width regardless of the height will reduce the *ID* to zero. Nor does it exhibit *H* and *W* duality since decreasing the height to a infinitesimal size will not increase the *ID* (Figure 2(a)).

In the case of the $ID_{\min}$ model (Equation (3)), the *H* and *W* duality and limit tasks properties are satisfied, however continuity is not. In strict mathematical terms, even though the $ID_{\min}$ model is a continuous function in terms of *W* and *H*, the derivative is not continuous. The $ID_{\min}$ model predicts that *H* does not affect the time as soon as it becomes greater than *W*. Similarly, the model is not affected by *W* as soon as it is larger than *H*. Thus, this model does not account for data reported by Sheikh and Hoffman [1994] that showed that it is harder to acquire a square than a rectangle with equal height but larger width (Figure 2(b)).

Another problem with the $ID_{\min}$ model is that it ignores the angle of approach (Figure 2(c)). Last, the $ID_{\min}$ model is not *W*-asymmetric since it allows the width and height factors to be interchanged without changing the *ID* (Figure 2(d)).

This unsatisfactory state of affairs led Accot and Zhai [2003] to develop and experimentally validate a weighted Euclidean model which satisfies all of the given properties. We refer to this model as ID_{WtEuc}

$$ID_{WtEuc} = \log_2\left(\sqrt{\left(\frac{A}{W}\right)^2 + \eta\left(\frac{A}{H}\right)^2} + 1\right), \tag{4}$$

where η is empirically determined. This model considers $(A/W, A/H)$ to be the "constraint vector", and by taking a weighted norm of this vector, they incorporate both variables into an "appropriate distance in a two-dimensional space" [Accot and Zhai 2003]. The addition of the parameter η allows the model to weight the effect of the height differently from the effect of the width. This ID_{WtEuc} model is a significant improvement over the $ID_{\min}$ model since it allows the larger dimension to still have an effect on the movement time. However, this model does not completely account for all pertinent factors of a general two-dimensional pointing task. First, it does not take into account the angle of movement towards the target. A two-dimensional pointing model should definitely consider this factor, particularly since previous work [Boritz et al. 1991; Hancock and Booth 2004] provides evidence that the movement time will depend on the direction of movement. Second, it only considers rectangular targets. It would be highly desirable for a model to be valid for any two-dimensional shape since interface targets can, and often have, varied shapes.

Our attempt at modeling two-dimensional pointing will be based on the spread of hits when pointing at targets. The spread of hits has been previously observed and described [Welford 1968; Schmidt et al. 1979] and has been used to compensate for error rates in pointing tasks [Welford 1968; MacKenzie 1992]. Welford [1968] observed that the spread of hits in a one-dimensional task resembled a normal distribution. The two-dimensional spread of hits for pointing has also been studied by Schmidt et al. [1979]. In a task where users moved to a target within a specified time, errors were measured in the dimensions parallel and perpendicular to the movement direction. It was found that both types of errors related linearly to the movement speed, and that errors perpendicular to the movement direction were about half the size of errors colinear with the movement direction. This produced an elliptical distribution of hits with the long axis in line with the direction of movement. In contrast to this approach, we study the effect of both the movement distance and movement angle on the spread of hits in a rapid two-dimensional pointing task. The findings of this study are then used to develop and evaluate a predictive model for two-dimensional target acquisition based on the proportion of this spread which the target of interest covers.

3. GOALS AND DIRECTIONS OF THE CURRENT STUDY

The goal of our work is to resolve some of the outstanding issues surrounding the present theoretical models which describe pointing tasks in two dimensions. We wish to establish a predictive model that accounts for more of the pertinent factors in two-dimensional pointing and which can be used to direct the design of user interfaces. We build on previous work in two key directions: manipulation of experimental parameters and theoretical modeling.

3.1 Manipulation of Experimental Parameters

Target Dimensions and Amplitude. Previous work by Accot and Zhai [2003] showed that increasing W beyond H will reduce movement time, while increasing H beyond W has no effect. Their analysis was performed by averaging over all values of A. We wish to verify this result when the conditions are broken up by A. We hypothesize that the vertical scatter of hits will deviate by a constant angle from the start point when moving towards a target. Thus, for distant targets, increasing the height past the width could still significantly reduce movement time as long as it is within this scatter of hits. If this is the case, the effect of increasing the height of a target would not only depend on

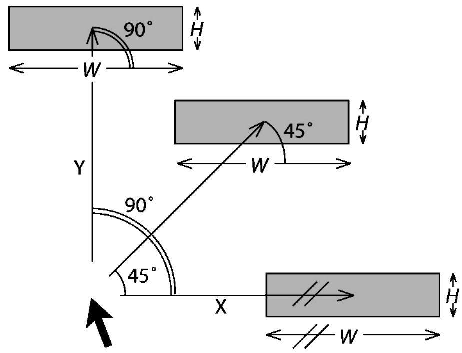

Fig. 3. Movement and approach angles.

the width, but also how much effect the vertical constraint has at that target's distance. As in Accot and Zhai [2003], we will treat the ratio between H and W as a controlled independent variable, allowing us to study the interaction between these two dimensions. Our analysis of the effect of this ratio will be broken up by amplitude, allowing us to also study the interaction between the ratio and amplitude.

Movement and Approach Angles. As mentioned earlier, Accot and Zhai [2003] limited their study to cursor movements along the X-axis. We wish to extend their work by studying the effects of varying the movement angle since two-dimensional targets can be positioned anywhere in a two-dimensional environment. For the present study, we explore movement angles of 0° (i.e., X-axis), 22.5°, 45°, 67.5°, and 90° (i.e., Y-axis). While the movement angle defines the human user's axis of movement, the *approach angle* defines the angle between the movement vector (defined by θ) and the axis parallel to the width of the target. We will always have the width of the target colinear with the X-axis and the height of the target colinear with the Y-axis. Thus, the approach angle will always equal the movement angle (Figure 3). This is similar to the setup used in MacKenzie and Buxton [1992], however, they only tested angles of 0°, 45°, and 90°. Although we do not completely cross the two variables, the design will allow us to infer whether or not our model will have the ability to accommodate the effects of both varying the movement angle and varying the approach angle. Evidence that varying the movement angle will affect MT has been seen in Boritz et al. [1991]; Hancock and Booth [2004] and is partially due to differences in the muscle groups required to affect movements in different directions. Varying the approach angle should also affect MT since it is, in essence, varying the shape of the target.

3.2 New Model Candidate

We now describe the new model which we propose and evaluate in this article. The key idea is to use a function to map the probability of hitting a target to an index of difficulty value. We will first introduce and define the probability of hitting a target in a one-dimensional task. We then generate the function using the defined probabilities and well-accepted index of difficulty values for one-dimensional pointing. Finally, we define the probability of hitting a target in a two-dimensional task, and map these probabilities to index of difficulties using the generated function.

Hoffman and Sheikh [1994] suggest that "only when the target height is less than the natural vertical scatter of hits on the target is there likely to be any effect of vertical constraint". We attempt to identify the nature of this "natural scatter" of hits both in the vertical and horizontal directions. As an analogy, consider throwing darts towards the bull's-eye of a dartboard. Some darts will be close, and some will be further, and, at the end, there will be a spread of hits where the darts fell. Similarly, if a user repeatedly attempts to quickly move towards a target and stop directly over it without using any corrective movements, there will be a spread of hits where the cursor stopped.

Based on this spread of hits, we propose a new approach to modeling pointing behavior. Previous work has attempted to make the index of difficulty a function of any variable which can affect the movement time. For one-dimensional pointing, the *ID* is a function of width and amplitude (Equation (1)). For two dimensions, Accot and Zhai [2003] made it a function of width, height, and amplitude (Equation (4)). To incorporate varying movement angles, it would need to be a function of this variable as well. It becomes cumbersome to incorporate all these variables into the index of difficulty as their number increases. It would be preferable to find a formula for the index of difficulty which can be generalized to any target shape, size, orientation, location, and dimension.

Our solution to this problem is to first calculate $P_{R,S}(hit)$, the probability of pointing inside the region R defined by the target, based on the spread of hits S. Using the dart board analogy, if you have a spread of hits on the dart board where the darts landed when being aimed at the bull's-eye, you could use that information to estimate the probability of a dart hitting a target of any shape or size. The probability will thus be a function of R as a smaller target will be harder to hit, and S since a larger spread will mean fewer points fall within the target region. The actual spread of hits S will be affected by a number of variables including A which we will discuss shortly.

We propose using a universal function F which maps the probability of hitting a target, its single parameter, to an index of difficulty:

$$ID_{\mathrm{Pr}} = F(P_{R,S}(hit)). \tag{5}$$

The general idea is that a target's index of difficulty can be completely determined by calculating the probability that the target will be hit by an open-loop movement. The higher the probability, the easier the target will be to acquire. It should be noted that by open-loop movement, we mean the initial ballistic impulse towards the target without any feedback-guided final adjustments [MacKenzie 1992].

Let's first consider the one-dimensional case. If we assume that the spread of hits is normally distributed which is supported by observations in Welford [1968], with a mean value of zero corresponding to the center of the target, the index of difficulty becomes:

$$ID_{\mathrm{Pr}} = F\left(P\left(-\frac{W}{2} \leq X_{N(0,\sigma)} \leq \frac{W}{2}\right)\right). \tag{6}$$

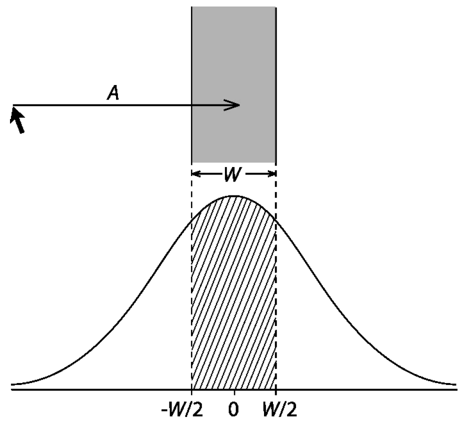

Fig. 4. Target in relation to normal distribution.

where $X_{N(0,\sigma)}$ is a random variable in the normal distribution with mean 0 and standard deviation σ (Figure 4).

As stated earlier, the spread should be affected by A. This variable is incorporated by setting σ to be a function of A, $\sigma(A)$, since the spread of hits should diverge the further the distance is to the target. Reverting to the dart board analogy, the points that the darts hit will become more spread out if they are thrown from further away. This is a different approach than Schmidt et al. [1979] who investigated the relation between the spread and movement speed. We will exploit the property of scale independence to determine the exact nature of the function $\sigma(A)$. We want the index of difficulty, and thus the probability value, to remain the same if the width and amplitude are both multiplied by a constant k:

$$P\left(-\frac{W}{2} \leq X_{N(0,\sigma(A))} \leq \frac{W}{2}\right) = P\left(-\frac{kW}{2} \leq X_{N(0,\sigma(kA))} \leq \frac{kW}{2}\right). \tag{7}$$

By converting X to the standard normal distribution $Z_{N(0,1)}$, we get:

$$P\left(-\frac{W}{2\sigma(A)} \leq Z_{N(0,1)} \leq \frac{W}{2\sigma(A)}\right) = P\left(-\frac{kW}{2\sigma(kA)} \leq Z_{N(0,1)} \leq \frac{kW}{2\sigma(kA)}\right). \tag{8}$$

Solving this equation gives:

$$\sigma(kA) = k\sigma(A), \tag{9}$$

so we can conclude that:

$$\sigma(A) = cA, \tag{10}$$

for some constant c which is determined empirically. Because the formula for the index of difficulty is widely accepted for one-dimensional tasks (Equation (1)), once c is determined, we can create a table of values for the function F based on the equivalence:

$$\log_2\left(\frac{A}{W} + 1\right) = F\left(P\left(-\frac{W}{2} \leq X_{N(0,cA)} \leq \frac{W}{2}\right)\right). \tag{11}$$

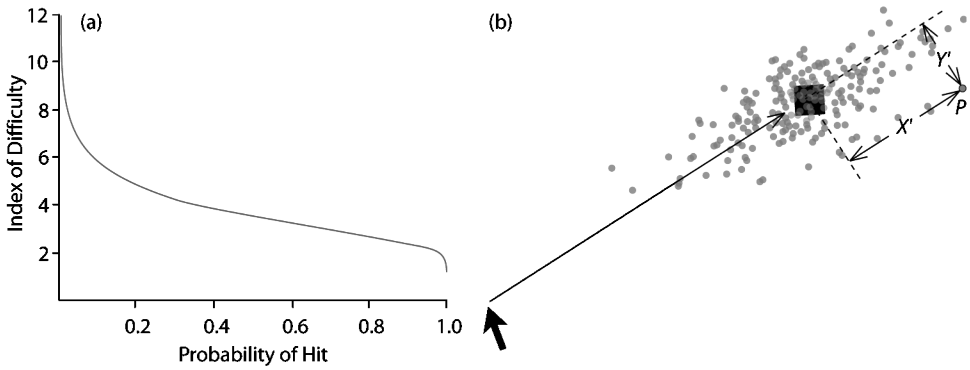

Fig. 5. (a) Example of the universal function F for $c = 0.07$. (b) Example of the spread of hits in two-dimensional pointing. X' is the error parallel to, and Y' is the error perpendicular to, the line of movement.

Stated in terms of the standard normal distribution gives:

$$\log_2\left(\frac{A}{W}+1\right) = F\left(P\left(-\frac{W}{2cA} \leq Z_{N(0,1)} \leq \frac{W}{2cA}\right)\right). \tag{12}$$

Letting $B = A/W$ gives:

$$\log_2(B+1) = F\left(P\left(-\frac{1}{2cB} \leq Z_{N(0,1)} \leq \frac{1}{2cB}\right)\right). \tag{13}$$

Finally, using ϕ for the cumulative distribution function of the standard normal distribution, we get:

$$\log_2(B+1) = F\left(\phi\left(\frac{1}{2cB}\right) - \phi\left(-\frac{1}{2cB}\right)\right). \tag{14}$$

Figure 5(a) shows the function F with c set to 0.07, a value which we have found to be in the possible range through experimentation. Because there is no closed formula for the cumulative normal distribution function as it contains an unsolvable integral, we compute F numerically. We generate the function in Figure 5(a) by substituting different values of B into the left-and right-hand sides of Equation (14).

The idea now is to take any pointing task condition, obtain the probability of pointing inside the region defined by the target, and map it to its index of difficulty using the universal function F. For two-dimensional pointing, we examine the spread of hits $P = (X', Y')$, where X' is the error parallel to the line of movement, and Y' is the error perpendicular to the line of movement (Figure 5(b)). We assume the spread of hits is a bivariate normal distribution, $N(\mu_{X'}, \mu_{Y'}, \sigma_{X'}, \sigma_{Y'}, \rho_{X'Y'})$. The means $(\mu_{X'}, \mu_{Y'})$, again, are zero, corresponding to the center of the target. The standard deviation $\sigma_{X'} = cA$, for some constant c, measures from the center of the target colinear with the direction of movement. The standard deviation $\sigma_{Y'} = dA$, for some constant d, measures from the center of the target, perpendicular to the direction of movement. For simplicity, we will assume X' and Y' to be independent, so $\rho_{X'Y'} = 0$. The bivariate normal density function thus becomes:

$$bndf(X', Y') = \frac{1}{cA\sqrt{2\pi}} e^{-\left(\frac{X'^2}{2(cA^2)}\right)} \frac{1}{dA\sqrt{2\pi}} e^{-\left(\frac{Y'^2}{2(dA^2)}\right)}. \tag{15}$$

So, the probabilistic index of difficulty for two-dimensional pointing is:

$$ID_{\text{Pr}} = F\left(\int_R \int bndf(X', Y')dY'dX'\right), \tag{16}$$

where R is the region defined by the target. The term inside the brackets is just the integral with respect to $X'$and Y'over the 2D region R. The model which we therefore propose and validate in this article is:

$$MT = a + bID_{\text{Pr}}. \tag{17}$$

As usual, a and b are empirically determined constants. This equation will provide index of difficulties for targets of any shape or orientation by simply integrating over a different region. In our study, the region is a rectangle rotated by the approach angle θ.

It is important to notice that this model possesses all of the properties desirable in defining general two-dimensional pointing. First, it has scale independence because of how the standard deviations are calculated. The limit tasks property also holds by the properties of the bivariate normal distribution because it regresses to the univariate normal-distribution if one of the dimensions is integrated over infinity. The dominance effect holds because increasing or decreasing one dimension will sweep out a greater area of the spread of hits, the larger the other dimension is. H and W duality clearly holds since the region is defined by both H and W. Continuity holds by the nature of the bivariate normal distribution since increasing a dimension size will gradually stop increasing the probability. Finally, the function will be W-asymmetric as long as the spread of hits parallel to the line of movement is greater than the spread of hits perpendicular to the line of movement ($\sigma_x > \sigma_y$). This is an expected effect based on empirical data [Schmidt et al. 1979].

It should also be noted that by diverging from the standard formulations of the index of difficulty, we also diverge from the explanatory reasoning of these models. The standard models, where the index of difficulty is based on the logarithm of the amplitude and dimension ratios, are analogous to the calculation of the information capacity of a communications channel based on the signal strength and noise power. Our model also serves as an explanatory model, albeit in a different manner. The justification of our model is that the less likely a target is to be selected using open-loop movements, the more closed-loop corrective movements will be required. This, in turn, increases the index of difficulty. Consistent with previous models, our index of difficulty can still be interpreted as "a measure of the average number of movements (or movement corrections) required to acquire the target or, in other words, the number of times the main human-machine processing loop is executed" [Ware and Balakrishnan 1994].

In the following sections, we will present two experiments. Although we explicitly label the sections as Experiment 1 and Experiment 2, they were conducted in parallel and make up the two major parts of our study. The first part of the study will explore the exact nature of the spread of hits in an open-loop pointing action and how it is affected by target distance and movement angle. This will provide the data needed to build our proposed model for two-dimensional pointing. The second part of the study records behavioral data for a number of conditions so that we can test the model which we built using the collected data.

4. EXPERIMENT 1

The goal of this experiment is to investigate the spread of hits resulting from ballistic or open-loop two-dimensional pointing actions. The results will serve to determine constants c and d (Equation (15)) for our proposed model (Equation (17)).

4.1 Apparatus

The experiment was conducted on a 2.4Ghz Pentium 4 PC using a WACOM tablet and puck for input. The display was a 19″ CRT monitor with 1280×1024 resolution. The computer ran Windows XP and OpenGL for graphics. The puck acceleration was set to 0 with a control-display gain of two. It is important to note that the puck operates on the tablet in absolute mode, removing a potential confound that exists in experiments which use a relative mouse whose origin can be reset during a trial by clutching the device. All dimensions will be measured in units (1 unit $\approx$ 0.2 cm).

4.2 Participants

Three female and seven male volunteers participated in the experiment. Participants ranged in ages from 20 to 25, all were right-handed and controlled the input device and, consequently, the cursor with their right hand.

4.3 Procedure

The task was reciprocal two-dimensional rapid *approximate* pointing which required participants to point towards two fixed-sized targets back and forth in succession. Subjects were told to move quickly towards the target and click wherever this movement led them, not using any corrective motions. In other words, the cursor did not actually have to end up *inside* the target, it was required only that participants point *towards* it. Although it is plausible that subjects would use the visual feedback of the target to continuously guide their movements, the task description made such effects minimal. The targets were rendered as solid squares, equidistant from the centre of the display in opposite directions along the given axis of movement. The squares were aligned with the principle screen axes. The target to be selected was green and the other brown. When participants clicked the puck button, the targets would swap colors as an indication that the participant had to now move to and point at the other target.

4.4 Design

A repeated measures within-participant design was used. The independent variables were amplitude A(32, 64, 96 units), and movement angle θ (0°, 22.5°, 45°, 67.5°, 90°). A fully-crossed design resulted in 15 combinations of A and θ. The target size was kept constant at 1×1.

The experiment was performed in one session lasting approximately 25 minutes. The session was broken up into movement angles, and two blocks of trials were completed for each movement angle. In each block, participants would complete trial sets for each of the 3 values of A presented in random order. A trial set consisted of 21 clicks (i.e., twenty reciprocal movements between the two targets).

Before each session, participants were given practice trials to familiarize themselves with the task. The practice lasted until participants were comfortable with the task, usually about 2 minutes. Participants were randomly divided into 5 groups of 2 each. Assignment of movement angle order to groups was counterbalanced using a balanced Latin square.

4.5 Performance Measures

The dependent variables were X'—defined as the distance between the point clicked and the center of the target measured along the line of movement and Y'—defined as the distance between the point clicked and the center of the target measured along the axis perpendicular to the line of movement (Figure 5(b)).

4.6 Results

Outliers were removed based on *MT* and accuracy. Any data point further than 2 standard deviations away from its condition's mean (by *MT*, or by accuracy, defined as distance between the click and the target center) was removed. A total of 5% of the data were outliers and removed.

Figure 6 shows the spread of hits for each of the 15 conditions in screen unit coordinates. We analyze $\sigma_{X'}$ and $\sigma_{Y'}$, the standard deviations of X' and Y', respectively. Multiple-means comparison of variances within each approach angle showed that $\sigma_{X'}$ significantly increases when the amplitude is increased from 32 to 64 and from 64 to 96 ($p < .0001$). Similarly, $\sigma_{Y'}$ increased (all $p < .01$), except for when the amplitude was increased from 32 to 64 for $\theta = 90°$ ($p = .208$) and for $\theta = 67.5°$, where $\sigma_{Y'}$ actually decreased.

To use our proposed model, we wish to find constants c and d such that $\sigma_{X'} = cA$ and $\sigma_{Y'} = dA$. Linear regression with no intercept was performed on each approach angle. Table I shows the results. We wanted to find the value of c and d for each approach angle since previous evidence [Boritz et al. 1991; Hancock and Booth 2004] indicates that performance is affected by movement direction. If our c and d values were uniform throughout approach angles, then our model would not capture this property. As expected, $c > d$ for all angles. Averaging over all angles, $c = 2.06d$ which resembles the results in Schmidt et al. [1979].

5. SAMPLE CALCULATIONS

We can now use the empirically determined c and d values to calculate ID_{Pr} values for various two-dimensional pointing trial conditions. For example, consider a target with the values $A = 64$, $W = 4$, $H = 4$, and $\theta = 0°$. We begin with

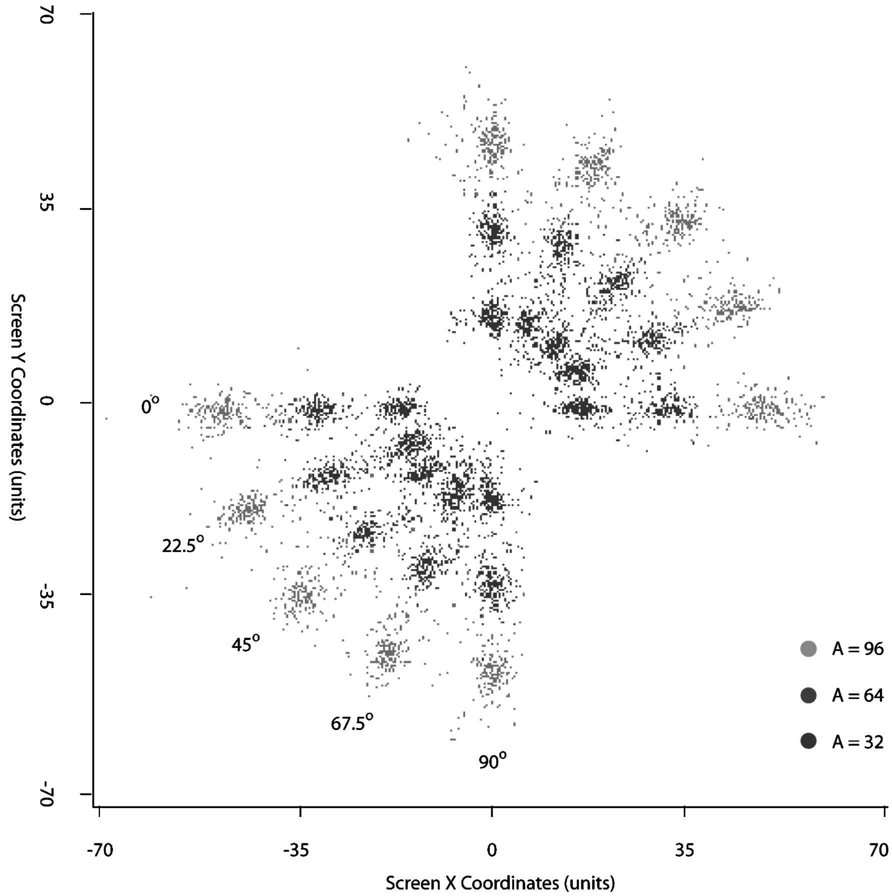

Fig. 6. Spread of hits for all 15 A, θ combinations in Experiment 1.

Table I. Linear Regression Results for Each θ

	c			*d*		
	Estim.	*Std. Er.*	R^2	*Estim.*	*Std. Er.*	R^2
0°	0.0717	0.0018	0.998	0.0284	0.0046	0.9243
22.5°	0.0686	0.0027	0.995	0.0304	0.0043	0.9413
45°	0.0634	0.0047	0.984	0.0344	0.0061	0.9117
67.5°	0.0582	0.0071	0.957	0.0331	0.0035	0.9668
90°	0.0665	0.0028	0.995	0.0345	0.0078	0.8610

Equation (16), noting that for a rectangular target the region R is defined by its width and height:

$$ID_{\mathrm{Pr}} = F\left(\int_{-W/2}^{W/2}\int_{-H/2}^{H/2} bndf(X', Y')dY'dX'\right). \tag{18}$$

Expanding the bivariate normal density function (Equation (15)) gives:

$$ID_{\mathrm{Pr}} = F\left(\int_{-W/2}^{W/2}\int_{-H/2}^{H/2} \frac{1}{cA\sqrt{2\pi}}e^{-\left(\frac{X'^2}{2(cA)^2}\right)}\frac{1}{dA\sqrt{2\pi}}e^{-\left(\frac{Y'^2}{2(dA)^2}\right)}dY'dX'\right). \tag{19}$$

Table II. ID_{Pr} Calculated Using Equation (14) for Various Values of B

$B = A/W$	$P(Hit)$	$ID_{\mathrm{Pr}} = F(P(Hit))$
.	.	.
.	.	.
22.2	0.24657	4.536
22.3	0.24550	4.542
.	.	.
.	.	.

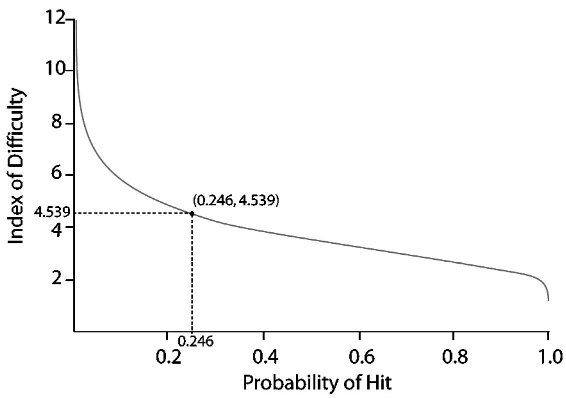

Fig. 7. Calculating $F(0.246)$. Based on values of P and ID_{Pr} calculated in Table II, linear interpolation can be used over the points corresponding to $B = 22.2$ and $B = 22.3$ to find $F(0.246)$.

We now substitute in A, W, and H, and the c and d values which we obtained for $\theta = 0°$:

$$ID_{\mathrm{Pr}} = F\left(\int_{-2}^{2}\int_{-2}^{2}\frac{1}{0.0717 \cdot 64\sqrt{2\pi}}e^{-\left(\frac{X'^2}{2(0.0717 \cdot 64)^2}\right)}\frac{1}{0.0284 \cdot 64\sqrt{2\pi}}e^{-\left(\frac{Y'^2}{2(0.0284 \cdot 64)^2}\right)}dY'dX'\right). \tag{20}$$

The integral can be easily evaluated using computational methods, giving:

$$ID_{\mathrm{Pr}} = F(0.246). \tag{21}$$

We will use the universal F function created with $c = 0.0717$ which was the value found in Experiment 1 when $\theta = 0°$. We construct a table of values using Equation (14) based on various values of B (Table II) and then use linear interpolation to get our desired value.

To find $F(0.246)$, we linearly interpolate between the points $F(0.24657)$, corresponding to $B = 22.2$, and $F(0.24550)$, corresponding to $B = 22.3$, giving the result (Figure 7):

$$ID_{\mathrm{Pr}} = 4.539 \tag{22}$$

If we next consider the same condition, except the width is cut by half ($A = 64$, $W = 2$, $H = 4$, and $\theta = 0°$), we only need to change the region of integration. Specifically, we cut in half the interval which X' ranges over:

$$ID_{\mathrm{Pr}} = F\left(\int_{-1}^{1}\int_{-2}^{2}\frac{1}{0.0717\cdot 64\sqrt{2\pi}}e^{-\left(\frac{X'^2}{2(0.0717\cdot 64)^2}\right)}\frac{1}{0.0284\cdot 64\sqrt{2\pi}}e^{-\left(\frac{Y'^2}{2(0.0284\cdot 64)^2}\right)}dY'dX'\right). \tag{23}$$

This results in a lower probability since less area is covered:

$$ID_{\mathrm{Pr}} = F\left(0.126\right). \tag{24}$$

In turn, this increases the index of difficulty since the universal F function is decreasing:

$$ID_{\mathrm{Pr}} = 5.493. \tag{25}$$

If, instead of decreasing the width, we cut the height in half ($A = 64$, $W = 4$, $H = 2$, and $\theta = 0^\circ$), the index of difficulty becomes:

$$ID_{\mathrm{Pr}} = F\left(\int_{-2}^{2}\int_{-1}^{1}\frac{1}{0.0717\cdot 64\sqrt{2\pi}}e^{-\left(\frac{X'^2}{2(0.0717\cdot 64)^2}\right)}\frac{1}{0.0284\cdot 64\sqrt{2\pi}}e^{-\left(\frac{Y'^2}{2(0.0284\cdot 64)^2}\right)}dY'dX'\right), \tag{26}$$

$$ID_{\mathrm{Pr}} = F(0.141), \tag{27}$$

$$ID_{\mathrm{Pr}} = 5.331. \tag{28}$$

Again, the probability is lowered, therefore increasing the index of difficulty. It should be noted that the index of difficulty is increased to a greater extent when the width was reduced since the model is W-asymmetric.

The calculation of the index of difficulty for conditions at other approach angles is very similar, only requiring minor changes to the method. Consider the case where the variables are $A = 64$, $W = 2$, $H = 4$, and $\theta = 67.5^\circ$. The first change is that the region of integration is now rotated. We still model the spread of hits as a bivariate normal distribution but rotated by 67.5° (Figure 8(a)). If we rotate the scene, we can integrate the nonrotated bivariate normal distribution over the region R defined by a rectangle with width 2, height 4, rotated by -67.5° (Figure 8(b)). The second change is that we use the c and d values found in Experiment 1 when $\theta = 67.5^\circ$.

Thus, the index of difficulty can now be calculated as follows:

$$ID_{\mathrm{Pr}} = F\int_{R}\int\left(\frac{1}{0.0582\cdot 64\sqrt{2\pi}}e^{-\left(\frac{X'^2}{2(0.0582\cdot 64)^2}\right)}\frac{1}{0.0331\cdot 64\sqrt{2\pi}}e^{-\left(\frac{Y'^2}{2(0.0331\cdot 64)^2}\right)}dY'dX'\right). \tag{29}$$

Again, we evaluate the integral and then use the F function to find the index of difficulty:

$$ID_{\mathrm{Pr}} = F(0.147), \tag{30}$$

$$ID_{\mathrm{Pr}} = 5.271. \tag{31}$$

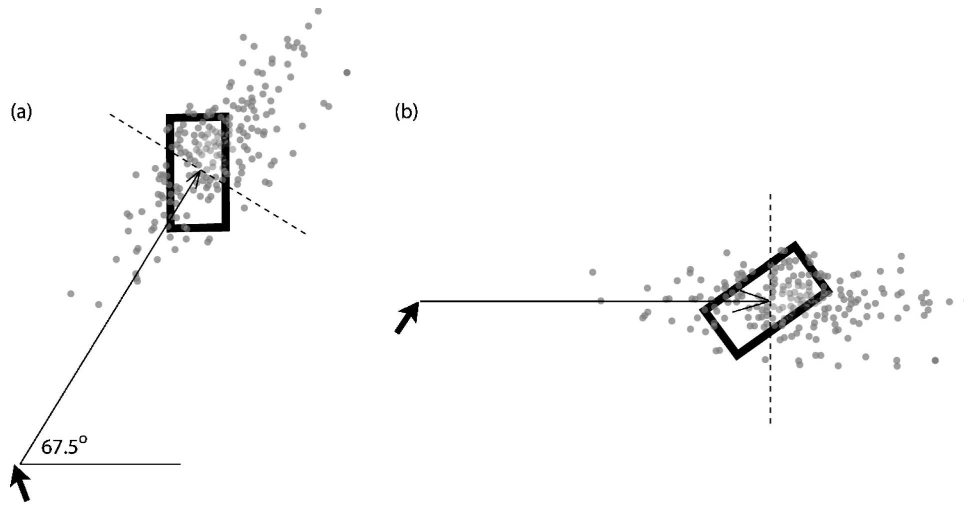

Fig. 8. (a) Integrating the bivariate normal distribution rotated by 67.5° over the region defined by a rectangle. (b) Integrating the nonrotated bivariate normal distribution over the region defined by a rectangle rotated by −67.5°.

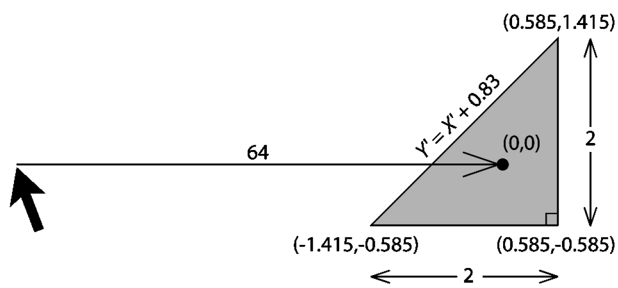

Fig. 9. Acquisition of a right triangle shaped target.

As stated earlier, we can also calculate the index of difficulties for targets of arbitrary shapes. The calculation follows the same procedure except that the region of integration is defined by the shape of the target. For example, consider the acquisition of a right triangle with a base and height of 2, and $\theta = 0°$. We will also let $A = 64$, defined as the distance to the triangle's incenter, the point where the angle bisectors of the triangle meet (Figure 9).

The calculation is procedurally identical to the case of a rectangle except that the region R is now a triangle:

$$ID_{\mathrm{Pr}} = F\left(\int_R \int \frac{1}{0.0717 \cdot 64\sqrt{2\pi}} e^{-\left(\frac{X'^2}{2(0.0717 \cdot 64)^2}\right)} \frac{1}{0.0284 \cdot 64\sqrt{2\pi}} e^{-\left(\frac{Y'^2}{2(0.0284 \cdot 64)^2}\right)} dY' dX'\right). \tag{32}$$

This can be computed by taking the double integral with the following bounds:

$$ID_{\mathrm{Pr}} = F\left(\int_{-1.415}^{0.585}\int_{-0.585}^{X'+0.83} \frac{1}{0.0717\cdot 64\sqrt{2\pi}} e^{-\left(\frac{X'^2}{2(0.0717\cdot 64)^2}\right)} \frac{1}{0.0284\cdot 64\sqrt{2\pi}} \times e^{-\left(\frac{Y'^2}{2(0.0284\cdot 64)^2}\right)} dY' dX'\right), \quad (33)$$

$$ID_{\mathrm{Pr}} = F(0.036), \quad (34)$$

$$ID_{\mathrm{Pr}} = 7.246. \quad (35)$$

6. EXPERIMENT 2

The results of Experiment 1 allowed us to compute c and d for our proposed model. We next conduct a second experiment where we vary more parameters to obtain a comprehensive dataset on two-dimensional pointing in order to verify the effectiveness of our new model. This experiment will involve rectangles of varying widths and heights with varied movement angles. We leave conditions of arbitrary target shapes to future work.

6.1 Apparatus and Participants

Experiment 2 was conducted on the same apparatus by the same 10 participants as Experiment 1.

6.2 Procedure

The task was reciprocal two-dimensional target acquisition requiring participants to point to two targets back and forth in succession. Subjects were told to select the targets as quickly as possible while maintaining an overall error rate of no more than 4%. The targets were rendered as solid rectangles, and, as in Experiment 1, were equidistant from the centre of the display in opposite directions along the given axis of movement. The width of the target was always colinear with the X-axis of the screen, and the height was along the Y-axis of the screen. Again, the target to be selected was green and the other brown. Unlike Experiment 1 where positive selection of the target was not enforced, in this experiment participants had to successfully select the target before the colors would swap even if it required multiple clicks. This effectively removes the possibility that participants may try to race through the experiment by clicking anywhere. For timing and error calculations, however, we consider the first click as the end point.

6.3 Design

A repeated measures within-participant design was used. The first 3 independent variables were the minimum target size (1, 2, 4), the target width and height ratio (1, 1.5, 2, 4), and smaller-of dimension (W, H). Table III shows all $3 \times 4 \times 2 - 3 = 21$ tested W-H combinations (the 3 conditions where $W = H$ need not be repeated). These were fully crossed with the remaining two

Table III. W-H Combinations Tested in Experiment 2

$W = H$	$W = H = 1$	$W = H = 2$	$W = H = 4$
$W < H$	$W = 1$	$H = \{1.5, 2, 4\}$	
	$W = 2$	$H = \{3, 4, 8\}$	
	$W = 4$	$H = \{6, 8, 16\}$	
$W > H$	$H = 1$	$W = \{1.5, 2, 4\}$	
	$H = 2$	$W = \{3, 4, 8\}$	
	$H = 4$	$W = \{6, 8, 16\}$	

independent variable values, amplitude A(32, 64, 128 units), and movement angle θ (0°, 22.5°, 45°, 67.5°, 90°), for a total of $21 \times 3 \times 5 = 315$ conditions.

The experiment was performed in two sessions. The first session, lasting approximately 40 minutes, was conducted immediately following the completion of Experiment 1. The second session, lasting approximately 60 minutes, was conducted on a separate day. In the first session, participants completed all trials for 2 approach angles. In the second session, subjects completed the trials for the remaining 3 approach angles. All trials for one angle were completed before moving on to the next. Within each angle, there were two blocks consisting of all W-H-A combinations presented in random order. For each condition, participants performed 8 reciprocal trials, resulting in a total of $315 \times 2 \times 8 =$ 5040 total trials per subject. At the end of a trial set, a message was displayed informing the participant of the error rate for the previous trial set and the overall error rate, allowing them to adjust their movement speeds if they were exceeding 4% errors. Prior to the sessions, the subjects were given practice trials to familiarize themselves with the new task. The participant groups from Experiment 1 were the same, and the same Latin square design was employed to counter-balance the movement angles.

6.4 Performance Measures

We used two dependent variables. Movement time MT is defined as the time between the first click which begins the trial and the next click which may not necessarily end the trial if it was an error. The error rate was the other dependent variable and is defined as the average number of errors per trial. Errors occurred when participants clicked first when the cursor was outside the target. A maximum of one error was counted per trial even if multiple clicks were made before the target was finally selected.

6.5 Results

Outliers were removed based on MT. Again, any data point further than 2 standard deviations away from its condition's mean was removed, resulting in 6.9% of the data being removed as outliers.

6.5.1 *Movement Time Analysis: Main Effects.* Analysis of variance showed that the independent variables A($F_{2,198} = 16625$, $p < .0001$), minimum target size ($F_{2,198} = 20174$, $p < .0001$), W and H ratio ($F_{2,198} = 239.74$, $p < .0001$), smaller-of dimension ($F_{1,99} = 88.51$, $p < .0001$), and θ ($F_{4,36} = 150.08$, $p < .0001$) all had a significant main effect on MT. The effect of θ is of special interest.

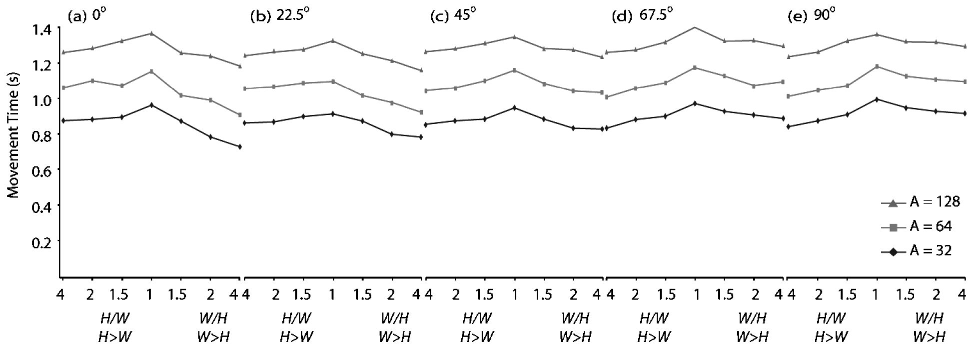

Fig. 10. Effect of W and H ratio on MT.

Recall that θ affects both the movement and approach angles. Multiple means comparisons showed no significant difference in MT between $\theta = 0°$ and $\theta = 22.5°$ (p = .11), but significantly higher MT for $\theta = 45°$, and even higher for $\theta = 67.5°$ (both p < .0001). There was no significant difference in MT for $\theta = 67.5°$ and $\theta = 90°$ (p = 1.0). It should be noted that for $\theta = 90°$, if W and H are swapped then the approach angle is equal to that of $\theta = 0°$ so it must be the movement angle causing the significant difference. This matches the results of previous studies [Langolf et al. 1976; Boritz et al. 1991; Card et al. 1991; Balakrishnan and MacKenzie 1997; Hancock and Booth 2004] which found that the use of smaller muscle groups (hand, wrist) required for horizontal movement will result in better performance in a pointing task than the larger muscle groups (upper arm, shoulder) required for vertical movement.

6.5.2 *Movement Time Analysis: W and H Ratio.* Previous work [Accot and Zhai 2003] showed that when movement was horizontal, the target width is more critical than the target height. It is reasonable to assume that the reason for this is because the width is colinear with the line of movement, while the height is perpendicular to the line of movement. However, in our experiment, this is not always the case since we varied the approach angle. This leads us to examine the effect of the ratio broken down by approach angle. Figure 10 shows how the W and H ratio affect MT broken down by θ and the three amplitudes tested. It can be seen that the weight gradually transfers from W to H as the approach angle goes from 0° to 90°. The ratio had significant interactions with both amplitude A ($F_{14,2277} = 986.45$, p < .0001) and approach angle θ($F_{28,351} = 30.51$, $p < .0001$).

Post hoc multiple means comparison using Tukey-Kramer adjustment was performed on the data illustrated in Figure 10, and we now summarize some of the interesting effects, broken down by θ.

$\theta = 0°$. For $W \geq H$ increasing the ratio from 1 to 1.5 was significant at all values of A ($p < .001$ for all significant effects reported in this subsection). Increasing from 1.5 to 2 was only significant for $A = 64$ and increasing from 2 to 4 was only significant for $A = 32$. For $W \leq H$, increasing the ratio from 1 to 2 was significant for all values of A. Thus, we see that increasing H, *even when W is the constraining factor*, can still significantly reduce MT. Further increase

Table IV. Fit of ID_{WtEuc} and ID_{Pr} Models to Experiment 2 Data

	a		b		η		
Model	*Estim.*	*Std. Err.*	*Estim.*	*Std. Err.*	*Estim.*	*Std. Err.*	R^2
ID_{WtEuc} $\theta = 0^\circ$	−66.32	23.89	225.63	4.66	0.38	0.04	0.974
ID_{Pr} $\theta = 0^\circ$	313.52	19.37	131.54	3.22			0.964
ID_{Pr}	356.00	9.696	127.08	1.61			0.952

to 4 was not significant. Of special interest is that for $A = 128$, when the ratio was 2, there was no difference between when $W > H$ and when $W < H$. This was the only case where the ratio was symmetric. In other words, doubling the height and width had similar effects at the furthest distance.

$\theta = 45^\circ$. In this case, there is complete symmetry through all ratios and A values. This is consistent with our expectations since increasing the height or width will sweep out the exact same area of the bivariate distribution since the target is simply reflected through the Y-axis.

$\theta = 90^\circ$. For $W \geq H$, increasing the ratio from 1 to 4 was significant for $A = 128$, and from 1 to 2 was significant for $A = 64$ and 32. Again, we see the dimension perpendicular to the line of motion having an impact on *MT*, even when greater than the dimension parallel to the line motion. For $W \leq H$, increasing the ratio from 1.5 to 2 was significant for $A = 128$. For $A = 64$ and 32, the increase was significant from both 1 to 1.5 and 1.5 to 4.

6.5.3 *Movement Time Analysis: Fit of the Model.* We fit the *MT* data to our candidate model ID_{Pr} and compared it to results of the weighted Euclidian model ID_{WtEuc} (Equation (4)) proposed in Accot and Zhai [2003]. Since Accot's model does not account for varying approach angles or movement angles, we limit the comparison to the condition $\theta = 0^\circ$, but we do a full fit of data for all θ for our model.

To calculate the probabilities for ID_{Pr} (Equation (16)), we used the c and d values calculated in Experiment 1 (Table I). Once a probability was calculated, we used linear interpolation on the universal F function to approximate the corresponding ID value. The c value used for the F function was 0.0717 which was the value found in Experiment 1 when $\theta = 0^\circ$. It should be understood that the c and d values which were calculated from Experiment 1 describe the spread of hits from open-loop target acquisition movements. This spread of hits is bound to be different in a target acquisition task where closed-loop movements are used. In addition, varying the target dimension will undoubtedly affect the spread of hits. However, we still use the c and d values from Experiment 1 because we want to determine how much deviation from an open-loop movement will be required to actually select a target. In other words, we will establish how much of the open-loop spread of hits is encompassed in the region defined by the target.

Table IV shows the results using a least-squares fit method. The table shows parameter estimates where applicable and the corresponding standard errors. The last column provides the R^2 values for the regression. We see that the Euclidean and probabilistic models give very similar results when $\theta = 0^\circ$. The Euclidean model has a slightly higher fit but also exhibits slightly higher

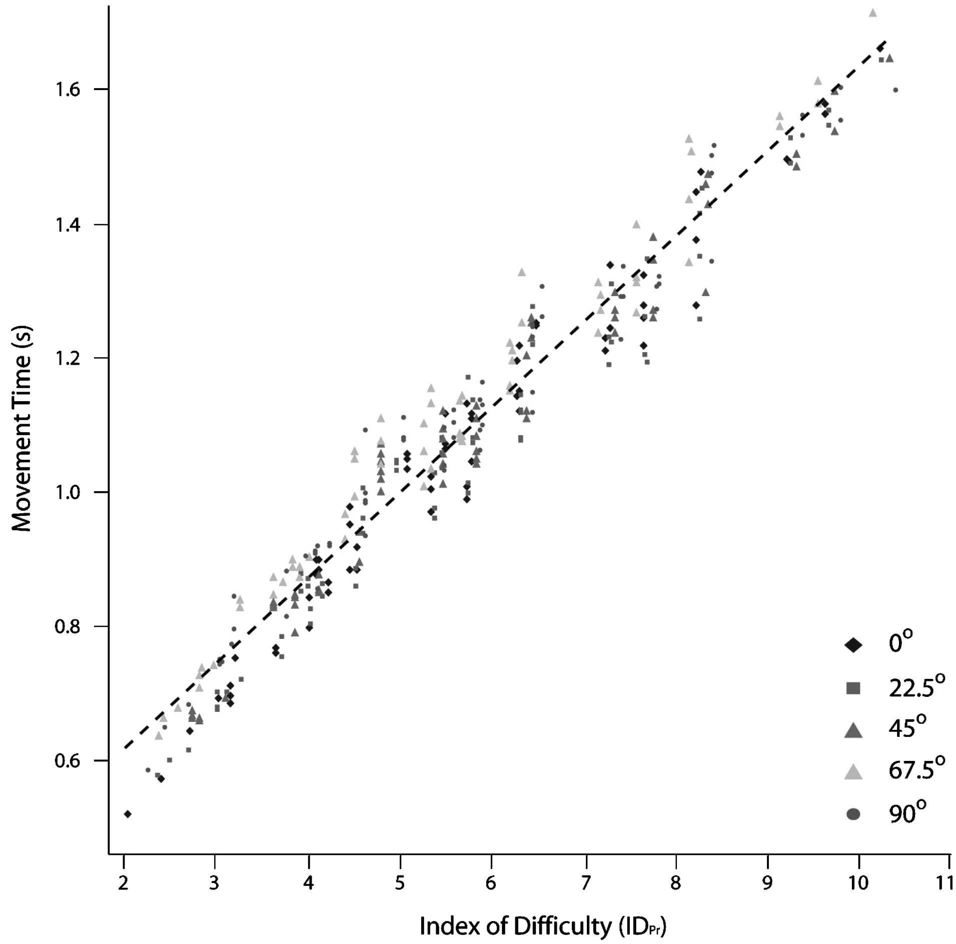

Fig. 11. Scatterplot and regression for all values of θ, using ID_{Pr}.

standard errors on parameter estimates. Furthermore, the Euclidian model requires an extra parameter which has been computationally calculated to optimize the R^2 value. The main benefit of the probabilistic model is that it can provide the index of difficulties for various approach angles. In contrast to the Euclidean model which only works for $\theta = 0^{\circ}$, the probabilistic model accounts for the entire data set for all values of θ, with a high correlation of 0.952 (Figure 11).

6.5.4 *Error Analysis.* For all participants, there were a total of 1322 errors and 45,571 correct trials. This is a 2.9% error rate, only slightly lower than the standard criterion for Fitts' law experiments. Error rate was significantly affected by A($F_{2,198} = 5.85$, p = .0029), minimum target size ($F_{2,198} = 74.73$, p < .0001), W and H ratio ($F_{2,198} = 67.5$, p < .0001), smaller-of dimension ($F_{1,99} = 28.19$, p < .0001) and θ ($F_{4,36} = 4.51$, p < .0012).

7. IMPLICATIONS FOR USER INTERFACE DESIGN

The results of our study can be used to guide the development of user interfaces from both design and theoretical perspectives. As in Accot and Zhai [2003], we found an asymmetrical impact of the target dimensions parallel and perpendicular to the line of movement. Accot and Zhai [2003] give a good account of the implications of this result for design. Our results show that this asymmetry also holds for varying movement angles. In contrast to Accot and Zhai [2003], we also found that increasing the height of a target can make it easier to select even when it is greater than the width. This new finding tells designers that it would not be in vain to design buttons with greater height than width. Another finding which could be utilized is that the height and width of rectangular targets have equal impact on movement time when moving at a 45° angle. Thus, icons placed at the corners of a display can be elongated in either direction with equally beneficial results since the acquisition of such targets will on average be at a 45° angle. In essence, the probabilistic model allows designers to understand exactly how modifying a target's shape or size will affect acquisition times as an alternative to simply following more general, but possibly inaccurate, rules of thumb. For example, it may be incorrectly assumed that increasing the width of a target will not reduce acquisition times once the width is twice as large as the height. Using the probabilistic model, however, analysis may show that increasing the width from two times to three times the target height will result in a possible 5% decrease in acquisition time or something along these lines. This will have special value when working with unconventional input and display devices. For example, when directly interacting with the surface of a wide-format (16:9) display, the spread of hits will likely be drastically elongated along the X-axis. As a result, designers may find it beneficial to have targets that are five times as wide as they are high. In contrast, when using a stylus on a PDA, the spread of hits will be condensed in both directions, and so it may be found that increasing the width to even two times the height will have negligible benefit. From a theoretical standpoint, designers can use our model to make accurate predictions about their interfaces when the user will be moving at different angles to point at targets, and pointing at nonrectangular buttons. For example, the stylus-based virtual keyboard optimization in Zhai et al. [2000] could be extended to optimize the shape of the keys and account for angles between them in performance predictions.

8. DISCUSSION AND FUTURE WORK

Some issues surrounding the probabilistic model which we have proposed and validated should be clarified. The critical step in applying our model is defining what the probability of a hit is. We do this by calculating how much of the bivariate normal distribution is covered by the target when centered within the distribution. While the results of our experiment show that this method is good enough to accurately predict movement times, it is possible that the prediction accuracy could be further increased by refining the underlying assumptions. First, the use of the bivariate normal distribution to model the open-loop spread of hits, while a reasonable first approximation, could be somewhat of a simplification. We did not validate that the spread did indeed take on this distribution, and it is possible that the spread could be skewed in various manners such that other types of distributions would be more appropriate for modeling its nature. A possible line of future work would be to perform a more in-depth analysis of the open-loop spread of hits and finding a best fit distribution which models it. Another assumption we made was that the center of the target was in the center of the distribution when calculating probabilities. In other words, we assumed that the user is aiming for the center of the target in an acquisition task. While this may be true for small targets, it is plausible that, as the target size grows, users will begin to aim closer to the side of the target first crossed by the pointer. If this were the case, the center of the distribution should be placed closer to this side. Furthermore, it is unclear where the center of the distribution should be placed in irregular-shaped targets. We discussed the example of a triangle and used the incenter which is a reasonable first approximation. It is less obvious how to determine the center of an irregular-shaped target which could be completely asymmetric and might even contain holes or gaps. It would be useful to explore methods of defining this center for any target shape. A related line of future work would be to validate the model for targets of various shapes. It would also be valuable to understand how the c and d coefficients in ID_{Pr} behave under varying conditions. We use empirical data to establish their values for discrete movement angles. It would be interesting to develop a method to determine their values for a general movement angle. It would also be valuable to understand how these values change and how well the model applies when task conditions such as the input device and control gain ratio are altered. For control purposes, our experiments were conducted using a puck and tablet in absolute mode. Understanding how the model behaves when using a mouse, a relative input device, would be of interest. It would also be interesting to explore less conventional display-input configurations such as pens on tablet displays and touch on table top displays. It is likely that the c and d coefficient values would compensate for such variations.

9. CONCLUSION

We have presented experimental work that investigated how target dimensions, movement angles, approach angles, and their interactions affect the selection of two-dimensional targets. In an initial study, we investigated the distribution of hits when using ballistic movements to point at a two-dimensional target. We found that at all movement angles, the distribution has a spread approximately twice as great in the direction of movement as in the direction perpendicular to movement. We also found that this spread increases in both directions by a constant factor with the distance to the target and empirically determined these constant factors c and d. In a second study, we observed that the dimension of a target, colinear with the line of movement, is the most critical; that with an approach angle of 45°, width and height have equal effect on movement time. And in contrast to previous findings, we found that the dimension perpendicular to the line of movement can still affect movement time even when greater than the dimension parallel to movement. This result was in contrast to our own hypothesis as well as the effect was seen over all amplitudes not just the longest. We introduced and validated a probabilistic model for pointing to targets that satisfies all of the desirable properties of such a model. The model calculates probabilities by integrating the bivariate normal distribution with standard deviations calculated from the c and d values determined in the first experiment, over the region defined by the target. The probability is then mapped to an index of difficulty using a function which was constructed based on well accepted one-dimensional index of difficulties. We have shown that this model can predict performance time as accurately as the best previous model when the movement angle is zero. Unlike the previous model, ours can account for varying movement angles and approach angles and be generalized to different target shapes and dimensions. We discussed the simplifying assumptions which we made and the future improvements to the probabilistic model which these assumptions leave room for. Finally, we have discussed the implications of our results to user interface design.

ONLINE APPENDIX

The data set from our experiments, including a complete table of ID_{Pr} for that data, is available at http://www.dgp.toronto.edu/~tovi/idpr/.

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