A Probabilistic Load Flow Method Considering Branch Outages

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***Abstract*—This paper proposes a probabilistic load flow method considering random branch outages as well as uncertainties of nodal power injections. Branch outages are simulated by fictitious power injections at the corresponding nodes. A unified procedure is given to deal with random branch outages, generating unit outages, and load uncertainties by their moments and cumulants. The variations of nodal voltages and line flows produced by normally and discretely distributed input variables are handled separately. The method proposed by Von Mises is employed to solve the discrete distribution part of each state and output variable. The final distribution of a desired variable is obtained by simply convoluting its continuous and discrete distribution part. Results of 24-bus IEEE Reliability Test System are analyzed and compared to those obtained by Monte Carlo simulation. A numerical test on a real power system shows the effectiveness of the proposed method.**

***Index Terms*—Branch outage, cumulant, probabilistic load flow (PLF), probability distribution function (PDF), Von Mises method.**

I. Introduction

OPEN access of transmission systems has resulted in more highly stressed and unpredictable operating conditions. Uncertain factors bring great challenges to power system planning and operation. Probabilistic analysis tools are needed urgently in the fields, such as transmission system expansion [1], [2] and real-time operation [3]. Software that is developed for online security assessment and can be integrated with the existing EMS systems considering load uncertainties and random outages is valuable [4]. However, the traditional deterministic load flow only finds nodal voltages and line flows under a specified operating condition. On the other hand, the probabilistic load flow (PLF) [5]–[18] or stochastic load flow [19]–[22] can be used to assess adequacy indexes, such as the probability of a nodal voltage being outside acceptable levels and the probability of a line flow being greater than its thermal rating, under load uncertainties and random contingencies. Hatziargyriou and Karakatsanis have applied PLF methods to reactive power control and voltage instability assessment [23]–[25].

The PLF was proposed by Borkowska for evaluation of power flow considering uncertainties [5]. Many papers [6]–[21] have been published on this interesting and challenging area, especially by Allan and Leite de Silva [6]–[14]. Most PLF formulations use linear load flow equations. By linearizing ac load flow equations around the expected input value region, the state and output variables can be represented as a linear combination of input variables. Assuming independence among all the variables, convolution techniques and methods based on moments and cumulants [15]–[17] are commonly applied to obtain the probability density functions (PDF) or cumulative distribution functions (CDF) of desired variables. Test results show that the performance of the linear model is good [8], and the quadratic items are small enough to reside [18] within a certain uncertainty range of input variables. The basic convolution operation or fast Fourier transform method [5]–[8] for the PLF is computationally intensive with large storage requirements. By using cumulants, the computation burdens can be greatly relieved [17].

Practically, correlations may exist between nodal powers [13]. The multivariate Gram–Charlier Type A series in [27] can be employed to the PLF problem in this case. Based on joint PDF of nodal powers, Sauer and Hoveida deduced full formulas to calculate moments of output variables [21]. Because it is difficult to model the exact correlations in power injection variations, only linear dependence has been considered in [9], [14], and [22]. Dispatch strategies are also considered in [9] and [18].

Monte Carlo simulation (MCS) techniques are widely used in power system computations, especially in reliability assessment. MCS can be employed to solve the PLF problem straightforwardly by repeated simulations. It can provide considerably accurate results, but the computation is time consuming for large systems. Techniques that combine MCS with multilinearized power flow equations are proposed to reduce computational burden in [10] and [11]. Results obtained by MCS can be set as benchmarks for comparison purposes.

So far, only a few works have considered random branch outages in the analytical PLF formulations [12], [26]. This may be due to the following two reasons.

1) Unlike the case when only considering random power injections, the network structure is also “uncertain” after incorporating random branch outages.
2) Branch outages may have much greater influence on system state than nodal power injection uncertainties.

The formulation in [26] uses a dc model. The method in [12] obtains the PDF of desired variables from a weighed sum of density function evaluated for each possible network configuration. So the computational burden could be very heavy when a large number of network configurations are required to deal with.

In this paper, random branch outages are simulated by fictitious power injections with 0–1 distributions at the corresponding nodes. The organization of this paper is as follows. Section II describes the formulation of the proposed PLF

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method. Section III introduces some mathematical backgrounds about moments, cumulants, and the Von Mises method [22], [29]. Section IV provides the distribution functions of desired random variables and the computation procedure. In Section V, tests on two systems are illustrated and discussed. Section VI concludes.

II. Formulation

A. Linear Load Flow Equations

The two sets of nonlinear load flow equations for a power system can be expressed as

$$\boldsymbol{Y} = \boldsymbol{g}(\boldsymbol{X}) \tag{1}$$

$$\boldsymbol{Z} = \boldsymbol{h}(\boldsymbol{X}) \tag{2}$$

where $\boldsymbol{Y}$ (the boldface symbol denotes a vector or matrix in this paper) is the input vector of real and reactive power injections, $\boldsymbol{X}$ is the state vector of nodal voltages and angles, $\boldsymbol{Z}$ is the output vector of line flows, and $\boldsymbol{g}$ and $\boldsymbol{h}$ are the nodal power and line flow functions, respectively.

Uncertainties of nodal power injections considered in this paper include random load variations and unit forced outages. They are defined by PDFs. The expected values of real and reactive power injections at all nodes are known. Running a conventional load flow at the expected value of $\boldsymbol{Y}$ and linearizing (1) and (2) at the solution point gives

$$\Delta\boldsymbol{X} = \boldsymbol{X} - \boldsymbol{X}_0 = \boldsymbol{S}_0\boldsymbol{\Delta Y} \tag{3}$$

$$\Delta\boldsymbol{Z} = \boldsymbol{Z} - \boldsymbol{Z}_0 = \boldsymbol{L}_0\boldsymbol{\Delta Y} \tag{4}$$

where $\boldsymbol{X}_0$ and $\boldsymbol{Z}_0$ are expected values of $\boldsymbol{X}$ and $\boldsymbol{Z}$, respectively; $\boldsymbol{S}_0$ is the inverse of the Jacobian matrix $\boldsymbol{J}_0$

$$\boldsymbol{J}_0 = \left.\frac{\partial \boldsymbol{g}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right|_{\boldsymbol{X}=\boldsymbol{X}_0} \tag{5}$$

$$\boldsymbol{L}_0 = \left[\left.\frac{\partial \boldsymbol{h}(\boldsymbol{X})}{\partial \boldsymbol{X}}\right|_{\boldsymbol{X}=\boldsymbol{X}_0}\right]\boldsymbol{S}_0. \tag{6}$$

If nodal power injections are independent, one can obtain the nodal voltages and line flows as a linear combination of input variables. When the variances of load uncertainties are not large, the error raised by linearization may be acceptable. On the other hand, the discrete disturbances may have much greater influence on state and output variables; therefore, errors introduced by the linearization should be studied carefully.

B. Discrete Disturbances

If random discrete disturbances and load uncertainties with continuous distributions are independent, then their influence on nodal voltages and line flows can be handled separately. The variations of $\boldsymbol{X}$ and $\boldsymbol{Z}$ produced by the continuous distribution of $\boldsymbol{Y}$ are called continuous parts in this paper. The discrete variations of $\boldsymbol{X}$ and $\boldsymbol{Z}$ deviated from $\boldsymbol{X}_0$ and $\boldsymbol{Z}_0$ that are produced by discrete disturbances are called discrete parts.

1) Branch Outage: Different from random power injections, the configuration of a power network is changed after a branch outage. To make use of the linear equations (3) and (4), each line outage is simulated by injecting fictitious powers at both ends of the line. When the injected powers are equal to powers leaving from each end of the line, the state of the system is equivalent to that after the line outage [30].

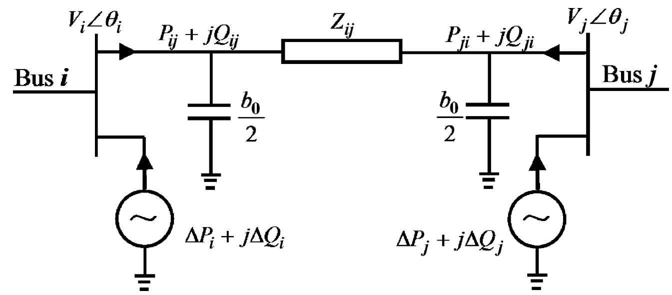

Fig. 1. Branch outage simulation by fictitious injection powers.

Suppose powers leaving from the two ends before the line outage are $P_{ij} + jQ_{ij}$ and $P_{ji} + jQ_{ji}$, respectively (see Fig. 1). The fictitious power injections $\Delta P_i + j\Delta Q_i$ and $\Delta P_j + j\Delta Q_j$ make them change $\Delta P_{ij} + j\Delta Q_{ij}$ and $\Delta P_{ji} + j\Delta Q_{ji}$, respectively. The post-outage system state should satisfy the following equations:

$$\Delta P_i + j\Delta Q_i = (P_{ij} + \Delta P_{ij}) + j(Q_{ij} + \Delta Q_{ij}) \tag{7}$$

$$\Delta P_j + j\Delta Q_j = (P_{ji} + \Delta P_{ji}) + j(Q_{ji} + \Delta Q_{ji}). \tag{8}$$

They can be rewritten in matrix form as

$$\begin{bmatrix} P_{ij} \\ Q_{ij} \\ P_{ji} \\ Q_{ji} \end{bmatrix} = \begin{bmatrix} \Delta P_i \\ \Delta Q_i \\ \Delta P_j \\ \Delta Q_j \end{bmatrix} - \begin{bmatrix} \Delta P_{ij} \\ \Delta Q_{ij} \\ \Delta P_{ji} \\ \Delta Q_{ji} \end{bmatrix} = \left[\boldsymbol{I}_{4\times4} - \boldsymbol{L}'_{4\times4}\right] \begin{bmatrix} \Delta P_i \\ \Delta Q_i \\ \Delta P_j \\ \Delta Q_j \end{bmatrix} \tag{9}$$

where $\mathbf{I}_{4\times4}$ is a 4-by-4 identity matrix, and

$$\boldsymbol{L}'_{4\times4} = \begin{bmatrix} \frac{\partial P_{ij}}{\partial P_i} & \frac{\partial P_{ij}}{\partial Q_i} & \frac{\partial P_{ij}}{\partial P_j} & \frac{\partial P_{ij}}{\partial Q_j} \\ \frac{\partial Q_{ij}}{\partial P_i} & \frac{\partial Q_{ij}}{\partial Q_i} & \frac{\partial Q_{ij}}{\partial P_j} & \frac{\partial Q_{ij}}{\partial Q_j} \\ \frac{\partial P_{ji}}{\partial P_i} & \frac{\partial P_{ji}}{\partial Q_i} & \frac{\partial P_{ji}}{\partial P_j} & \frac{\partial P_{ji}}{\partial Q_j} \\ \frac{\partial Q_{ji}}{\partial P_i} & \frac{\partial Q_{ji}}{\partial Q_i} & \frac{\partial Q_{ji}}{\partial P_j} & \frac{\partial Q_{ji}}{\partial Q_j} \end{bmatrix}. \tag{10}$$

It follows that

$$\begin{bmatrix} \Delta P_i \\ \Delta Q_i \\ \Delta P_j \\ \Delta Q_j \end{bmatrix} = \left[\boldsymbol{I}_{4\times4} - \boldsymbol{L}'_{4\times4}\right]^{-1} \begin{bmatrix} P_{ij} \\ Q_{ij} \\ P_{ji} \\ Q_{ji} \end{bmatrix}. \tag{11}$$

$\boldsymbol{L}'_{4\times4}$ is actually a submatrix of $\boldsymbol{L}_0$, which is available after linearizing the load flow equations. When $\Delta P_i + j\Delta Q_i$ and $\Delta P_j + j\Delta Q_j$ are solved using (11), the random branch outage can be simulated by two random power injections with 0-1 distributions at each end of the branch. Thus, the linear load flow equations can be used. Multiple branch outages can also be simulated in a similar way.

The above simulation method is based on linear relationships between input and output vectors. Some line outages have great influence on system state; errors raised by using the linear load flow (3) and (4) may not be negligible. To balance between speed and accuracy, a threshold ΔC_f is defined. After the fictitious power injections are obtained, (3) is then used to calculate the voltage angle variation $\Delta\theta_i$ and $\Delta\theta_j$ at each end of the line. If the absolute value of $\Delta\theta_i$ or $\Delta\theta_j$ is greater than ΔC_f, a full load flow should be run with all nodal power injections set equal to their expect values.

2) Unit Outage and Load Uncertainty: Random generating unit outages are generally considered as power injection uncertainties in the PLF formulations. In order to reduce the error introduced by linearization, unit outages can be handled in a similar way as line outages if reactive limits and the power balance problem are not considered. For each unit outage, calculate its nodal voltage angle variation after its outage using (3). If the variation is smaller than ΔC_f, the linear load flow equations will be used, and the swing bus is responsible for matching the balance of power. Otherwise, an exact load flow will be run to obtain the post-outage state. The "lost" power is compensated by other units. Dispatch strategies can also be incorporated. The "blackout" of a power plant with no more than two units is also considered.

A bus with several discrete load levels is similar to a node with several generating units because they all have several different power injection levels. So load uncertainties with discrete distributions can be handled in the similar way as random unit outages.

To sum up, a unified approach can be used to deal with the discrete distributions of load uncertainties, unit outages, and branch outages.

C. Correlation and Dispatch Strategy

The aforementioned model assumes total independence between all disturbances. Practically, correlations may exist between nodal power injections (including loads and generator outputs) and line outages. If only linear correlations among components of $\boldsymbol{Y}$ are considered, these correlations can be handled by the method proposed in [9] and [22]. Jointly, normal distributions can also be considered using a covariance matrix [21].

It is very complex to include dispatch strategies in the analytical PLF method. The swing bus absorbs all changes due to load uncertainties in the proposed method. However, the difference between composite reliability evaluation and the PLF should be noted [11]. In the long-term or mid-term planning, the load uncertainties may be significant. It is better to take into account different dispatch or unit commitments strategies under different load levels. The multilinearization methods proposed in [10] and [11] may be tailored to solve this problem.

III. Mathematical Background

A. Moment

For a random variable ξ with a continuous distribution function $F_\xi(x)$, the definition of its rth $(r \geq 0)$ moment is given by [28]

$$m_r = E(\xi^r) = \int_{-\infty}^{+\infty} x^r dF_\xi(x) \tag{12}$$

where $E(\cdot)$ is the mathematical expectation operator.

If a random variable ξ is discretely distributed and each possible point x_i and the corresponding probability p_i are known, the definition of its rth moment is

$$m_r = E(\xi^r) = \sum_i p_i x_i^r. \tag{13}$$

The following descriptions will focus on the moments and cumulants of continuously distributed random variables and suppose their moments exist. The definitions and properties can be easily extended to discrete distribution by degenerating integration to summation.

B. Cumulant

For a random variable ξ, its characteristic function is defined as [28]

$$\varphi_\xi(t) = E(e^{it\xi}) = \int_{-\infty}^{+\infty} e^{itx} dF_\xi(x) \tag{14}$$

where $i = \sqrt{-1}$.

Applying a logarithm operator to (14) and expanding by a MacLaurin series for a small value of t gives

$$\ln \varphi_\xi(t) = \sum_{r=1}^{s} \frac{k_r}{r!}(it)^r + o(t^s) \tag{15}$$

where the coefficients k_r are called cumulants or semi-invariants, and $o(t^s)$ is the error of the expansion.

Property 1: If ξ is the sum of n independent random variables ξ_i, then its rth $(r > 0)$ cumulant is equal to the sum of the rth cumulant of all ξ_i.

Supposing that ξ is the sum of two independent random variables ξ_1 and ξ_2, the following equation can be obtained according to the property of mathematical expectation:

$$\varphi_\xi(t) = \varphi_{\xi_1}(t) \cdot \varphi_{\xi_2}(t). \tag{16}$$

Therefore

$$\ln \varphi_\xi(t) = \ln \varphi_{\xi_1}(t) + \ln \varphi_{\xi_2}(t). \tag{17}$$

From (15) and (17), one can see that the convolution of two independent random variables can be transformed into the summation of their cumulants.

Property 2: If $\xi' = a\xi$, then the rth $(r > 0)$ cumulant of ξ' is equal to the rth $(r > 0)$ cumulant of ξ times a^r.

Supposing that $\xi' = a\xi + b$, the characteristic function of ξ' is

$$\varphi_{\xi'}(t) = e^{itb}\varphi_\xi(at) \tag{18}$$

$$\ln \varphi_{\xi'}(t) = (itb) + \ln[\varphi_\xi(at)] = \sum_{r=1}^{s} \frac{k'_r}{r!}(it)^r + o(t^s) \tag{19}$$

where k'_r is the rth-order cumulant of ξ'. According to (15), it can be observed that

$$\sum_{r=1}^{s} \frac{k'_r}{r!}(it)^r + o(t^s) = (itb) + \sum_{r=1}^{s} \frac{k_r}{r!}(iat)^r + o(at^s). \tag{20}$$

Therefore

$$k'_r = \begin{cases} a^r k_r + b & r = 1 \\ a^r k_r & r > 1. \end{cases} \quad (21)$$

C. Relationship Between Cumulants and Moments

The cumulants are not, like the moments, directly ascertainable by integrative processes. To find them, it is usually necessary to find the moments first. The following recursive relationship is very useful [31]:

$$k_1 = m_1 \quad (22)$$

$$k_{r+1} = m_{r+1} - \sum_{j=1}^{r} C_r^j m_j k_{r-j+1} \quad (23)$$

$$m_1 = k_1 \quad (24)$$

$$m_{r+1} = k_{r+1} + \sum_{j=1}^{r} C_r^j m_j k_{r-j+1} \quad (25)$$

where C_r^j are the binomial coefficients.

D. Determining a Discrete Distribution by its Moments

Von Mises proposed a method to determine a discrete distribution with r impulses by its first $2r-1$ moments in [29].

Define determinants as follows:

$$D_0 = m_0 \quad D_1 = \begin{vmatrix} m_0 & m_1 \\ m_1 & m_2 \end{vmatrix} \cdots$$

$$D_{r-1} = \begin{vmatrix} m_0 & m_1 & \cdots & m_{r-1} \\ m_1 & m_2 & \cdots & m_r \\ \vdots & & & \\ m_{r-1} & m_r & \cdots & m_{2r-2} \end{vmatrix}. \quad (26)$$

It can be proved that D_0 to D_{r-1} are all greater than 0 for a discrete distribution with not less than r impulses. The roots of the following equation are equal to the abscissas of the r points:

$$x^r + c_{r-1}x^{r-1} + \cdots + c_1 x + c_0 = 0. \quad (27)$$

The coefficients c_i are determined by the following linear equations:

$$[\boldsymbol{D}_{r-1}][\boldsymbol{c}] = [\boldsymbol{M}] \quad (28)$$

where $[\boldsymbol{D}_{r-1}]$ is a matrix with the same elements as determinant D_{r-1}, and

$$[\boldsymbol{M}] = [-m_r, -m_{r+1}, \cdots, -m_{2r-1}]^T. \quad (29)$$

It can be proved that the roots of (27) are real numbers and different from each other. When each x_i of a discrete distribution is known, the corresponding possibility p_i can be solved by the linear equations given in (30) based on the moment definition (13)

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_r \\ x_1^2 & x_2^2 & \cdots & x_r^2 \\ \vdots & \vdots & & \vdots \\ x_1^{r-1} & x_2^{r-1} & \cdots & x_r^{r-1} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_r \end{bmatrix} = \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ \vdots \\ m_{r-1} \end{bmatrix}. \quad (30)$$

Now, a discrete distribution with r impulses is completely determined. If a distribution has more than r discrete points, the solved distribution with r discrete points approximates to it because they have the same first $2r-1$ moments [28].

IV. Distribution of Desired Random Variable and The Computation Procedure

The continuous part and the discrete part of each desired random variable are treated separately in this paper. The continuous distribution considered here is the normal distribution. Since the result of convoluting normal distributions is still a normal distribution, so the continuous parts of voltages and line flows are all normally distributed when using linearized equations. After running a deterministic load flow at the expected value of $\boldsymbol{Y}$, cumulants of the desired variables can be obtained using (3) and (4). For a normal distribution, all cumulants higher than the second order are zero. So only the first- and the second-order cumulants are required to calculate. The unified procedure described in Section II-B is employed to calculate discrete part cumulants of each desired variable.

To completely determine the discrete part of each desired variable with r impulses, the first $2r-1$ moments should be calculated. If there are a large number of impulses, the computational burden will be huge. A discrete distribution with $\tilde{r}$ ($\tilde{r} < r$) impulses obtained by the Von Mises method can serve as an approximation. If $\tilde{r}$ is equal to 5, only the first nine moments are required. This is similar to the Gram–Charlier expansion using several items. The difference is that the accuracy will be improved by using more moments when employing the Von Mises method, while the Gram–Charlier expansion does not ensure higher accuracy with more items. It also should be noted that the desired random variables may have divergent cumulants when random branch outages are considered. Errors introduced by the Gram–Charlier expansion may be significant.

A desired random variable R is the sum of a normally distributed random variable R_c and a discretely distributed random variable R_d, i.e.,

$$R = R_c + R_d. \quad (31)$$

According to the definition of convolution, the PDF of R is

$$f_R(x) = \int_{-\infty}^{+\infty} f_{R_c}(x - x_d) f_{R_d}(x_d) dx_d = \sum_{i=1}^{\tilde{\gamma}} p_i f_{R_c}(x - x_i) \quad (32)$$

where p_i, x_i, and $f_{R_d}(\cdot)$ are the probabilities (impulses), variate-values (abscissas), and the PDF of R_d, respectively; $f_{R_c}(\cdot)$ is the PDF of the normally distributed R_c; and

$$f_{R_c}(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad (33)$$

where μ and σ are the expectation and the standard deviation of R_c, respectively.

The CDF of a standard normal distribution is

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{t^2}{2}\right) dt. \quad (34)$$

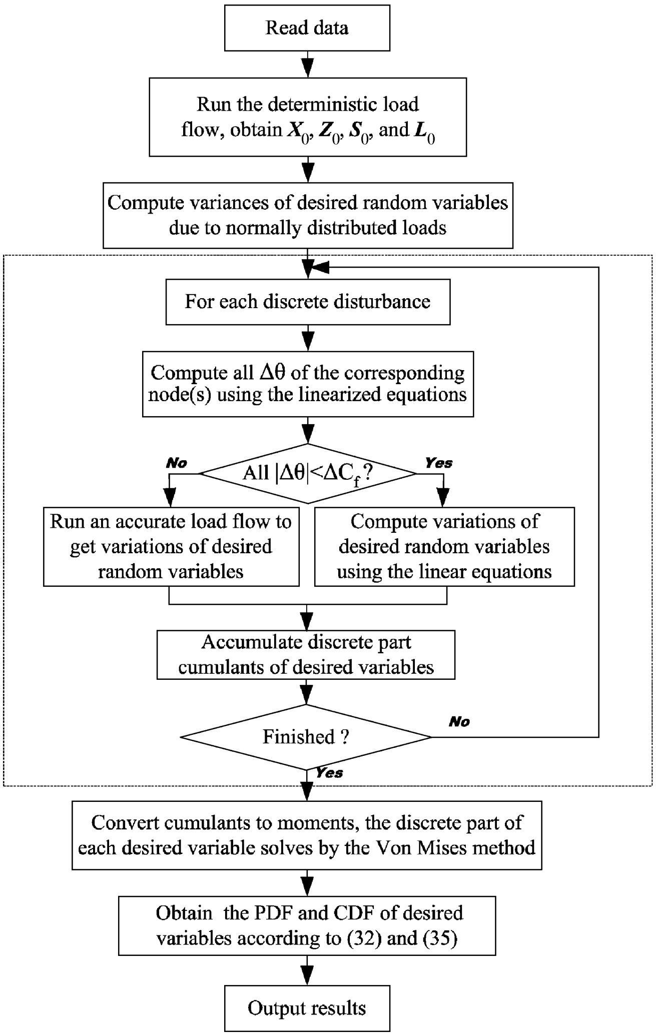

Fig. 2. Flowchart of the proposed PLF method.

Thus, the CDF of an output or state variable R is

$$F_R(x) = \sum_{i=1}^{\tilde{\gamma}} p_i \phi \left(\frac{x - (x_{d,i} + \mu)}{\sigma} \right). \tag{35}$$

The final distributions of desired random variables can be obtained according to (32) and (35). The flowchart of the proposed PLF method is shown in Fig. 2. Output data include expected values, variances, and distribution functions of nodal voltages and line flows. The part contained in the dashed line box highlights the method described in Section II-B for dealing with discrete disturbances. $\Delta\theta$ means voltage angle variation of each related node after a discrete disturbance.

V. Simulated Test Results

A. 24-Bus IEEE RTS

The system data can be found in [32]. The base power is 100 MVA. All loads are assumed to be normally distributed, their expected values are equal to the annually peak loads, and their standard deviations are all equal to 10% of their expected values. All branch outages are considered, except those that make the system separate. Random outages of four pair lines that have common structures are also considered. Rates of line outages are obtained according to

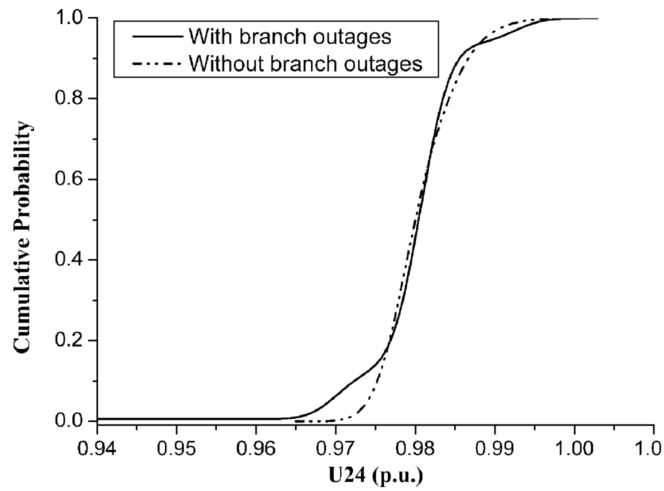

Fig. 3. Influence of random branch outages on the CDF of U24.

$$P_l = \alpha \cdot \lambda p \cdot \frac{Dur}{8760} \tag{36}$$

where λp is permanent outage rate (outages/year), and Dur is permanent outage duration (hours). α is a scaling factor. If $\alpha = 1$, the average value of P_l is 0.04%. To make the influence of branch outages more obvious, α is set equal to 10. ΔC_f is set equal to 0.1 rad. The first nine cumulants of each random variable are calculated.

The allowable voltage range is (0.95, 1.05) per unit. All voltage distributions are within this range without considering random branch outages. When branch outages are considered, voltages of buses 3 and 24 may violate the lower limit. Fig. 3 shows the cumulative distribution curves of the voltage at bus 24 (U represents nodal voltage magnitude). The main difference between these two curves lies in the lower voltage side, especially in the range (0.965, 0.975) per unit.x

The branches' continuous ratings given in [32] are considered as their thermal ratings. Except for line 7–8, no branch overloads without random branch outages. When random branch outages are taken into account, lines 1–5, 15–16, 15–24, and 16–19 could violate their thermal ratings. The PDFs of the active power of line 15–24 are shown in Fig. 4. One can see that the influence of branch outages on the distribution is notable. No voltage or line flow violates its limit under $N-1$ criteria. However, when load uncertainties, generating units, and line outages are all considered together, the weak buses and lines can be found. This demonstrates that PLF considering random branch outages can provide more comprehensive information about the system condition.

The results of the PLF and MCS are compared. Only crude MCS is used in this paper, and the sampling size is 50 000. Average root mean-square (ARMS) error used in [17] is calculated using the result of MCS as a reference. ARMS is defined as

$$\text{ARMS} = \frac{\sqrt{\sum_{i=1}^{K} (MC_i - PR_i)^2}}{K} \tag{37}$$

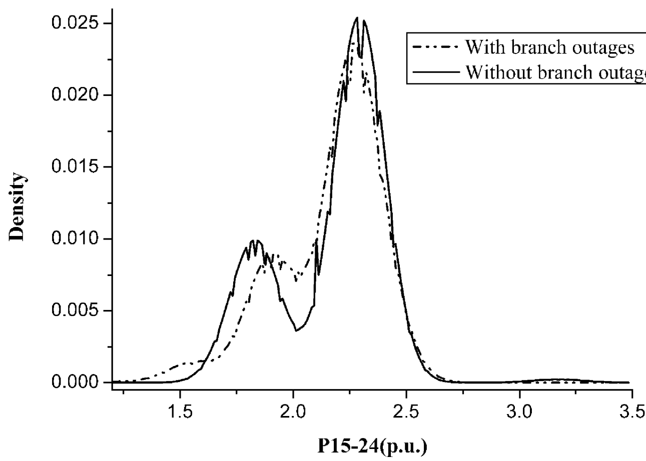

Fig. 4. Influence of random branch outages on the PDF of P15–24.

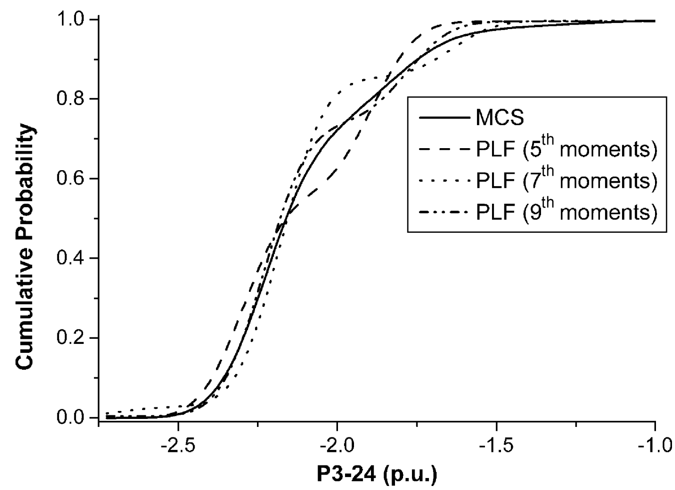

Fig. 5. Cumulative density curves of P3–24 obtained by using different order of moments.

TABLE I
ARMS OF NODAL VOLTAGES

Voltage	*dEV (%)*	*ARMS (%)*	Voltage	*dEV (%)*	*ARMS (%)*
U3	0.03	0.021	U10	0.03	0.031
U4	0.03	0.034	U11	0.02	0.026
U5	0.02	0.018	U12	0.03	0.014
U6	0.03	0.021	U19	0.01	0.036
U8	0.02	0.026	U20	0.01	0.029
U9	0.03	0.023	U24	0.01	0.027

TABLE II
LINE FLOW INTERVALS BETWEEN 5% AND 95% CONFIDENCE LEVEL

Branch	Active power (p.u.)		Reactive power (p.u.)	
	MCS	PLF	MCS	PLF
1-3	(-0.27,0.20)	(-0.30,0.21)	(0.16,0.33)	(0.14,0.34)
3-24	(-2.44,-1.67)	(-2.51,-1.62)	(-0.02,0.21)	(-0.05,0.26)
7-8	(0.35,1.71)	(0.35,1.72)	(0.18,0.51)	(0.13,0.51)
14-16	(-3.58,-2.34)	(-3.73,-2.29)	(-0.29,0.04)	(-0.36,0.12)
16-17	(-3.56,-1.56)	(-3.66,-1.63)	(-0.58,-0.23)	(-0.58,-0.22)

where K is the number of selected points, and MC_i, PR_i represent the ith value on the CDF obtained by MCS and PLF, respectively. The statistical points are evenly chosen from the range of CDF obtained by MCS. For nodal voltages, the intervals between each two neighboring chosen points are 0.0001 per unit. The errors of expected values (represented by dEV) and ARMS of some nodal voltages are shown in Table I. One can see that the differences between the results obtained by the two methods are very small.

When using the first five, seven, and nine moments, the discrete part of each desired random variable obtained by the Von Mises method consists of three, four, and five impulses, respectively. The more moments are used, the more accurate results will be obtained. Fig. 5 shows the CDF of the active power of branch 3–24. The results of the proposed PLF method are satisfactory when moments of the first nine moments are used in this test.

Take branch 3–24, 14–16, 16–17 with heavy loading, line 7–8 with moderate loading, and line 1–3 with light loading, for example. Table II lists their power flow intervals between 5% and 95% confidence level. The results obtained by the PLF are consistent with those by MCS. Among them, power flow of line 14–16 has the largest difference. This is due to this line's position in the system. Figs. 6 and 7 show the probability density curves obtained by the PLF and MCS. The distributions given by the two methods are close. There are visible differences in some local part of the distributions. The errors are mainly caused by linear approximations. Another important reason is that only five impulses are used to approximate the discrete part of each desired random variable. Thus, the results given by the proposed method cannot exhibit full detailed distribution information of the desired variables.

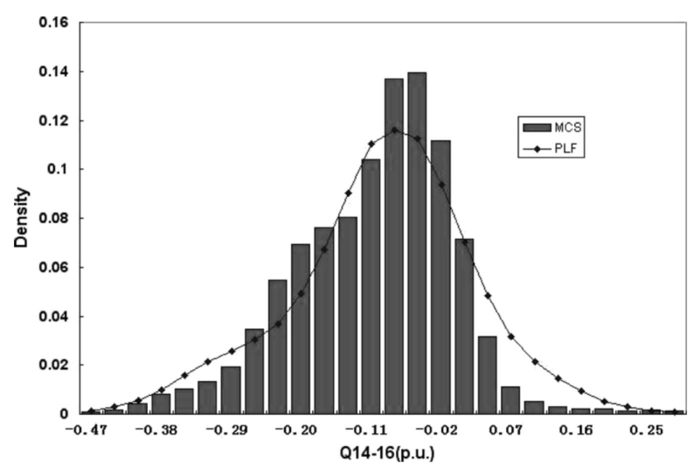

Fig. 6. Comparison between the PDF of Q14–16 obtained by the PLF and MCS.

The proposed PLF method and the MCS method are implemented by the C++ programming language. They are tested using a P4 1.8-GHz/512-MB RAM PC. The average computing times are listed in Table III. Let T_P stand for the total active power flow of all branches of the test system. The second termination criterion of MCS is the standard deviation of expectation estimate of T_P less than 3 MW. MCS converges after 9420 trials on average. The proposed PLF method is about 30 times faster than the MCS method using the second termination criterion. An accurate load flow is run for every discrete disturbance when setting $\Delta C_f = 0$. The slight computation time savings by using linear load flow equations to deal with some disturbances (by setting $\Delta C_f = 0.1$ rad) can be seen from Table III.

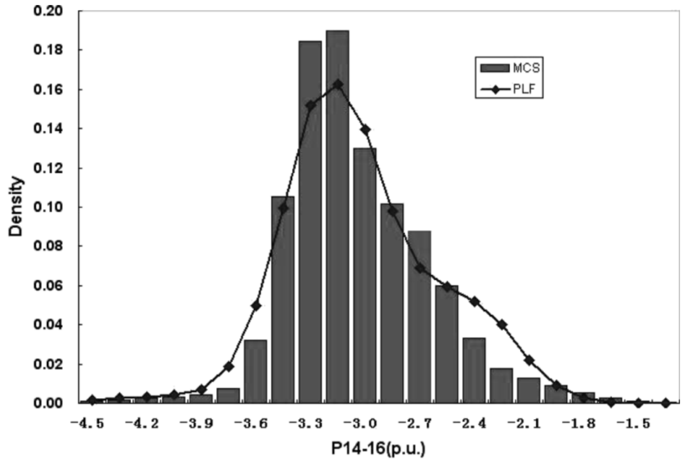

Fig. 7. Comparison between the PDF of P14–16 obtained by the PLF and MCS.

TABLE III
COMPUTATION TIME COMPARISON

Methods	Computation Time (s)
MCS (9420 trials)	45.32
MCS (50000 trials)	228.73
PLF(7th cumulants, ΔC_f=0.1)	1.45
PLF(9th cumulants, ΔC_f=0.1)	1.52
PLF(7th cumulants, ΔC_f=0)	1.48
PLF(9th cumulants, ΔC_f=0)	1.60

TABLE IV
PROBABILITIES OF VIOLATING VOLTAGE LOWER BOUNDS BEFORE AND AFTER COMPENSATION

Node	Probability (%)		Node	Probability (%)	
	Before	After		Before	After
7	86.81	1.52	157	28.88	0.40
139	19.38	0.84	158	98.23	5.32
140	1.20	0.01	161	27.76	0.72
141	60.51	2.50	162	98.02	4.65
156	83.62	1.49	163	98.20	5.02

B. 682-Bus System

This is a real power system in China with 682 buses, 973 branches, and 130 generating units. The system condition is the operation state of a peak load hour of year 2004. Suppose that all loads are normally distributed, and their standard deviations are equal to 3% of their expected values. Random outages of 134 lines are considered. Generating unit forced outages are not taken into account. Set $\Delta C_f = 0.05$ radian. The aim of this case study is to check the adequacy of the transmission system under the peak load condition.

The given nodal voltage range is (0.90, 1.02) per unit. Results of the PLF show that voltages of some buses in a province violate the lower bound with large probabilities. The voltages of several buses are already lower than 0.9 per unit without any uncertainty, so more reactive power supports are required. If 200 MVar of capacitors were installed at bus 163 and switched on, the voltages would be greatly improved. Table IV lists part of the results.

Line flow distributions obtained by the PLF are also analyzed. A critical line from bus 161 to 163 may violate its thermal limit with a possibility of 16.93%. Because there is no available generating capacity, shedding load at bus 150 is unavoidable when this line is overloading. The possibility of overloading of line 161–163 will decrease to 1.66% after shedding a 100-MW load.

TABLE V
COMPUTATION TIMES AND ARMS UNDER DIFFERENT THRESHOLDS

ΔC_f (rad)	Run full load flow times	Time/ Time saving (s)	ARMS of P161-163 (%)
0.00	134	51.1/0.0	0.0149
0.01	102	45.0/6.1	0.0150
0.02	84	40.4/10.7	0.0153
0.05	48	34.3/16.8	0.0244
0.10	19	28.9/22.2	0.0514
0.15	5	23.0/28.1	0.0517
0.30	0	22.4/27.7	0.0519

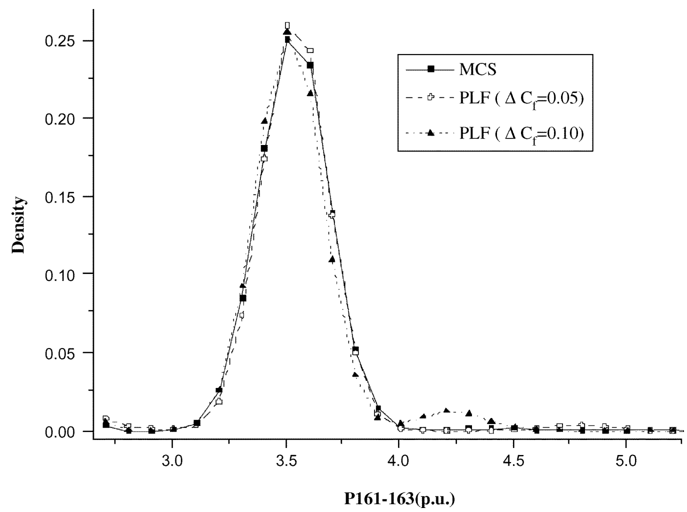

Fig. 8. Probability density curves of P161–163 obtained under different conditions.

The MCS consumes 604 s when using the second termination criterion as the last test case, while the proposed PLF method only needs 34 s. The computation time savings by using the linearized load flow equations to deal with some line outages are shown in Table V. The second column in Table V lists times of rerunning full load flow when dealing with random line outages. If the threshold ΔC_f were greater than 0.3 rad, all random line outages would be handled using (11) and the linear load flow (3) and (4). Although the computation time would be reduced remarkably, the error might be great. The ARMS of active power flow of line 161–163 (P161–163) increases about 3.4 times when this threshold is augmented from 0.01 to 0.15 rad (see the last column of Table V). Fig. 8 shows the distribution curves of P161–163 obtained under three different conditions. There will be discernible errors when the threshold is increased to 0.1 rad, especially in the tail part of the distribution. Although reasonable value of this threshold is system-dependent, 12% computation time would be saved, even if it was set equal to 0.01 rad.

VI. CONCLUSION

The general probabilistic load flow methods do not include random branch outages in their formulations. This paper proposes a unified approach to handle random branch outages along with discrete load uncertainties and unscheduled generating unit outages. PDFs of voltages and line flows are obtained. Moments and cumulants are used to simplify the convolution of random variables. Cumulants of desired random variables caused by uncertainties with continuous or discrete distributions are treated separately. The discrete part of each desired random variables is determined by the method proposed by Von Mises.

Test results of the 24-bus IEEE RTS indicate that random branch outages have considerable influence on the distribution of output and state variables, though their occurrence possibilities are small. The proposed method is also tested on a power system with 682 buses to check the adequacy indexes of nodal voltages and line flows under a heavy load condition. Possible low-voltage buses and overload transmission lines are found. The validity of corrective measures is also verified by the proposed method. The distributions of nodal voltages and line flows obtained by the proposed method are in reasonable agreement with those by MCS in the case studies. The calculation speed of the proposed method is much faster than that of MCS.

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