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Program Abstracts/Algorithms

A program for rotation to smooth functions

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Description. In an application of principal components analysis to a set of learning curves, Tucker (1966) advanced the following criterion for identifying meaningful components. The variables were consecutive trials, so the loadings of any meaningful component should form a smooth curve when plotted against trial number. Any particularly jagged curve would, according to this line of reasoning, probably be associated with random error. Similar considerations would lead us generally to expect that any meaningful component would be smooth in appearance when the variables consisted of successive measurements taken from some presumably continuous process. Other examples might be found in the principal components analysis or factor analysis of electrophysiological responses sampled over time, of economic time series, or of data from studies of growth or development.

Tucker used the smoothness criterion in evaluating the meaningfulness of components already chosen according to some other criterion (contribution to total sum of squares). The present program performs a transformation of an arbitrary set of components into a new set that are as smooth as possible. The measure of smoothness associated with a vector $y' = (y_1, y_2, \ldots, y_m)$ is taken to be:

$$\sum_{i=2}^{m}(y_{i-1} - y_i)^2 \Big/ \sum_{i=1}^{m} y_i^2$$

For a vector of length one, the measure of smoothness is simply the sum of the squares of the differences between adjacent elements. A procedure designed to rotate a matrix of component loadings so as to minimize this smoothness criterion has been developed by the authors (Arbuckle & Friendly, Note 1) and is implemented in the present program. Given an arbitrary matrix of component loadings, the program performs a transformation to a new ordered set of components so that (a) the loadings on the first component are as smooth as possible, and (b) loadings on the ith component ($i > 1$) are as smooth as possible subject to the constraint that they be orthogonal to the loadings on each of the first i-1 components.

Program structure. The program is designed for either stand-alone use or for incorporation into a larger program as a subprogram. The distribution version reads input data and produces output as described below. The rotation procedure is isolated in subroutines that can easily be called from another program, such as a component analysis or factor analysis program.

Input. The user furnishes either a matrix of component loadings or a matrix of latent vectors together with associated latent roots.

Output. (a) Smoothness measures for each of the untransformed components. (b) The transformation matrix. (c) Loadings on the transformed components. (d) Smoothness measures for each of the transformed components. (e) Sums of squares accounted for by each of the transformed components.

Computer and language. This FORTRAN IV program runs in 22K core on the CDC-6400 operating under SCOPE 3.4.1. Only standard FORTRAN features have been used, and no difficulties in conversion to other machines are anticipated.

Limitations. As the program is currently dimensioned, the number of components must not exceed 10, and the number of variables must not exceed 30. Directions for increasing the capacity of the program are included in the form of comment cards.

Availability. A source listing of the program may be obtained without charge from James Arbuckle, Department of Psychology, Temple University, Philadelphia, Pennsylvania 19122.

REFERENCE NOTE

1. Arbuckle, J., & Friendly, M. L. On rotating to smooth functions. Unpublished manuscript, 1975.

REFERENCE

Tucker, L., R. Learning theory and multivariate experiment: Illustration by determination of generalized learning curves. In R. B. Cattell (Ed.), *Handbook of multivariate experimental psychology*. Chicago: Rand McNally, 1966.

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