From the right-hand sides of (23), (29a), and (29b), the group velocity is evaluated as

$$\frac{S}{w} = \hat{z}\frac{k_z c^2}{\omega} \tag{30}$$

which, as is to be expected, is identical to that deduced in (28). It is interesting to note that w and S are independent of v_d and are, therefore, unaffected by the drift of the electrons. Hence, the lengthy expressions for the energy density in (13) and the power flux density in (22) are invariant under the Lorentz transformation in the same manner as the simple dispersion relation (27) for the special case of plane waves propagating in the direction of the beam.

The constitutive relations (4) and (5) describing an electron beam as a magnetoelectric medium and the expressions (13) and (22) for the energy and the power flux densities are useful in the solution of problems involving transversely bounded electron beams.

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A Property of Digital Quadrature Filters

ENRICO DEL RE

Abstract—A relationship is derived for digital linear-phase quadrature filters of even length. It is shown that their impulse responses are simply related through an alternate sign inversion operation.

I. INTRODUCTION

Quadrature filters, i.e., filters related through the Hilbert transformation, are used in many aspects of signal processing and communications, as, for example, in the generation of the analytic signal, in the complex envelope extraction of signals and in the SSB modulation [1], [2]. In particular the design and implementation of digital quadrature filters are much more feasible than their analog counterparts, as many of the difficulties related to the realization of analog Hilbert transformers have been overcome using digital techniques. Specifically techniques and programs exist for an efficient design and implementation of digital finite-impulse-response (FIR) low-pass, bandpass, and high-pass filters and of their associated quadrature filters [3]. A common feature of these design techniques is that they do not exploit any possible general relation between the two filter types, so that quadrature filters (e.g., a bandpass filter and its associated quadrature filter) are obtained through two separate filter designs. Of course, this approach does not guarantee *a priori* that their frequency responses are well matched. An exception is the relation between digital half-band low-pass filters and symmetric wide-band Hilbert transformers found by Jackson [4] for odd-length linear-phase FIR filters.

In this paper a general and simple relationship is derived for the impulse responses of FIR linear-phase even-length digital quadrature filters. This property can be conveniently used in the quadrature filter designs and allows considerable savings in the computational load and memory locations required by the quadrature filter implementation in certain applications.

II. DERIVATION OF THE RELATIONSHIP

A FIR digital linear-phase (low-pass, bandpass, or high-pass) filter $H(e^{j\omega})$, ω being the radian frequency normalized to the sampling frequency, with magnitude response $A(\omega)$,

$$H(e^{j\omega}) = A(\omega)\, e^{-j\omega(N-1)/2}, \quad |\omega| < \pi \tag{1}$$

has an associated quadrature linear-phase FIR filter [3] defined by

$$\hat{H}(e^{j\omega}) = \begin{cases} A(\omega)\, e^{-j\omega(N-1)/2} e^{-j\pi/2}, & 0 < \omega < \pi \\ \hat{H}^*(e^{-j\omega}), & -\pi < \omega < 0 \end{cases} \tag{2}$$

* indicating complex conjugation.

The operation of multiplying the coefficients $h(n)$ of the impulse response of the digital filter (1) by the sequence $(-1)^n$ transforms $H(e^{j\omega})$ into the filter

$$\bar{H}(e^{j\omega}) = \begin{cases} A(\omega - \pi) e^{-j\omega(N-1)/2} e^{-j\pi(N-1)/2}, & 0 < \omega < \pi \\ \bar{H}^*(e^{-j\omega}), & -\pi < \omega < 0. \end{cases} \tag{3}$$

For N even two cases are possible: $N = 2(2k+1)$ and $N = 2(2k)$, with k any integer.

From (3) it follows:

$$\bar{H}(e^{j\omega}) = \begin{cases} A(\omega - \pi) e^{-j\omega(N-1)/2} e^{\mp j\pi/2}, & 0 < \omega < \pi \\ \bar{H}^*(e^{-j\omega}), & -\pi < \omega < 0 \end{cases} \tag{4}$$

where the − sign in the second exponential is for $N = 2(2k+1)$ and the + sign is for $N = 2(2k)$.

Hence $\bar{H}(e^{j\omega})$, for N even not multiple of four, or $-\bar{H}(e^{j\omega})$, for N multiple of four, is the linear-phase quadrature filter associated to the filter with magnitude response $A(\omega - \pi)$, i.e., to the FIR linear-phase filter which is the "mirror" image of $H(e^{j\omega})$ with respect to $\pi/2$.

Therefore, from (4) we conclude that, by multiplying the impulse response $h(n)$ of a FIR linear-phase filter of even length N by $(-1)^n$, if $N = 2(2k+1)$, or by $(-1)^{n+1}$, if $N = 2(2k)$, we exactly obtain the linear-phase quadrature filter associated with the "mirror" filter. Obviously, this procedure can be inverted to derive a linear-phase filter starting from the impulse response of its associated "mirror" quadrature filter.

As an example these conclusions can be easily verified for the impulse responses $h(n)$ and $\hat{h}(n)$ of linear-phase quadrature filters of even length N obtained by straightforward inverse transformation of their ideal frequency responses truncated to N samples. They are given by

$$h(n) = \frac{1}{\pi\left(n - \frac{N-1}{2}\right)}\left[\sin\omega_2\left(n - \frac{N-1}{2}\right) - \sin\omega_1\left(n - \frac{N-1}{2}\right)\right]$$

$$\hat{h}(n) = \frac{1}{\pi\left(n - \frac{N-1}{2}\right)}\left[\cos\omega_1\left(n - \frac{N-1}{2}\right) - \cos\omega_2\left(n - \frac{N-1}{2}\right)\right],$$

$$0 \leqslant n \leqslant N-1 \tag{5}$$

ω_1 and ω_2 being the normalized lower and upper cutoff radian frequencies, respectively.

As a particular case the multiplication by $(-1)^n$, or by $(-1)^{n+1}$, of its impulse response transforms a filter with a frequency response symmetric with respect to $\pi/2$ into its associated quadrature filter and vice versa.

III. CONCLUSIONS

A general property of FIR linear-phase digital filters of even length has been derived, which simply relates the impulse responses of quadrature filters. By an alternate sign inversion of a given filter impulse response, we obtain the impulse response of the quadrature filter associated to the "mirror" filter, i.e., to the filter symmetric to the original one with respect to one-fourth of the sampling frequency. This property allows programs and tables for designing low-pass, bandpass, or high-pass filters to be used for the appropriate quadrature filter designs and vice versa. Moreover in applications where a bank of "mirror"

couples of quadrature filters are required, as for example in the envelope detection of multiple transmission frequencies [5] or in the implementation of transmultiplexer equipments using quadrature channel filter banks [6], this property can allow considerable savings in the system total number of arithmetic operations and storage locations for the coefficient memory.

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Evaluation Method of Device Noise Figure and Gain Through Noise Measurements

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Abstract—A new method of evaluating the noise figure and available gain of a linear two-port device only from noise measurements is proposed. In this method, there is no need for tuning at the output port of a device under test (DUT) by the use of an excess noise injection through a circulator to the device output port. Hence it gives a much simplified procedure and an improvement of accuracy.

A method of evaluating the noise and gain parameters of a linear two-port device solely from noise figure measurements was proposed [1]. However, the simple application of the Friis formula to a cascaded network, where the effect of mismatch between stages is not taken into consideration, leads to some error [2]. In this paper, a new method is proposed to overcome such a difficulty.

The noise figure and gain measuring system proposed here is schematically shown in Fig. 1. In this system, a circulator with another well-matched noise source is connected to the output port of a device under test (DUT). The noise figure of the noise measuring stages (NMS) subsequent to the DUT is dependent on the output admittance Y_{out} of the DUT, and denoted here by $F_2(Y_{out})$.

Let the excess noise temperature ratio of the above-mentioned noise source be t_{ex} when evaluated at the input port (port 1) of the circulator with a characteristic admittance of Y_0. Also, let F' be the value of $F_2(Y_{out})$ under the condition of $t_{ex} = 0$ and $Y_{out} = Y_0$. Then, $F_2(Y_{out})$ is given by (refer to the Appendix)

$$F_2(Y_{out}) = \frac{t_{ex}\rho^2 + F'}{1 - \rho^2} \tag{1}$$

where $\rho(\equiv |(Y_0 - Y_{out})/(Y_0 + Y_{out})|)$ is the magnitude of reflection coefficient looking into the output port of the DUT from the circulator.

Therefore, the overall system noise figure F_m can be expressed by

$$F_m(Y_s) = F(Y_s) - \frac{1}{G_a(Y_s)} + \frac{t_{ex}\rho^2 + F'}{G_a(Y_s)(1 - \rho^2)} \tag{2}$$

where $F(Y_s)$ and $G_a(Y_s)$ are the noise figure and available power gain of the DUT, respectively, as functions of the source admittance Y_s.
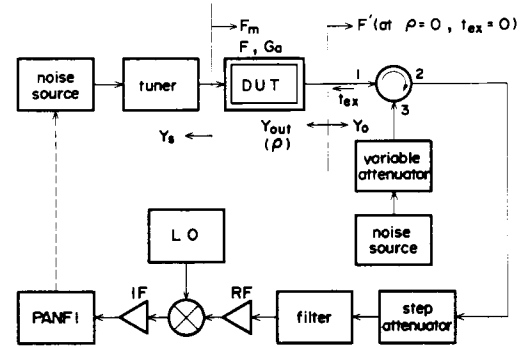
Fig. 1. The proposed noise figure and gain measuring system.

F, G_a, and ρ^2 can be obtained through measurements of F_m for three different sets of F' and t_{ex} values, as described below in detail.

Now, let the values of F_m be F_{m1}, F_{m2}, and F_{m3} under three conditions of 1) ($F' = F'_0$, $t_{ex} = 0$), 2) ($F' = F'$, $t_{ex} = 0$) and 3) ($F' = F'_0$, $t_{ex} = t_{ex}$), respectively. Then, $F(Y_s)$, $G_a(Y_s)$, and ρ^2 are derived from (2) as follows:

$$F(Y_s) = F_{m1}\frac{F'-1}{F'-F'_0} - F_{m2}\frac{F'_0 - 1}{F'-F'_0} - \frac{F_{m3} - F_{m1}}{t_{ex}} \tag{3}$$

$$\frac{1}{G_a(Y_s)} = \frac{F_{m2} - F_{m1}}{F' - F'_0} - \frac{F_{m3} - F_{m1}}{t_{ex}} \tag{4}$$

$$\rho^2 = \frac{F_{m3} - F_{m1}}{F_{m2} - F_{m1}} \cdot \frac{F' - F'_0}{t_{ex}}. \tag{5}$$

Here, it is interesting to note how the effect of mismatch between the DUT and the NMS appears in (3) to (5).

The value of F' can be changed by resetting the step attenuator in the NMS. Also, the value of t_{ex} be set by turning on and off the noise source and/or adjusting the precision variable attenuator. The calibration of the t_{ex} value can be easily performed by connecting a short to the circulator input port (port 1).

As stated above, the device noise figure and available gain can be easily evaluated through a simple procedure, that is, noise figure measurements only, without any tuning at the DUT output port regardless of Y_s values. To achieve better accuracy, it is desirable to select the value of t_{ex} comparable to G_a.

APPENDIX
DERIVATION OF (1)

We will first derive the noise figure $F_c(Y_{out})$ and the insertion loss $L_c(Y_{out})$ of an ideal circulator with a noise source (Fig. 1). The noise input power from the signal source is

$$kT_0B(1 - \rho^2)$$

and the total equivalent noise input power which involves the contribution from the noise source connected to the circulator is

$$kT_0B(1 - \rho^2) + kT_0B(1 + t_{ex})\rho^2.$$

From the definition of noise figure, we have

$$F_c(Y_{out}) = \frac{1 + t_{ex}\rho^2}{1 - \rho^2}. \tag{A1}$$

Also, $L_c(Y_{out})$ is simply given by

$$L_c(Y_{out}) = \frac{1}{1 - \rho^2}. \tag{A2}$$

On the other hand, F' is equal to the noise figure of the stages subsequent to the circulator, since F' is the noise figure of the NMS under