A PROPOSED "MIRROR TRANSIT-CIRCLE"

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Summary

A fundamental transit-circle is proposed, in which the usual moving telescope is replaced by a moving plane mirror, with its normal sweeping out the meridian. Observation is through either one of two horizontal telescopes, lying in the meridian in the positions at present occupied by the two collimators; these telescopes also act as collimators when required.

A number of advantages of this method are listed, and also a few disadvantages; the latter, however, appear relatively unimportant.

It is shown that if the rear face of the mirror is roof-shaped, half of it being inclined laterally towards the east by about 15°, and half towards the west, extra-meridian observations of declination can be used to obtain fundamental azimuths in a period of about four hours instead of twelve.

Equations are developed for the position of the point observed, and the tilt of the star's path at apparent transit, for the case when this mirror-tilt cannot be treated as a small quantity; it appears that the transit does in fact occur nearly horizontally over a considerable range of declinations fairly near the zenith, and that the accuracy of azimuth determinations should be satisfactory.

The general operation of the instrument is discussed, including the determination of the collimation; the collimation error cannot be represented in the form $c \sec \delta$, and the necessary expression is derived from the equations already obtained, by making the approximations appropriate for very small tilts. The instrument has a considerable number of self-checks which are not possible with the ordinary type, and allows a particularly fundamental method of obtaining both its pivot errors and the personal equation of the observer.

The mounting of the mirror is discussed, and it is suggested that the final form of the instrument would be with both telescopes broken towards the east, so that the two observing positions are close together, with a permanent pivot-telescope between them; that the focal lengths might be about 30 feet with the magnification not much changed from the present figure; that the telescopes could be very elaborately jacketed; and that if desired some quite different and much more bulky arrangement than a screw could be used for the micrometer drives.

In the course of some recent work at Aberdeen, Maryland, U.S.A., it occurred to me that many of the serious difficulties which still beset the transit-circle ought to be diminished, or even eliminated altogether, if one substituted a moving plane mirror for the moving telescope. The mirror would be mounted with its plane parallel to the existing rotation axis, so that its normal would sweep out the meridian, and one would observe stars reflected in it, through one or the other of two fixed telescopes, situated in the meridian; these might very well in fact be the existing collimators, if they were fitted with suitable driven-micrometer eyepieces.

The advantages to be expected from such an arrangement are really very considerable. We may note, in particular, the following:—

1. There will be practically complete freedom from variable flexure (droop) due to varying weight stresses. The telescopes will not now move at all, and the

mirror, if in fact it were to bend perceptibly, could cause no first-order effects except an astigmatism; in practice, the flexure of any reasonably thick mirror would be quite inappreciable. It would, of course, have to be very well mounted, and this question is considered below.

2. There will be practically complete freedom from telescope-flexure due to temperature differences. Although such distortions can be determined by repeated collimation and nadir observations, it is very time-consuming to have to do this at all frequently; but the effects can certainly be too large to ignore by modern standards, and may also vary rapidly. Fixed telescopes can be shielded to any desired extent, including the use of circulating oil-baths etc. if necessary.

3. The weight relations in the micrometer will be constant. There is evidence that the shifting relations inseparable from a moving telescope can cause variable collimation and/or nadir depending on the altitude. With a fixed telescope, the micrometer can be designed with special reference to the one direction in which gravity will act.

4. The observing position will be constant, including that for observations of levels and nadir. This should make the personal equation more constant; moreover, since the position can be made a permanently convenient one, it should diminish fatigue.

5. It would be easy to arrange the two driven micrometers so that one acted as a personal-equation machine for the other. Observers would thus determine their own personal equation with the actual instrument, and in the actual observing position, which they also used for the observations.

6. Very much shorter piers can be used; they need only be tall enough to accommodate the circles. This will greatly decrease the changes of level caused (e.g. when the wind changes) by changing temperature differences between the piers; it will decrease any azimuth changes that may be caused by one pier inclining more north than the other (for example if its south face is warmed); and if it is desired to observe stars after reflection in a mercury pool, the size (or the range of displacement) of the pool can be kept small.

7. A very much longer focal length can be used, with or without any corresponding increase in the magnification. Since the aperture would probably not be increased very much, the lens corrections would be more satisfactory, especially those for spherical aberration, coma, and curvature. In addition, backlash, screw errors, etc., would become less important. The magnification should probably not, in fact, be much increased, so that the eyepiece focal length would also be made considerably greater; this would give more room for reversing prisms, etc., and less trouble with the parallax, or focus difference, between the two moving wires; moreover, the wires themselves would appear thinner.

8. Any desired modifications to the micrometer system can be planned without having to keep a strict eye on considerations of weight and space. For example, the position of the wire could control a rotating mirror, and a fixed scale could be photographed every second by reflection in the mirror; such an arrangement would practically eliminate backlash.

9. The aperture can also be increased without difficulty, if this is in fact desired. A very small increase would take care of the loss of light due to the single reflection in the mirror.

10. The connections for the micrometer drive, field illumination, chronograph, etc., will be simplified; neither slip-rings nor flexible cables will be needed any more, and all leads can be permanently run with fixed cable.

11. Reversing will be simplified, since no leads at all have to be disconnected; moreover, the body to be reversed will be much smaller.

12. Nadir and levels can be taken in either collimator. The differences between the two determinations must check with the division errors, in the one case, and with the pivot errors in the other.

13. A considerable range of near-zenith stars can also be observed in either collimator. The differences must again check with the division or pivot errors.

14. A particularly fundamental method of obtaining the pivot errors themselves can be provided by squaring, figuring and silvering a small patch on one or both edge faces of the mirror block, and taking autocollimation on this through the corresponding pivot. If flexure of the axis or movement of the mirror contributes in any way to the effective pivot errors, its contribution will automatically be exactly included in a determination made in this way.

15. As will be shown below, it is in principle possible to give the mirror a tilted rear face, making extra-meridian observations possible, and to use declinations obtained in this way so as to determine absolute azimuths fundamentally in a period of about three or four hours instead of twelve. This would be a major advantage if the practical difficulties can be overcome. Since the necessary observations do not involve any sub-pole measurements, it would thus become possible to do fundamental astronomy in the tropics without relying on low-altitude measurements, and indeed the method is in principle most sensitive (or alternatively, most rapid) in low latitudes. It has, however, some drawbacks.

In view of these apparent advantages, I was surprised to learn, through discussion with various astronomers at Aberdeen and elsewhere, that the idea of a mirror transit-circle in any form seemed unfamiliar. On my return here, however, Mr Martin pointed out that it is not in fact new; essentially the same proposal was made by Turner in 1894 *, but it has apparently lain disregarded since then. It is true that Turner did not propose to use the collimators as observing telescopes; he proposed two or more special telescopes, to be arranged so that the observer might be in a comfortable warmed room. This does not appear advisable, judged by modern standards of accuracy, since it would introduce an awkward temperature gradient into the optical path; moreover, the collimators are indispensable in any case, and it is clearly desirable to have as few independent components as possible. Turner also did not discuss the actual process of collimation, and did not give any of the necessary formulae; nevertheless, the idea was in its essence proposed by him, and it does appear surprising that in spite of his prestige, and the many apparent advantages (of which he listed a considerable number himself), no attempt has yet been made to put the proposal into practice.

A mirror transit-circle has of course some disadvantages of its own, and we will now consider these briefly. On the whole, they appear insignificant by comparison with the advantages. The following seem to be the principal ones:

1. An error in the circle-readings, or in the determination of the division-errors of the circle, will have twice as much effect on the declinations as with a

* H. H. Turner, *M.N.*, **54**, 412, 1894.

telescope. However, these errors are not at present the limiting factor even for the internal consistency, and it appears that the expected improvement in systematic accuracy should hardly be impaired by doubling them.

2. An error in the determination of the pivot errors will have an increased (but never quite doubled) effect on the right ascensions; the determination of pivot errors has not always been sufficiently thorough in the past, but it can in fact be made with more accuracy than that with which the star can be observed, so that here also doubling the uncertainty could perhaps be tolerated.

3. There will be a small reflection-loss of light; but, as already mentioned, this can be taken up by a very slight increase in aperture, since weight is no longer a consideration.

4. There will be a lower limit to the altitudes observable; below this, the cell of the telescope lens will interfere with the incoming beam. The lowest altitude, A, free from interference is given by $\tan A = d/D$, where d is the diameter of the objective (more accurately, the mean of the clear lens-diameter and the outside cell-diameter), and D is the distance from the bottom edge of the lens to the (nearest point of the) mirror. It is clear that even if d were as much as 25 cm., altitudes as low as 6° could be observed with the two collimator lenses separated by only 5 m.; this is a smaller separation than is often found, and it may be questioned whether astronomical measurements at lower altitudes than 6° have any general reliability.

5. It is not immediately clear how azimuth-marks can be viewed, since in general it would be undesirable to place them at 6° elevation, and the observer's body would now be in the way at 6° depression. This question is considered further below.

6. The operation of determining collimation must still include sighting the one collimator on the other. It would be undesirable to have the mirror so far away from the rotation axis that when it was laid horizontally there was a clear view through, above or below it, and there thus seems no alternative to raising it a few inches every time collimation is taken. However, it would be easy to design the weight-relieving apparatus so that this short lift was carried out by it rather than by the reversing apparatus; and it does not appear likely that any difficulty or inconstancy would then result from such repeated raisings. If it does, it would presumably do so with a telescope-transit also, but would be less readily noticed.

It may well be that there remain other disadvantages which have been overlooked; but as far as the above survey goes, there can be little doubt that the proposed instrument would be considerably more satisfactory, in respect of systematic errors and also in respect of several accidental ones, than the existing type, which has in fact suffered practically no fundamental improvement, except the introduction of the impersonal micrometer, for over a century.

We will now consider the operation of the instrument. The process of observing a star will be exactly as with a telescope-transit, except only that the observer's own line of sight is always horizontal. The choice between observing in the north collimator or the south one, for all stars within (say) 20° of the zenith, is new, and one would in fact arrange a suitable amount of alternation, both for the programme of any one night and for the total schedule for any one star. In setting on a star, the circle must be set with only half the previously allowable tolerance to bring it into the field of view; and in moving from one star to another, without change of collimator, the circle moves only half as far as before. The level error and nadir may of course be obtained by reflection in a mercury pool as before, by turning the mirror so that its normal is 45° away from vertically downwards. In the absence of collimation error, if we trace a central ray backwards from telescope to mirror to pool, it will hit the pool vertically if there is no level error; and if there is a level error it will hit the pool at an angle differing from vertical by once (not twice) this error; thus the numerical value of the level is the same, in terms of micrometer readings, as it would be for a telescope-transit of equal focal length. Difference in level on reversal of the instrument will give the difference in pivot-diameter, as at present. An azimuth-error of the instrument as a whole will also appear from the star observations in exactly the same way as before, i.e. without any factor of two or other correction. Collimation, however, is radically different from collimation with a telescope-transit, and we must study this in detail.

We have here in fact an entire additional parameter, beyond those which occur with a telescope-transit. There the collimation was simply the angle, at the lens, between the line through the micrometer-wire in what may be called the "position of the standard contact" and the line perpendicular to the rotation axis; with the mirror-transit we shall have to consider in addition to this angle the angle between the plane of the mirror and the rotation axis. Both angles are of course readily obtained, the procedure being as follows.

Using a Bohnenberger autocollimating eyepiece, one first obtains for each collimator the setting that gives autocollimation in R.A. in the mirror. Let these readings be N_0 and S_0 for the north and south collimators respectively. Then if we measure azimuths from the direction perpendicular to the rotation axis (clockwise from "south"), and if the angle between mirror and axis is β_1 (positive if the normal to the mirror is tipped west of the "meridian" plane), the north collimator is pointing in an azimuth $-\beta_1$, and the south one in an azimuth $\pi+\beta_1$, when the readings are N_0 and S_0. Now raise the mirror, and sight the north collimator on the south one when the south one reads S_0. Let the north setting for this case be N_1; for positive β_1, N_1 will be greater than N_0 if the N micrometer readings increase as the wire moves east. At the setting N_1, the north collimator is evidently pointing in an azimuth $\pi+(\pi+\beta_1)=\beta_1$, and the setting which gives an "azimuth" of zero, i.e. perpendicular to the rotation axis, is $(N_1+N_0)/2$. If the values of N are in mm., and if the focal length is F_N mm., the value of β_1 in seconds of arc is

$$\beta_1 = 206265\,\frac{N_1-N_0}{2F_N}. \qquad (1)$$

Similarly, the setting of the south collimator which gives an "azimuth" of π, i.e. perpendicular to the rotation axis, is $(S_1+S_0)/2$ if S_1 is the setting of the S collimator when it is sighted on the N one reading N_0; and

$$-\beta_1 = 206265\,\frac{S_1-S_0}{2F_S} \qquad (2)$$

if the S micrometer readings increase as the wire moves west. The two determinations of β_1 should of course agree, and a further obvious check is that when the north collimator reads $(N_1+N_0)/2$ and the south one $(S_1+S_0)/2$, they must appear set on each other, whether or not $F_N=F_S$.

In the analysis which follows, we will also consider that the collimators are horizontal; this can obviously be arranged by comparing nadir readings with autocollimation readings, adjusting the zero of the Z.D. micrometer until the circle readings differ by 45° in the two positions. When both collimators have been adjusted in this way, if the mirror is again raised, the horizontal wires should appear set on each other, as well as the vertical ones.

We may note in passing that these operations, which will in fact form part of the standard routine, give us three independent checks, to which we may add a fourth from the comparison of the level as obtained in both collimators. Of these four, two have their counterparts in similar checks with a telescope-transit (one is used for flexure determinations); but the two independent nadir determinations, and two independent level-determinations, are peculiar to the mirror-transit.

We will now consider the geometrical relations which hold, and the corrections which must be applied, when β_1 is not zero; in view of the possible application to a mirror with a deliberately tilted rear face, we will develop the analysis rigorously, i.e. without at first making any of the approximations that are admissible when β_1 is very small. For the front of the mirror, it should in fact be possible to reduce the tilt to a very few seconds, and an analysis restricted to the first power of β_1 will be amply accurate; but the tilts required for the back of the mirror may be 10 or 15 degrees.

We will continue, for the present, to speak of "azimuths" as though the instrument as a whole were correctly oriented, i.e. we shall measure them from the direction perpendicular to the rotation axis. "North", "west", "meridian", etc., are to be understood in this sense.

Consider now spherical polar coordinates with the poles on the observer's horizon, at the east and west points. Let "latitudes", β, on this system be measured positive to the west, from the meridian, and "longitudes", λ, positive up from the south horizon, so long as we are concerned with the south collimator. Then if the mirror is rotated through a "longitude" angle λ_1, up from the south autocollimation position, the coordinates of the normal are (λ_1, β_1). Let the coordinates of the star which is central in the field of view in this position be (λ_2, β_2). Then if θ is the angle from the south point to (λ_1, β_1), it is also the angle from (λ_1, β_1) to (λ_2, β_2); moreover, these three points lie on a great circle. In Fig. 1, O is the star; N, W, S, the north, west, and south points of the horizon; and M the normal to the mirror. Then from the triangle MWS we have

$$\cos\theta = \cos\lambda_1 \cos\beta_1 \tag{3}$$

$$\sin\theta\cos\psi = \sin\lambda_1\cos\beta_1 \tag{4}$$

$$\sin\theta\sin\psi = \sin\beta_1 \tag{5}$$

where the angles are as marked in the figure.

From the triangle OWS we have

$$\sin\beta_2 = \sin 2\theta \sin\psi \tag{6}$$

$$\sin\lambda_2\cos\beta_2 = \sin 2\theta\cos\psi \tag{7}$$

$$\cos\lambda_2\cos\beta_2 = \cos 2\theta$$

$$= 2\cos^2\lambda_1\cos^2\beta_1 - 1 \tag{8}$$

by applying (3).

From (3), (5) and (6),

$$\sin\beta_2 = \cos\lambda_1 \sin 2\beta_1 \tag{9}$$

and from (3), (4) and (7),

$$\sin \lambda_2 \cos \beta_2 = \sin 2\lambda_1 \cos^2 \beta_1. \tag{10}$$

Any two of the equations (8), (9) and (10) give the coordinates of the star in terms of the mirror angles λ_1 and β_1.

The track given by these equations, with β_1 constant and λ_1 variable, is not a small circle. It evidently passes through $\lambda_2 = 0$, $\beta_2 = 2\beta_1$, and through $\lambda_2 = \pi$,

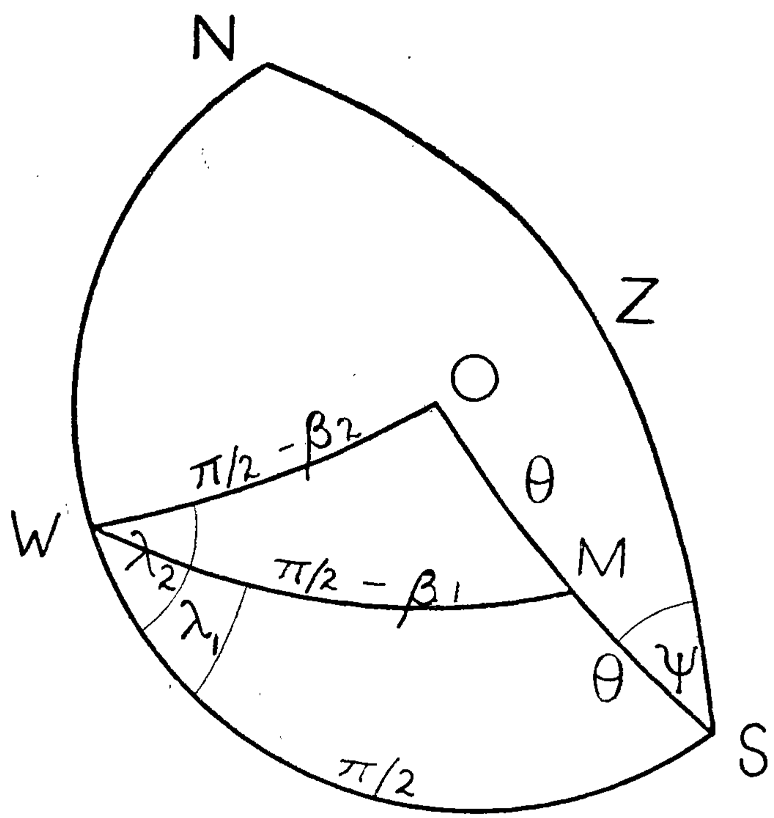

FIG. 1.

$\beta_2 = 0$; any small circle through these two points must culminate at an azimuth $\pi/2 + \beta_1$, but the actual track culminates at an azimuth A given by

$$\cos A \sqrt{(1 + \cos^2 \beta_1)} = -\sin \beta_1$$

instead of

$$\cos A = -\sin \beta_1.$$

In fact the track still departs from a small circle by quantities of the first order in β_1 when β_1 is small. Thus it cannot even in that case be adequately represented by any combination of *constant* fictitious level, azimuth, and collimation errors.

The hour angle H and declination δ of a star at "transit" may readily be obtained for any β_1, for the case in which the azimuth error of the instrument as a whole is zero. In Fig. 2, P is the celestial pole, and the other points have the same significance as before. From the triangle POS we have, if ϕ is the latitude,

$$\sin \delta = -\cos 2\theta \cos \phi + \sin 2\theta \sin \phi \cos \psi \tag{11}$$

$$\sin H \cos \delta = \sin 2\theta \sin \psi \tag{12}$$

$$\cos H \cos \delta = \cos 2\theta \sin \phi + \sin 2\theta \cos \phi \cos \psi \tag{13}$$

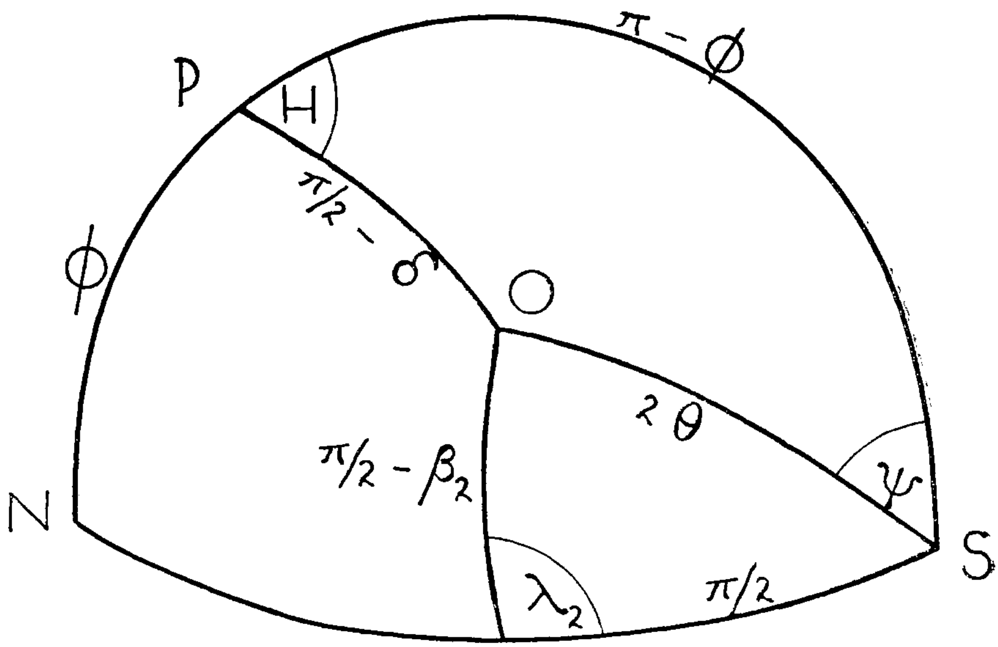

Fig. 2.

and thus from (11), (8), (7) and (10),

$$\sin\delta = \sin^2\beta_1\cos\phi - \cos^2\beta_1\cos(2\lambda_1+\phi); \qquad (14)$$

from (12), (6) and (9),

$$\sin H\cos\delta = \cos\lambda_1\sin 2\beta_1; \qquad (15)$$

and from (13), (8), (7) and (10),

$$\cos H\cos\delta = \cos^2\beta_1\sin(2\lambda_1+\phi) - \sin^2\beta_1\sin\phi. \qquad (16)$$

Any two of the equations (14), (15), and (16) give the hour angle and declination in terms of the mirror angles and the latitude.

The largest value of δ that can be observed, δ_1 say, is evidently given by the value of λ_1 which satisfies the equation

$$\frac{d\delta}{d\lambda_1} = 0 = 2\sec.\delta_1\cos^2\beta_1\sin(2\lambda_1+\phi)$$

from which $2\lambda_1+\phi=\pi$, and so from (14)

$$\sin\delta_1 = \sin^2\beta_1\cos\phi + \cos^2\beta_1 \qquad (17)$$

which is naturally less than unity unless either $\beta_1=0$ or $\phi=0$. Since at the maximum $\cos\lambda_1 = \sin\frac{1}{2}\theta$, the corresponding hour angle H_1 is given from (15) and (16) by

$$\cot H_1 = -\tan\beta_1\cos\tfrac{1}{2}\theta. \qquad (18)$$

When β_1 differs considerably from zero, the stars will in general transit obliquely. If the true azimuth and level of the instrument are both reasonably close to zero, the apparent direction of transit may be obtained as follows. The angle ψ is the angle between the vertical plane through the collimator axis and the plane through the mirror-normal and the collimator axis; if the normal, i.e. the plane SMO, were visible through the telescope, it would be seen tilted at this angle to the vertical. Any star moving along the arc SMO on the sphere produces a reflected ray still moving along this arc at the S point; i.e. this direction of motion in the sky gives an apparent direction of motion at the angle ψ in the telescope. Relative directions around O in the sky are reproduced without distortion in the telescope field, and in particular, as appears from Fig. 3, the image of the hour circle through O appears as a line at the angle η from the ψ direction, where η is equal to POS. Thus the image of the small circle of constant declination through O makes an angle $\eta+\psi+\pi/2$ with the vertical, or $\eta+\psi$ with the horizontal. Now from the triangle POS we have

$$-\cos\phi = \sin\delta\cos 2\theta + \cos\delta\sin 2\theta\cos\eta \tag{19}$$

and

$$\sin\eta = \frac{\sin\phi\sin H}{\sin 2\theta}. \tag{20}$$

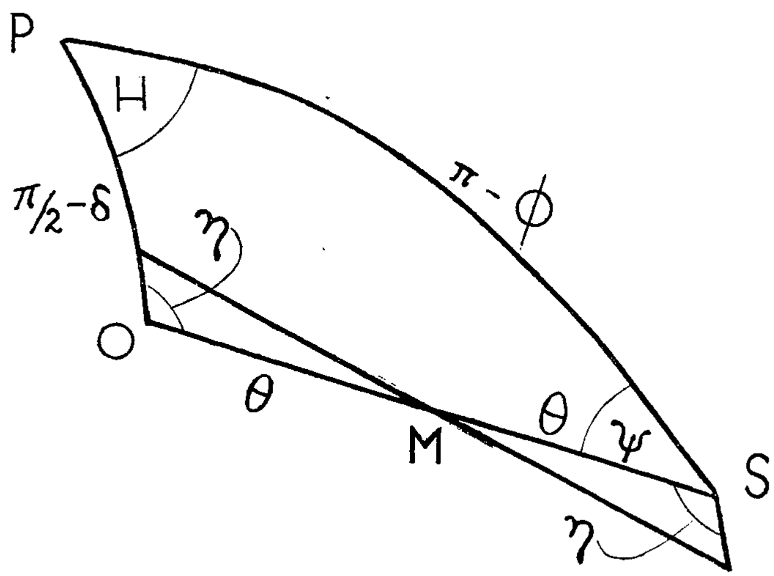

FIG. 3.

From (4) and (5) we have

$$\cot\psi = \sin\lambda_1\cot\beta_1 \tag{21}$$

and from (19) and (20)

$$\cot\eta = -\frac{\cos\phi + \sin\delta\cos 2\theta}{\sin\phi\sin H\cos\delta} \tag{22}$$

from which, using (14), (8) and (15), we obtain after some tedious reductions

$$\cot(\eta+\psi) = \sin\lambda_1\cot\beta_1 + \sec(\lambda_1+\phi)\frac{\sin\phi}{\sin 2\beta_1}. \tag{23}$$

Thus $\eta+\psi$, the angle from the horizontal at which a star appears to transit, can be tabulated in terms of λ_1, for any given values of ϕ and β_1.

An alternative formula for the tilt may be obtained by using one of the standard expressions for spherical excess. Since, in the triangle POS,

$$\eta+\psi+H = \pi+E \tag{24}$$

if E is the spherical excess, and since we can just as well consider either $\pi-(\eta+\psi)$ or $\eta+\psi$ to be the tilt, we may say that the tilt is $H-E$. We have then

$$\frac{\tan\frac{1}{2}(H-E)}{\tan\frac{1}{2}H} = \frac{\cos\frac{1}{2}\left(\pi-\phi+\frac{\pi}{2}-\delta\right)}{\cos\frac{1}{2}\left(\pi-\phi-\frac{\pi}{2}+\delta\right)} \tag{25}$$

or

$$\tan \tfrac{1}{2}(H-E) = \frac{\sin \tfrac{1}{2}\left(\phi+\delta-\dfrac{\pi}{2}\right)}{\sin \tfrac{1}{2}\left(\phi-\delta+\dfrac{\pi}{2}\right)} \tan \tfrac{1}{2} H. \qquad (26)$$

The equivalence of the expressions (23) and (26) appears a little cumbrous to demonstrate trigonometrically; it can of course be verified arithmetically for particular cases without difficulty, and this provides, in fact, a useful check on the computations of H and δ.

It is evident from (23) that $\eta+\psi$ will be zero (or π), i.e. the star will after all transit horizontally, if $\lambda_1=\pi/2-\phi$; in this case, the normal to the mirror points towards the equator, whatever the tilt β_1 may be. It is then easily verified also that $\delta=\pi/2-\phi=\lambda_1$, and $H=2\beta_1$, whatever the value of β_1; provided, of course, that δ does not exceed the limit given by (17), in which case no horizontal transit occurs. For moderate latitudes (say between 35° and 55°) the stars in question culminate not far from the zenith; these are, fortunately, quite suitable stars for azimuth determinations by this method.

The following table, calculated for latitude 50° 52′ 20″, shows for various β_1 the tilt T, hour angle H, and declination δ, corresponding to various circle-settings λ_1. For small λ_1, the rate of change of tilt with λ_1 is nearly constant and nearly proportional to β_1; by the time the tilt has reached zero, its rate of change has begun to rise a little, but linear interpolation will still give a good first estimate. H is comparable with $2\beta_1$, from $\lambda_1=10°$ up to a little beyond $\eta+\psi=0$, and δ changes by roughly twice the change in λ_1, in this same range.

If now we observe a star at about the declination at which the transits are horizontal, it will be possible to follow it with the Z.D. micrometer, and to record this micrometer's readings at the instants when the star passes various fixed vertical wires; the collimations of these wires being known, the Z.D. micrometer reading can be obtained for the instant when the star passed the fictitious wire of zero collimation. Since the movement of the micrometer will be very slow, the various instants do not have to be obtained with high accuracy. The declination corresponding to the circle-reading is obtained from (14); and the Z.D. micrometer reading, reckoned from the zero already established, and reduced to seconds of arc, is directly added to this declination. (Strictly, the Z.D. micrometer reading should first be multiplied by the cosine of the tilt.) Thus the declination can be obtained in the tilted rear face of the mirror, just as it can in the front, and with equal accuracy.

Now let us suppose that the entire instrument has a small azimuth error a; this will be much too small to affect the tilt itself visibly, but it will affect the apparent declination. We may study the question as though the mirror had been exactly set on the star, i.e. λ_1 was so skilfully chosen that the Z.D. micrometer reading at "transit" was exactly zero. Then there are two points in the sky that we are concerned with; one is the point defined by λ_1 on the assumption that $a=0$, and the other is the actual point observed, with the same λ_1 but with the azimuth error a. It is clear that these two points have the same zenith distance z, and that their azimuths differ by a. The length of the arc joining them is thus $a \sin z$, and resolving this in the directions of increasing declination and increasing hour angle we readily obtain from Fig. 4

TABLE I

β_1		λ_1 0°	10°	20°	30°	39° 07′ 40″	40°	50°	60°	70°
5°	T	− 8° 02′	− 5° 07′	− 3° 16′	− 1° 41′	0	+ 0° 12′	+ 3° 50′	+20° 45′	+153° 50′
	H	$0^{h}51·2^{m}$	$0^{h}41·6^{m}$	$0^{h}37·6^{m}$	$0^{h}37·1^{m}$	$0^{h}40·0^{m}$	$0^{h}40·5^{m}$	$0^{h}52·7^{m}$	$1^{h}59·2^{m}$	$10^{h}51·6^{m}$
	δ	−38° 25′	−18° 41′	+ 1° 08′	+21° 00′	+39° 08′	+40° 52′	+60° 39′	+79° 57′	+ 78° 21′
10°	T	−15° 33′	−10° 03′	− 6° 26′	− 3° 20′	0	+ 0° 24′	+ 7° 25′	+34° 31′	+132° 16′
	H	$1^{h}40·5^{m}$	$1^{h}22·7^{m}$	$1^{h}15·0^{m}$	$1^{h}14·2^{m}$	$1^{h}20·0^{m}$	$1^{h}21·0^{m}$	$1^{h}44·4^{m}$	$3^{h}30·7^{m}$	$10^{h}01·8^{m}$
	δ	−36° 22′	−17° 23′	+ 1° 56′	+21° 23′	+39° 08′	+40° 49′	+60° 01′	+77° 35′	+ 76° 17′
15°	T	−22° 08′	−14° 37′	− 9° 27′	− 4° 54′	0	+ 0° 35′	+10° 33′	+41° 42′	+115° 48′
	H	$2^{h}26·6^{m}$	$2^{h}02·8^{m}$	$1^{h}52·3^{m}$	$1^{h}51·4^{m}$	$2^{h}00·0^{m}$	$2^{h}01·5^{m}$	$2^{h}34·5^{m}$	$4^{h}35·9^{m}$	$9^{h}32·5^{m}$
	δ	−33° 08′	−15° 17′	+ 3° 14′	+22° 00′	+39° 08′	+40° 45′	+59° 01′	+74° 28′	+ 73° 27′
20°	T	−27° 36′	−18° 41′	−12° 12′	− 6° 21′	0	+ 0° 44′	+13° 07′	+44° 46′	+102° 57′
	H	$3^{h}09·0^{m}$	$2^{h}41·6^{m}$	$2^{h}29·3^{m}$	$2^{h}28·7^{m}$	$2^{h}40·0^{m}$	$2^{h}41·9^{m}$	$3^{h}22·5^{m}$	$5^{h}25·1^{m}$	$9^{h}17·9^{m}$
	δ	−28° 55′	−12° 27′	+ 5° 00′	+22° 51′	+39° 08′	+40° 40′	+57° 41′	+71° 01′	+ 70° 13′

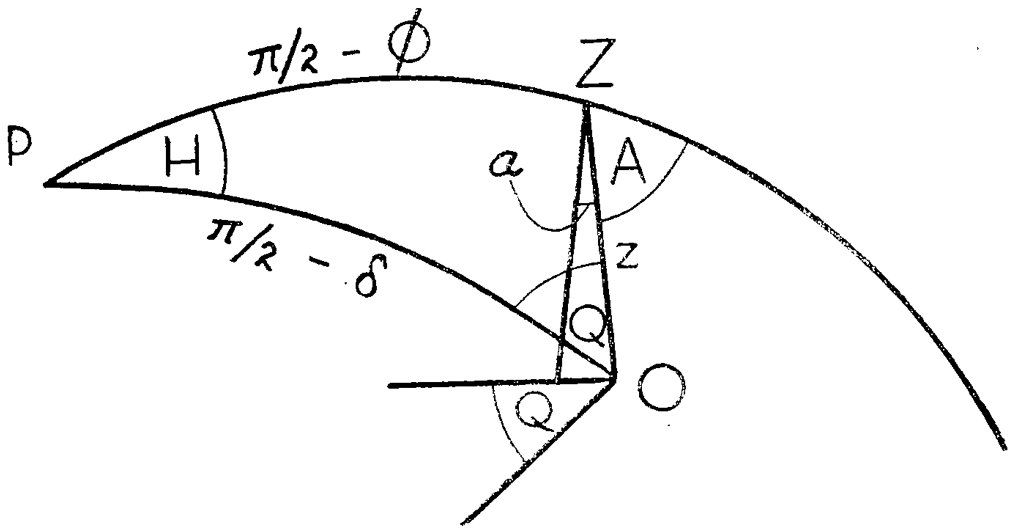

FIG. 4.

$$d\delta = a \sin z \sin Q, \tag{27}$$

$$\cos\delta \,.\, dH = a \sin z \cos Q, \tag{28}$$

where Q is the parallactic angle. We have

$$\sin z \sin Q = \sin H \cos\phi, \tag{29}$$

$$\sin z \cos Q = \sin\phi \cos\delta - \cos\phi \sin\delta \cos H, \tag{30}$$

so that

$$d\delta = a \sin H \cos\phi, \tag{31}$$

$$dH = a \sec\delta[\sin(\phi-\delta) + \cos\phi \sin\delta(1-\cos H)]. \tag{32}$$

If we pick a value of β_1 that makes H about 2 hours, we have, for $\phi \approx 50°$

$$d\delta \approx a/3.$$

It would appear desirable in fact to give the back of the mirror a roof shape, with half of it tilted east and half west. The apparent difference in declination if the same star were observed in both extra-meridian positions, i.e. two hours before and two hours after meridian passage, would then be $2a/3$ approximately, so that each star observed in this way would give a fundamental azimuth with an accuracy approaching quite reasonably close to that with which the difference of the two closely-equal "declinations" could be determined. Refraction, and most of the other likely sources of error, would cancel out in general. It is true that only half the effective aperture can be used in this way, but even so a large number of stars will be bright enough. The diffraction pattern will also in principle be a little unsymmetrical; but the asymmetry will cancel out as between the two observations. From Table I we see that, for the latitude in question and taking $H=2$ hours (i.e. $\beta_1=15°$), the angle of tilt, in the neighbourhood of zero tilt, changes by about 1° for each 1·7° of declination change; if we estimate that the method would be practicable so long as the tilt does not exceed $\pm 3°$, we can make use of the belt of stars bounded by $\delta=\pi/2-\phi\pm 5°$, i.e. a ten-degree wide belt. This will clearly contain ample stars; however, it would be necessary to discover by trial whether a 3° tilt could in practice be accepted. Extra-meridian observations have of course often been made in the past, but perhaps not quite with the accuracy we are now demanding.

We may also enquire as to the declination at which the stars transit nearly vertically, since the R.A. micrometer might be used on these in a similar manner. From (23), the condition for vertical transit is

$$2 \sin \lambda_1 \cos^2 \beta_1 + \sec(\lambda_1 + \phi) \sin \phi = 0$$

or

$$\sin(2\lambda_1 + \phi) + \sin \phi \tan^2 \beta_1 = 0. \tag{33}$$

For the value of ϕ in question, λ_1 lies between 60° and 70°, as is also clear from Table I; this gives rather a large angle of incidence. From equation (16) we now have

$$\cos H \cos \delta = -2 \sin^2 \beta_1 \sin \phi, \tag{34}$$

so that H is always a little greater than 6 hours, and the method is not much more rapid (if we use both tilts of the mirror-back) than the conventional one. However, it might perhaps still be valuable for comparison and check; it is clear from (31) that there would be some increase of accuracy as compared with a 4-hour interval, provided the large angle of incidence can be accepted.

Two important questions have so far been passed over in this discussion. The first is how the value of β_1 can be determined when it is, as it now will be, much too large for autocollimation in the tilted face to be possible. The second is how the azimuth of the mirror *in its cell* can be determined; this is, in effect, the same as asking what are the circle-readings when the tilted faces are vertical. It is these zeros from which λ_1 must in fact be reckoned, in applying equations (14), (23), (31), etc. If the azimuth of the mirror in its cell were zero, i.e. if the "ridge" of the roof were exactly perpendicular to the rotation axis, when one tilted face was vertical the other would be so also, though the front face would not necessarily be quite vertical as well, and though the two values of β_1 (for a roof-shaped back) might also be unequal. The values of β_1 could presumably be obtained by goniometer methods with sufficient accuracy; for example, the mirror might be rotated in its cell, round its own normal, through 90°, and the circle might then be read for autocollimation in the three faces. Alternatively, it may be obtained from R.A. measurements, using equation (15). The question of the zero of λ_1 is a little more difficult.

The effect of rotating the mirror through a small azimuth in its cell will be to increase the zero value from which λ_1 is measured, in the case of one half of the roof, and to decrease it for the other. If the mirror is rotated through an angle γ the change in λ_1 will be $\gamma \sin \beta_1$; there will also, in principle, be a change in the effective value of β_1 itself, but this is of the second order and can be neglected. From (14) we thus have, if $d_2\delta$ is the effect on δ of an azimuth error of the *mirror*,

$$\cos \delta \,.\, d_2\delta = 2 \cos^2 \beta_1 \sin(2\lambda_1 + \phi) d\lambda_1$$

or

$$d_2\delta = \sin 2\beta_1 \sec \delta \,.\, \gamma \cos \beta_1 \sin(2\lambda_1 + \phi). \tag{35}$$

If, on the other hand, $d_1\delta$ is the effect on δ of an azimuth-error of the instrument, we have from (31) and (15)

$$d_1\delta = \sin 2\beta_1 \sec \delta \,.\, a \cos \lambda_1 \cos \phi. \tag{36}$$

The dependence on λ_1 is different in (35) and (36), so that if stars can in fact be observed over a wide range of λ_1 the two effects are separable. Indeed they are in principle separable even with a single star, provided it can be observed in both collimators. However, any such separation, whether using one collimator and two stars, or two collimators and one star, will in general involve considerably tilted transits. It would require trial before one could say with certainty what accuracy could be obtained with large tilts, and we may therefore note that there are at least two other methods of attacking the same problem. On the one hand, we might employ any one of various known pieces of auxiliary apparatus to determine directly when a tilted face was vertical; and on the other, we might determine γ by comparison between the intrumental azimuths obtained by extra-meridianal declinations and the azimuths obtained in the standard manner from sub-pole right ascensions observed in the front face of the mirror.

It is clear that the mounting of the mirror must be such as to prevent any change in γ; however, if the method of pivot-testing already proposed is adopted, any change either in β_1 or γ will in fact be immediately observable.

Naturally, the pivot errors must be applied to these measurements. We may note in passing that pivot errors are sometimes tabulated in terms of their horizontal and vertical components, but only the component parallel to the altitude of the telescope is relevant in the case of a telescope-transit; the component perpendicular to this merely rotates the field (quite imperceptibly, of course). The same applies to a mirror-transit working in the meridian; the component perpendicular to the mirror-normal merely rotates the mirror, and is thus without effect even in principle. If, however, we are working off the meridian, both components matter; the one parallel to the normal (of the front face) effectively changes β_1, and the other one, when multiplied by $\sin\beta_1$, is directly a correction to λ_1, just as if the mirror actually rotated in its cell.

The whole question of determining fundamental azimuths by extra-merdianal observations of declination is of course a large one, and we shall not pursue it further at present, though we may note that, obviously, the extra-meridian formulae for level and for diurnal aberration would have to be employed. The proposal to use a mirror-transit is not really conditional upon the practicability of this particular application; it appears to have so many other advantages that it would be worth trying even if this one were found to be quite illusory. We will therefore return now to the main question, i.e. we shall confine ourselves from now on to the case when β_1 is very small.

The scale of the chronograph record will evidently vary in just the same way, with declination, as in the case of a telescope-transit. Thus the collimation correction, due to the fact that when the micrometer is in the position of the "standard contact" the line of collimation is not perpendicular to the rotation axis, is of the form $c \sec\delta$ just as before, c being the difference between the micrometer reading for the "standard contact" and the zero obtained as on p. 295 (both in seconds of arc). In addition, there is now the correction due to the fact that β_1 is not zero; from (15) we have

$$dH = \cos\lambda_1 \sin 2\beta_1 \sec\delta \approx 2\beta_1 \cos\lambda_1 \sec\delta. \tag{37}$$

Since now for small β_1, (14) becomes

$$\sin\delta \approx -\cos(2\lambda_1+\phi)$$

we have

$$2\lambda_1+\phi \approx \pi/2+\delta \tag{38}$$

as is of course also geometrically obvious. Thus this correction is also expressible for any given β_1 purely as a function of δ, though it does not, of course, vary simply as $\sec\delta$. Accordingly, it is only necessary to replace the usual correction table, which gives $c\sec\delta$, by one table for $c\sec\delta$ and a second one for $2\beta_1\cos\lambda_1\sec\delta$, and the complete collimation is allowed for essentially as before.

One undeniable drawback to the mirror-transit is that the telescope, and still more the observer himself, tends to obstruct the line of sight to any possible azimuth mark. It is true that one might in principle arrange a mark at 6° depression; if the observer could be so placed that not only was there still a clear view back past his body, but also no deleterious heating of the air by the proximity of his body, a depression of 6° could in fact be accepted; indeed, it would even perhaps be desirable in so far as the optical path could then be underground and so shielded from the heat of the Sun. However, unless the telescopes themselves are broken, a clear optical path would not be easy to arrange.

An alternative would be to provide a second pair of Y's on the main piers but about a foot higher than the mirror's pair, and somewhat further apart. If an axle (either rectangular or round) were laid in these, it could support a right-angle prism above the mirror in the manner sketched in Fig. 5, and azimuth marks could be observed by reflection in one or the other face of this prism.

It seems probable that the two sets of Y's would change their azimuth together,

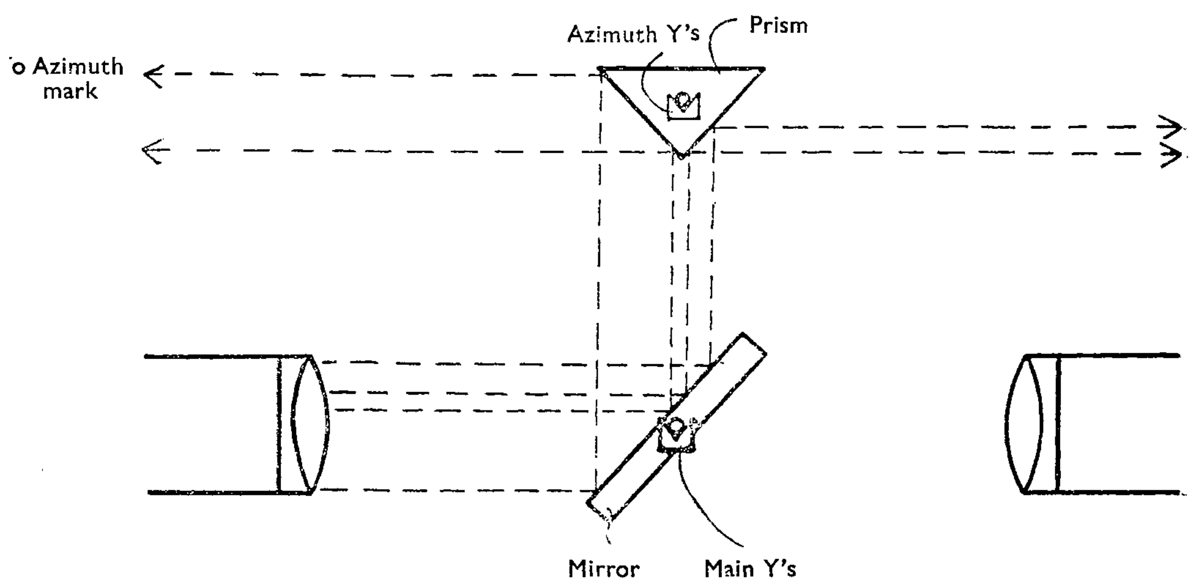

Fig. 5.

though admittedly if a change were due to one pier leaning over more north than the other it would have a larger effect on the azimuth Y's than on the main ones. Each collimator could observe, at the same time, one azimuth mark with the upper half of its aperture and the other with the lower; but it does not appear that this fact provides any additional check of value. The method is not altogether attractive; in particular, there is no obviously satisfactory place for this axle and prism when it is desired to observe near-zenith stars.

Of course, if it is in fact practicable to keep a running check on *fundamental* azimuths, by extra-meridianal observations, the need for azimuth marks largely disappears. They do, however, in favourable cases also provide a check on the cross-component of the polar movement, and they are therefore valuable even if there is no diurnal azimuth variation as such. It would clearly be preferable to have them if possible.

We are thus led to consider the use of broken telescopes. Clearly, if we can afford the necessary additional reflection, we can permanently "break" each of the telescopes, or break the beams even before we come to the telescopes; the line of sight to the azimuth mark, though it would still have to be depressed about 6°, would then be free from all interference by the observer's body. It is therefore worth noting that there is one other appreciable advantage in the use of broken telescopes.

If, namely, the telescopes are both broken towards the same side, say east, and both by slightly more than a right angle, it becomes possible to bring both observing positions close together. Modern methods of recording (for example on the 6″ transit circle at Washington) provide for all circle- and micrometer-readings to be taken by photography, so that one observer can do what previously needed two; but if the two collimator lenses are separated by 15 feet, and if the two focal lengths (as is perhaps not excessive) are now to be 30 feet, the observer will have to spend a good deal of his time moving between two observing positions 75 feet apart if the telescopes are not broken. It would clearly be better to have the positions close together, especially if the circle-settings were also made by a motor. One might, indeed, mount a long-focus pivot-telescope permanently between the other two, and take pivot-errors regularly after collimation, if desired; a few stars well observed are undoubtedly better than many poor ones. It should, however, be noted that there will be a tendency for the "main" mirror and the "breaking" mirror to act like a pair of crossed Nicols; if the mirrors were both unsilvered, there would in fact be serious extinction beyond the zenith. With silvered surfaces the effect is of course relatively much smaller, but the loss will still be larger than if polarization could be neglected.

In conclusion, we may note a few points in connection with the mounting of the mirror. We have seen that if it either rotates about its normal or tips laterally in its cell, the movement can be detected if we observe through the pivot-telescope; nevertheless, it is highly desirable to eliminate these movements if possible. In addition, it is absolutely essential to eliminate any rotation about an axis parallel to the main rotation axis of the instrument. It seems likely that the only satisfactory procedure would be to apply counterpoises so that the pressure of the mirror on each of the six ideally necessary restraining-points did not vary as the circle was rotated. If we are to be able to use both the front and the back of the mirror, only the marginal regions of these faces could be used for positioning; however, with a reasonably thick mirror this should not result in any appreciable flexure. The counterpoises might be attached to a belt running all round the edge, preferably fitted into a groove cut in the edge except where the small pivot-testing flats come; they should work with no slap or idle range at all, going over smoothly from pull to push as the sign of the load changes. If they are not to do any absolute positioning of the mirror themselves, it seems likely that they would have to be designed so that, in the absence of the six restraining-points, each one of them was in a neutral equilibrium rather than stable; but there does not seem to be any disadvantage in this. The necessary pressure on the six points would of course be provided by springs, not weights, and it should be possible to keep it quite small if the counterpoises are properly designed.

The six points themselves would probably be three on the front of the mirror near its edges, two on one of the side faces perpendicular to the axis (if the mirror is rectangular) and one on one side face parallel to the axis. If in fact the mirror were round, it would be necessary to form special steps on its sides.

In any case, each of the six "points" should be so shaped (very nearly flat to the glass) that the area in contact, due to the elastic deformation of glass and "point", is as large as possible at the pressures used.

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