

# Bundle methods with approximate subgradient linearizations

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## Abstract

We discuss proximal bundle methods for minimizing  $f(u)$  subject to  $h(u) \leq 0$ ,  $u \in C$ , where  $f$ ,  $h$  and  $C \subset \mathbb{R}^m$  are convex. We only require evaluating  $f$ ,  $h$  and their subgradients with an accuracy  $\epsilon \geq 0$ , which is fixed but possibly unknown. The methods employ an exact penalty function with an updated coefficient, or a combination of the classic method of centers' improvement function with an exact penalty function, without needing a feasible starting point. They asymptotically find points with at least  $\epsilon$ -optimal  $f$ -values that are  $\epsilon$ -feasible. When applied to the solution of LP programs arising in column generation approaches to integer programming problems, they allow for  $\epsilon$ -accurate solutions of column generation subproblems.

Our talk is partly based on joint research with Claude Lemaréchal, INRIA, France.

**Keywords:** nondifferentiable optimization, convex programming, proximal bundle methods, approximate subgradients, Lagrangian relaxation, column generation.

## References

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