A Pure Production-Based Asset Pricing Model[*]

Frederico Belo[†]

November 20, 2007

Abstract

This paper explores the implications of the producers' first order conditions for asset pricing and provides an explanation of the cross-sectional variation in expected stock returns. I recover a stochastic discount factor for asset returns from the equilibrium marginal rate of transformation, the rate at which a producer can transform output in one state of nature into output in another state. Empirically, I show that the marginal rate of transformation captures well the risk and return trade-off of several portfolio sorts, including the 25 Fama-French portfolios sorted on size and book-to-market. The returns on small stocks and value stocks have a large negative covariance with the marginal rate of transformation, which explains their high average returns relative to big stocks and growth stocks.

JEL Classification: E23, E44, G12

Keywords: Production-Based Asset Pricing, Production Under Uncertainty, Macro-Factor model, Cross-Sectional Asset Pricing, Equity Premium, Size Premium, Value Premium

---

The goal of this paper is to do for the production side the exact analog of the consumption-based asset pricing paradigm. In a consumption-based model we use the consumers' first order conditions to recover a discount factor for asset returns from the equilibrium marginal rates of substitution. The goal of a *pure* production-based model is to use the producers' first order conditions to recover a discount factor from the equilibrium marginal rates of transformation, the rate at which a producer can transform output in one state of nature into output in another state. As we need a utility function to measure marginal rates of substitution from consumption data, in this approach we need to specify a production function to obtain marginal rates of transformation from production data.

Alas, standard representations of the technology of a firm that operates in an uncertain environment don't let us do that. In standard representations, there is nothing the producer can do to transform output across states of nature and thus the marginal rates of transformation are not well defined. To see this, consider a typical production function of the form

$$Y(s) = \epsilon(s)F(K_t) \tag{1}$$

where K_t is the input (chosen today), $Y(s)$ is the output and $\epsilon(s)$ is an exogenous productivity level, which are a function of the state of nature s (tomorrow). The producer can only transform output today into output tomorrow in *fixed* proportions across states of nature. In order to produce more in one state (by increasing the use of the input K_t), it must produce more in *all* the other states as well. Thus standard representations of the technology are Leontief across states of nature, as illustrated in the left panel of Figure 1. The bold lines in this figure represent the production possibilities frontier generated by this standard technology for a given amount of inputs K_t. By definition, the marginal rate of transformation is given by the slope of the production possibilities frontier at a given point such as A. Since there is a kink in the production possibilities frontier, the marginal rate of transformation is not well defined.[1]

[Insert Figure 1 here]

To address this issue, I consider an alternative representation for the firm's technology, first proposed in Cochrane (1993), in which the producer can transform output across states of nature. In this representation, the producer has access to a standard technology such as (1) but is allowed to choose the state-contingent productivity level $\epsilon(s)$ in order to produce more in high-value states and less in low-value states, subject to a constraint set. This technology has a smooth (differentiable) production possibilities frontier across states of nature and thus well defined marginal rates of transformation, as illustrated in the right panel of Figure 1.

I consider the production decision problem of a producer that has access to the smooth production technology described above and maximizes the contingent-claim value of the firm. The producer is competitive and takes as given a market-determined stochastic discount factor $M(s)$ to value the cash-flows produced by the firm. The first order conditions with respect to the state-contingent productivity level $\epsilon(s)$ imply

$$M(s) = \text{MRT}\left(\text{production data}; \theta(s), \ s, \ b\right), \tag{2}$$

---

[1]This result also holds in more general representations of the technology in which some inputs such as capital or labor utilization are allowed to be adjusted after the state of nature is realized. Naturally, once a state of nature is realized, no transformation of output across states is possible by definition.

where MRT is the marginal rate of transformation in state of nature s. The marginal rate of transformation is a function of production variables such as the productivity level and GDP, and is parameterized by $\theta(s)$, a random variable that specifies how difficult it is to produce in the state of nature s, and by b, the vector of parameters of the smooth production technology. Equation (2) is the crucial condition for all the empirical work in this paper. It states that in order to maximize the contingent claim value, the producer equates its marginal rate of transformation to the stochastic discount factor. It therefore allows us to recover a stochastic discount factor from the producers' first order conditions without any information about consumers' preferences, in strict analogy to a consumption-based model.

The hypothesis that the producer has some control over its state-contingent productivity level $\epsilon(s)$, and hence its state-contingent output, is plausible. According to the evidence provided in Sheffi (2005) and, more generally, in the literature on operational risk management (e.g. Apgar (2006)), firms respond to uncertainty by adjusting their production practices. For example, Federal Express Corporation puts two empty planes in the air each night in order to be able to reach any airport with a grounded plane if an unexpected delivery service occurs. This production procedure effectively transforms output across states of nature by decreasing output in one state (no delivery service occurs) into increased output in another state (unexpected delivery service occurs). Likewise, several firms (e.g. Compaq) have multiple production units in several regions of the world in order to, among other reasons, control the exposure of its *aggregate* output (the sum of the output in all its production units) to local shocks. In this case, by shifting inputs such as labor and capital across the different production units, the producer can effectively control the distribution of its aggregate output across states of nature. More formally, Cochrane (1993) shows that smooth production sets across states of nature can occur when one aggregates standard production functions that are not smooth. Appendix A-I explains this construction in detail.

In the empirical section, I test whether the standard moment condition $\mathbb{E}[M_t R_t^e] = 0$ holds for the excess returns R_t^e of several portfolio sorts, using the marginal rate of transformation (2) as the stochastic discount factor M_t. Since the estimated vector of parameters b is related to the parameters of the technology of the firm, I provide an economic interpretation of the estimates and check whether the model fits the data with theoretically plausible parameter values. This is a desirable feature of any asset pricing model as emphasized by Lewellen et al (2006) and many others, and is a characteristic that distinguishes the production-based model from other macro-factor models where the factor loadings are usually free parameters.

The main empirical findings in this paper can be easily summarized. The marginal rate of transformation captures well the risk and return trade-off of many portfolio sorts. The returns on small stocks and value stocks have a large negative covariance with the marginal rate of transformation, which explains their high average returns relative to big stocks and growth stocks. The model explains about 75% of the cross-sectional variation in the returns of the 25 Fama-French size and book-to-market portfolios (the standard benchmark portfolios used in the empirical asset pricing literature) and about 88% of the cross-sectional variation in the returns of 9 risk-sorted portfolios. Finally, the performance of the production-based model also compares favorably with the standard consumption-based asset pricing model and the empirical Fama-French (1993) three factor model, on the 25 Fama-French portfolios sorted on size and book-to-market.

The paper proceeds as follows. Section I discusses the related literature. Section II presents the

3

production-based model, derives the asset pricing implications and proposes a procedure to measure the marginal rates of transformation in the data. Section III tests the production-based model on the cross-section of expected stock returns of several portfolio sorts under two alternative empirical specifications and compares the performance of the production-based model with that from other asset pricing models. Section IV concludes.

I.   Related literature

This paper is related to a growing literature on production-based asset pricing. The goal of this approach is to link stock returns to production variables thus providing an alternative framework for interpreting the well documented relationship between stock returns and macroeconomic events. Standard work in asset pricing has analyzed this relationship by watching the consumption decisions of the consumers and focusing on the properties of utility functions. Campbell (2003) and Cochrane (2005) provides a review of the consumption-based asset pricing literature. In contrast, the production-based approach interprets the relationship between stock returns and macroeconomic events by watching the production decisions of the firms and focusing on the properties of production functions. Cochrane (1991, 1993 and 2005) and Jermann (2007) provide a detailed motivation for the production-based asset pricing approach.

The central novel contribution of this paper is to estimate an asset pricing model based on operating marginal rates of transformation. Many successful macro-factor models of the form of (2) have been evaluated, including Chen Roll and Ross (1986), Li, Vassalou and Xing (2003), Cochrane (1996), and Jagannathan and Wang (1996). However, the theoretical motivation in these papers relied on consumers' first order conditions and the estimated parameters b (the factor risk prices) are estimated as free parameters. One can view the contribution of the paper as providing a theory behind successful empirical work, as well as in extending that empirical work to the precise specification of (2). In turn, the strategies for identifying the unobserved random variable $\theta(s)$ in the data are the central innovation that let this empirical work go through.

The work most closely related to mine is Cochrane (1993) and Jermann (2007). Cochrane (1993) proposes the smooth production technology that I use in this paper. Cochrane does not provide an empirical evaluation of the ability of a marginal rate of transformation (2) to price assets since this requires data on the unobserved random variable $\theta(s)$, an important variable that controls the ability of the producer to produce in each state of nature. In this paper, I solve this identification problem by assuming a factor structure for this variable. As I show in this paper, if the unobserved random variable $\theta(s)$ is a function of a small number of common factors, I am able to identify this variable from the optimal production decisions of at least two producers in the economy. Jermann (2007) calibrates a two-sector economy with two states of nature exploiting the disaggregated description of technology described in Appendix A-I rather than the aggregated smooth technology I use. With only two states of nature in the economy, Jermann recovers the contingent-claim prices from the investment returns in the two sectors and is able to replicate some interesting stylized facts. In contrast to the approach I follow in this paper, Jermann (2007) focus on the evaluation of the model through simulation and does not provide an empirical test of the model in the data. My work thus differs from Jermann's in that I directly model the marginal rates of transformation, allow for an arbitrary number of states of nature in the economy and I provide a set of moment conditions for the cross section of expected stock returns which I test in

the data.

Most of the existing production-based asset pricing literature focus on studying the firms' optimal *investment* decisions. With linearly homogenous production functions (average q equals marginal q) the investment return should be equal to the market return on a claim to the firm's capital stock. Cochrane (1991) finds that investment returns, a function of investment and output data, are highly correlated with stock returns. Cochrane (1996) and Li, Vassalou and Xing (2003) extend this approach to the study of the cross section of equity returns and Gomes, Yaron and Zhang (2006) incorporate costly external finance into this framework. But the individual production functions used in these studies have no cross sectional asset pricing implications since there is nothing the producer can do to transform output across states of nature. Therefore, these studies study the hypothesis that the investment returns are factors for asset returns, but this is not a direct prediction of these models.

Balvers and Huang (2006) extend Cochrane (1991) approach and show that under the standard neoclassical assumptions about preferences and production functions, the equilibrium stochastic discount factor in the economy is a function of the investment return. Balvers and Huang model is not pure production-based since it imposes restrictions on the preference side by ruling out features such as durable goods, habit formation or preference shocks. In addition, Balvers and Huang are able to recover a stochastic discount factor in their model due to the ability of the consumers, not the firms, to substitute consumption across states.

This paper is also related to the literature that studies the asset pricing implications of nontrivial production functions in general equilibrium models. These papers establish an endogenous link between returns and production variables. Examples of this approach include Brock (1982), Rouwenhorst (1995), Jermann (1998), Berk, Green and Naik (1999), Boldrin, Christiano and Fisher (2001), Gomes, Kogan and Zhang (2003), Gourio (2005), Gala (2005), Gomes, Kogan and Yogo (2006), Panageas and Yu (2006) and Papanikolaou (2007). These studies are also not pure production-based since the production functions in these models do not have well defined marginal rates of transformation of output across states. Therefore, these models still rely on the consumer' first order conditions to find marginal rates of substitution or a discount factor across states of nature to obtain the equilibrium conditions.

Finally, this paper is also related to a vast literature on production under uncertainty. One example of this literature is Chambers and Quiggin (2000) (and references therein) who argue that a state-contingent production approach is a realistic description of the production process of firms that operate in an uncertain environment. According to these authors, if the different inputs used in the production process are subject to different productivity shocks, the choice of the mix of inputs is equivalent to a state-contingent choice of output. This approach is not operational since we don't observe all the different inputs used by the firm in the data, but it provides theoretical support for the aggregate smooth technology that I use in this paper.

II. A Pure Production-Based Model

A. Technology

Each producer in the economy has access to a production technology that is smooth (differentiable) both across time and across states of nature. I use the analytically tractable specification of a smooth

technology proposed in Cochrane (1993). In this specification, a producer, indexed by the subscript i, produces output Y_{it} using a standard technology of the form

$$Y_{it} = \epsilon_{it} F^i (K_{it})$$

where $F^i(.)$ is an increasing and concave function of the inputs K_{it}, but is also allowed to choose the state-contingent productivity level ϵ_{it}, subject to a constraint set. In defining this constraint set, Cochrane (1993) proposes a standard CES aggregator

$$\mathbb{E}\left[\left(\frac{\epsilon_{it}}{\theta_{it}}\right)^{\alpha_i}\right]^{\frac{1}{\alpha_i}} \leq 1 \tag{3}$$

where $\alpha_i > 1$ is a parameter and $\theta_{it} > 0$ is an exogenously given random variable that makes it easier to produce in some states of nature relative to others. In this technology, the ability of the producer to transform output across states of nature is captured by the curvature parameter α_i. This parameter is related to the elasticity of substitution of output across states which is defined as $\sigma_i = (\alpha_i - 1)^{-1}$. When $\alpha_i \rightarrow \infty$, the firm has effectively no ability to transform output across states of nature (as in standard representations) since the chosen productivity level ϵ_{it} must converge to θ_{it} state-by-state in order to satisfy the restriction (3). The choice $\epsilon_{it} = \theta_{it}$ is always feasible and as α_i decreases, it becomes easier for the firm to transform output across states. Thus this restriction can be interpreted as follows: nature hands the firm an *underlying* state-contingent productivity level θ_{it}, which the firm distorts into a new state-contingent productivity level ϵ_{it} in order to produce more in some states at the expense of producing less in other states. As an example, consider $\alpha_i = 2$ and $\theta_{it} = \theta$ (constant). According to restriction (3), the producer can choose any state-contingent productivity level whose second moment is less than θ^2, including $\epsilon_{it} = \theta$. Appendix A-II explains the derivation of the constraint set (3) in detail.

B.   The Producers' Maximization Problem

Each producer $i = 1, ... N$ in the economy has access to one technology to produce one differentiated good. In what follows, I take the good produced with technology $i = 1$ to be the numeraire and I consider the maximization problem of producer i. The producer is competitive and takes as given the market-determined stochastic discount factor M_t, measured in units of good 1, to value the cash-flows arriving at the end of period t. I assume markets are complete, in which case the (unique) stochastic discount factor is the contingent-claim price divided by the probability of the corresponding state of nature. The existence of a strictly positive stochastic discount factor is guaranteed by a well-known existence theorem if there are no arbitrage opportunities in the market (see for example, Cochrane (2001), chapter 4.2).

The producer makes its production decisions in order to maximize the contingent claim value of the firm. Output is realized at the end of each period. The producer then chooses the current period investment I_{it-1}, the next period state-contingent productivity level ϵ_{it} in each technology i and distributes the total realized output minus investment costs as dividends D_{it-1} to the owners of the firm.

To derive the first order conditions, it is useful to state the problem recursively. Define the vector of state variables as $x_{t-1} = (K_{it-1}, \epsilon_{it-1}, p_{it-1}, \tilde{M}_t, \tilde{\theta}_{it})_{i=1}^2$ where K_{it-1} is the current period stock of capital, ϵ_{it-1} is the current period productivity level, $P_{it-1} = p_{it-1}/p_{1t-1}$ is the current period relative price of

good i with respect to good 1, $\tilde{M}_t$ is the next period distribution of the stochastic discount factor in units of the first good and $\tilde{\theta}_{it}$ is the next period distribution of the underlying productivity level. Let $V(x_{t-1})$ be the contingent claim value of the firm at the end of period $t-1$ given the vector of state variables x_{t-1}. The Bellman equation of the firm is

$$V(x_{t-1}) = \max_{\{I_{it-1}, \epsilon_{it}\}} \left\{ D_{it-1} + \mathbb{E}_{t-1}\left[M_t V(x_t)\right] \right\}$$

subject to the constraints,

$$
\begin{aligned}
D_{it-1} &= P_{it-1}\left(Y_{it-1} - I_{it-1}\right) & \text{(Dividend)} \\
Y_{it-1} &= \epsilon_{it-1}F^i\left(K_{it-1}\right) & \text{(Output)} \\
1 &\geq \mathbb{E}_{t-1}\left[\left(\frac{\epsilon_{it}}{\theta_{it}}\right)^{\alpha_i}\right]^{\frac{1}{\alpha_i}} & \text{(Productivity Level)} \\
K_{it} &= (1-\delta_i)K_{it-1} + I_{it-1} & \text{(Capital Stock)}
\end{aligned}
$$

for all dates t. $\mathbb{E}_{t-1}[.]$ is the expectation operator conditional on the firms' information set at the end of period $t-1$ and δ_i is the depreciation rate of the capital stock in technology i. I ignore capital adjustment costs and the choice of labor inputs since, under some assumptions and as I discuss below, these features do not affect the equilibrium marginal rate of transformation across states of nature.

C.   First-Order Conditions

The first order condition for the state-contingent productivity level ϵ_{it} is given by (all the algebra is in Appendix B)

$$\epsilon_{it} = \phi_{it-1}^{\frac{1}{1-\alpha_i}} M_t^{\frac{1}{\alpha_i-1}} P_{it}^{\frac{1}{\alpha_i-1}} \theta_{it}^{\frac{\alpha_i}{\alpha_i-1}} \tag{4}$$

where ϕ_{it-1} is a variable pre-determined at time t.[2] Intuitively, this condition states that the firms' optimal choice of the productivity level is determined by prices and technological constraints. Since $\alpha_i > 1$, the firm chooses a higher productivity level in states of nature in which output is more valuable, high M_t states, and in states of nature in which it is easier to produce, high θ_{it} states.

We can invert the first order condition (4) to recover the stochastic discount factor from the firms' optimal choice of the productivity level. Rearraging terms, we have

$$M_t = \phi_{it-1} P_{it}^{-1} \epsilon_{it}^{\alpha_i-1} \theta_{it}^{-\alpha_i} \tag{5}$$

This condition states that in order to maximize the contingent claim value of the firm, the producer equates the stochastic discount factor M_t to the marginal rate of transformation in each state of nature. Thus with this condition we can recover the stochastic discount factor from the producers' decisions without any information about preferences in the same way that we recover a discount factor in the

---

[2]This variable is $\phi_{it-1} = \mathbb{E}_{t-1}\left[M_t P_{it}\right]/\mathbb{E}_{t-1}\left[\epsilon_{it}^{\alpha_i-1}\theta_{it}^{-\alpha_i}\right]$. I don't solve for this variable since this variable only affects the mean of the stochastic discount factor. Since in this paper I only look at the implications of the model for excess returns, the mean of the stochastic discount factor is not identified since it does not affect the pricing errors in the estimation of the model.

consumption-based model from the consumers' first order conditions without any information about the technologies.

For empirical purposes, it is convenient to express the stochastic discount factor in terms of stationary and directly observed variables (up to the underlying productivity level θ_{it} which is discussed below). Using the fact that output is given by $Y_{it} = \epsilon_{it} F^i(K_{it})$ and that $F^i(K_{it})$ is pre-determined at time t, we can express the stochastic discount factor in (5) as

$$M_t = \bar{\phi}_{it-1} \left(\frac{P_{it}}{P_{it-1}} \right)^{-1} \left(\frac{Y_{it}}{Y_{it-1}} \right)^{\alpha_i - 1} \theta_{it}^{-\alpha_i} \tag{6}$$

where $\bar{\phi}_{it-1}$ is again a variable pre-determined at time t.[3] Representing the stochastic discount factor in terms of output instead of the unobserved productivity level ϵ_{it} simplifies the empirical implementation of the model. Although the productivity level ϵ_{it} can be measured in the usual way as a Solow residual, this procedure is subject to possible misspecification errors in the functional form of the production function $F(.)$, as discussed in Burnside, Eichenbaum and Rebelo (1996), for example.

The first order condition for physical investment is given by

$$\mathbb{E}_{t-1}[M_t R_{it}^I] = 1, \tag{7}$$

where

$$R_{it}^I = (1 - \delta_i) + P_{it} \epsilon_{it} F_k^i (K_{it}) \tag{8}$$

is the (stochastic) investment return. This is the standard condition that the investment return is correctly priced. According to this condition, the firm removes arbitrage opportunities from the physical investment and whatever assets the firm has access to.

I abstract from capital adjustment costs since, these costs only affect the investment returns and not the across-states predictions in (6) that I explore. For the same reason, I also abstract from the choice of labor inputs by the firm. If labor is included, the marginal rate of transformation is still given by (6) provided that labor inputs are chosen *before* the state of nature is realized. In this case, the production function with labor, $F^i(K_{it}, L_{it})$, is still pre-determined at time t and thus the algebra step from (5) to (6) is unchanged. This is no longer true if the firm is allowed to adjust labor in response to the realized productivity shock. In this case, labor is another source of variation across states of nature and, to recover the marginal rate of transformation, it would be necessary to identify the movements in output across states that are due to variation in the productivity level across states or due to variation in the use of labor inputs across states. This is an interesting generalization that I don't pursue in this paper in order to keep the model simple and transparent and to emphasize the role of the choice of the productivity level in the results. It should be pointed out however, that allowing labor to adjust is not a substitute mechanism to transform output across states of nature. Once a state of nature is realized, it is not possible to transfer output across states by definition. To measure a marginal rate of transformation it is necessary to have a decision in which more in one state costs less in another. The mere option to adjust something ex-post does not tell us anything about the rate at which a producer give up one thing in one state to get it in another.

---

[3] This variable is $\bar{\phi}_{it-1} = \mathbb{E}_{t-1}\left[M_t P_{it}/P_{it-1} \right] / \mathbb{E}_{t-1}\left[(Y_{it}/Y_{it-1})^{\alpha_i - 1} \theta_{it}^{-\alpha_i} \right]$.

D. Identification

In order to take the model to the data we need to measure the unobserved underlying productivity level θ_{it} in (6). The simplest approach to this identification problem would be to assume that θ_{it} =constant across states of nature. In economic terms, this assumption specifies that it is not easier to produce in one state of nature relative to another state. This assumption makes the estimation of the model particularly easy since, for excess returns, any variable that is constant across states can be ignored since these variables affect the level of *all* returns. This is the approach suggested in Cochrane (1993). However, this assumption leads the model to produce wrong predictions. If the underlying productivity level θ_{it} was constant, the first order condition (4) implies that firm chooses a higher productivity level, and hence produce more, in states of nature with high values of the stochastic discount factor M_t. However, it is well known that states with high values of the stochastic discount factor are associated with less output, not more. Thus to match the real world it must be true that the underlying productivity level θ_{it} does vary across states of nature and it is higher in states with low values of the stochastic discount factor. This follows naturally from general equilibrium and it is not an assumption about the stochastic discount factor. Consumers who eat the output would place an higher value for the stochastic discount factor M_t in states of nature with low output.

To solve the identification problem, I assume the underlying productivity level is *related* across technologies. I specify a factor structure for the underlying productivity level in each technology as stated in Assumption 1. This assumption imposes a strong restriction on the performance of the model thus providing testable empirical content to it.

Assumption 1 (Identification):
The underlying productivity level in each technology $i = 1, .., N$ has the following factor structure

$$\alpha_i \bar{\theta}_{it} = \sum_{j=1}^{J} \lambda_j^i \bar{\theta}_{jt}^c \quad i = 1, .., N$$

where $\bar{\theta}_{it} = log(\theta_{it})$ and $\bar{\theta}_{jt}^c$ is the jth common productivity factor, with $j = 1, .., J$ and λ_j^i are the loadings of the underlying productivity level of technology i on the common productivity factor j. The loadings for technology 1 are normalized to $\lambda_j^1 = 1 \ \forall j$.

This assumption is motivated by the well documented existence of common factors in production technologies. Aggregate production possibilities are higher for one firm when they are higher for another; that is, business cycles have common components. However, some industries are more cyclically-sensitive than others and thus the loadings of each firms' underlying productivity level on the common components, here captured by the loadings λ_j^i, may vary across firms.

Technically, Assumption 1 provides one additional condition which, combined with the producers' first order conditions (6), allows me to infer the underlying productivity level θ_{it} in each technology from output and price data and without any information about the stochastic discount factor M_t. For example, for the case of one common factor $(J = 1)$, we can use the first order conditions for any two technologies (here technologies 1 and 2) and use the condition in Assumption 1 to solve the log common productivity factor $\bar{\theta}_{1t}^c$ and obtain

$$\bar{\theta}_{1t}^c = \left(\lambda_1^2 - 1\right)^{-1} \left[\log\left(\bar{\phi}_{2t-1}/\bar{\phi}_{1t-1}\right) - \Delta p_{2t} + (1-\alpha_1)\Delta y_{1t} + (\alpha_2 - 1)\Delta y_{2t}\right] \tag{9}$$

where lowercase variables are the log of the corresponding uppercase variables and Δ is the first difference operator. In turn, the previous equation allows me to infer the equilibrium marginal rate of transformation from observed output and price data. This conclusion is stated in Proposition 1.

Proposition 1 *Under Assumption 1 and with $J \geq 1$ common productivity factors, the equilibrium marginal rate of transformation can be identified from output and price data in $J + 1$ technologies. The marginal rate of transformation is given by*

$$M_t = \varsigma_{t-1} \prod_{i=1}^{J} \left(\frac{P_{it}}{P_{it-1}}\right)^{b_i^p} \prod_{i=1}^{J+1} \left(\frac{Y_{it}}{Y_{it-1}}\right)^{b_i^y}, \tag{10}$$

where Y_{it} is the output in technology i, P_{it} is the relative price of good i with respect to good 1, ς_{t-1} is a variable pre-determined at time t and the factor risk prices b_i^y and b_i^p are related to the parameters of the production technologies (see Appendix C for exact formula). For the one common productivity factor case ($J = 1$), the factor risk prices are

$$\begin{bmatrix} b_2^p \\ b_1^y \\ b_2^y \end{bmatrix} = \begin{bmatrix} 1/(\lambda_1^2 - 1) \\ (\alpha_1 - 1)\lambda_1^2/(\lambda_1^2 - 1) \\ (1 - \alpha_2)/(1 - \lambda_1^2) \end{bmatrix}$$

Proof. See Appendix C. ∎

This Proposition shows that a macro-factor asset pricing model follows from a pure production-based asset pricing setup.

The factor risk prices b in (10) are related to the technology of the firms in this economy. This fact allows me to address some puzzling findings in the empirical macro-factor asset pricing literature. For example, Li, Vassalou and Xing (2003) and Cochrane (1996) find that the factor risk prices in their models have typically opposing signs, even for factors that are strongly positively correlated. Cochrane (1996, table 9) obtains this result when domestic and non-domestic investment growth are used as pricing factors. The estimated pattern of the risk prices is not explained in these models since the factor loadings are free parameters. In contrast, for the output factor and with one common productivity factor, this finding is a prediction of the production-based model provided that λ_1^2 is positive but smaller than one, which turns out to be the empirically relevant case in the specification used in the paper.

E. Asset Pricing Implications

In this paper, I focus on *excess* returns, which allows me to consider a simplified version of the marginal rate of transformation defined in Proposition 1. Since for a vector of excess returns (R_t^e) of tradable assets any valid discount factor M_t satisfies

$$\mathbb{E}_{t-1}\left[M_t R_t^e\right] = 0, \tag{11}$$

the conditional mean of the discount factor is not identified from data on excess returns. Therefore, we can set $\varsigma_{t-1} = 1$ in the marginal rate of transformation defined in Proposition 1 without affecting the pricing errors of the model. This implies that a valid discount factor for excess returns from the production-based model is given by

$$M_t^* = \prod_{i=1}^{J} \left(\frac{P_{it}}{P_{it-1}} \right)^{b_j^p} \prod_{i=1}^{J+1} \left(\frac{Y_{it}}{Y_{it-1}} \right)^{b_j^y}, \tag{12}$$

where the factor risk prices $b's$ are specified in Proposition 1. This discount factor is *proportional* to the true marginal rate of transformation in the model: it measures the component of the marginal rate of transformation that varies across states of nature and therefore has pricing implications for excess returns. Substituting this discount factor into an unconditional version of the moment condition (11) and some algebra, yields the standard asset pricing condition

$$\mathbb{E}[R_t^e] = -\frac{\text{Cov}(M_t^*, R_t^e)}{\mathbb{E}[M_t^*]}.$$

This equation tells us that cross sectional variation in stock returns is explained by cross-sectional variation in the level of risk. The main proposition of the production-based model is that the risk of any asset can be measured by the covariance of its returns with the marginal rate of transformation. An asset is risky if it delivers low returns in states of nature in which the marginal rate of transformation is high and thus it must offer higher expected returns in equilibrium as a compensation for its level of risk.

III. The Production-Based Model in Practice

In this section I estimate and test the production-based model. To establish the robustness of the empirical findings, I test the production-based model on several portfolio sorts and I compare the performance of the production-based model with the standard consumption-based model and the empirical Fama-French (1993) three factor model.

A. Data and Empirical Specification

I use annual data from 1947 to 2006, which is the frequency and the sample size available for the macro-variables used in this paper. I identify the data for each technology as a NAICS (North American Industry Classification System) two-digit sector. I only consider goods-producing sectors. At this level of aggregation, NAICS defines four goods-producing sectors: Agriculture, Mining, Construction and Manufacturing. I thus exclude from the analysis all the services-producing sectors. Output in each technology is measured by the real gross value added. Data for gross value added is from the Bureau of Economic Analysis (BEA) website, GDP-by-Industry accounts, table Real Value Added by Industry, lines 3, 6, 11 and 12. The price data for each sector is also from the BEA, GDP-by-the-Industry accounts, table Chain-Type Price Indexes for Value Added by Industry, lines 3, 6, 11 and 12.

The asset market data is standard. Data for the Fama-French three factors, namely Market (Market excess return), SMB (Small-Minus-Big) and HML (High-Minus-Low), the Fama-French 6 benchmark portfolios, the 25 Fama-French portfolios sorted on size and book-to-market and the risk-free rate is from

Prof. Kenneth French's webpage. The data to construct the 9 risk-sorted portfolios is from the Centre for Research in Security Prices (CRSP). Appendix D provides a more detailed explanation of the asset data used as well as a description of the additional macro data used.

In matching returns with output growth, I report results for the following two conventions: (i) contemporaneous matching: match returns at time t with output at time t and (ii) lagged matching: match returns at time t with output growth at time $t + 1$. A convention is needed because the level of output is a flow during a year rather than a point-in-time observation as the returns; that is, output data are time averaged. The contemporaneous matching assumes that output data for year t measures the output at the end of the year. In this case, output growth for a given year is this year output divided by last year output. The lagged matching follows the Campbell's (2003) (on consumption) beginning of the period timing convention and assumes that output data for year t measures the output at the beginning of the year. In this case, output growth for a given year is next year output divided by this year output. Given that there is no consensus regarding the appropriate matching assumption, I report results for both timing conventions.[4]

I estimate and test the production-based model under two alternative empirical specifications. In the first specification I assume the existence of only one common productivity factor in the underlying productivity level. This specification is appealing since it makes the analysis more tractable by keeping the number of pricing factors small and thus avoiding parameter proliferation. It also facilitates the economic interpretation of the results. I then consider a more general specification that allows for the existence of possibly many common productivity factors in the underlying productivity level. This extension is interesting since it allows me to incorporate in the estimation of the model information from a larger cross-section of technologies. The number of pricing factors in this specification increases with the assumed number of common productivity factors in the economy which complicates the analysis. To maintain the tractability of the empirical model, I reduce the number of pricing factors through a principal components analysis.

A.1 One common productivity factor specification

According to Proposition 1, under the assumption of one common productivity factor, the equilibrium marginal rate of transformation M_t^* is

$$M_t^* = \left(\frac{P_{2t}}{P_{2t-1}} \right)^{b_2^p} \left(\frac{Y_{1t}}{Y_{1t-1}} \right)^{b_1^y} \left(\frac{Y_{2t}}{Y_{2t-1}} \right)^{b_2^y} \tag{13}$$

where the factor risk prices b are related to the parameters of the underlying technologies. This specification thus requires price and output data for two production technologies. I interpret the two technologies as the mining and the other goods-producing sectors in the US economy. The other goods-producing sector includes all the goods-producing sectors excluding the mining sector. Naturally, the specification of the identity of the technologies is an additional modeling choice. This specification is convenient since, as I show below, the time series of the output growth in these two sectors reveal that both sectors fluctu-

---

[4]Jagannathan and Wang (2007) show that even though the standard consumption-based model does not perform well with annual averages, it performs significantly better when annual consumption growth is measured based only on the fourth quarter of each year. Jagannathan and Wang's paper emphasizes the effect of different matching assumption between returns and macroeconomic variables on asset pricing tests.

ate according to the business cycle, but the output in the other goods-producing sector is more cyclical sensitive than in the mining sector. This makes the application of assumption 1 to these sectors plausible.

[Insert Table I here]

[Insert Figure 2 here]

Table I reports the descriptive statistics of selected macroeconomic variables. I report the descriptive statistics for these variables across the whole sample period and during expansions and recessions. I define a year as being a recession if there are at least five months in that year that are defined as being a recession by the NBER. In addition, Figure 2 plots the time series of the output growth in the mining and other goods-producing sectors as well as the growth rate of consumption. Two important features of the data reported in Table I and Figure 2 are worth emphasizing. First, the output growth in these two sectors has a clear business cycle component: naturally, output growth in both sectors tends to be higher in expansions than in recessions. Second, the two sectors seems to have different sensitivities to the business cycle: output growth in the other goods sector is on average 6.84% higher in expansions than in recessions while the output growth in the mining sector is only 2.29% higher in expansions than in recessions. For comparison, consumption growth has an even lower business cycle pattern, with average consumption growth during expansions only 1.62% higher than in recessions. As is well known, this lack of business cycle variation of consumption growth is one of the reasons for the empirical difficulties of the standard consumption-based model with consumption growth of non-durables and services as the single pricing factor.

A.2 Multiple common productivity factors specification

With multiple common productivity factors in the economy, I need to use information from more than just two technologies. As specified in Proposition 1, with $J > 1$ common productivity factors, the marginal rate of transformation can be identified from price and output data in $J + 1$ technologies. In this case, the marginal rate of transformation M_t^* is given by

$$M_t^* = \prod_{i=1}^{J} \left(\frac{P_{it}}{P_{it-1}} \right)^{b_i^p} \prod_{i=1}^{J+1} \left(\frac{Y_{it}}{Y_{it-1}} \right)^{b_i^y} \tag{14}$$

where again the factor risk prices b are related to the parameters of the underlying technologies. With a possibly large number of common productivity factors, using the marginal rate of transformation (14) as a stochastic discount factor is not feasible in practice. Output growth are highly correlated across sectors (Murphy, Shleifer and Vishny (1988)), which creates multicollinearity problems and makes inference unreliable. In addition, the numbers of pricing factors and thus the number of parameters to be estimated increases with the number of common productivity factors. For example, with $J = 5$ common productiviy factors, the marginal rate of transformation contains nine pricing factors: the growth rate of output in five technologies and the growth rate of relative prices in four technologies. Clearly, a stochastic discount factor with such a large number of factors would not be feasible in practice.

To overcome this problem and reduce the number of pricing factors, I first linearize the marginal rate of transformation defined in (14) and then I use principal components analysis to summarize the information contained in the cross section of output and relative prices growth in a small number of

orthogonal variables-principal components- that by construction retain most of the information of the original variables. Mardia, Kent and Bibby (1979) provide a textbook treatment of principal components analysis. This procedure also allow me to identify the components of the cross section of output and relative prices growth that are relevant for pricing. Appendix D provides a brief description of the procedure.

To do a principal components analysis, I first linearize the marginal rate of transformation (14) by a first order Taylor expansion around $(Y_{it}, P_{it})_{i=1}^{J+1} = (Y_{it-1}, P_{it-1})_{i=1}^{J+1}$. Normalizing the constant in the marginal rate of transformation to one (since the mean is not identified from the estimation of the model on excess returns) yields

$$M_t^* \approx 1 + \sum_{i=1}^{J} b_j^p \Delta p_{it} + \sum_{i=1}^{J+1} b_j^y \Delta y_{it} \tag{15}$$

where $\Delta y_{it} = (Y_{it}/Y_{it-1}) - 1$ and $\Delta p_{it} = (P_{it}/P_{it-1}) - 1$. I then do a separate principal components analysis of the cross section of relative price growth and of the cross section of output growth. Once the principal components have been extracted, each pricing factor in (15) can specified as a linear combination of the principal components as

$$\Delta p_{it} = \sum_{j=1}^{J} \gamma_{ij}^p PPC_j \tag{16}$$

$$\Delta y_{it} = \sum_{j=1}^{J+1} \gamma_{ij}^y YPC_j \tag{17}$$

where PPC_j is the jth principal component of the cross section of relative prices growth, YPC_j is the jth principal component of the cross section of output growth and γ_j^P and γ_j^y are the loadings of each pricing factor on the corresponding principal components.

In this specification, I consider all the four goods-producing sectors at the NAICS two-digit level, namely Mining, Agriculture, Construction and Manufacturing and I specify the output from the Mining sector as the numeraire. Thus, by using data from four sectors, I'm considering three common productivity factors. Naturally, this analysis can be extended to any arbitrary number of common productivity factors.

Table II presents the results of the principal components analysis of the cross–section of output growth and relative price growth. Each principal component is a linear combination of the corresponding pricing factors. The top part in Panel A report the loadings of each principal component of the cross section of output growth on the output growth in each sector. The first principal is almost a "level" factor. It moves all sectors in the same direction but puts considerably less weight in the Agriculture sector. The top part in Panel B reports the cumulative percentage in the variation in the cross section of output growth that is explained by the first $k = 1, .., 4$ principal components. The first principal component alone explains almost 62% of the total variance and the first two principal components explain together approximately 75% of the total variance. The bottom part in Panels A and B repeat the same analysis for the cross-section of relative price growth. Clearly, the first principal component of the cross section of relative price growth is a "level" factor moving all sectors in the same direction. Thus the price first principal component is approximately the same as the average relative price growth. The correlation between the average relative price growth and the first principal component is 0.99. The bottom part

14

in Panel B shows that almost 90% of the total variance in the cross section of relative price growth is explained by the price first principal component alone.

In the empirical section, I only use the first principal components of the output growth and of the relative price growth in the asset pricing tests. I label the first principal component of the cross-section of the relative price growth as PFPC_t (price first principal component) and the first principal component of the cross-section of output growth as OFPC_t (output first principal component). A series of asset pricing tests suggests that only the first principal components of the output and price factors are relevant for pricing (results available upon request). Thus, in the empirical section, I use the following linear approximation of the marginal rate of transformation

$$M_t^* \approx 1 - b^p \text{PFPC}_t - b^y \text{OFPC}_t \tag{18}$$

as the stochastic discount factor.[5] One limitation of this approach is that, since I only use two factors, I cannot identify the technological parameters (α_i) in each technology. This limits the economic interpretation of the results since in this case the sign and the value of the factor risk prices (b^p and b^y) are not restricted by the theory. Thus, in this empirical specification, I evaluate the model by examining the overall ability of the approximate marginal rate of transformation (18) in explaining the cross-sectional variation in the expected returns of several portfolio sorts.

B.   Estimation Methodology

I estimate and test the production-based model using the standard unconditional moment condition for a vector of excess returns R_t^e,

$$\mathbb{E}[M_t^*(b)R_t^e] = 0 \tag{19}$$

where $M_t^*(b)$ is the marginal rate of transformation, as defined in Proposition 1 and b is the vector of parameters to be estimated. Estimation is by the Generalized Method of Moments (GMM), following the methodology developed by Hansen and Singleton (1982). The moment conditions used in the estimation are the sample counterpart of the population pricing errors (19). The GMM estimates are formed by choosing the parameters b that minimizes a quadratic form of the sample pricing errors. I report second stage (efficient) estimates. In the first stage I use the identity matrix as the weighting matrix, $W_T = I$, while in the second stage I use $W_T = S^{-1}$ where S is the Newey-West estimate of the covariance matrix of the sample pricing errors in the first stage. In the estimation of this matrix, I use one period lag to account for the possibility of time aggregation in output data (see Hall (1988) on consumption data).

To test the production-based model, I use the J-test (Hansen (1982)) of overidentifying restrictions. As additional measures of the goodness of fit of the model, I report the cross-sectional R-squared (R^2) and the mean absolute pricing error (MAE). The R^2 is obtained from an OLS regression of the realized excess returns on the predicted excess returns by the model and including a constant. The MAE is obtained by first computing the pricing error of each asset i, $\alpha_i = E[R_i^e]^{\text{observed}} - E[R_i^e]^{\text{Predicted}}$, and then take the average across assets of the absolute value of the pricing errors to obtain MAE$= \frac{1}{N}\sum_{i=1}^{N}\text{Abs}(\alpha_i)$, where N is the number of test assets and $\text{Abs}(\alpha_i)$ is the absolute value of the pricing error of asset i.

---

[5] To obtain this representation: using only the first principal component, the output and price factors in (16) and (17) are approximated by $\Delta p_{it} \approx \gamma_{i1}^p PFPC_t$ and $\Delta y_{it} = \gamma_{i1}^y OFPC_t$. Substituting this (15) yields equation (18).

C. Empirical Results for the One Common Productivity Factor Model

In this section I examine if the production-based model is able to explain the variation in the average returns of the Fama-French six benchmark portfolios sorted by size (breakpoints at the median) and book-to-market equity (breakpoints at the 30th and 70th percentile). The choice of these portfolios is motivated by a large empirical literature who found that size and value premia capture a large fraction of the cross-sectional variation in expected returns. In addition, by focusing on a relatively small number of test assets I can do efficient (second stage) GMM. Efficient GMM usually performs poorly when the number of moment conditions relative to the number of data points is large. In addition, the efficient weighting matrix S^{-1} is difficult to estimate precisely when a large number of test assets is used and the time series dimension of the data is relatively small, which may lead to spurious results. Since I have 58 data points in my sample the concern with the small sample size if a large number of test assets is used seems warranted.

[Insert Table III here]

Table III, reports the second stage GMM tests and measures of the goodness of fit of the production-based model on these portfolios as well as estimates of the factors risk prices and the implied parameters of the technologies in the two sectors. Panel A reports the results under the contemporaneous matching assumption and Panel B reports the results under the lagged matching assumption. Clearly, the matching assumption has a significant impact in the estimation of the model. The estimation results under the contemporaneous matching assumption (Panel A) show that the production-based model is unable to explain the cross-sectional variation in the returns on these portfolios. The model is rejected at the 1% confidence level. Finally, none of the factor risk prices seems to be statistically significant at the 5% level and the estimates of the curvature parameter α_i in the mining sector has the wrong sign. The analysis changes completely under the lagged matching assumption (Panel B). The model is comfortably not rejected by the $J-$ test of overidentifying restrictions (p-value of 60%). In addition, all the factor risk prices are statistically significant at the 5% level and all the technological parameters have the correct sign in both sectors ($\alpha_i > 1$). Finally, the model completely captures the variation in the average returns across these portfolios, with a cross sectional R^2 of the predicted vs. realized excess returns of around 90% and low annual mean absolute pricing error of 0.82% (annualy).

[Insert Figure 3 here]

Figure 3 plots the predicted versus realized excess returns implied by the first stage GMM estimates of the model. The straight line is the $45°$ line, along which all the assets should lie. The deviations from this line are the pricing errors which provides the economic counterpart to the statistical analysis. This figure provides a visual description of the overall good fit of the production-based model on these portfolios since all portfolios lie along the $45°$ line.

The estimates of the technological paramaters reveal interesting information about the characteristics of the technology in each of the two sectors. First, the point estimate of the curvature parameters α_g in the other goods-producing sector is slightly greater than the curvature parameter α_m in the mining sector, which suggests that it is harder to substitute output across states in the other goods-producing sector, although the difference is not statistically significant. The curvature parameter is related to the

elasticity of subtitution of output across states, defined as $\sigma_i = (\alpha_i - 1)^{-1}$, which is the production-based analogue of the coefficient of relative risk aversion in the standard consumption-based model. Given the point estimates of the curvature parameters, the elasticity of substitution of output in the two sectors are $\hat{\sigma}_g = 0.36$ in the other goods-producing sector and $\hat{\sigma}_m = 0.4$ in the mining sector. Unfortunately, since these parameters are new in the literature, there is no benchmark to compare these values with. Finally, the parameter that controls the sensitivity of the underlying productivity level of the mining sector to the common productivity factor (parameter λ_1^g) is positive but smaller than one. According to this estimate, the underlying productivity level in the two sectors are positively correlated but the underlying productivity level in the mining sector is less sensitive to the common productivity factor. This fact might explain why the output growth in the mining sector is less cyclical than the output growth in the other goods-producing sector.

[Insert Figure 4 here]

Given the second stage GMM point estimates of the technological parameters, we can recover the time series of innovations in the marginal rate of transformation. This time series is interesting since it provides information about the realized time series of the contingent-claim prices in the US economy. To obtain this time series, I assume that the states of nature are independent and identically distributed (i.i.d.). This assumption is not required for any of the asset pricing tests reported, but is necessary in order to recover the expected value of the marginal rate of transformation and hence of the pricing factors. The innovations in the log marginal rate of transformation are given by[6]

$$\omega_{mt} = (1 - \lambda_1^2)^{-1} \left[\omega_{p_2 t} + (\alpha_1 - 1)\lambda_1^2 \omega_{y_1 t} + (1 - \alpha_2)\omega_{y_2 t} \right], \tag{20}$$

where $\omega_{xt} = x_t - \mathbb{E}[x_t]$ is the innovation in variable x_t. The time series of the innovations in the log marginal rate of transformation implied by the second stage GMM estimates is plotted in Figure 4, Panel A. In this Figure, the shaded bars are NBER recession years. As expected, state-contingent claim prices, as measured by the marginal rate of transformation, tend to be high during recessions. The mean log innovation in the marginal rate of transformation is 0.59 in recessions and -0.15 in expansions. Interestingly however, the estimated innovations in the marginal rate of transformation reveal recession states that are not captured by the NBER-designated business cycle recessions dates. For example, in 1988 and 1991 we observe large innovations in the marginal rate of transformation and hence high contingent claim prices, but these years are not classified by the NBER as recession years. In addition, not all NBER recessions were equally important. According to the production-based model, the recessions in 1970 and 1980 were particularly severe since they correspond to the realization of states of nature with very high state-contingent claim prices. In short, the estimated marginal rate of transformation captures information about recessions that transcends the NBER-designated business cycle recessions dates.

---

[6]To obtain this equation, note that from equation (10) in Proposition 1 that, for the one common productivity factor case, the log marginal rate of transformation is given by

$$m_t = \log(\varsigma_{t-1}) + (1 - \lambda_1^2)^{-1} \left[\Delta p_{2t} + (\alpha_1 - 1)\lambda_1^2 \Delta y_{1t} + (1 - \alpha_2)\Delta y_{2t} \right].$$

If the states of nature are iid, the conditional expected values of $\mathbb{E}_t[\Delta p_{2t}]$, $\mathbb{E}_t[\Delta y_{1t}]$ and $\mathbb{E}_t[\Delta y_{1t}]$ are constant over time. Computing the inovation as $\omega_{mt} = m_t - \mathbb{E}[m_t]$ yields equation (20).

It is also interesting to examine the time series of the innovations in the common productivity level. In the i.i.d case, the innovations in the log common productivity factor are given by (take the log of equation (9))

$$\omega_{\theta t} = \left(1 - \lambda_1^2\right)^{-1} \left[\omega_{p_2 t} - (1 - \alpha_1)\omega_{y_1 t} - (\alpha_2 - 1)\omega_{y_2 t}\right],$$

where, as before, $\omega_{xt} = x_t - \mathbb{E}[x_t]$ is the innovation in variable x_t. Figure 4, Panel B plots the time series of the inovations in the log common productivity factor. Interestingly, the plot reveals that the common productivity factor tends to be particularly low in recessions. The mean innovation in the log common producivity factor is -0.74 in recessions and 0.2 in expansions. Since a large value of the common productivity factor corresponds to states of nature in which it is easier to produce, this result suggest that recessions corresponds to the realization of states of nature in which it is difficult to produce.

Figure 4 also shows that the innovations in the marginal rate of transformation and the innovations in the common productivity factor are almost the mirror image of each other. The correlation between the two innovations is -0.99 and the volatility of the common productivity factor is approximately equal to the volatility of the marginal rate of transformation. This result is not surprising given the low volatility of output growth in these sectors compared with the required volatility of any valid discount factor that prices assets in the US economy. To link the two variables, recall that the marginal rate of transformation in the numeraire sector (here, the goods-producing sector) in an i.i.d. world can be written as

$$M_t = \bar{\phi} \left(\frac{Y_{gt}}{Y_{gt-1}}\right)^{\alpha_i - 1} \theta_{gt}^{-\alpha_i}, \tag{21}$$

which follows from equation (6). From Table I, the standard deviation of annual output growth in the other goods-producing sector is approximately 5%. In addition, the Sharpe ratio in the US economy in the post-war period is approximately 0.4 which implies that the standard deviation of the discount factor must be at least 40% on annual data.[7] To be consistent with these values, and given the low point estimates of the curvature parameter α_g, equation (21) implies that we need a volatile common productivity factor that is highly negatively correlated with the stochastic discount factor in order to be consistent with the observed relatively low volatility of output growth.

D. Empirical Results for the Multiple Common Productivity Factors Model

As discussed in section III-A.2, with multiple common productivity factors, the marginal rate of transformation can be approximated by the following linear representation

$$M_t^* \approx 1 - b^p PFPC_t - b^y OFPC_t \tag{22}$$

---

[7]This analysis follows from the basic pricing equation for excess returns (R^e)

$$0 = E[MR^e] = E[M]E[R^e] + \rho[M, R^e]\sigma[M]\sigma[R^e]$$

we have

$$\sigma[M] = -\frac{E[M]}{\rho[M, R^e]}\frac{E[R^e]}{\sigma[R^e]}.$$

The Sharpe ratio in the US postwar data is about $E[R^e]/\sigma[R^e] = 0.4$ annualy. Thus, even if the discount factor and returns are perfectly correlated $(\rho[M, R^e] = 1)$ we need $\sigma[M] = 40\%$ annually.

where $PFPC_t$ is the first principal component of the cross-section of the relative price growth and $OFPC_t$ is the first principal component of the cross-section of output growth. This specification implies the following linear factor model[8]

$$E[R_{it}^e] = b^p Cov(R_{it}^e, PFPC_t) + b^y Cov(R_{it}^e, OFPC_t). \tag{23}$$

In this section, I examine if this linear factor model is able to explain the cross sectional variation in the returns of three sets of portfolios: (i) the 25 Fama-French portfolios sorted on size and book-to market; (ii) 9 risk-sorted portfolios; and (iii) the previous 34 portfolios together. The 9 risk-sorted portfolios are formed based on the "pre-ranking" price and output first principal components betas of each individual stock. Appendix E-III explains the construction of these portfolios in detail.

I examine the 25 Fama-French portfolios since these portfolios are the standard benchmark portfolios currently used in the empirical asset pricing literature (results are similar if I use the 6 Fama-French benchmark portfolios). In addition, I examine the 9 risk-sorted portfolios since sorting on beta provides a rigorous test for asset pricing models by creating a large spread in the post-formation betas or covariances. Table IV shows that this procedure achieves its goal. In each row (similarly for column) the ex-post covariance of the high pre-ranking beta portfolio is subtantially higher than the ex-post covariance of the low pre-ranking beta portfolio. In addition, consistent with the hypothesis that these factors are important risk factors, this sorting procedure generates a large spread in average returns. The average excess return of the high-high pre-ranking beta portfolio is 5.1% higher than the average return of the low-low pre-ranking beta portfolio. In addition, the relationship between average returns and the corresponding covariances with the factors is almost monotonic.

[Insert Table IV here]

Testing the production-based model on the 9 risk-sorted portfolios, in addition to the 25 Fama-French portfolios, also allows me to address Daniel and Titman (2005) and Lewellen et al (2006) critiques. These authors criticize the procedure of focusing exclusively on the 25 Fama-French portfolios in testing asset pricing models. Since the 25 Fama-French portfolios have a strong factor structure (the average R^2 of the Fama and French (1993) three factors explains more than 90% of the time-series variation in the returns of these portfolios), these authors argue that any factor that is slighlty correlated with HML and SMB will appear successful in explaining the cross section of the 25 Fama-French portfolios. The 9 risk-sorted portfolios address this concern by attempting to relax the tight factor structure of the 25 Fama-French portfolios.

D.1   Fama-French 25 Portfolios Sorted on Size and Book-to-Market and 9 Risk Portfolios

[Insert Table V here]

Table V reports the second state stage GMM tests and estimates of the production-based model on these portfolios. I only report results for the lagged matching assumption given the evidence in favor of

---

[8]From the standard pricing equation, $0 = E[MR^e] = E[M]E[R^e] + Cov(M, R^e)$. Substituting the marginal rate of transformation (22) in the previous equation and rearranging terms yields the linear asset pricing model (23).

this assumption in the previous section. Overall, the results show that this specification of the production-based model is able to explain the cross-sectional variation in the returns across these portfolios as well. The model is comfortably not rejected by the $J-$ test of overidentifying restrictions independently of the test assets used. In addition, the estimates of the factor risk prices are comparable across test assets which helps to make the results robust. Finally, the cross sectional R^2 is high, approximately 71% when all assets are considered, and the annual mean absolute pricing errors are low, approximately1.1% annually.

[Insert Table VI here]

To help in the interpretation of the good fit of the production-based model on the 25 Fama-French portfolios sorted on size and book-to-market, Table VI, Panel A reports the average annual excess returns on the 25 Fama-French and Table VI, Panel B reports the opposite of the cross sectional covariances between the fitted marginal rate of transformation and returns on these portfolios. The pattern of the covariances in Panel B is consistent with the pattern of excess returns reported in Panel A, which explains the good fit of the production-based model on these portfolios. The opposite of the covariances of the value stocks are on average almost twice that of the growth stocks thus explaining the *value* premium. Small stocks tend to have higher opposite covariances than big stocks thus explaining the *size* premium.

In the evaluation of any asset-pricing model it is important to understand which facts in the data are driving the results. Table VI, Panels C and D reports the covariance of the output and price first principal components with the returns on the 25 Fama-French portfolios. In Panel C, there is large spread in the covariances of the price first principal component with the returns on these portfolios, especially along the book-to-market dimensions, and the pattern of the covariances matches that of the average returns reported in Panel A. In Panel D, there is also a reasonable spread in the covariances of the output first principal component with the returns on these portfolios along both the size and the book-to-market dimension. Taken together, these results suggests that the price first principal component is mostly capturing variation along the book-to-market dimension and the output first principal component is capturing variation along the size dimension. In turn, this fact explains why both factors are statitically significant pricing factors in the estimation of the production-based model on these portfolios.

D.2   Comparison With Other Asset Pricing Models

In order to evaluate the production-based model, it is also important to compare it to close competitors rather than simply reject or fail to reject it on the basis of statistical tests. In fact, it is not hard to statistically reject any of the current popular models if one uses a sufficiently rich set of test assets or a data sample covering a long period. I compare the production-based model with the Lucas (1978) and Breeden (1979) standard consumption-based model and the empirical Fama-French (1993) three factor model. I include the standard consumption-based model since it is a natural theoretical benchmark for the production-based model. I also include the Fama-French three factor model since this model has been very successful in pricing several portfolio sorts thus also providing an interesting benchmark. Appendix F provides a complete description of these two models.

[Insert Table VII here]

Table VII presents the second stage GMM tests and estimates the three asset pricing models using the 25 Fama-French portfolios sorted on size and book-to-market as test assets. Overall, these results support the production-based model. None of the three model is rejected but the production-based model has the lowest annual mean absolute pricing errors and highest cross sectional R^2. The results for the consumption-based model (C-CAPM) confirm the difficulty of this model in explaining the cross sectional variation in the returns of these portfolios. The annual mean absolute pricing error of the consumption-based model is 1.64% which is higher than the 1.13% obtained in the production-based model. In addition, the consumption-based model requires an implausibly high coefficient of relative risk aversion of 101.6 to price these portfolios. This is the re-statement of the equity premium puzzle using cross sectional data. The results for the Fama-French three factor model confirm the well known good fit of the model on these portfolios and, consistent with previous literature, only the returns on the HML and the Market portfolio seem to be significant pricing factors.

[Insert Figure 5 here]

Figure 5 plots the predicted versus realized excess returns implied by the first stage GMM estimates of the three models. Again, this figure shows the overall good fit of the production-based model on these portfolios: most of the 25 Fama-French portfolios lie along the 45° line. Interestingly, the production-based model is able to price the small-growth portfolio (portfolio 11 in the figure) which is known to be notorious hard to price. D'Avolio (2002) and Lamont and Thaler (2003) suggests that short sale constraints are binding on a typical small-growth stock which creates limits to arbitrage which might explain why several asset pricing model (here the Fama-French 3 factor model and the standard consumption-based model) cannot price this portfolio. It is thus interesting that the frictionless production-based that I consider here can price these stocks.

D.3 Testing the Model With a Longer Time Series

The output and price data used in the tests of the production-based model covers the period between 1947 and 2006. To construct a longer time series and test the model over a larger sample, I follow Breeden, Gibbons and Litzenberger (1989) and Malloy, Moskowitz and Vissing-Jorgensen (2005) (on consumption) and project the macro-data used in the production-based model on a constant and the excess returns of a set of tradable assets. Since stock return data is available for a longer period than the macro data, I can use the factor portfolios weights estimated in sample to project a time-series of returns for the macro-factors portfolios out of sample. The longer time-series may help improve the accuracy of the findings. In addition, if there is measurement error in the macro data that is uncorrelated with the asset returns of the base assets, the macro-factors mimicking portfolios may contain less measurement error than the actual macro-data.

I construct factor mimicking portfolios of the price and output first principal components. The price first principal component mimicking portfolio, which I label PMP_t (price mimicking porfolio) is obtained by first estimating the following regression,

$$\text{PFPC}_t = a + b'R_t^e + \varepsilon_t \tag{24}$$

where PFPC_t is the price first principal component and R_t^e are the excess returns on the base assets. The coefficients b can be interpreted as the weights in a zero-cost portfolio. The return on the PMP_t is then

$$\text{PMP}_t = b' R_t^e \tag{25}$$

which is the minimum variance combination of assets that is maximally correlated with the price first principal component. Assuming that the coefficients b are relatively stable over time I use Equation (25) to extend the sample before 1949. The base test assets I employ are the Fama-French 6 benchmark portfolios. An identical procedure is used to obtain the output first principal component mimicking portfolio, which I label as OMP_t (output mimicking portfolio).

In the same spirit of Malloy, Moskowitz and Vissing-Jorgensen (2005), the projection of these macro-factors on the Fama-French 6 base assets (equation (24)) can be interpreted as a way of forming factors related to size and value that employs the output and price first principal components as an economic guide to determine the weights that should be placed on the base assets. For example, Fama and French (1993) HML factor is constructed by going long a dollar in a portfolio of value stocks and short a dollar in a portfolio of growth stocks but these weights are not dictated by theory.

[Insert Table VIII here]

Regression (24) (and similarly for the output growth first mimicking portfolio) is estimated using annual data from 1947 to 2006, for which annual output data at the sectoral level is available. Table VIII Panel A reports the coefficient estimates of the loadings b for both factor mimicking portfolios and the corresponding standard errors. Panel B reports the correlation between the output and price first principal components with the corresponding component mimicking portfolios as well as with the Fama-French three factors (Market, SMB and HML) for comparison. The correlation between first principal component of each factor and the corresponding mimicking portfolio is 0.62 for the price factor and 0.65 for the output factor which suggest that the mimicking portfolios are capturing a large component of the information in the original factors. Interestingly, the output mimicking portfolio is highly correlated with the market portfolio (correlation of 0.91). The price mimicking portfolio is reasonably correlated with the HML factor (correlation of 0.54).

[Insert Table IX here]

Using the two factor mimicking portfolios as factors in the linear factor model (23) instead of the original price and output first principal components, I examine if the production-based model can explain the cross-sectional variation of the returns of the Fama-French 25 portfolios sorted on size and book-to-market and of the 9 risk sorted portfolios over a longer time series between 1933 and 2006. Table IX reports the second stage GMM estimates of the model. Overall, the results are very similar to the ones reported for the model estimated using the original factors. The model is still not rejected despite the larger sample size and thus the higher power of the test. In addition, the model is able to explain the cross-sectional variation of the returns on these portfolios well, with a cross-sectional R^2 of 75% and a MAE of 1.22% (annually) when all assets are considered. Finally, the factor risk prices are also comparable to the ones estimated before, although they are slightly larger now. In short, the production-based model is also able to explain cross-sectional variation of the returns over the larger sample.

IV. Conclusion

I find empirical support for a production-based approach to asset pricing. The central insight of the production-based model is that the marginal rate of transformation of output across states of nature is an appropriate measure of risk in the economy. According to the model, an asset is risky if it tends to deliver low returns in states of nature in which the marginal rate of transformation is high. Therefore, these assets must offer higher expected returns in equilibrium as a compensation for its level of risk. I test this prediction by developing a procedure to measure the equilibrium marginal rates of transformation in the data. I show that the marginal rate of transformation captures well the risk and return trade-off of many assets. The returns on small stocks and value stocks have a large negative covariance with the marginal rate of transformation, which explains their high average returns relative to big stocks and growth stocks. The model explains about 75% of the cross-sectional variation in the returns of the 25 Fama-French size and book-to-market portfolios and about 88% of the cross-sectional variation in the returns of 9 risk-sorted portfolios.

Appendix A: Smooth production sets

In this section I show that a smooth (differentiable) production possibilities frontier can be justified by an aggregation result of individual production functions that are not smooth. I also derive the restriction set for the choice of the state-contingent productivity level $E\left[\left(\frac{\epsilon_t}{\theta_t}\right)^{\alpha}\right] < 1$ that I use in the paper. This section is based on Cochrane (1993).

I.   Aggregation

A smooth production possibilities frontier across states of nature can occur when one *aggregates* standard production functions which are not smooth. This is analogous to the standard result that an aggregate of Leontief production functions can produce a smooth production function such as a Cobb-Douglas. As a simple example to illustrate this claim, consider a two-state world, in which a farmer can plant in two fields (technologies). Let the technology of field i have the following standard form

$$y_i(s) = \epsilon_i(s)\sqrt{k_i} \quad s = \text{wet}, \text{dry and } i = 1, 2$$

where $y_i(s)$ is the output in field i in state s, and k_i is the number of seeds planted in field i. In addition, consider the following simple structure for the shocks in each field

$$\epsilon_1(s) = \begin{cases} 1 \text{ if s = wet} \\ 0.5 \text{ if s = dry} \end{cases} \quad \text{and} \quad \epsilon_2(s) = \begin{cases} 0.5 \text{ if s = wet} \\ 1 \text{ if s = dry} \end{cases}$$

so that field one is relatively more productive if the weather is wet and field two is relatively more productive if the weather is dry. The left panel in Figure 6 plots the production possibilities frontier in each one of this standard technologies for the case $k_i = 1$ in each technology.

Total output is $Y(s) = y_1(s) + y_2(s)$ and the number of seeds available to the farmer are constrained to be

$$K = k_1 + k_2$$

We only observe the aggregates K and $Y(s)$ but we know that the farmer can vary the amount of seeds in each of the two fields. This structure implies that *aggregate* production possibilities frontier that relates the total inputs of the firm (K) to its *total* output (Y) across states is a smooth set. This is illustrated in the right panel in Figure 6 which plots the production possibilities frontier across states, when $K = 1$ and as we vary the amount of seeds in each of the two fields subject to the constraints $k_1 + k_2 \leq 1$, $k_1, k_2 \geq 0$. As the figure shows, even though the individual production technologies have kinks, the aggregate technology is smooth. The farmer *can* shape the risk exposure of his total output to weather by varying the amount planted in each of the two fields.

[Insert Figure 6 here]

II.   A tractable representation of a smooth technology

In the previous section, in order to construct a smooth production set across states of nature the producer needs to have access to as many technologies as states of the nature. In addition, the individual technologies $y_i(s)$ are not observable to economists. Therefore, instead of aggregating the production function of

many technologies, I follow Cochrane (1993) and simply posit an aggregate, smooth production set with an analytically tractable functional form. In particular I impose that output across states of nature s_t must satisfy the restriction,[9]

$$\left[\sum_{s_t} [\mathrm{a}(s_t)Y(s_t)]^\alpha \right]^{\frac{1}{\alpha}} \leq F(K_t) \tag{1}$$

i.e. a CES transformation curve for the output across states. Here, $\alpha > 1$ and $\mathrm{a}(s_t)$ are parameters and $F(.)$ is the (certain) production function which is increasing and concave in the input K_t (capital). The restriction $\alpha > 1$ guarantees that the set of feasible outputs in each state lies along a strictly concave transformation curve defined by (1). Therefore, in order to increase output in one state of nature the producer must decrease output in the other states of nature and at an increasing rate. This sensible property of the production function reflects diminishing returns to scale in the production of output in each state of nature.

In a continuous state-space, the transformation curve (1) can be expressed as

$$\left[\int dM(\omega) y(\omega)^\alpha \right]^{\frac{1}{\alpha}} \leq F(K_t)$$

In this representation, $\mathrm{a}(s_t)$ or dM are not necessarily a probability measure. Since it is convenient to use a probability measure in order to take this technology to the data, we can use the Radon-Nikodym derivative and express the previous transformation curve as

$$\left[\int d\Pr(\omega) \, (y(\omega)/\theta\,(\omega))^\alpha \right]^{\frac{1}{\alpha}} \leq F(K_t)$$

or

$$\mathbb{E}_{t-1}\left[\left(\frac{Y_t}{\theta_t} \right)^\alpha \right]^{\frac{1}{\alpha}} \leq F(K_t) \tag{2}$$

where the expectation is conditional on the information set in period $t-1$. Since $F(K_t)$ is pre-determined at time t, we can express the previous production function as

$$Y_t = \epsilon_t F(K_t) \tag{3}$$

$$\mathbb{E}_{t-1}\left[\left(\frac{\epsilon_t}{\theta_t} \right)^\alpha \right]^{\frac{1}{\alpha}} \leq 1 \tag{4}$$

where I divided both sides of (2) by $F(K_t)$ to obtain (4). This is the representation of the technology that I use in the paper.

---

[9]Feenstra (2003) proposes a similar transformation curve. However, instead of choosing the output across states, he considers the choice of different output varieties.

Appendix B: Solving the producer's maximization problem

Define the vector of state variables as $x_{t-1} = (K_{it-1}, \epsilon_{it-1}, p_{it}, \tilde{M}_t, \tilde{\theta}_{it})_{i=1}^2$ where K_{it-1} is the current period stock of capital, ϵ_{it-1} is the current period productivity level, $P_{it} = p_{it}/p_{1t}$ is the current period relative price of good i with respect to good 1, $\tilde{M}_t$ is the next period distribution of the stochastic discount factor in units of the first good and $\tilde{\theta}_{it}$ is the next period distribution of the underlying productivity level. Let $V(x_{t-1})$ be the contingent claim value of the firm at the end of period $t-1$ given the vector of state variables x_{t-1}. The Bellman equation of producer i is given by

$$V(x_{t-1}) = \max_{\{I_{it-1}, \epsilon_{it}\}} \{D_{t-1} + \mathbb{E}_{t-1}[M_t V(x_t)]\}$$

subject to the constraints,

$$
\begin{aligned}
D_{t-1} &= P_{it-1}(Y_{it-1} - I_{it-1}) \\
Y_{it-1} &= \epsilon_{it-1} F^i(K_{it-1}) \\
1 &\geq \mathbb{E}_{t-1}\left[\left(\frac{\epsilon_{it}}{\theta_{it}}\right)^{\alpha_i}\right]^{\frac{1}{\alpha_i}} \\
K_{it} &= (1-\delta_i)K_{it-1} + I_{it-1}
\end{aligned}
\tag{5}
$$

$\mathbb{E}_{t-1}[.]$ is the expectation operator conditional on the firms' information set at the end of period $t-1$, δ_i is the depreciation rate of capital and $F^i(.)$ is the (certain) production function, which is increasing and concave.

Substitute the law of motion for capital in the value function and let λ_{it-1} be the Lagrange multiplier associated with the technological constraint (5), the first order conditions are

$$\frac{\partial}{\partial I_{it-1}} : \mathbb{E}_{t-1}[M_t V_k(x_t)] = 1 \tag{6}$$

$$\frac{\partial}{\partial \epsilon_{it}} : M_t V_{\epsilon_i}(x_t) = \lambda_{it-1}\mathbb{E}_{t-1}\left[\left(\frac{\epsilon_{it}}{\theta_{it}}\right)^{\alpha_i}\right]^{\frac{1}{\alpha_i}-1} \epsilon_{it}^{\alpha_i-1}\theta_{it}^{-\alpha_i}$$

Since in equilibrium the restriction (5) is naturally binding, we have $\mathbb{E}_{t-1}\left[\left(\frac{\epsilon_{it}}{\theta_{it}}\right)^{\alpha_i}\right] = 1$. Substituting this in the previous equation, we can write the first order condition for the optimal choice of the productivity level ϵ_{it} as

$$\frac{\partial}{\partial \epsilon_{it}} : M_t V_{\epsilon_i}(x_t) = \lambda_{it-1}\epsilon_{it}^{\alpha_i-1}\theta_{it}^{-\alpha_i} \tag{7}$$

The envelope conditions are

$$V_{k_i}(x_{t-1}) = P_{it-1}\epsilon_{t-1}F_{k_i}^i(K_{it-1}) + \mathbb{E}_{t-1}[M_t V_{k_i}(x_t)](1-\delta_i) \tag{8}$$

$$V_{\epsilon_i}(x_{t-1}) = P_{it-1}F^i(K_{it-1}) \tag{9}$$

Using equation (6), the envelope condition (8) is

$$V_{k_i}(x_{t-1}) = P_{it-1}\epsilon_{it-1}F_k^i(K_{it-1}) + (1-\delta_i) \tag{10}$$

Substituting the envelope condition (9) at time t in (7) yields

$$M_t P_{it} F^i(K_{it}) = \lambda_{it-1} \epsilon_{it}^{\alpha_i - 1} \theta_{it}^{-\alpha_i} \tag{11}$$

Taking expectations on both sides of the previous equation yields

$$\mathbb{E}_{t-1}[M_t P_{it}] F^i(K_{it}) = \lambda_{it-1} \mathbb{E}_{t-1}\left[\epsilon_{it}^{\alpha_i - 1} \theta_{it}^{-\alpha_i}\right] \tag{12}$$

This equation defines the Lagrange multiplier. Substitute λ_{it-1} from (12) back in (11) yields

$$M_t P_{it} F^i(K_{it}) = \mathbb{E}_{t-1}[M_t P_{it}] F^i(K_{it}) / \mathbb{E}_{t-1}\left[\epsilon_{it}^{\alpha_i - 1} \theta_{it}^{-\alpha_i}\right] \epsilon_{it}^{\alpha_i - 1} \theta_{it}^{-\alpha_i} \tag{13}$$

Rearranging terms

$$M_t = \phi_{it-1} P_{it}^{-1} \epsilon_{it}^{\alpha_i - 1} \theta_{it}^{-\alpha_i}$$

where $\phi_{it-1} = \mathbb{E}_{t-1}[M_t P_{it}] / \mathbb{E}_{t-1}\left[\epsilon_{it}^{\alpha_i - 1} \theta_{it}^{-\alpha_i}\right]$. This is equation (5) in the text. Solving for the productivity level yields

$$\epsilon_{it} = \phi_{it-1}^{\frac{1}{1-\alpha_i}} M_t^{\frac{1}{\alpha_i - 1}} P_{it}^{\frac{1}{\alpha_i - 1}} \theta_{it}^{\frac{\alpha_i}{\alpha_i - 1}}$$

This is equation (4) in the text.

Finally, to obtain the expression for investment returns, substitute (10) at time t back in (6) to obtain

$$\mathbb{E}_{t-1}[M_t R_t^I] = 1$$

where

$$R_t^I = (1 - \delta_i) + P_{it} \epsilon_{it} F_k^i(K_{it})$$

is the (random) investment return. These are equations (7) and (8) in the text.

The second order conditions are satisfied by the assumptions on the production technology, i.e., $\alpha_i > 1$ and $F^i(.)$ increasing and concave.

Appendix C: Proof of Proposition 1

The proof is mainly algebra. From the producers' i first order condition (see equation (6) in the text) we have

$$M_t = \bar{\phi}_{it-1} \left(\frac{P_{it}}{P_{it-1}}\right)^{-1} \left(\frac{Y_{it}}{Y_{it-1}}\right)^{\alpha_i - 1} \theta_{it}^{-\alpha_i} \tag{14}$$

Since market are complete, the SDF M_t is unique. This implies that at an interior solution, the marginal rate of transformation is equalized across time and states across all technologies. Taking the log of both sides of the previous equation we have

$$m_t = \gamma_{i,t-1} - \Delta p_{it} + (\alpha_i - 1) \Delta y_{i,t} - \alpha_i \bar{\theta}_i \text{ for } i = 1, .., N \tag{15}$$

where lowercase variables are the log of the corresponding uppercase variable, $\gamma_{i,t-1} = \ln\left(\bar{\phi}_{it-1}\right)$ and Δ

is the first difference operator. The identification Assumption 1 implies

$$\alpha_i \bar{\theta}_{it} = \sum_{j=1}^{J} \lambda_j^i \bar{\theta}_{jt}^c \quad i = 1, .., N \tag{16}$$

where J is the number of common productivity factors in the economy.

Substituting (16) in (15) yields

$$m_t = \gamma_{i,t-1} - \Delta p_{it} + (\alpha_i - 1) \Delta y_{i,t} - \sum_{j=1}^{J} \lambda_j^i \bar{\theta}_{jt}^c \quad \text{for } i = 1, .., N \tag{17}$$

Now take the difference between equation (17) for technology $i = 2, ..., N$ and technology 1 (the numeraire). This yields

$$0 = \left[\gamma_{i,t-1} - \gamma_{1,t-1}\right] - \Delta p_{it} + \left[(\alpha_i - 1) \Delta y_{i,t} - (\alpha_1 - 1) \Delta y_{1,t}\right] - \sum_{j=1}^{J} \left(\lambda_j^i - 1\right) \bar{\theta}_{jt}^c \quad \text{for } i = 2, .., N \tag{18}$$

where I've used the fact that, for technology 1, $\Delta p_{1t} = 0$ and $\lambda_j^1 = 1 \ \forall j$ as specified in Assumption 1. From now on, it is convenient to write all the $i = 2, .., N$ equations defined in (18) in matrix form. Rearranging terms we have

$$A\bar{\theta}_t^c = \Omega_{t-1} - I\Delta P_t + B\Delta Y_t \tag{19}$$

where I is a $(N-1) \times (N-1)$ identity matrix and

$$\Omega_{t-1[(N-1)\times N]} = \begin{bmatrix} \gamma_{2,t-1} - \gamma_{1,t-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_{N,t-1} - \gamma_{1,t-1} \end{bmatrix}$$

$$A_{[(N-1)\times J]} = \begin{bmatrix} \left(\lambda_1^2 - 1\right) & \cdots & \left(\lambda_J^2 - 1\right) \\ \vdots & \ddots & \vdots \\ \left(\lambda_1^N - 1\right) & \cdots & \left(\lambda_J^N - 1\right) \end{bmatrix}$$

$$B_{[(N-1)\times N]} = \begin{bmatrix} (1 - \alpha_1) & (\alpha_2 - 1) & 0 & \cdots & 0 \\ (1 - \alpha_1) & 0 & (\alpha_3 - 1) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (1 - \alpha_1) & 0 & 0 & \cdots & (\alpha_N - 1) \end{bmatrix}$$

$$\Delta P_{t[(N-1)\times 1]} = \begin{bmatrix} \Delta p_{2t} \\ \vdots \\ \Delta p_{Nt} \end{bmatrix}, \ \Delta Y_{t[N\times 1]} = \begin{bmatrix} \Delta y_{1t} \\ \vdots \\ \Delta y_{Nt} \end{bmatrix} \text{ and } \bar{\theta}_{t[J\times 1]}^c = \begin{bmatrix} \bar{\theta}_{1t}^c \\ \vdots \\ \bar{\theta}_{Jt}^c \end{bmatrix}$$

Now, with $J \geq 1$ common productivity factors, we can identify these common factors from price and output data from $N = J + 1$ technologies. In this case, the matrix A is square matrix and, provided it

28

has full rank, is invertible. It this case we can solve (19) for the matrix $\bar{\theta}_t^c$ to obtain

$$\bar{\theta}_t^c = A^{-1}\Omega_{t-1} - A^{-1}\Delta P_t + A^{-1}B\Delta Y_t \tag{20}$$

For the case of one common productivity factor $(J = 1)$ we have $A = \left[\lambda_1^2 - 1\right], B = \left[\begin{array}{cc} (1 - \alpha_1) & (\alpha_2 - 1) \end{array}\right]$ and thus the common productivity factor can be recovered from

$$\bar{\theta}_{1t}^c = c_{t-1} - \left(\lambda_1^2 - 1\right)^{-1}\left[\Delta p_{2t} - (1 - \alpha_1)\Delta y_{1t} - (\alpha_2 - 1)\Delta y_{2t}\right]$$

where c_{t-1} is a variable pre-determined at time t.

To express the actual marginal rate of transformation in terms of observed price and output data, substitute (20) in the marginal rate of transformation (15) for firm 1 to obtain

$$m_t = \gamma_{1,t-1} + (\alpha_1 - 1)\Delta y_{1,t} - \iota_J\left[A^{-1}\Omega_{t-1} - A^{-1}\Delta P_t + A^{-1}B\Delta Y_t\right]$$

where ι_J is a row vector of ones. Finally, the previous equation can be written as

$$m_t = \lambda_{t-1} + C\Delta P_t + D\Delta Y_t \tag{21}$$

where $\lambda_{t-1} = \gamma_{1,t-1} - \iota_J A^{-1}\Omega_{t-1}$ is a variable pre-determined at t, and

$$\begin{aligned} C_{1\times(N-1)} &= \iota_J A^{-1} \\ D_{1\times N} &= \left[\begin{array}{cccc} (\alpha_1 - 1) & 0 & \cdots & 0 \end{array}\right] - \iota_J A^{-1}B \end{aligned}$$

Finally, taking the exponential on both side of (21) yields

$$M_t = \varsigma_{t-1}\prod_{i=1}^{J}\left(\frac{P_{it}}{P_{it-1}}\right)^{b_i^p}\prod_{i=1}^{J+1}\left(\frac{Y_{it}}{Y_{it-1}}\right)^{b_i^y}$$

where b_i^p is the ith element in the row vector C , b_i^y is the ith element in the row vector D and $\varsigma_{t-1} = \exp(\lambda_{t-1})$.

For the case of one common productivity factor $(J = 1)$, we have

$$\begin{aligned} C &= \left[\lambda_1^2 - 1\right]^{-1} \\ D &= \left[\begin{array}{cc} (\alpha_1 - 1) & 0 \end{array}\right] - \left(\lambda_1^2 - 1\right)^{-1}\left[\begin{array}{cc} (1 - \alpha_1) & (\alpha_2 - 1) \end{array}\right] \end{aligned}$$

and thus

$$m_t = \lambda_{t-1} + (\lambda_1^2 - 1)^{-1}\Delta p_{2t} + \left(1 - \left(\lambda_1^2 - 1\right)^{-1}\right)(\alpha_1 - 1)\Delta y_{1,t} - \left(\lambda_1^2 - 1\right)^{-1}(\alpha_2 - 1)\Delta y_{2t}$$

and rearranging terms we obtain

$$m_t = \lambda_{t-1} + b_2^p\Delta p_{2t} + b_1^y\Delta y_{1,t} + b_2^y\Delta y_{2t}$$

where the factor risk prices are given by

$$
\begin{bmatrix} b_2^p \\ b_1^y \\ b_2^y \end{bmatrix} = \begin{bmatrix} 1/(\lambda_1^2 - 1) \\ (\alpha_1 - 1)\,\lambda_1^2/(\lambda_1^2 - 1) \\ (1 - \alpha_2)/(\lambda_1^2 - 1) \end{bmatrix}
$$

Appendix D: Principal Components Analysis

To construct the principal components of the cross section of output growth (similarly for the cross-section of relative prices growth), define ΔY_t as the $1 \times N$ vector containing the realizations of the net output growth rate in each sector $i = 1, .., N$ at time t. The variance-covariance matrix of ΔY_t can be written as

$$
var(\Delta Y_t) = \Omega \Lambda \Omega^\top
$$

where Λ is a diagonal matrix of eigenvalues of the matrix $var(\Delta Y_t)$ and Ω is an orthogonal matrix (i.e. $\Omega^\top = \Omega^{-1}$) whose columns are standardized eigenvectors. The vector $1 \times N$ of principal components pc_t is then defined by

$$
PC_t = (\Delta Y_t - \Delta \bar{Y})\Omega \tag{22}
$$

where $\Delta \bar{Y} \in \mathbb{R}^n$ is a vector with the sample means of the output growth rate. The variance of the kth principal component is equal to Λ_k, the kth eigenvalue of $var(\Delta Y_t)$. Moreover, the total variation in the cross section of output growth $\text{tr}(var(\Delta Y_t))$ is equal to the total variation of principal components $\text{tr}(\Lambda)$, where tr denotes trace. Thus, the percentage variation in output growth explained by the first k principal components is

$$
100 \times \frac{\sum_{i=1}^{k} \Lambda_i}{tr(\Lambda)}
$$

By construction, the first principal component is the orthogonal component that explains most of the variation in output growth in all sectors, the second component explains most of the part not explained by the first component and so forth.

Appendix E: Additional description of the data

I.   Macro data

The consumption of nondurable and services data, the population and the consumption price index data used in the tests of the standard consumption-based asset pricing model is from the BEA. Nondurable plus Services consumption is obtained from Table 2.3.5, sum of lines 6 and 13. Population is from Table 2.1, line 38. Real consumption is obtained by deflating nominal consumption by the Consumer Price Index, obtained from Table 2.3.4, line 2. Per capita real consumption is obtained by dividing real consumption by the population.

II.   Asset data

The data for the three Fama-French factors (SMB, HML and Market excess returns) and the six Fama-French factors is from Prof. Kenneth French's webpage. The three factors are: (i) the Market excess return on a value-weighted portfolio of NYSE, AMEX, and Nasdaq stocks minus the T-bill rate; (ii) SMB which is the return on the Small-minus-Big portfolio; and (iii) HML, which is the return on the High-minus-Low portfolio. The SMB and HML portfolios are based on the six Fama-French benchmark portfolios sorted by size (breakpoint at the median) and book-to-market equity (breakpoints at the 30th and 70th percentiles). The SMB return is the difference in average returns between three small and three big stock portfolios. The HML return is the difference in average returns between two high and two low book-to-market portfolios. See Fama and French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds," Journal of Financial Economics, for a complete description of these factor returns.

III.   Description of the Portfolios used

I consider the following sets of portfolios as test assets: (i) the Fama-French six benchmark portfolios (described above); (ii) the 25 Fama-French Portfolios sorted on size and book-to-market; and (iii) 9 Risk-sorted portfolios. The data for the 25 portfolios is from Prof. Kenneth French's webpage. The data used to compute the 9 Risk-sorted portfolios is from CRSP, available at the Wharton Research Data Services (WRDS) website. Excess returns are computed by subtracting the risk free rate, as measured by the US treasury bill return rate, from the CRSP. The description of each set of portfolio sorts is the following:

25 portfolios sorted on size and book-to-market: according to the description provided at Prof. Kenneth French's webpage, these portfolios, which are constructed at the end of each June, are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoints for year t are the NYSE market equity quintiles at the end of June of t. BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of $t-1$. The BE/ME breakpoints are NYSE quintiles.

9 risk-sorted portfolios (double sorted on "pre-ranking" PMP and OMP betas): the output and price first principal components are only available at annual frequency and thus it is infeasible to use this these variables to create "pre-ranking" betas due to the small sample size. To address this issue, I use the price mimicking portfolio (PMP_t) and the output mimicking portfolio (OMP_t) (section III-D.2 in the text explains the construction of these portfolios in detail) Then following Fama and French (1992) I create nine risk-sorted portfolios of NYSE, AMEX and NASDAQ stocks as follows. For every calendar year, I first estimate the PMP and the OMP betas for each firm, using 24 to 60 months of past return data. As in Fama and French (1992), I denote this beta as the "pre-ranking" PMP and OMP beta estimate. I then do the following double sorting procedure: first I sort stocks into three bins (cutoffs at the 33 and 66 percentile) based on their "pre-ranking" PMP beta and then, within each PMP bin, I sort stocks based on their "pre-ranking" OMP beta. This gives 9 portfolios. I then compute the return on each of these portfolios for the next 12 calendar months by an equally weighted average of the returns of the stocks in the portfolio. This procedure is repeated for each calendar year.

Appendix F: Description of the benchmark models

Lucas-Breeden standard consumption-based model: The consumption-based model is based on a measure of consumer's marginal rate of substitution whereas the production-based model is based on a measure of the firm's marginal rate of transformation. Thus this model is a natural benchmark for the production-based model. In a SDF representation, this model is described by

$$M_t = \beta \left(C_{t+1}/C_t \right)^{b_1}$$

where $b_1 = -$Coefficient of Relative Risk Aversion and C_t is consumption of nondurable and services goods and β is the subjective discount factor. I set $\beta = 1$ because the mean of the SDF is not identified from data on excess returns. In matching returns and consumption growth, I follow Campbell (2003) timing convention as in the production-based model. Thus I match the returns at time t with the consumption growth at time $t + 1$. This timing convention improves the fit of the consumption-based model thus providing a better benchmark for the production-based model.

Fama-French (1993) three factor model: This model uses the returns on three factor mimicking portfolios to explain expected returns. In a SDF representation, this model is described by

$$M_t = 1 - b_1 Market - b_2 SMB - b_3 HML$$

where Market is the excess returns on the market portfolio, SMB is the returns on the Small-minus-Big portfolio, and HML is the returns on the High-minus-Low portfolio. The excess market return is the return on a value-weighted portfolio of NYSE, AMEX, and Nasdaq stocks minus the one-month T-bill rate. See Appendix C-III for an additional description of these portfolios.

References

[1] Apgar, David, 2006, *Risk Intelligence: Learning to Manage What We Don't Know* (Harvard Business School Press)

[2] Balvers, Ronald.and Dayong Huang, 2006,Productivity-Based Asset Pricing: Theory and Evidence,*Journal of Financial Economics, forthcoming*

[3] Berk, Jonathan B, Richard C. Green and Vasant Naik, 1999, Optimal Investment, Growth Options and Security Returns, *Journal of Finance* 54, 115 − 1607

[4] Boldrin, Michele, Lawrence J. Christiano and Jonas Fisher, 2001, Habit Persistence, Asset Returns and the Business Cycle, *American Economic Review* 91, 149 − 166

[5] Breeden, Douglas T., 1979, An Intertemporal Capital Pricing Model with Stochastic Investment Opportunities, *Journal of Financial Economics* 7, 265 − 296

[6] Brock, William, 1982, Asset Prices in a Production Economy, in John J. McCall, ed.: *The Economics of Information and Uncertainty* University of Chicago Press

[7] Burnside, Craig, Marting Eichenbaum, and Sergio T. Rebelo, 1996, Sectoral Solow Residuals, *European Economic Review* 40, 861 − 869

[8] Campbell, John, 2003, Consumption-Based Asset Pricing, in George Constantinides, Milton Harris, and Rene Stulz, ed.: *Handbook of the Economics of Finance*, Vol. IB, Chapter 13 , North-Holland, Amsterdam, 803 − 887, 2003

[9] Chambers, Robert, and John Quiggin, 2000, *Uncertainty, Production, Choice, and Agency-The State-Contingent Approach* (Cambridge University press)

[10] Chen, Nai-Fu, Richard Roll and Steven Stephen A. Ross, 1986, Economic Forces and the Stock Market, *Journal of Business* 59, 383 − 403

[11] Cochrane, John, 1991, Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations, *Journal of Finance* 461, 209 − 237

[12] _____, 1993, Rethinking Production Under Uncertainty, Working paper, University of Chicago

[13] _____, 1996, A Cross Sectional Test of an Investment Based Asset Pricing Model, *Journal of Political Economy* 1043, 572 − 621

[14] _____, 2002, *Asset Pricing* (Princeton University Press, NJ)

[15] _____, 2005, Financial Markets and the Real Economy, in Richard Roll, ed.: *The International Library of Critical Writings in Financial Economics*, forthcoming

[16] Daniel, Kent and Sheridan Titman, 2005, Testing Factor-Model Explanations of Market Anomalies, Working Paper, Northwestern University and University of Texas, Austin

[17] D'Avolio, Gene, 2002, The market for borrowing stock, *Journal of Financial Economics* 25, 23 − 49

[18] Fama, Eugene F., and Kenneth R. French, 1992, The Cross-Section of Expected Stock Returns, *Journal of Finance* 47, 427 − 466

[19] _____, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3 − 56

[20] Feenstra, Robert C., and Hiau Looi Kee, 2004, Export Variety and Country Productivity, Working paper, NBER, No. 10830

[21] Gala, Vito, 2005, Investment and Returns: Theory and Evidence, Working paper, University of Chicago GSB

[22] Gomes, Joao F., Leonid Kogan, and Lu Zhang, 2003, Equilibrium Cross Section of Returns, *Journal of Political Economy* 111, 693 − 732

[23] Gomes, Joao F., Leonid Kogan, and Motohiro Yogo, 2006, Durability of Output and Expected Stock Returns, Working paper, Wharton School of the University of Pennsylvania

[24] Gomes, Joao F., Amir Yaron, and Lu Zhang, 2006, Asset Pricing Implications of Firms's Financing Constraints, *Review of Financial Studies*

[25] Gourio, Francois, 2005, Operating Leverage, Stock Market Cyclicality and the Cross-Section of Returns, Working paper, University of Chicago

[26] Hall, Robert E.,1988, Intertemporal Substitution in Consumption, *Journal of Political Economy* $96, 339 - 357$

[27] Hansen, Lars Peter and Kenneth J. Singleton, 1982, Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models, *Econometrica* 55, $1163 - 1198$

[28] Jagannathan, Ravi and Zhenyu Wang, 1996, The Conditional CAPM and the Cross-Section of Expected Returns, *Journal of Finance* $51, 3 - 53$

[29] Jermann, Urban, 1998, Asset Pricing in Production Economies, *Journal of Monetary Economics* $41, 257 - 275$

[30] _____, 2007, The Equity Premium Implied by Production, Working paper, Wharton School of the University of Pennsylvania

[31] Lamont, Owen and Richard H. Thaler, 2003, Can the Market Add and Subtract? Mispricing in Tech Stock Carve-Out, *Journal of Political Economy* 111, $227 - 268$

[32] Lewellen, Jonathan, Stefan Nagel and Jay Shanken, 2006, A Skeptical Appraisal of Asset-Pricing Tests, NBER working paper No. 12360

[33] Li, Erica X. N., Dmitry Livdan, and Lu Zhang, 2007, Anomalies, Working paper

[34] Li, Qing, Maria Vassalou, and Yuhang Xing, 2003, Sector Investment Growth Rates and the Cross-Section of Equity Returns,Working paper, Columbia University

[35] Liu, Laura X. L., Toni M. Whited, and Lu Zhang, 2007, Regularities, Working Paper

[36] Lucas, Robert E. Jr, 1979,Asset Prices in an Exchange Economy, *Econometrica* $46, 1426 - 1446$

[37] Panageas, Stavros, and Jianfeng Yu, 2006, Technological Growth, Asset Pricing, and Consumption Risk, Working paper, Wharton School of the University of Pennsylvania

[38] Papanikolaou, Dimitris, 2007, Investment-Specific Technology Shocks and Asset Prices, Working paper, Massachusetts Institute of Technology

[39] Rouwenhorst, Geert, 1995, Asset Pricing Implications of Equilibrium Business Cycle Models, in T. F.Cooley, ed.: *Frontiers of Business Cycle Research*

[40] Sheffi, Yossi, 2005, *The Resilient Enterprise: Overcoming Vulnerability for Competitive Advantage* (MIT Press)

Table I
Descriptive Statistics for Selected Macroeconomic Variables

This table reports the descriptive statistics of the growth rate of output (ΔY_i) in sectors $i = g$ (other goods-producing) and m (mining), the growth rate in the relative price in the mining sector, the first principal components of the cross section of relative price growth (PFPC) and of output growth (OFPC), and the growth rate in the per capita consumption of non-durables and services (ΔC) for comparison. The data are annual and the sample is $1947 - 2006$.

Var	Total			NBER expansions		NBER recessions	
	Mean	S.D.	Autocorrel.	Mean	S.D.	Mean	S.D.
	(%)	(%)		(%)	(%)	(%)	(%)
ΔY_g	2.87	4.78	0.06	4.28	3.86	-2.56	4.09
ΔY_m	1.06	5.44	-0.15	1.53	5.18	-0.76	6.23
ΔP_m	3.33	13.49	0.27	1.77	12.84	9.31	14.83
ΔC	2.34	1.18	0.33	2.68	1.01	1.06	0.87
PFPC	0	1.62	0.26	0.26	1.55	-0.83	1.69
OFPC	0	1.36	0.12	0.32	1.20	-1.38	1.10

Table II
Principal Component Analysis of the Cross-Section of Output Growth and Relative Price Growth

Panel A reports the loadings of each principal component of the cross section of output growth on the growth rate of output in each sector as well as the loadings of each principal component of the cross section of relative price growth on the relative price growth in each sector. Panel B reports the cumulative percentage variation in the cross section of output growth and relative price growth that is explained by the first $k = 1, .., 4$ principal components. The data are annual and the sample is $1947 - 2006$.

	Panel A: Loadings					Panel B: Explained Variance			
	Principal Component (Output)					Principal Component (Output)			
k	1	2	3	4	k	1	2	3	4
Mining	0.49	-0.32	0.72	0.36	Percentage	62	73	92	100
Agriculture	0.12	-0.91	-0.39	-0.10					
Construction	0.57	0.22	-0.57	0.55					
Manufacturing	0.65	0.14	-0.02	-0.75					

	Principal Component (Price)					Principal Component (Price)			
k	1	2	3	4	k	1	2	3	4
Agriculture	0.55	0.84	-0.04	–	Percentage	89	98	100	–
Construction	0.59	-0.35	0.72	–					
Manufacturing	0.59	-0.42	-0.69	–					

Table III

GMM Estimation of the Production-Based Model on the Fama-French 6 Benchmark Portfolios

This table reports the second stage GMM estimates and tests of the production-based model. The moment conditions are $0 = \mathbb{E}[M_t R_t^e]$ in which R_t^e is a vector with the excess returns on the six Fama-French base factors, $M_t = (P_{mt}/P_{mt-1})^{b_m^p} (Y_{gt}/Y_{gt-1})^{b_g^y} (Y_{mct}/Y_{mct-1})^{b_m^y}$ with $b_m^p = 1/(\lambda_1^m - 1)$, $b_g^y = (\alpha_g - 1)\lambda_m^2/(\lambda_m^2 - 1)$, $b_m^y = (1 - \alpha_m)/(\lambda_m^2 - 1)$ and Y_{it} is the output in sector $i = g$ (other goods-producing) and m (mining). Panel A reports results under the contemporaneous matching assumption and Panel B reports results under the lagged matching assumption. The table reports measures of the goodness of fit and tests of the model: the GMM first stage R-squared (R^2), obtained from an OLS regression of the predicted average returns on the realized excess returns and including a constant, the mean absolute pricing error (MAE, in %) and the J-test of overidentifying restrictions with the corresponding p-value (in %). In addition, it reports the estimates of the factor risk prices b_m^p, b_g^y and b_m^y and the corresponding standard errors. Values with (*) are statistically different from zero at the 5% level. Finally, the table reports the estimates of the curvature parameters α_g, α_m and the sensitivity of the underlying productivity level in the mining sector to the common productivity factor (λ_1^m) implied by the second stage GMM estimates of the risk prices with the corresponding standard errors obtained by the delta-method. The data are annual and the sample is $1947 - 2006$.

	Panel A: Contemporaneous Matching					Panel B: Lagged Matching			
	Tests					Tests			
	R^2 %	MAE %	J_t	p-val(J_t)		R^2 %	MAE %	J_t	p-val(J_t)
value	-10.40	12.03	12.35	1.49	value	90.51	0.82	1.87	59.96
	Risk Prices					Risk Prices			
	b_m^p	b_g^y	b_m^y			b_m^p	b_g^y	b_m^y	
estimate	-0.91	-6.39	1.92		estimate	-14.33^*	-37.28^*	32.93^*	
std errors	(4.15)	(12.29)	(12.35)		std errors	(6.94)	(11.88)	(16.37)	
	Technological Parameters					Technological Parameters			
	λ_1^m	α_g	α_m			λ_1^m	α_g	α_m	
estimate	-0.10	-67.53	3.12		estimate	0.93	3.80	3.30
std errors	(5.05)	(30.45)	(16.73)		std errors	(0.03)	(1.76)	(1.60)

Table IV
Risk Sorted Portfolios

This figure shows the mean excess returns and the post-ranking covariances with the price first principal component (PFPC) and output first principal component (OFPC) of 9 risk sorted portfolios formed by pre-ranking PFPC and OFPC factor betas. Portfolio "High" is a portfolio of stocks whose pre-ranking beta of the corresponding factor is in the top 33 percentile and portfolio "Low" is a portfolio with stocks whose pre-ranking beta of the corresponding factor are on the bottom 33 percentile. The portfolios are rebalanced annually. The data are annual and the sample is $1947 - 2006$.

	Panel A: Average Excess Returns (%)				Panel B: PFPC Covariance $\times 10^{-2}$			
	PFPC				PFPC			
OFPC	High	Medium	Low	avg.	High	Medium	Low	avg.
High	13.4	12.8	10.2	12.2	8.8	5.2	0.4	4.8
Medium	11.2	10.5	9.6	10.4	7.9	4.8	1.4	4.7
Low	9.4	8.9	8.3	8.9	9.0	4.5	-1.0	4.2
avg.	11.4	10.7	9.4		8.6	4.8	0.3	0.0
	Panel C: OFPC Covariance $\times 10^{-2}$							
High	19.6	18.7	19.0	19.1				
Medium	16.0	15.7	14.3	15.3				
Low	11.9	13.1	12.1	12.4				
avg.	15.8	15.8	15.1	0.0				

Table V

GMM Estimation of the Production-Based Model on the Fama-French 25 Portfolios and 9 Risk Sorted Porfolios

This table reports the second stage GMM estimates and tests of the production-based model. The moment conditions are $0 = \mathbb{E}[M_t R_t^e]$ in which $M_t^* = 1 - b^p PFPC_t - b^y OFPC_t$, where $PFPC_t$ is the price first principal component, $OFPC_t$ is the output first principal component and R_t^e is a vector of excess returns of the following portfolios sorts: (i) the 25 Fama-French portfolios sorted on size and book-to-market; (ii) 9 Risk Sorted Portfolios and (iii) all the previous 34 portfolios together. Appendix E-III provides a description of these portfolios. Panel A provides measures of the goodness of fit and tests of the model: it reports the GMM first stage R-squared (R^2), obtained from an OLS regression of the predicted average returns on the realized excess returns and including a constant, the mean absolute pricing error (MAE) and the J-test of overidentifying restrictions with the corresponding p-value (in %). Panel B reports the estimates of the factor risk prices b^p and b^y and the corresponding standard errors. Values with (*) are statistically different from zero at the 5% level. The data are annual and the sample is $1947 - 2006$.

	Panel A: Tests					Panel B: Risk Prices	
	Portfolios: 25 Size and Book-to-Market						
	R^2 %	MAE %	J_t	p-val(J_t)		b^p	b^y
value	75	1.13	21.9	53	estimate	0.54*	0.41*
					std errors	(0.13)	(0.11)
	Portfolios: 9 Risk Sorted						
	R^2 %	MAE %	J_t	p-val(J_t)		b^p	b^y
value	88	0.49	7.4	39	estimate	0.32*	0.43*
					std errors	(0.17)	(0.12)
	Portfolios: All previous 34						
	R^2 %	MAE %	J_t	p-val(J_t)		b^p	b^y
value	71	1.12	29.9	57	estimate	0.57*	0.34*
					std errors	(0.11)	(0.08)

Table VI
Average Returns, Fitted Marginal Rate of Transformation Covariances and Price and Output First Principal Components Covariance of the 25 Fama-French Portfolios

Panel A reports the average annual excess returns on the 25 Fama-French portfolios sorted on size and book-to-market equity (BE/ME). Panel B reports the opposite of the covariance between the fitted marginal rate of transformation and each one of the 25 Fama-French Portfolios. The fitted marginal rate of transformation is given by $M_t^* = 1 - b^p \text{PFPC}_t - b^y \text{OFPC}_t$, where PFPC_t is the price principal component and OFPC_t is the output first principal component, and the factor risk prices b are the GMM first stage estimates of the production-based model on the 25 Fama-French portfolios as test assets. Avg. is the corresponding row or column average. The data are annual and the sample is $1947 - 2006$.

| | Panel A: Average Excess Returns (%) | | | | | | Panel B: MRT Covariance $\times -10^{-2}$ | | | | | |
| | Size | | | | | | Size | | | | | |
BE/ME	Small	2	3	4	Big	avg.	Small	2	3	4	Big	avg.
Growth	5.8	6.6	7.4	8.3	7.6	7.1	7.6	8.4	7.9	6.4	9.1	7.9
2	12.1	10.1	10.6	8.5	7.8	9.8	11.1	10.6	11.7	9.5	7.3	10.0
3	12.1	13.0	11.0	12.0	9.4	11.5	11.7	13.1	12.3	10.8	8.7	11.3
4	15.0	13.9	13.1	12.2	9.5	12.7	12.5	14.3	14.0	12.1	11.5	12.9
Value	17.0	15.4	14.6	13.4	10.2	14.1	14.6	14.4	14.7	14.0	13.8	14.3
avg.	12.4	11.8	11.3	10.9	8.9		11.5	12.1	12.1	10.5	10.1	
	Panel C: PFPC Covariance $\times 10^{-2}$						Panel D: OFPC Covariance $\times 10^{-2}$					
Growth	−2.9	−0.3	−0.0	−0.7	3.8	−0.0	18.8	17.0	15.7	13.6	13.3	15.7
2	1.9	4.2	6.2	3.8	2.0	3.6	19.6	15.9	15.4	14.0	11.9	15.4
3	4.8	7.6	6.9	4.6	2.9	5.4	17.2	16.5	15.8	15.6	13.6	15.7
4	6.1	9.0	8.7	6.9	5.2	7.2	17.1	17.0	16.8	15.4	16.2	16.5
Value	5.9	7.3	8.9	5.9	7.4	7.1	21.6	19.3	18.0	20.2	18.1	19.5
avg.	3.2	5.6	6.1	4.1	4.3		18.9	17.1	16.3	15.8	14.6	

Table VII
Comparison of Three Asset Pricing Models on the 25 Fama-French Portfolios

This table presents the second stage GMM estimates and tests of three asset pricing models. The moment conditions are $0 = \mathbb{E}[M_t R_t^e]$ in which R_t^e is the vector of excess returns of the 25 Fama-French Portfolios sorted on size and book-to market equity. The three models are: (i) Production–based model (PBM); (ii) Consumption-CAPM (C-CAPM); and (iii) the Fama-French three factor model (FF3F). Appendix F provides a description of these models. The stochastic discount factor representation of each model is the following: In the PBM, $M_t^* = 1 - b^p \text{PFPC}_t - b^y \text{OFPC}_t$, where PFPC_t is the price first principal component and OFPC_t is the output first principal component; In the C-CAPM, $M_t = (C_t/C_{t-1})^{b_1}$ where C_t is real per capita consumption of nondurables+services; and in the FF3F, $M_t = 1 - b_1 \text{Market}_t - b_2 \text{SMB}_t - b_3 \text{HML}_t$ where Market, SMB and HML are the three Fama-French factors. Panel A provides measures of the goodness of fit and tests of each model: it reports the GMM first stage R-squared (R^2), obtained from an OLS regression of the predicted average returns on the realized excess returns and including a constant, the mean absolute pricing error (MAE) and the J-test of overidentifying restrictions with the corresponding p-value (in %). Panel B reports the estimates of the factor risk prices b and the corresponding GMM standard errors in parenthesis. Values with (*) are statistically different from zero at the 5% level. The data are annual and the sample is $1947 - 2006$.

	PBM	C-CAPM	FF3F
Panel A: Tests			
MAE (%)	1.13	1.64	1.37
R^2 (%)	75	43.88	68.08
J-Test	21.90	8.49	21.40
p-value(J_t) (%)	52.57	99.85	49.64
Panel B: Risk Prices			
Factor			
PFPC	0.54*		
	(0.13)		
OFPC	0.41*		
	(0.11)		
ΔC		-101.6^*	
		(47.33)	
MARKET			2.14*
			(0.58)
SMB			0.25
			(0.95)
HML			2.74*
			(0.80)

Table VIII

Estimation of the Price and Output First Principal Component Mimicking Portfolios.

Panel A reports the estimates of the loadings (*b*) and corresponding standard errors obtained from the regression of the price first principal component (PFPC) and the output first principal component (OFPC) on the Fama-French 6 Base Assets. These loadings are used to construct the price mimicking portfolio (PMP) and the output mimicking portfolio (OMP). Panel B reports the correlation between each factor and the corresponding mimicking portfolio as well with the Fama-French three factors (MKT, SMB and HML) for comparison. The sample period is annual data between 1947 and 2006.

Panel A: Factor Mimimicking Portfolio loadings

Base Assets

	Small			Big		
	Value	Neutral	Growth	Value	Neutral	Growth
b(PMP)	−6.23	3.68	4.49	4.30	−2.19	−1.92
std errors	(2.24)	(3.99)	(3.92)	(2.35)	(2.44)	(2.89)
b(OMP)	−0.26	−4.23	4.93	2.30	1.07	0.76
std errors	(1.33)	(3.38)	(2.56)	(1.46)	(1.87)	(1.59)

Panel B: Correlations

	PMP	OMP	PFPC	OFPC	MKT	SMB	HML
PMP	1.00						
OMP	0.49	1.00					
PFPC	0.61	0.20	1.00				
OFPC	0.32	0.65	0.23	1.00			
MKT	0.27	0.91	0.09	0.61	1.00		
SMB	0.05	0.22	0.02	0.16	0.26	1.00	
HML	0.54	0.20	0.16	0.11	−0.17	−0.03	1.00

Table IX

GMM Estimation of the Production-Based Model on the Fama-French 25 Portfolios and 9 Risk Sorted Porfolios Using Factor Mimicking Portfolios Over a Longer Time Series

This table reports the second stage GMM estimates and tests of the production-based model. The moment conditions are $0 = \mathbb{E}[M_t R_t^e]$ in which $M_t^* = 1 - b^p \text{PMP}_t - b^y \text{OMP}_t$, where PMP_t is the price mimicking portfolio, OMP_t is the output mimicking portfolio and R_t^e is a vector of excess returns of the following portfolios sorts: (i) the 25 Fama-French portfolios sorted on size and book-to-market; (ii) 9 risk-sorted portfolios and (iii) all the previous 34 portfolios together. Appendix E-III provides a description of these portfolios. Panel A provides measures of the goodness of fit and tests of the model: it reports the GMM first stage R-squared (R^2), obtained from an OLS regression of the predicted average returns on the realized excess returns and including a constant, the mean absolute pricing error (MAE) and the J-test of overidentifying restrictions with the corresponding p-value (in %). Panel B reports the estimates of the factor risk prices b^p and b^y and the corresponding standard errors. Values with (*) are statistically different from zero at the 5% level. The data are annual and the sample is $1933 - 2006$.

	Panel A: Tests					Panel B: Risk Prices	
	Portfolios: 25 Size and Book-to-Market						
	R^2 %	MAE %	J_t	p-val(J_t)		b^p	b^y
value	88	0.88	31.55	10.9	estimate	0.86*	0.19
					std errors	(0.19)	(0.15)
	Portfolios: 9 Risk						
	R^2 %	MAE %	J_t	p-val(J_t)		b^p	b^y
value	77	1.13	9.02	25.1	estimate	0.56*	0.29
					std errors	(0.22)	(0.15)
	Portfolios: All previous 34						
	R^2 %	MAE %	J_t	p-val(J_t)		b^p	b^y
value	75	1.22	33.12	41.2	estimate	0.70*	0.20*
					std errors	(0.22)	(0.09)

Figure 1
Production Possibilities Frontier Across States of Nature

This figure plots the production possibilities frontier across states of nature (bold line) for a standard representation of technology (left panel) and for a smooth (differentiable) representation of technology (right panel) in a two states of nature economy. The firm is producing at point A.
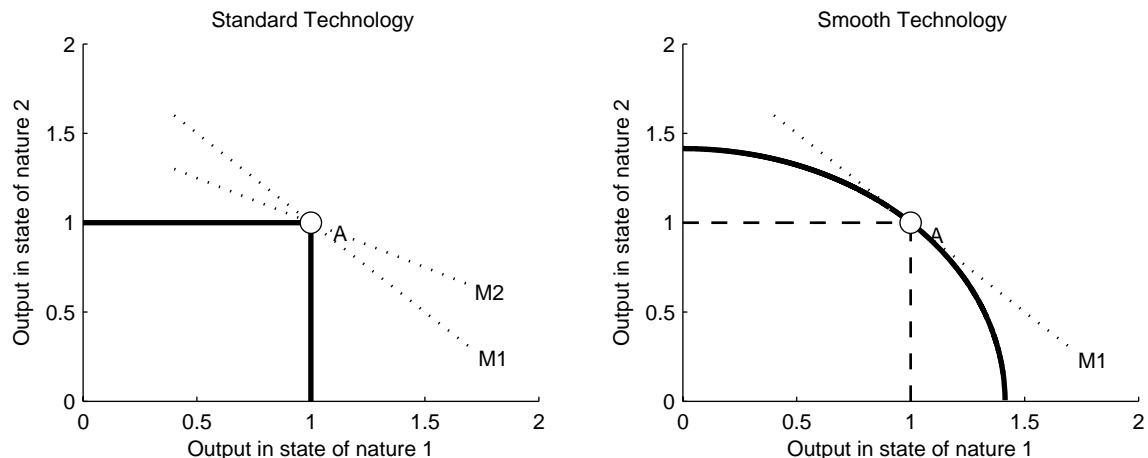

Figure 2
Time Series of the Output Growth in the Mining and in the Other Goods-Producing Sector and Consumption Growth of Non-Durables and Services

This figure plots the time series of the output growth in the mining and in the other goods-producing sector and the per capita consumption growth of non-durables and services. Shaded bars are NBER recession years. The data are annual and the sample is $1947 - 2006$.
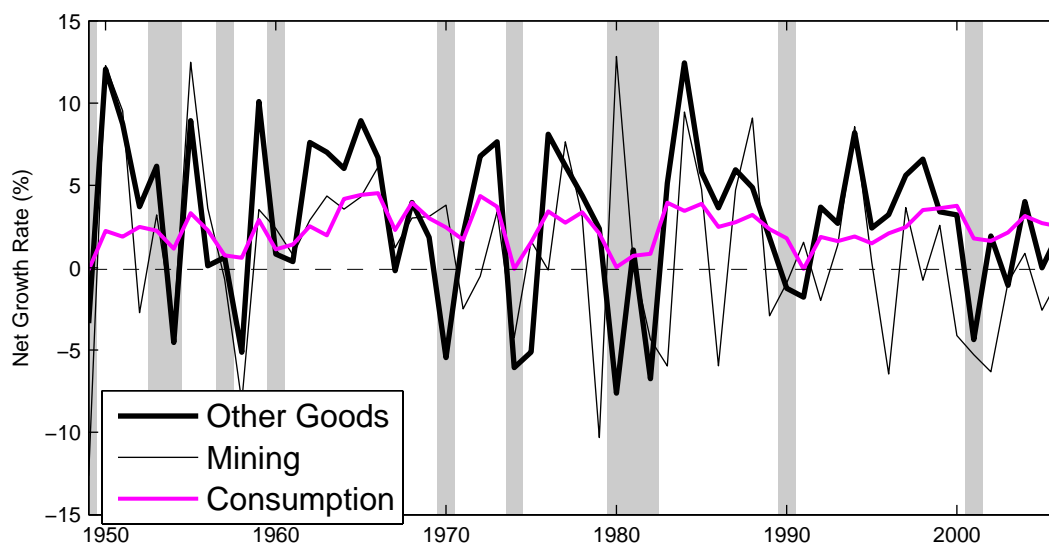

43

Figure 3
Predicted vs Realized Excess Returns of the Production Based Model on the Fama-French Six
Benchmark Portfolios

The figure shows the plot of realized versus predicted excess returns (per year) for the six Fama-French benchmark portfolios
sorted on size and book-to-market equity implied by the first stage GMM estimation of the one common productivity factor
production-based model (PBM). The R-squared (R^2) is obtained from an OLS regression of the predicted average returns
on the realized excess returns and including a constant. The data are annual and the sample is $1947 - 2006$.
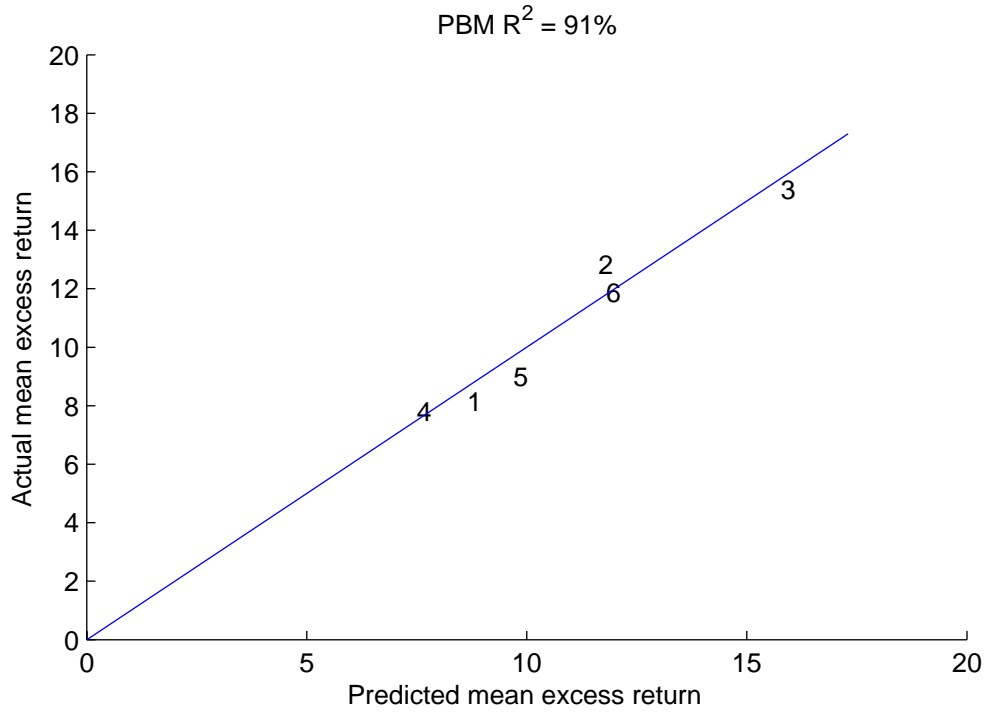
Figure 4

Times Series of the Inovations in the Marginal Rate of Transformation and in the Common Productivity Factor

Panel A plots the inovations in the log marginal rate of transformation and Panel B plots the inovations in the log common productivity factor implied by second stage GMM estimates of the production-based model on the Fama-French six benchmark portfolios. Shaded bars are NBER recession years. The data are annual and the sample is $1947 - 2006$.
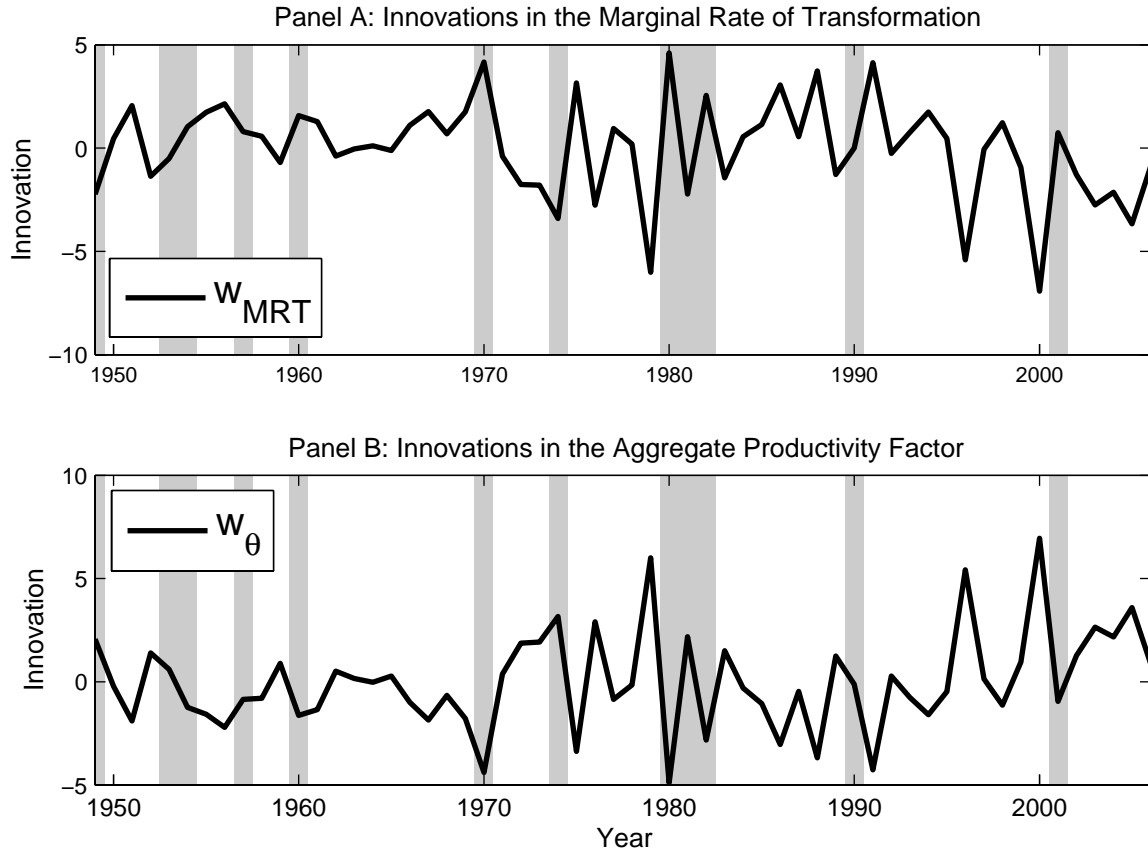

Panel A: Innovations in the Marginal Rate of Transformation

Panel B: Innovations in the Aggregate Productivity Factor

Figure 5
Predicted vs Realized Excess Returns of Three Asset Pricing Models on the 25 Fama-French Portfolios

The figure shows the plot of realized versus predicted excess returns (per year) for the 25 Fama-French portfolios sorted on size and book-to-market equity implied by the first stage GMM estimation of the following models : (i) top left: production-based model (PBM); (ii) top right: consumption-based model (C-CAPM) and (iii) lower left: Fama-French three factors model (FF3F). In the figure, each digit number represents one portfolio. The first digit refers to the size quintiles (1 indicating the smallest firms, 5 the largest), and the second digit refers to book-to-market quintiles (1 indicating the portfolio with the lowest book-to-market ratio). The R-squared (R^2) is obtained from an OLS regression of the predicted average returns on the realized excess returns and including a constant and MAE is the mean absolute pricing error. The data are annual and the sample is $1947 - 2006$.
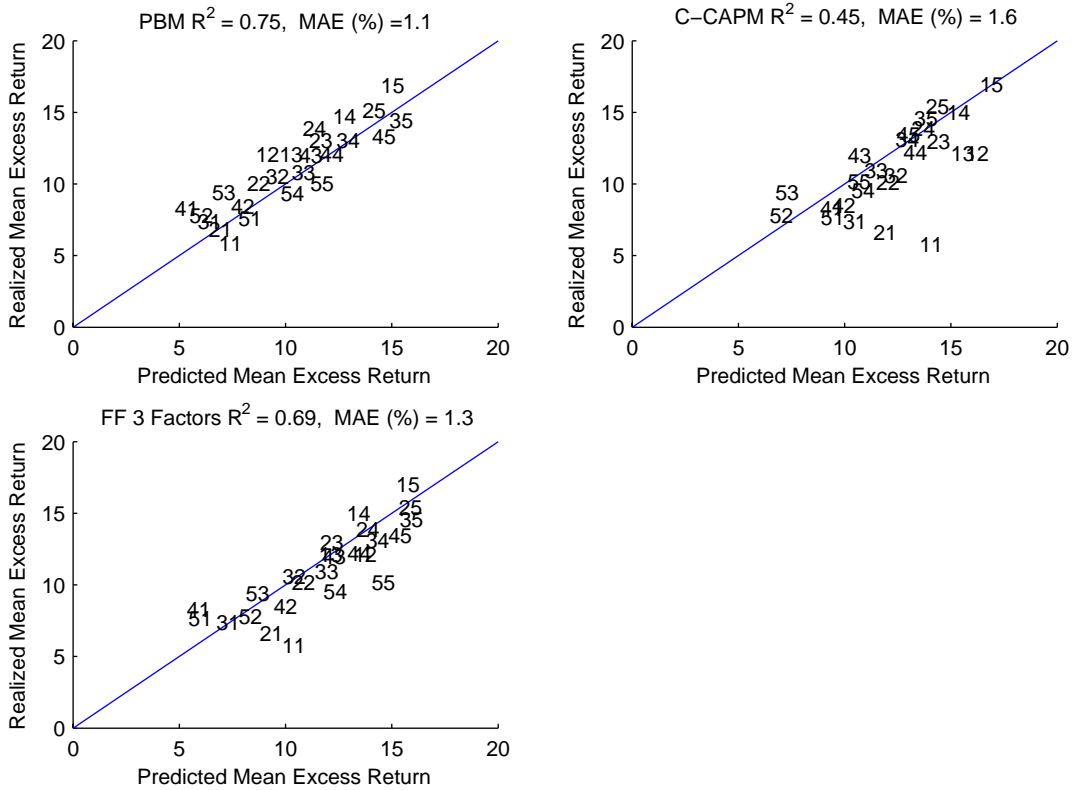
Figure 6
Smooth Production Possibilities Frontier Across States of Nature: an Aggregation Result

This left panel in the figure plots the production possibilities frontier across states of nature for two standard representations of the technology of the form $y_i(s) = \epsilon_i(s)\sqrt{k_i}$ where $k_i = 1$, $s = wet, dry$, $\epsilon_1(wet) = \epsilon_2(dry) = 0.5$ and $\epsilon_1(dry) = \epsilon_2(wet) = 1$. The right panel plots the production possibilities frontier of the resulting aggregate production function $Y = y_1 + y_2$, in which $k_1 + k_2 \leq 1$.