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A QUADRILATERAL-FREE ARRANGEMENT OF SIXTEEN LINES[1]

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n lines in general position in the real projective plane determine $(n^2-n+2)/2$ convex polygons. It has been observed ([**1**], [**2**]) that for almost all arrangements of $n \geqq 3$ lines that one or more of these polygons must be quadrilaterals. In fact, only three arrangements, one each for $n=3$, 6 and 10, have been reported in which there are no quadrilaterals. Because of this, Branko Grünbaum has recently conjectured [**3**] that for all $n \neq 3$, 6 or 10 at least one quadrilateral will be formed by every possible arrangement of n lines in the projective plane.

The figure shows that this conjecture is not true by displaying an

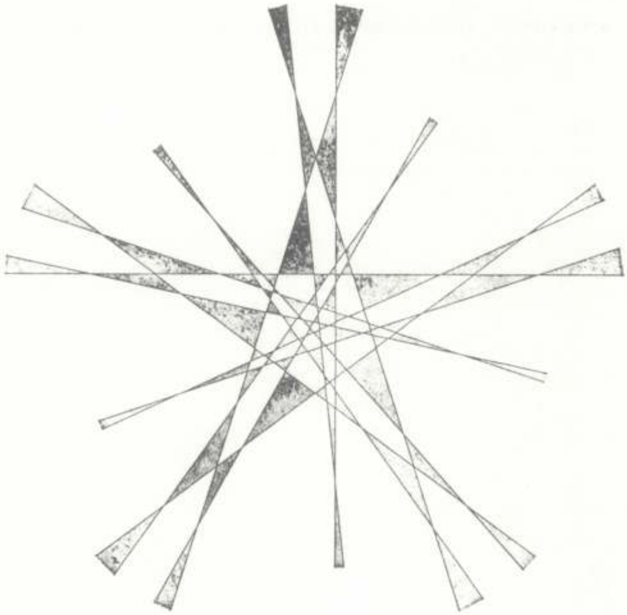

FIGURE 1

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arrangement of fifteen lines which when taken with the line at infinity (any finite line of the plane such that all of the closed polygons in the figure are on the same side of it is equivalent to the line at infinity for this construction) forms 80 triangles, no quadrilaterals, 11 pentagons, 25 hexagons and 5 heptagons.

The question of whether there exists other quadrilateral-free arrangements of n lines in the real projective plane remains open.

References

1. B. Grünbaum, *Convex polytopes*, Pure and Appl. Math., vol. 16, Interscience, New York, 1967, pp. 397–400. MR **37** #2085.

2. S. Roberts, *On the figures formed by the intercepts of a system of straight lines in a plane, and on analogous relations in space of three dimensions*, Proc. London Math. Soc. **19** (1889), 405–422.

3. B. Grünbaum, "Problem 23," in *Combinatorial structures and their applications*, edited by R. Guy, H. Hanani, N. Sauer and J. Schonheim, Gordon and Breach, New York, 1970, p. 501.

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