A qualitative study of cosmic fireballs and γ-ray bursts

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Summary. If a large amount of energy is suddenly converted into a concentrated burst of (MeV) γ rays, the prolific creation of electron–positron pairs will inhibit the escape of photons until they have been degraded below the pair-production threshold. This sets general constraints on the possible luminosities of rapidly varying γ-ray sources and suggests why the observed γ-ray bursts have an approximately standardized and 'soft' spectrum.

1 Introduction

The phenomenon of γ-ray bursts implies that there must be some astrophysical mechanism capable of creating a sudden 'fireball' of high-energy photons. The consensus view is that these bursts result from energetic processes near a compact object, but as yet the data are insufficient to decide between the numerous specific proposals in the literature (reviewed by Ruderman 1975). Some of the outbursts in quasars may be larger-scale manifestations of a similar physical mechanism.

In this paper we outline in a general way, the chain of events that ensues when a large amount of energy is suddenly converted into high-energy (MeV) photons. Even if other opacity is absent, pair production by $\gamma + \gamma \rightarrow e^+ + e^-$ is inevitable if the photon density is large enough. This provides opacity and extra cooling processes which rapidly degrade the typical photon energy until this falls below the pair-production threshold. We find that a broad variety of initial conditions yield qualitatively similar final spectra and suggest that this is the reason why γ-ray bursts are found to have rather standardized spectra. We also discuss general constraints on the timescale, dimensions and luminosity of compact γ-ray sources.

We first summarize the relevant processes (Section 2) and then (Section 3) outline the behaviour of an idealized fireball. Finally (Section 4) the relevance to γ-ray bursts and other cosmic fireballs is briefly discussed.

2 Physical processes

We will now list the most important mechanisms, giving also the appropriate expressions and values for the relevant cross-sections. We assume that ϵ_i, the initial photon energy measured

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in $m_e c^2$ units, is below the threshold for meson production and consider interactions between two photons (or particles) whose energies are similar.

2.1 PAIR PRODUCTION

2.1.1 Photon–photon collisions

The effective threshold for pair production will be approximately at photon energies $\epsilon = 1$, and the cross-section will be reasonably well approximated by $\sigma(\epsilon) \simeq \sigma_T \epsilon^{-2}$, where σ_T is the Thompson cross-section. Logarithmic terms will be systematically neglected in the following. For $\epsilon \gg 1$, the average energy of the electron–positron pair produced in such a collision will again be ϵ.

2.1.2 Photon–proton collisions

The threshold value will be $\epsilon \simeq 2$ and, again neglecting logarithmic terms, the total cross-section will be $\alpha \sigma_T$. The average energy of the produced pair will be of the order of $\frac{1}{2}\epsilon$.

2.1.3 Particle–particle collisions

The cross-section is of the order of $\alpha^2 \sigma_T$ and this pair-production mechanism will be consistently neglected in our study.

2.2 PHOTON PRODUCING MECHANISMS

2.2.1 Pair annihilation

If the pairs have a Maxwellian distribution with temperature $\sim \epsilon m_e c^2/k$, then, for $\epsilon \gg 1$, the cross-section is well approximated by σ_T/ϵ^2. For $\epsilon < 1$, on the contrary, the cross-section will diverge, but in such a way as to keep the annihilation rate constant, and given by $\sigma_T c n_e$, i.e. essentially the rate for Thompson scattering.

2.2.2 Bremsstrahlung

All bremsstrahlung cross-sections will be approximated by $\alpha \sigma_T$. For $\epsilon < 1$ the electron–electron bremsstrahlung will lack a dipole contribution and will thus be a factor $(v/c)^2$ less that the electron–positron and the electron–proton cross-sections. For $\epsilon \gg 1$, on the contrary, all three cross-sections will be of the same order of magnitude.

2.3 SCATTERING MECHANISMS

2.3.1 Single Compton scattering

When an electron or positron with energy ϵ scatters a photon of similar energy, the cross-section is approximately $\sigma_T \epsilon^{-2}$, and there is on average no exchange of energy. But when a 'soft' photon (of energy $< \epsilon^{-1} m_e c^2$) is scattered, its energy is boosted by a factor $\sim \epsilon^2$. Thus the electrons and positrons in an ultrarelativistic plasma will share their energy with any soft photons, created by (e.g.) bremsstrahlung, on a timescale of only a few times $(n_e \sigma_T c)^{-1}$.

2.3.2 *Double Compton scattering*

Double Compton scattering essentially is a photon-producing mechanism. The ratio of double to single scattering cross-sections is α for $\epsilon > 1$ and $\alpha\epsilon^2$ for $\epsilon < 1$. As the Compton scattering cross-section is $\sigma_T\epsilon^{-2}$ for $\epsilon \gg 1$, one sees that this mechanism cannot compete with bremsstrahlung except when $\epsilon \simeq 1$.

2.4 TIMESCALES

The relative timescales associated with these processes are of interest. Thus $t_{\text{pair production}}$ and t_{brems} both exceed $(n_e\sigma_T c)^{-1}$. When $\epsilon \lesssim 10$ the timescale for photons and pairs to establish an equilibrium ratio is shorter than that for producing new photons by bremsstrahlung; at higher energies, bremsstrahlung has a relatively larger cross-section. But the timescale on which any new photons share their energy with the existing particles via Compton scattering is always shorter still.

3 The evolution of a γ-ray fireball

To illustrate the relevant points, we consider a homogeneous fireball of total energy $\mathscr{E}_f$, initial radius R_i, in which photons of energy $\epsilon_i m_e c^2$ are produced 'instantaneously' (i.e. on a timescale $\lesssim R_i/c$). Ionized plasma (assumed mainly hydrogen) of density n_i and total mass M_i may be mixed with the radiation. We write $\mathscr{E}_f = \eta M_i c^2$. The fireball is then radiation-dominated if $\eta > 1$ and matter-dominated otherwise. For instance, a lump of matter impacting on a neutron star, or colliding with a similar lump near a black hole, would yield a fireball with $\eta \gtrsim 0.1$; and magnetic flares, or effects involving pair production in strong magnetic fields, could in principle yield $\eta > 1$.

The ratio between N, the photon density, and n_i will be given by

$$N/n_i = (\eta/\epsilon_i)(m_p/m_e) = 1836\,\eta/\epsilon_i. \tag{1}$$

With these initial conditions a number of phenomena will then occur, with different relative importance according to the values of the initial parameters.

The relative importance of the physical mechanisms in Section 2 and the main features of the fireball evolution, can be expressed in terms of the parameter $\theta = \mathscr{E}/R^2 \simeq L/(Rc)$, which is related to all optical depths of interest. It is convenient to measure θ in units of $m_p c^2/\sigma_T = 2.26 \times 10^{21}$ erg/cm². We will use θ^* to denote the parameter θ measured in these units. In this system of units we have $\theta^* = \eta\tau_{\text{es, gas}}$, where $\tau_{\text{es, gas}}$ is the optical depth due to Compton scattering of photons on the electrons of the ionized gas (density n_i). The optical depth τ_{pp} to pair production will be given by the expression:

$$\tau_{\text{pp}} = \theta^*(m_p/m_e)\,\epsilon^{-3}. \tag{2}$$

We thus have $\tau_{\text{pp}}/\tau_{\text{es, gas}} = \eta(m_p/m_e)\,\epsilon^{-3}$.

The entire evolution of the fireball can be followed in the (θ^*, ϵ) plane. In this plane (Fig. 1) a number of interesting features appear. (For sake of definiteness we have used a value $\eta = 0.1$ in the diagram.) The line (a) marks the value $\tau_{\text{es, gas}} = 1$, or $\theta^* = \eta$. The line (b) marks the value $\tau_{\text{pp}} = 1$, or $\theta^* = (m_e/m_p)\,\epsilon^3$. (Actually, we do not expect precisely the value at $\epsilon = 1$, but rather a rounded shape, depending on the detailed expression for the pair-production cross-section.)
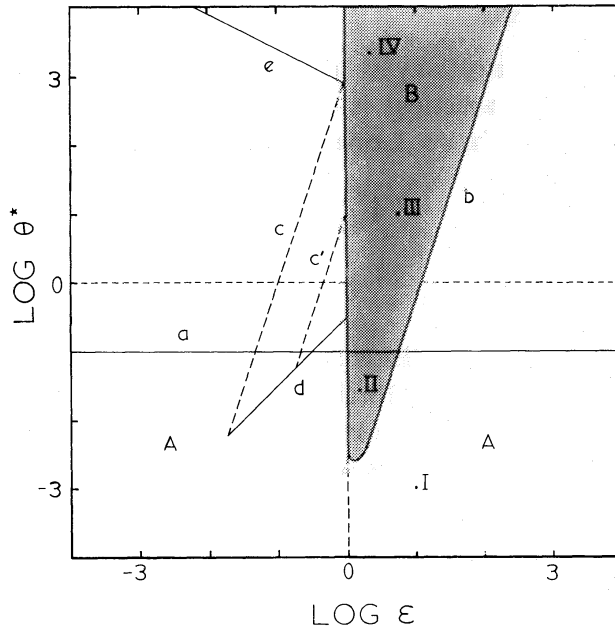
Figure 1. Plot of the (θ^*, ϵ) plane, for the value $\eta = 0.1$. In this diagram θ^* is the quantity $\mathcal{E}/R^2 \simeq L/(Rc)$, measured in units of $m_p c^2 \sigma_T^{-1}$; ϵ is the average photon energy, measured in units of $m_e c^2$. The shaded area (B) represents the region of abundant pair production. Points labelled I, II, III, IV represent typical starting conditions leading to the corresponding cases in the text. Higher-energy effects appearing for $\epsilon > 100$ have not been taken into account. See text for further explanations. Pair-production effects become important even when L is only $(\alpha^{-3} r_e/R)$ times that corresponding to a blackbody with radius R and $kT \simeq m_e c^*$. (r_e is the classical electron radius.)

There are four possible cases, depending essentially on the value of θ^*.

Case I. If injection occurs in the region labelled A, with $\theta^* < \min (\tau_{pp} = 1, \tau_{es, gas} = 1)$, there will be no opacity effects at all and radiation will escape without being substantially modified. The presence of narrow nuclear lines in the spectrum of an exploding source would therefore suggest that the initial energy was injected in region A.

Case II $(\tau_{pp} > 1, \tau_{es, gas} < 1)$. In the dashed zone (B), abundant pair production occurs, giving rise to the most interesting phenomena. The requirement $\tau_{pp} > 1$ may be fulfilled in actual physical situations. A typical γ-ray burst at a distance of D parsec will emit a total energy of $10^{33} D^2$ erg. If the initial photon energy is in the region $\epsilon > 1$, the opacity τ_{pp} will exceed 1 for $R < 2 \times 10^7 D\epsilon^{-3/2}$. The appropriate conditions may thus exist in practically all stellar sources at a distance of a few hundred parsecs.

If $\tau_{pp} > 1$, then on a timescale $t_{pair\, production}$ the process $\gamma + \gamma \leftrightarrow e^+ + e^-$ yields a photon–lepton gas, and on the same timescale the photons and the pairs establish a thermal Bose and Fermi distribution respectively at the same kinetic temperature (because the radiation is dilute compared to a blackbody at the same temperature, both these distributions will be essentially relativistic Maxwellian). On a timescale t_{brems}, new photons are produced, which share their energy with the photons and leptons already present. The fireball thus cools on a timescale $t_{brems}(\epsilon)$, until $\epsilon \simeq 1$, when the number of e^+–e^- pairs starts to fall below the number of photons.

If $\tau_{es, gas} < 1$ the fireball then becomes transparent and radiation emerges with average energy $\epsilon \simeq 1$.

In the (θ^*, ϵ) plane the path of the fireball is parallel to the axis of the abscissae, as $\mathscr{E}$ is constant; and R does not change appreciably, since the timescales for evolution of the spectrum are less than R/c.

Case III $(\tau_{\mathrm{pp}} > 1, \tau_{\mathrm{es,gas}} > 1)$. The situation changes again if there is enough plasma in the fireball to make $\tau_{\mathrm{es,gas}} > 1$. In this case, after photons have been degraded to $\epsilon = 1$ and pairs have annihilated, the material is still opaque. Unless this plasma is 'anchored' (by — for instance — a strong magnetic field) it will undergo adiabatic expansion before photons can escape. The total radiation energy decreases proportionally to R^{-1} until it can leak out. For $\tau_{\mathrm{es,gas}} \gg 1$ the expansion will attain a terminal speed $v \simeq \eta^{1/2} c$ $(\eta \ll 1)$, and radiation will escape when $\tau_{\mathrm{es,gas}} (\propto R^{-2})$ has fallen to a value of the order of c/v. This happens when:

$$R = R_{\mathrm{esc}} \simeq R_{\mathrm{i}} \tau_{\mathrm{i}}^{1/2} \eta^{1/4} \qquad (3)$$

where τ_{i} is $\tau_{\mathrm{es,gas}}$ at the injection time. If $\eta \gg 1$ the terminal velocity is relativistic with Lorentz factor $\sim \eta$, and there are high Doppler effects and relativistic corrections to the timescale.

In the absence of large relativistic corrections, the peak luminosity attained by the fireball will be reached for $R \simeq R_{\mathrm{esc}}$, its value being given by the ratio between the remaining energy after adiabatic expansion and the timescale for expansion, namely:

$$L_{\mathrm{max}} \simeq (R_{\mathrm{esc}}/v)^{-1} \mathscr{E}(R_{\mathrm{esc}}/R_{\mathrm{i}})^{-1} \simeq R_{\mathrm{i}} m_{\mathrm{p}} c^3 \eta \sigma_{\mathrm{T}}^{-1}. \qquad (4)$$

It is interesting to observe two facts: first of all L_{max} does not depend strictly on $\mathscr{E}$ (although it depends on the two unknown parameters η and R_{i}, both appearing in $\mathscr{E}$); secondly, if the energy is produced by the infall of matter on to a body of mass M, with subsequent release of gravitational energy and efficiency $(R/R_{\mathrm{s}})^{-1}$, where R_{s} is the Schwarzschild radius, then $L_{\mathrm{max}} \simeq L_{\mathrm{Edd}}$, the Eddington limiting luminosity.

Thus the fireball follows a new path, labelled (c) or (c′) in the (θ^*, ϵ) plane. Adiabatic expansion requires $\theta^* \propto \epsilon^3$. The point corresponding to the final values of θ^* and ϵ has coordinates $\epsilon_{\mathrm{f}} = (R_{\mathrm{i}}/R_{\mathrm{esc}})$ and $\theta_{\mathrm{f}}^* = \theta_{\mathrm{i}}^* (R_{\mathrm{i}}/R_{\mathrm{esc}})^3$. With the expression (3) for $R_{\mathrm{i}}/R_{\mathrm{esc}}$, and $\tau_{\mathrm{es,gas}} = \theta^* \eta^{-1}$, one easily obtains the result that all final points lie on the line $\theta^* = \eta^{1/2} \epsilon$, labelled (d) in Fig. 1. (Note that the length of the paths in Fig. 1 has nothing to do with the time it takes the fireball to cover them.)

Case IV $(\tau_{\mathrm{es,gas}} \gtrsim \alpha^{-1})$. If the bremsstrahlung cooling of the ionized gas is faster than the expansion characteristic time R/v, radiation will be thermalized. This occurs for $\epsilon \leqslant 1$, when the ratio of the radiation energy density $\mathscr{E}/R^3$ to the emissivity $j(T)$ is less than or equal to R/v, namely:

$$\mathscr{E}/R^3 \leqslant j(T)(R/c)\, \eta^{-1/2}. \qquad (5)$$

With the known expressions for $j(T) \simeq \alpha \sigma_{\mathrm{T}} c^2 m_{\mathrm{e}}^{1/2} (kT)^{1/2} n_{\mathrm{e}}^2$ and for $n_{\mathrm{e}} \simeq \mathscr{E}/(\eta R^3 m_{\mathrm{p}} c^2)$, one obtains:

$$\theta^* \geqslant \eta^{5/2} \alpha^{-1} (m_{\mathrm{p}}/m_{\mathrm{e}})\, \epsilon^{-1/2} = 2 \times 10^5\, \epsilon^{-1/2} \eta^{5/2}. \qquad (6)$$

The function $\theta^* = 2 \times 10^5 \epsilon^{-1/2} \eta^{5/2}$ is labelled (e) in Fig. 1. In the previous case (III) energy density and average energy were unrelated. Now, on the contrary, the radiation will assume a blackbody spectrum with temperature given by

$$\epsilon = kT/(m_{\mathrm{e}} c^2) = (\mathscr{E}/R^3/9 \times 10^{24})^{1/4}. \qquad (7)$$

Here the radiation energy density $\mathscr{E}/R^3$ is measured in erg cm^{-3}.

The blackbody will cool as the fireball expands: radiation will again escape when the optical depth drops to $\sim c/v$. The path in the (θ^*, ϵ) plane again follows a $\theta^* \propto \epsilon^3$ law, parallel to the previous, non-blackbody case. The final value of θ^* will be the same as for Case III, but the final value of ϵ will be related to the blackbody value and not to $\epsilon = 1$. This result reminds one of the classical theory of supernova explosions: the maximum luminosity will be reached when the optical depth of the remnant becomes approximately equal to c/v_{exp}.

4 Applications to γ-ray bursts

The spectrum of γ-ray bursts (if we disregard a possibly power-law high-energy tail with slope > 2) can be approximated by the expression:

$$dN/dE = KE^{-1} \exp(-E/E_c) \tag{8}$$

with similar values of $E_c \simeq 0.3\,\text{MeV}$ for most sources (Strong, Klebesadel & Evans 1975). Such a spectrum might suggest a thermal-bremsstrahlung origin in a plasma with temperature $T_c = E_c/k$, and many models do indeed make reference to a thermal origin (Ruderman 1975). On the other hand there is a possible difficulty if the energy production mechanism is located at the surface of a neutron star (or in the inner region of an accretion disc) because GMm_p/R is then $\simeq 100\,\text{MeV}$ ($\gg kT_c$). But our discussion demonstrates that such γ rays would have been degraded below the pair-production threshold and that it is no coincidence that $kT_c \lesssim m_e c^2$.

The actual form of the spectrum and the time structure of the observed bursts depend on the detailed radiative transfer and dynamics in the fireball, which we do not attempt to discuss here.

We mention, however, three effects:

4.1 HIGH-ENERGY TAIL

A high-energy tail will always exist. Suppose that a spherical distribution of energetic photons of energy $\epsilon_i m_e c^2$, with number density N_i, has been produced in the situation previously described as Case II. While the inner photons must interact with each other, thus starting the series of processes we have outlined, about one-half of the photons in the external layer will escape. The thickness of this layer is $\lambda = \tau_{pp}/R_i$, and the total energy is $E_i = 2\pi R_i^2 \lambda N_i \epsilon_i$. The total energy emitted in the process will be $E_t = (4/3)\,\pi R_i^3 N_i \epsilon_i$. Thus $E_t/E_i \simeq (2/3)\,R_i/\lambda$. The choice of appropriate values of ϵ_i and of R_i/λ will reproduce the high-energy tail of the source of interest. One can obtain crude determinations of the distance and size of specific bursts. Comparison of the emitted energies in the burst of 1972 April 27 (Metzger *et al.* 1974) gives $\epsilon_i \simeq 10$, and $\tau = R/\lambda \simeq 8$. But an upper limit to R is of the order of $c\,\Delta t$. Therefore the luminosity is given by:

$$L \simeq N_i R^2 \epsilon_i c m_p c^2/(4\pi) \simeq \tau \epsilon_i^3 R c m_e c^2/(4\pi\sigma_T) \lesssim \tau \epsilon_i^3 c^2 \Delta t m_e c^2 = 6 \times 10^{41}\,\Delta t\ \text{erg/s.} \tag{9}$$

For $\Delta t \simeq 30\,\text{ms}$ this yields $L < 2 \times 10^{40}\,\text{erg/s}$. Unless we invoke relativistic effects or special geometry, the distance is therefore less than 4 kpc, because $L \simeq 10^{33} D^2\,\text{erg/s}$, with D measured in pc.

4.2 OTHER COOLING PROCESSES

Apart from the high-energy tail, the emerging photons will generally result from annihilation when the fireball has cooled to $\epsilon \simeq 1$, but will have undergone repeated Compton scattering

during their escape. The cumulative amount of energy which they lose during these scatterings depends on the electron temperature at the time when $\tau_{es} \simeq 1$. In Case I this will still be $\sim \frac{1}{2}$ MeV. However, if the fireball were embedded in a strong magnetic field, then cyclotron cooling could reduce the electron temperature below $m_e c^2/k$ *before* annihilation, so that most annihilation photons are cooled by Compton recoil and the emergent spectrum would be softer. In Cases III or IV there are additional possibilities of adiabatic (and perhaps radiative) cooling. This, however, has the effect of converting energy into kinetic energy, which can be reconverted if the expanding fireball is braked by external matter.

4.3 IMPACT ON EXTERNAL MATERIAL

In cases where $\tau_i \gg 1$, the bulk of energy goes into kinetic energy. If the kinetic energy is reconverted into radiation by hitting external matter, then a higher γ-ray luminosity than (7) can result. The optimum case is when the fireball impacts on an equal amount of mass (initially at rest) at a radius R_{esc}. About one-half the kinetic energy will be randomized and will escape without further adiabatic losses. This gives a luminosity larger by a factor (R_{esc}/R_i) than the one previously calculated:

$$L \simeq R_{esc} m_p c^3 \eta \sigma_T^{-1} \simeq 5 \times 10^{40} \eta (R_{esc}/10^9 \, \text{cm}) \, \text{erg/s}. \tag{10}$$

Impact on lumps of matter of appropriate size might reproduce the temporal behaviour (multiple structure) of γ-ray bursts.

5 Conclusions

If a source with the energy and dimensions inferred for γ-ray bursts were to generate photons with $\epsilon \gg 1$, then pair-production effects would create a 'photosphere' which would prevent these γ rays from escaping directly; we would instead see their energy only after it had been transformed into photons below the pair-production threshold. We suggest this as an explanation for the standardized character of γ-ray burst spectra which should be incorporated in any plausible specific model for the phenomenon.

Although we have focused on one application, we note finally that some aspects of this work, particularly the constraints on L, Δt and R expressed in (4), (8) and (9), may be relevant to larger-scale outbursts such as those in quasars.

Acknowledgments

References

Metzger, A. E., Parker, R. H., Gilman, D., Peterson, L. E. & Trombka, J. I., 1974. *Astrophys. J.,* **194**, L27.
Ruderman, M., 1975. *Ann. N.Y. Acad. Sci.,* **262**, 164.
Strong, I. B., Klebesadel, R. W. & Evans, W. D., 1975. *Ann. N.Y. Acad. Sci.,* **262**, 145.