

A QUANTITATIVE-COMPARATIVE APPROACH TO ANALYSIS OF DISTORTION IN MENTAL MAPS

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A quantitative procedure for analyzing and comparing distortion in mental maps is presented. Using a microcomputer and digitizer, and by developing a distortion index, comparisons are made by direct analysis of mental maps. A case study of the mental maps of Israel of high school and university students showed a tendency to generalize shape to simple geometric forms, with foreshortening of north-south lines and exaggeration of east-west distances.

Key Words: mental maps, distortion, quantitative, analysis, comparison.

This paper presents a simple quantitative framework and procedure for analyzing and comparing distortion in mental maps. The comparisons are made by direct analysis of the raw maps using a digitizer and microcomputer and through development of a distortion index. A case study is provided by maps of Israel drawn by a sample of high school and university students.

Several methods have been used in cognitive mapping to elicit the organized representations of a given environment. These include the direct-mapping procedure adopted by Lynch [15] and others, construction of preference maps [5, 12, 13] and the use of questionnaires in more sophisticated statistical procedures [11], multidimensional scaling procedures [20], and computer scanning techniques [6]. The direct-mapping technique is marked by attempts on the part of the individual to create a map of an environment as mental image. It is thus distinguished from other methods involving maps constructed on the basis of information provided by respondents and drawn indirectly by the investigator.

Where a map is considered the most important element in the analysis, a free-mapping procedure is desirable. It represents a direct line to the respondent without much of the prompting involved in some questionnaire methods which can influence the research subjectively [29]. Although direct mapping techniques involve difficulties such as in differentiating drafting ability from place cognition [27], such methods continue to be widely used as they offer relatively uncomplicated access to the respondent and obviate the need for detailed questionnaires and sophisticated statistical and mathematical procedures.

Distortion in Mental Maps

Tobler [25, 26, 27] has been concerned with understanding the distortion in maps caused by different map projections. This interest has been extended to the implicit projections underlying maps distorted with little preconceived logic or purpose, such as medieval maps or mental maps. This concern presumes deviations in overall shape and its elements from some other given shape, whether this be the three-dimensional globe or a map drawn correctly to scale along some particular, predefined projection.

In the study of mental maps both the measurement of distortion of shape and the examination of ways in which such maps are constructed have attracted somewhat less attention in the geographic literature than the analysis of the content of such maps. Whereas in the analysis of names and places, the development of procedures for their cartographic representation is well-developed [7, 13, 15, 17, 22, 23], the measurement of distortion suffers from technical problems including the measurement of large numbers of lines or angles (distances and directions) on maps, and from its essential nature which demands mathematical, rather than statistical, approaches.

Although Tobler has discussed the geometry of mental maps [27], he offered little in the way of developing a method or technique for measuring the distortion of such maps. Practical experimental approaches have recently been adopted by Richardson [20] and Gale [10]. Others have approached the problem of techniques from the standpoint of empirical study and have adopted methods that are either unique to the area under study or which involve numerous calculations so that replication of the technique is cumbersome.

Sanders and Porter offered a technique for comparing the shapes of mental maps of Africa [24]. Their empirical study encountered the problem of scale in comparing such maps, a problem which Tobler noted when approaching the comparison problem from a mathematical and theoretical viewpoint [27]. The solution offered by Sanders and Porter used a standardization of scale in their map set, permitting comparisons of shape in mental maps drawn by students in Minnesota and Tanzania. Their method is useful only in continuously bounded areas where the boundaries are clearly recognized.

Pocock adopted a verbal approach in his study of mental maps of Durham [19]. Though useful for the individual case, it required even more manual drafting than the Sanders and Porter method. In an earlier study in Dundee [18], Pocock measured mean conceptualizations of distance and orientation estimates to landmarks from different areas in the city.

Measurement and Comparison of Distortion in Mental Maps

Both Sanders and Porter and Tobler have noted the difficulties in comparing distortions in maps drawn to different scales [24, 27]. Because the maps are each at different scales (and, as Tobler might add, at different projections), comparison is rendered difficult due to a lack of common points or lines from which to begin the comparison. The scale factor is further complicated by distortions in the maps themselves, the effect being to change the scale of the maps in different directions.

Orientation is encountered as an additional problem. When a set of maps is to be compared, what orientation should be given? Distortion affects scale and orientation. Sanders and Porter gave instructions that north was to be at the top of the map, although there is no guarantee that the respondents in the mapping exercise would or could carry out that particular instruction. Another problem involved a shift in the coordinate system used in the representation of the mental maps from map to map. Thus, horizontal and vertical shift of the co-ordinates must be considered in addition to scale and orientation before two maps can be compared.

An Analytic Approach to Mental Maps

Whereas in an ideal situation it might be desirable to compare the overall shape and distortion of the mental maps, given the infinite number of points which make up the total map outline and content, this is difficult from a practical point of view. In the specific case of mental maps, an additional problem lies in the adequate recognition of all segments of a given line. Thus, in order to permit a rapid comparison among two or more maps, reducing them to a finite and even small number of points is desirable. By transforming the coordinates of these points, correcting for rotation, coordinate shift and scale change, it is possible to measure the degree of association between the configurations and the distortion along any line or set of lines within the map or graph [27, 28]. Thus the object of the exercise is not to measure shape and shape distortion, itself a separate and important problem [3, 4], but to compare the distortion and overall relative deviations of a set of points with another set taken as true.

The approach used to neutralize the effect of rotation, translation and scale change involves a least-squares Euclidean transformation well known in the literature of statistics, cartography and geodesy [9, 14, 21]. The coordinates of the mental map undergo a transformation so that the sum of squares of the distances from the "true" or given points to the "transformed" points is minimal.

The required transformation is characterized by four quantities:

- α — the change of scale factor
- θ — angle of rotation of the coordinate axes
- A — shift of the x-coordinate
- B — shift of the y-coordinate

Denoting by $F(A, B, \alpha, \theta)$ the sum of the squares of the distances between the true points and the transformed points, we obtain four equations for A, B, α , θ by setting all four partial derivatives of F to 0. The computational procedure for A, B, α , θ , and F is provided in the Appendix. It can be proved that this "best-fit" solution is unique.

Setting $D = \sqrt{F}$, we see that D is in units of length and since D (or F) is the quantity minimized by the transformation, it is most suitable as a basis of comparison between different mental maps. D thus becomes the *distortion distance* of the mental map.

D by itself is limited to comparing different mental maps of the same true map. The value of D in no way indicates whether the distortion is "large" or "small" which raises the question of whether there is some form of measuring distortion in an "absolute" way, in the same sense that correlation in statistics is an absolute measure (between -1 and +1). It can be shown that there is a maximum value that D can achieve, D_{max} , and that D_{max} is obtained when all the points on the mental map coincide in a single point. The best-fit solution always brings the mean center of the mental map to the mean center of the true map, so D_{max} can be easily calculated (see Appendix).

The existence of D_{max} naturally suggests the definition of a *distortion index* of a mental map as the ratio of distortion distance D to D_{max} :

$$DI = 100 D/D_{max}$$

DI, which is always between 0 percent and 100 percent is a dimensionless value the size of which indicates the amount of distortion regardless of the scales of the true or mental maps.

As a further refinement of the distortion distance, we develop the concept of the *box of distortion* which indicates the orientation of maximum and minimum distortion, whether by "stretching" or by "compressing." This measure is analogous to Bachi's "cross of dispersion" and resembles Tissot's indicatrix as applied to the whole map rather than locally [14, 21].

The underlying idea is to identify the principal direction, if any, along which the mental map is distorted. This is done as follows:

Consider a straight line, L forming an angle, φ , with the x-axis. Let S be the straight line segment joining a true point (x_i, y_i) with its corresponding mental map point (a_i, b_i) after transformation (Figure 1). Denoting by P_i the length of the projection of S_i on L, we seek φ such that $\sum P_i^2$ is maximal. The solution is set out in the Appendix. Setting $D_1 = \sqrt{\sum P_i^2}$, we locate the box of distortion at the mean center, its long side D_1 forming an angle φ with the x-axis. Its short side, D_2 is $\sqrt{\sum Q_i^2}$, where Q_i is the length of the projection of S_i on a line forming an angle $\varphi + \pi/2$ with the x-axis. (When φ maximizes D_1 it also minimizes D_2 .)

The box of distortion provides more information than the distortion index about the way in which the mental maps are distorted from the true-scale map. It also provides the orientation of the primary distortion. Centered on the mean center, the box of distortion can be presented graphically. The box indicates orientation of distortion and the degree of distortion along a major axis. Its diagonal is the distortion distance D, as can be readily verified, so the box informs us at a glance of the amount of distortion and its primary orientation.

The geostatistical measures of central tendency and dispersion developed by Bachi and others [14] have been used to some extent in studying mental maps [9, 16]. In our study, it was possible to calculate the mean center and measures of dispersion for each of the control points of a set of mental maps. In this way, the mean center, standard distance, cross of dispersion, principal axes and oblongity index were calculated for each of the points in the

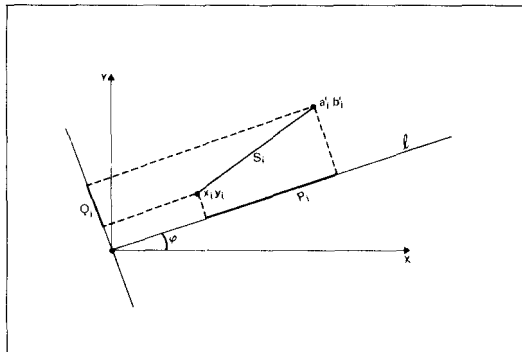


Figure 1. Points from true-scale map and transformed points from the best fit solution.

set of maps. Examination of the geostatistical parameters permits the recognition of the degree of error in the location of each of the control points. The cross of dispersion is analogous to the axes of the standard errors of the estimate of each point. In addition to measures of distortion and dispersion, simple distance distortion along any given set of lines can be calculated and compared with the true-scale map; these data can be presented either in graphical form or in tables.

Case Study

The accuracy of spatial information contained in mental maps of children has been shown to increase with age [12, 16]. As they get older, their information curves rise rapidly as they acquire and store away large amounts of information. Approaching adulthood, the rise in the information curve becomes less steep, eventually flattening out. It can be hypothesized that the smallest distortions in mental maps will occur in those drawn by more mature and experienced individuals and that differences will be smallest among maps of older individuals. It is also suggested that there will be a tendency to overestimate the size and importance of geographic features that are well known to the individual and to underestimate features that are less well known, and the exaggerations for features closest to the home area are greater among females [8].

As part of a more general study of mental maps of Israeli students, eight groups of pupils and students (approximately 300 individuals ranging in age from 12 to 40) were presented with blank sheets of paper measuring 34×22 cms and requested to draw a map of Israel within a limited period of time [1, 2]. No specific instructions on orientation, borders, size of map or type of information to be included were given. Included among the objects of the wider study was the measurement of map distortion.

Eight points, which could be easily recognized on all the maps, were used as surrogates for the complete map outline. Using the parameters of distortion, central tendency and dispersion

TABLE 1
DISTORTION INDICES AND RATIOS OF PERCEIVED DISTANCES TO TRUE DISTANCES FOR
EIGHT SAMPLE GROUPS

Group	Distortion Index	Distance Distortions ^a						
		Tel-Aviv	Kinneret		Dead Sea		Jerusalem	Eilat
			North	South	North	South		
Ein Harod VII	21.3	0.98	1.70	1.70	1.14	1.01	1.04	0.88
			1.29	1.27	1.42	1.12	1.22	0.87
Ein Harod XI	19.1	1.14	1.59	1.66	1.20	1.07	1.07	0.89
			1.28	1.28	1.43	1.14	1.36	0.84
Nazareth VII	31.2	0.95	1.82	1.80	1.14	0.97	1.03	0.75
			1.31	1.28	1.43	1.08	1.27	0.73
Nazareth XI	19.2	0.93	1.74	1.76	1.08	0.91	1.04	0.89
			1.21	1.24	1.38	0.99	1.36	0.88
Qiryat Haim VII	23.2	0.85	1.51	1.49	1.10	0.90	1.00	0.80
			1.15	1.09	1.38	1.02	1.35	0.81
Qiryat Haim XI	18.4	0.90	1.72	1.71	1.11	0.96	1.08	0.88
			1.25	1.26	1.41	1.08	1.45	0.90
University	13.4	0.89	1.61	1.60	1.05	0.93	0.96	0.94
			1.19	1.18	1.31	1.04	1.25	0.95
Teachers' College	13.6	0.79	1.50	1.51	0.97	0.89	0.88	0.95
			1.07	1.06	1.20	1.00	1.18	0.99

^a For distance distortions, upper figures represent lines drawn from Haifa, lower figures represent lines from Tel-Aviv.

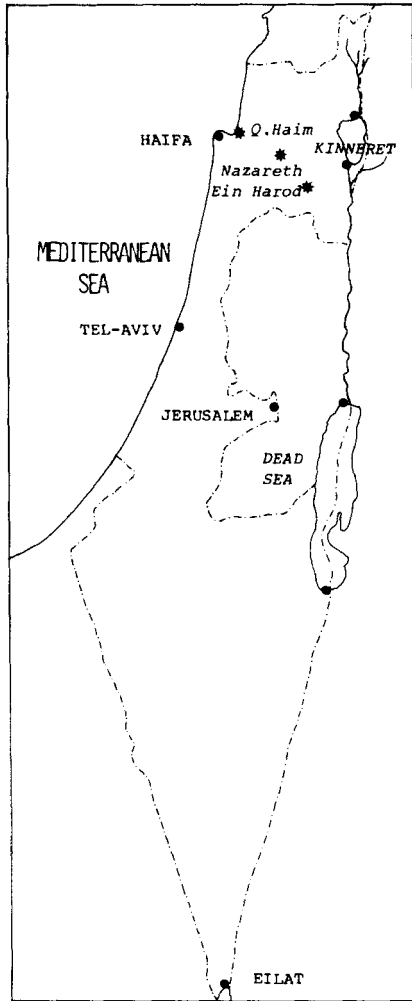


Figure 2. Israel, location of eight control points and sample areas.

outlined above, a rapid and simple technique was used in computation and plotting by using a microcomputer with graphic capabilities, plotter, digitizer and hard-copy unit. Although only eight common points were used as control points, there is no limit imposed on using only common points as control points. A larger number of points could have been chosen and the presence or absence of the corresponding points on the mental map noted. The DI would then be calculated on the points present and in respect to the individual map. For a set of maps, relative weights can be assigned on the basis of the number of times each of the control points is recognized in each set of maps.

The eight common points from a true-scale map were digitized and stored. These serve as the points from which comparisons are made and transformations calculated. The corre-

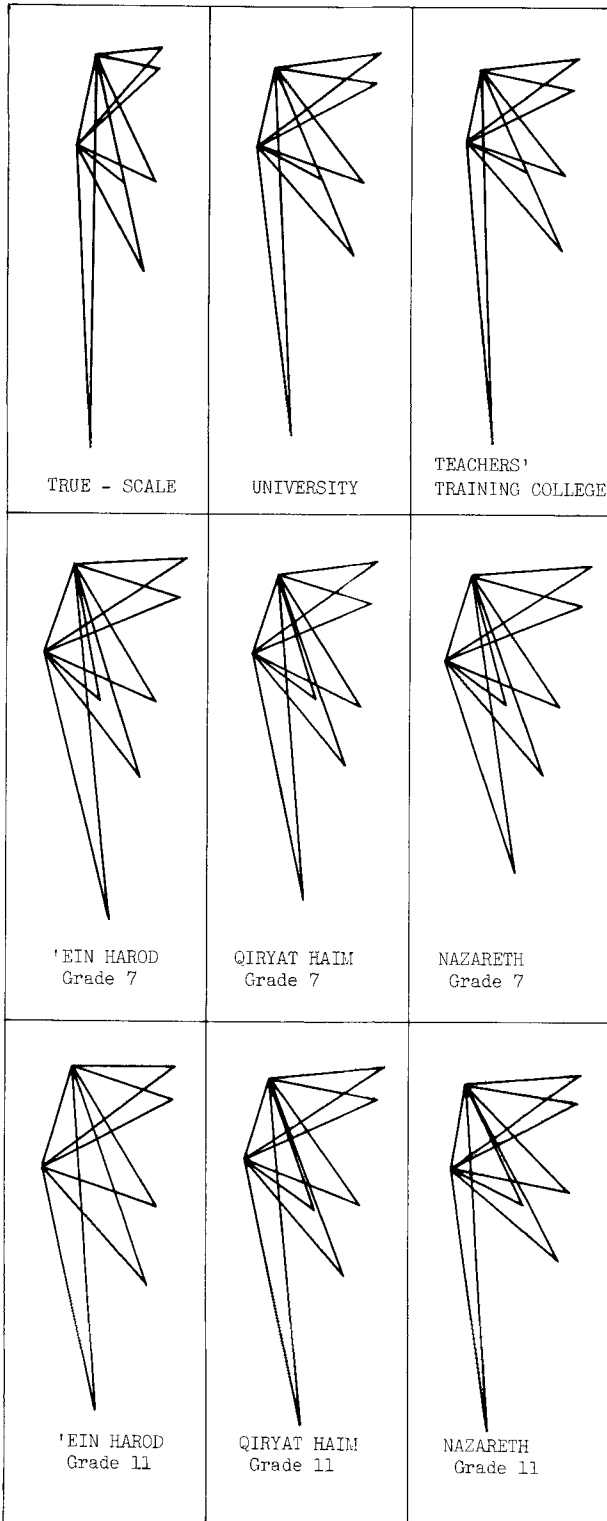


Figure 3. Distortion in mental maps of eight sample groups compared with true-scale map along thirteen selected lines (see Table 1).

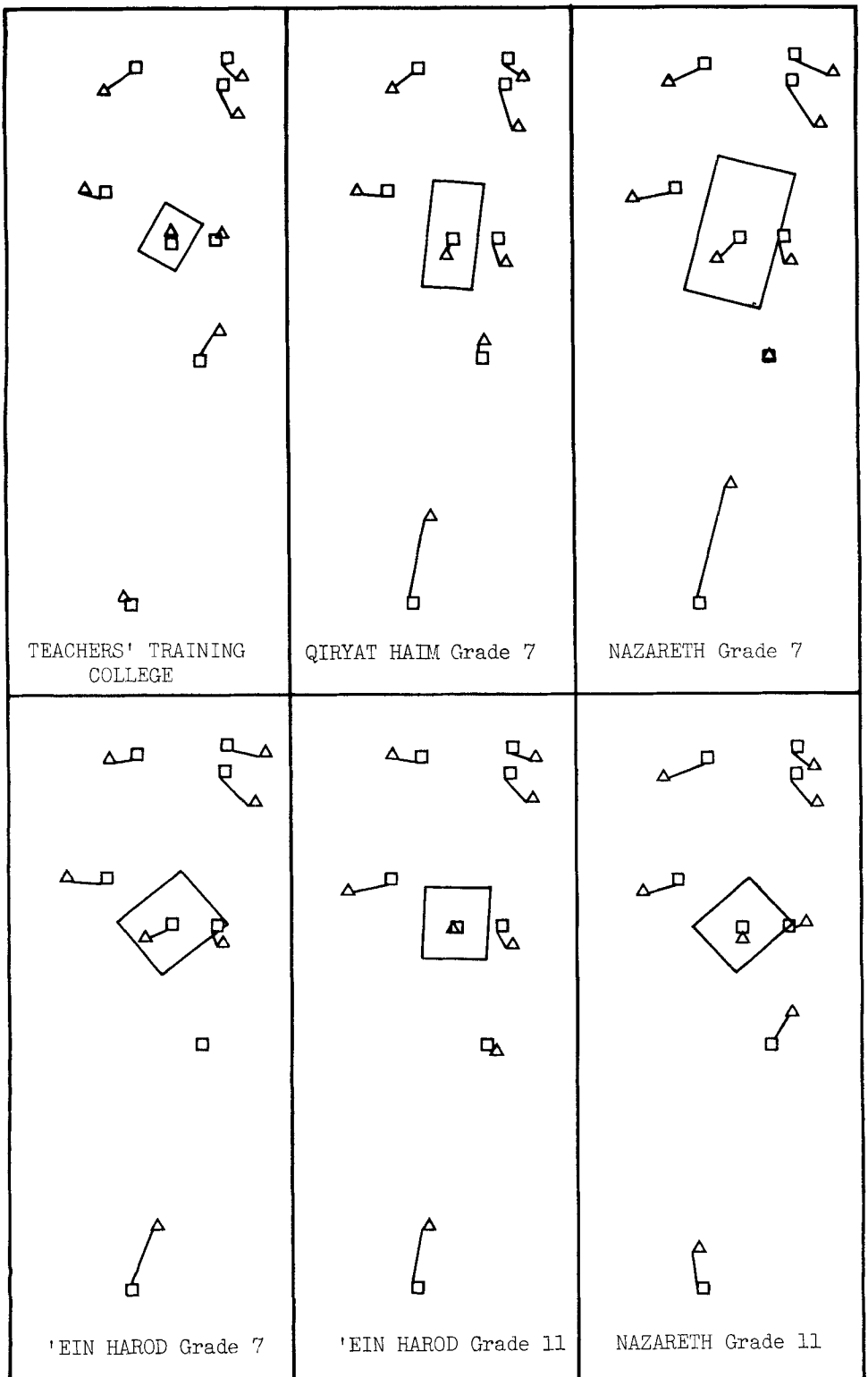


Figure 4. Location of eight control points on the mental maps as compared with true-scale map for six sample groups. Squares represent the locations of the points from the true-scale map; triangles represent the respective points from the mean mental map for the group. The "boxes of distortion" from the mean mental map are also indicated.

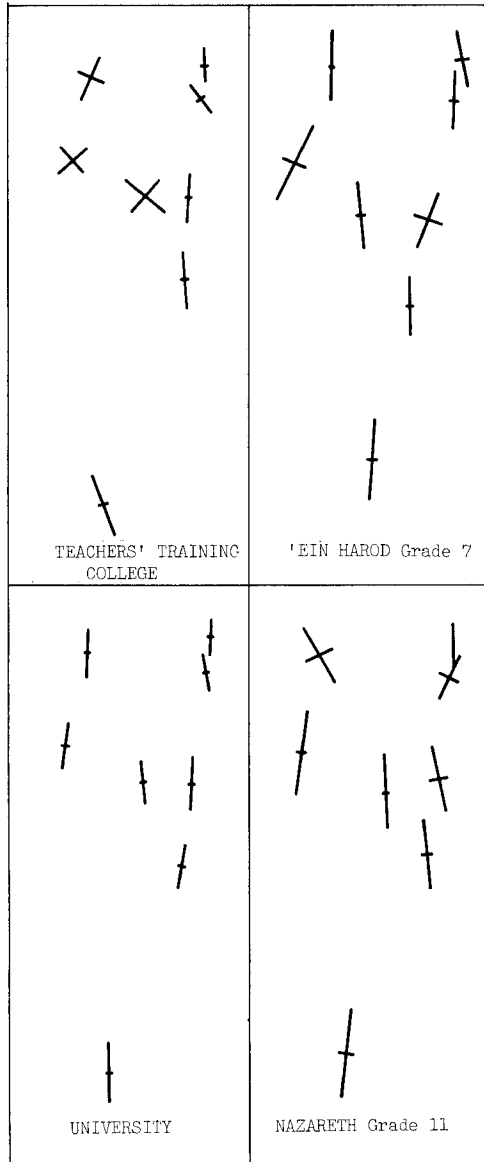


Figure 5. Crosses of dispersion for eight control points for four sample mean mental maps.

sponding n points from a set of m mental maps were then digitized. Transformations of these points are easily and rapidly calculated. The program developed permits optional summary diagrams and statistics for each individual map in addition to the storage of data until the complete set of maps has been entered and the display of mean maps and statistics for the whole set has been obtained. Total time for digitizing a set of thirty maps, computation and production of summary diagrams and tables in hard-copy form is under twenty minutes using raw data (the original maps produced by the respondents).

The samples analyzed included eight sets of maps produced by Grade 7 and Grade 11 students in each of three settlements—Qiryat Haim, a suburb of Haifa; Upper Nazareth, a development town in Lower Galilee and Ein Harod, a kibbutz in Northern Israel (Figure 2). Two further samples included were the maps of first-year geography majors at the University of Haifa and students at a teachers' training college north of Tel-Aviv.

The mean distortion indices for each of the eight sets ranged from 13.4 percent to 31.2 percent (Table 1). In general, the distortion is less exaggerated the more mature the group samples. Smallest distortions occurred with the samples from the teachers' training college and the university and greatest occurred with the Grade 7 pupils, in particular those from Upper Nazareth, a town with a high percentage of children from immigrant homes as compared with the two other settlements. These data tend to support the hypothesis stated above (Figures 3 and 4).

There was a general tendency to overestimate the distances of lines in an east-west direction. This is illustrated by showing the lines drawn between Haifa and Tel-Aviv (Figure 3) and each of the six points that appear in Table 1. The same data are shown in Figure 4; the relatively more peripheral location of the points from the mental maps as compared with the "true-scale" points is easily noted. The exaggeration is particularly pronounced in the north, as shown by the overestimates of the distance from Haifa to Lake Kinneret by a factor of 1.5–1.8. Part of this overestimate is the result of an exaggeration of the size of Lake Kinneret. There is also a foreshortening of the shape of the country in a north-south direction, which is brought out in the form of the underestimates of the distances between Tel-Aviv and Haifa and between both Haifa and Tel-Aviv and Eilat. The foreshortening ratios appear to be smaller than those of the corresponding east-west exaggerations, probably because the latter, especially in the north, involve shorter absolute distances. This apparent anomaly in brought out when comparing distortion along the lines and the "box of distortion," leading to the conclusion that the distortion is caused both by an overestimate of distances close to the place of residence and an underestimate of distances in more remote areas.

The geostatistical parameters show that much of the deviation within the standard distance occurred close to the north-south axis for each point (Figure 5). This finding indicates that there was greater variability in locating the north-south coordinate for each point on the map than the east-west coordinate. The points indicating the location of the Dead Sea, Lake Kinneret and Eilat supports this finding.

An analysis of differences between distance distortion on maps drawn by male and female respondents showed mixed results. In general, there was a tendency for female respondents to produce greater errors for both overestimates and underestimates of distance. These differences were most marked on the shortest lines on the maps (Haifa to Lake Kinneret), partially corroborating the findings of Driever [8]. Male-female differences appeared to be least clear among the university group (Table 2).

In general, there was a tendency to generalize shape to simple geometric forms and to exaggerate the close and familiar at the expense of the more distant and less familiar features. Questions about the need for improving the level of teaching elementary aspects of the national map have already been raised from analyses of the same data [1, 2].

Conclusions and Suggestions for Further Research

The technique outlined above illustrates a simple and rapid means for measuring and comparing distortions among individual mental maps and a series of mental maps. It can pick out major directions of distortion and the extent to which different sections of the map are dis-

TABLE 2

RATIO OF LINE DISTORTION ON MAPS DRAWN BY MALE AND FEMALE RESPONDENTS (MALE/FEMALE)

Group	Tel-Aviv	Kinneret		Dead Sea		Jerusalem	Eilat
		North	South	North	South		
Ein Harod VII (13/24)	1.10	0.98 ^a	1.10 ^a	1.07 ^a	1.10	1.15	1.03 ^b
		0.94 ^a	0.95 ^a	1.06 ^a	1.08 ^a	1.07 ^a	1.03 ^b
Ein Harod XI (9/21)	1.19 ^a	0.81 ^a	0.79 ^a	0.89 ^a	0.95	1.09	1.02
		1.06 ^a	1.07 ^a	0.94 ^a	0.87 ^a	0.89 ^a	0.96 ^b
Nazareth VII (16/12)	0.95 ^b	0.77 ^a	0.82 ^a	0.98 ^a	1.01 ^b	0.92	1.00 ^b
		0.78 ^a	0.78 ^a	0.85 ^a	0.95 ^a	0.84 ^a	1.00 ^b
Nazareth XI (3/27)		Too few males for comparison					
Qiryat Haim VII (14/12)	1.01 ^b	0.91 ^a	0.87 ^a	0.95 ^a	0.95 ^a	0.97 ^a	1.06 ^b
		0.96 ^a	0.95 ^a	0.87 ^a	0.90 ^a	0.86 ^a	1.06 ^b
Qiryat Haim XI (9/29)	1.06 ^b	1.03 ^a	1.01 ^a	0.92 ^a	0.90	0.93 ^a	1.05 ^b
		1.02 ^a	1.03 ^a	0.96 ^a	0.86 ^a	0.84 ^a	1.02 ^b
University (31/31)	1.11 ^b	0.96 ^a	0.94 ^a	1.01 ^a	0.98 ^b	0.96	1.00 ^b
		1.03 ^a	1.04 ^a	0.97 ^a	0.93	0.91 ^a	0.97 ^b

Upper line of figures represents ratios along lines measured from Haifa, lower line from Tel-Aviv.

^a Overestimate of true distance by males and females^b Underestimate of true distance by males and females

torted; the results indicate levels of misperception and misrepresentation of areas or parts of areas on the mental map.

There are several ways in which the basic idea behind the technique can be refined and made more flexible. Once the points have been transformed, several other methods of analysis and statistics, such as from a Von Mises distribution or circular histograms, can be computed, analyzed and displayed. The relative perceptions of the geography of their own countries could be compared among groups chosen from different states. Each student could be asked to draw a map including, say, 15 points each from his/her own country; the average distortion indices then could be compared. Comparisons of mental maps of different areas drawn by the same populations can be made and the distortion indices of a local area, a region and the national territory can be compared. Similarly, it is possible to test the level of perception of different true maps by the same population. The technique also allows the development of experimental work, facilitating measurement and analysis of which points or places, through their presence or absence and the accuracy of their location, have the greatest influence on the level and form of distortion on the cognitive map.

In summary, the technique and methodology outlined here presents the basis for rapid and easy comparisons among maps. It complements existing work dealing with map distortion from theoretical, experimental and empirical viewpoints. Furthermore, by using a graphic microcomputer with a digitizer, and by reducing the maps to a finite number of control points, the investigator has direct access to the raw maps, providing almost immediate summary measures for analysis, thereby reducing the time and effort involved in this work.

Appendix: Computational Procedure for Measuring Distortion

Let $(x_1, y_1), \dots, (x_n, y_n)$ be the coordinates of the true-scale map. Compute the following quantities:

$$x = \sum x_i, \quad y = \sum y_i, \quad z = \sum (x_i^2 + y_i^2), \quad D_{\max} = \sqrt{z - (x^2 + y^2)/n}$$

For each mental map with coordinates given by $(a_1, b_1), \dots, (a_n, b_n)$ compute:

$$\begin{aligned} a &= \sum a_i, & b &= \sum b_i, & c &= \sum x_i a_i \\ d &= \sum y_i b_i, & e &= \sum x_i b_i, & f &= \sum y_i a_i \\ & & g &= \sum (a_i^2 + b_i^2) \end{aligned}$$

$$R = ng - a^2 - b^2, \quad S = xb - ya + n(-e + f)$$

$$T = xa + yb - n(c + d)$$

$$\theta = \arctan(S/T) \quad (\text{if } T = 0 \text{ take } \theta = \pi/2)$$

$$\alpha = -(T \cos \theta + S \sin \theta)/R$$

(if $\alpha < 0$, redefine θ as $\theta + \pi$, and α as $-\alpha$)

θ is the angle of rotation and α is the change of scale factor. Mathematically, there are two solutions for θ and α , but they produce the same transformation of the coordinates because the two θ 's differ by π and one α is the negative of the other (rotation by π is equivalent to multiplying by a scale factor of -1). Of the two solutions, we choose that with $\alpha > 0$. Compute the coordinate shift:

$$A = (x - \alpha(a \cos \theta + b \sin \theta))/n$$

$$B = (y - \alpha(-a \sin \theta + b \cos \theta))/n$$

The coordinates of the mental map points after transformation are given by:

$$a'_i = A + \alpha (a_i \cos \theta + b_i \sin \theta)$$

$$b'_i = B + \alpha (-a_i \sin \theta + b_i \cos \theta)$$

The sum of the squares of the distances between the true points and the transformed points can be computed from:

$$\begin{aligned} F &= n(A^2 + B^2) + \alpha^2 g + 2A\alpha (a \cos \theta + b \sin \theta) \\ &\quad + 2B\alpha (-a \sin \theta + b \cos \theta) - 2\alpha ((e-f) \sin \theta + (c+d) \cos \theta) \\ &\quad - 2Ax - 2By + z \end{aligned}$$

F is also $\sum ((x_i - a_i)^2 + (y_i - b_i)^2)$, but the first formula involves no further summations, so it is preferable when n is large. The distortion *distance* of each mental map is $D = \sqrt{F}$ and the distortion *index* of the mental map is $DI = 100 D/D_{\max}$.

Computing the Box of Distortion

$$\begin{aligned} \text{Compute } U &= \sum (a'_i - x_i)^2, & V &= \sum (a'_i - x_i) (b'_i - y_i), \\ W &= \sum (b'_i - y_i)^2 \end{aligned}$$

(U, V, W can also be expressed in terms similar to F above).

$$\varphi = \arctan (2V/(U - W))/2 \quad (\text{if } U = W \text{ take } \varphi = \pi/4)$$

$$E = V \sin (2\varphi) + (U + W + (U - W) \cos (2\varphi))/2$$

The sides of the box are given by $D_1 = \sqrt{E}$ and $D_2 = \sqrt{F - E}$.

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