A Quantitative Model and Analysis of Information Confusion in Social Networks

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Abstract

Information consumers in online social networks receive information from multiple information providers, which results in confusion. The amount of confusion depends on three main factors- (a) attributes of the source, (b) characteristics of the consumer and (c) trust relation between the information provider and the consumer. While information confusion has been qualitatively observed in social networks, no quantitative model or analysis was presented. We present the first quantitative model to analyze confusion in the presence of multiple information providers. We address the following fundamental issues- (i) What is a good model for confusion? (ii) How does the quality of information degrade due to confusion? (iii) What are good strategies for the information providers to control the power or the intensity with which the information is transmitted? The scenario is modeled as a non-cooperative game with pricing, whose Nash equilibrium provides the solution to the questions posed above. We use data from Twitter (e.g., on full body scan in airports) and diabetes outreach networks to illustrate the analysis. We use the solution of the non-cooperative game to study the confusion levels of consumers, in terms of the aggressiveness and passiveness of the information providers. Results indicate that confusion levels are high in networks in networks in which all information providers are equally trusted. In networks where information providers are unequally trusted, the confusion levels are moderate.

Index Terms – Social networks, information, confusion, aggression, passiveness.

I. Introduction

Users using the Internet, or social networks, e.g., Twitter [1], or diabetes outreach networks [2], etc, seek information from specific information providers (e.g., by posing a question on the wall of a friend in Facebook or by searching on Google or from other friends). However, apart from the sought primary information provider, the consumer also receives supplementary information from the other direct or indirect information providers (e.g., other friends in Facebook who may respond to the same query on the wall or multiple links resulting due to a web search or information from multiple friends). The supplementary information can cause confusion to the consumer. Consider the set of messages in the experiment conducted by Paul Adams [3]. When a consumer sought information about a particular restaurant, he/she received the following messages.

1) *You should come to this restaurant, it is delicious*!
2) *The restaurant is average, service was slow.*
3) *Paul visits this restaurant three times a week.*

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The consumer may now be confused about the quality of the restaurant. We define information confusion as the confusion experienced by the consumers due to information received from multiple information providers. We present a quantitative analysis of presenting information with appropriate intensities to deal with confusion. We also present a quantitative study of the confusion levels of consumers in a network, in terms of the average aggression levels and the average levels of passiveness of different information providers.

As another example to illustrate the confusion, we collected tweets from Twitter users in different regions on full body scan in airports, using the architecture we developed in [4]. We counted the number of tweets in Twitter that were (a) supportive to full body scan, (b) opposed to full body scan and (c) neutral to full body scan using the sentiment analysis described in [5], [6]. A spatial snapshot of the data is tabulated in Table I. According to Table I, if a user sends a tweet to a friend who lives in the North American region to obtain information on full body scan, he/she is likely to obtain information which is supportive of full body scan. However, another user who lives in the Asian region may provide negative information on full body scan. This results in confusion to the consumer who sought information. Another example is that the web-site in [7] claims that social networking sites are the most popular influences for users where as [8] claims that Google is the most trusted site for users. Similarly, in the field of cellular telecommunications, some studies (e.g., [9]) suggest that there are no dangers of cancer due to the usage of cellular phones while other studies (e.g., [10]) suggest otherwise.

In general, a scenario with multiple providers of information can be depicted as shown in Fig. 1[1], where a consumer seeks information from a primary information provider but receives information not only from the primary information provider but also from $M-1$ other information providers. In the scenario in Fig. 1, the information from other information providers is treated as confusion/noise because they can add to the confusion levels of the information consumer. The quality of information a consumer obtains useful information is then degraded because of the confusion. It then becomes essential to study the usefulness of the information received by consumers when they also receive supplementary information from other information providers.

The concept of “noise” due to multiple providers of information has been qualitatively listed , e.g., [11], [12]. Kumar *et al* [11] acknowledged the high amount of noise present in information obtained in social networks and used graph theoretic models to extract authentic web pages. Olson [12] suggested qualitative techniques, e.g.,

[1]Although Figs. 1 and 2, represent a *single-hop* between the information provider and the information consumer, the actual number of hops could be more than one. The figures only indicate the information provider and the final destination and not the number of hops in between them.

clarity of goals, to deal with the noise. A survey by Brand Republic [13] mentions that only 10% of the information obtained is really useful and the rest is noise. While the literature has discussed the noise in social networks and its correlation to individual performance based on statistics (e.g., [14]), no quantitative means of computing or mitigating the noise has been studied. Game theoretic approaches have been used to study other issues in social networks, e.g., contribution strategy of players to perform tasks [15].

In this paper, we present a quantitative analysis of the relevance of information in the presence of confusion caused due to multiple providers of information. We present a term, *Information-to-Confusion Noise Ratio (ICNR)*, that quantitatively represents the quality of the information obtained by the consumers. The ICNR takes into account, the trust placed by the consumers on the different providers of information, the ability of information providers to present information with different intensities, the resources available to the information providers to influence the consumers and the natural ability the consumers have in processing information. We determine strategies for information providers to control the powers or intensities with which they transmit information, which results in maximum relevance of information for all the consumers. The optimal strategies are obtained as the Nash equilibrium solution of a non-cooperative game. We use data from Twitter (e.g., on full body scan in airports) and diabetes outreach networks to illustrate the analysis. Results indicate that information providers tend to transmit with maximum intensities (i.e., be too aggressive) or with minimum intensities (i.e., be too passive) in networks in which all information providers are equally trusted. In networks where information providers are unequally trusted, the aggression levels are moderate. *To the best of our knowledge, this is the first quantitative analysis of information confusion in social networks.*

The rest of the paper is organized as follows. Section II presents the basic formulation and mathematical representation of the information to confusion ratio (ICNR) (Section II-A) and the definition of the problem solved in this paper (Section II-B). The non-cooperative game with pricing and the discussion on the Nash equilibrium using the $\mathcal{M}-$ matrix based approach to the power control problem are presented in Section III. Section IV presents some applications of the proposed research in this paper. Numerical results are presented in Section V and conclusions are drawn in Section VI.

II. Mathematical Formulation

A. Information to Confusion Noise Ratio (ICNR)

Consider a system with M information information providers and N intended information consumers as shown in Fig. 2 (We first consider $M = N$ and later, we also specify how to address the case when $M \neq N$). In Fig. 2, Sources 1, 2, $\cdots$, M could represent information providers such as friends in Facebook, friends in Orkut, links to other web-sites in Twitter/Facebook/Orkut, etc., that provide information on a topic and the information consumers 1, 2, $\cdots$, N represent people who seek the information. The i^{th} information provider transmits information with a power or intensity, P_i. Intuitively, P_i could indicate (but not limited to) one or more of the following.

- The authenticity of the information (e.g., by referring a well known web page or a certified document).
- The aggression (i.e., presenting information in an imperative manner by exploiting any hierarchy like the personal relation (e.g., parental or friendship) or a professional relation (e.g., supervisor) with the end consumer. Aggression could also include arguing in an intense manner.
- Confidence (e.g., by leveraging past successes or by leveraging the knowledge about the consumer posing the question so that the solution can be tailored accordingly).
- Propaganda (e.g., if the information is political and the information providers are political parties).
- Advertisement (e.g., if the information sought is about a commercially available product or an opportunity).
- Intentional information manipulation by the information providers.

The i^{th} information consumer not only receives information from the i^{th} information source, but also receives confusing "noise" from the other information providers. The causes for confusion depend on

- the intensity or the power of the information obtained from other information providers
- the trust placed by the information consumer on the various information providers
- the natural dilemma or confusion a consumer has in processing information.

The i^{th} information consumer places a trust, h_{ji} $(0 \leq h_{ji} \leq 1)$ on the j^{th} information provider. The trust represents the quality of relationship between the j^{th} information provider and the i^{th} information consumer, which, in turn, could be due to friendliness or fear of authority or truthiness of the information provider. Trust could be a binary variable, i.e., taking values only in $\{0, 1\}$, e.g., [16], or could take real values, e.g., [17], or can take real values in $[0, 1]$ as we consider in this paper. Additional trust models can be found in [18]. A trust factor, $h_{ji} = 0$ represents no trust and a trust factor, $h_{ji} = 1$ represents 100% trust. The effective information received by the i^{th} consumer from the j^{th} information provider is then, $P_j h_{ji}$. The i^{th} information consumer therefore receives $P_i h_{ii}$ amount of useful information (i.e., from the main information provider).

Let ν_{ji} $(0 \leq \nu_{ji} \leq 1)$ represent the amount of contradiction between the information from information provider j and information provider i. A value of $\nu_{ji} = 0$ represents no contradiction while $\nu_{ji} = 1$ represents 100% contradiction. The effective information received by the i^{th} information consumer from the j^{th} information provider for $j \neq i$ is the confusion perceived by the i^{th} consumer due to the j^{th} information provider. The total confusion at the i^{th} information consumer due to information from all information providers, $j \neq i$) is then $\sum_{j \neq i} P_j h_{ji} \nu_{ji}$. The factor ν_{ji} is included because the information from information provider j is a cause for confusion to the i^{th} information consumer when the information is contradictory to that received from information provider i.

The contradiction between information providers using the data in Table I is measured as described below. The number of neutral tweets do not bring about any contradiction and hence, can be discarded. Thus, the percentage of tweets supporting full body scan in USA is 967/(967+753)=56.22%. The corresponding numbers for Europe and Canada are 69/(69+59)=53.91% and 14/(14+99)=42.42%, respectively. If an information consumer then obtains information from USA on body scan and the same consumer receives information from Europe then the contradiction factor, ν_{ji}, is the probability of obtaining opposite opinions, which is $0.5622(1 - 0.5391) + 0.5391(1 - 0.5622) = 0.4951$. Similarly, the contradiction factor between USA and Canada is 0.5094. Consider two regions, R_1 and R_2, such that all tweets from region R_1 supported full body scan and all tweets from region R_2 were opposed to full body scan. Then the contradiction factor between regions R_1 and R_2 is 1.

Let W_i represent the amount of auxiliary resources the i^{th} information provider can expend (in the form of efforts to convince the information consumer) which helps the i^{th} information consumer mitigate some of the confusion. The effort put in by the information provider could be in the form of providing reference material like books, or exploiting the knowledge about the past of the information consumer to remind the consumer. Alternatively, the auxiliary resources can represent the ability of the information provider to tailor its information to suit the needs of the information consumer. Larger amount of auxiliary resources represents a better ability of the consumer to deal with the confusion.

In order to understand the difference between the terms, P_i and W_i, consider the experiment by P. Adams in [3] mentioned in Section I. The statement, "*You must visit this restaurant*!" is imperative and if posted by the business owner, is representative of the intensity, P_i. However, the statement, "*Paul visits this restaurant three times a week*" is a referral about *another customer* and is like the auxiliary information, W_i. Other parameters that can increase P_i in this case include link to the restaurant's web page or pictures of the items in the menu, any certificates about the healthy practices of the hotel like the New York city restaurant grades [19] or the price advantages for customers.

The actual value of P_i can be a weighted combination of the number of links to the hotel, the number of pictures of the items in the menu, the number of discount offers and coupons. For instance, consider three restaurants, $R1$, $R2$ and $R3$. Let the parameters that could control the intensity for these restaurants be according to Table II. The normalized intensity (NI) corresponding to the number of links for $R1$ can be computed as $\frac{2}{2+5+3} = 0.2$. The NI corresponding to the discounts in $R1$ is $\frac{20}{20+25+15} = 0.33$. In a similar way, the NI corresponding to all the parameters are as listed in Table III. Let the weightages given to the number of links, health rating, number of pictures and discounts be ϕ_1, ϕ_2, ϕ_3 and ϕ_4, respectively, such that $\phi_1 + \phi_1 + \phi_3 + \phi_4 = 1$. Then, the normalized intensity for $R1$ can be obtained as $0.2\phi_1 + 0.33\phi_2 + 0.5\phi_3 + 0.33\phi_4$. Similarly, the normalized intensity for $R2$ is obtained as $0.5\phi_1 + 0.25\phi_2 + 0.33\phi_3 + 0.42\phi_4$ and that for $R3$ can be obtained as $0.3\phi_1 + 0.42\phi_2 + 0.17\phi_3 + 0.25\phi_4$.

W_i can be computed by obtaining the ratings of the consumers on the training resources. Alternatively, for information providers which are reputed journals, W_i can be measured using the impact factor of the journal. As an example, consider the study on diabetes outreach networks conducted by Corteville and Sun [2]. The authors obtained ratings provided by users on various information providers that help them obtain awareness on diabetes, on a scale of 1-7. Some of the results obtained in [2] are listed in Table IV. The ratings provide a means to compute W_i for various information providers.

In order to study the effect of W_i in mitigating confusion, we study variation of the information-to-confusion-noise ratio (ICNR) with respect to the amount of auxiliary resources. The detailed expression for the ICNR and its explanation is provided in (1). Consider an example of a social network with $M = 10$ information providers and information consumers. For simplicity we make $W_i = W$, $\forall\ i$. If W_i's are unequal, then it requires an $(M + 1)-$dimensional plot, which would be difficult to interpret. We compute the ICNR, x given by (1), for a particular consumer in this system. We generate uniform random variables in $(0, 1)$ to obtain h_{ji} and ν_{ji} in the expression in (1). We compute the ICNR, x, for $W_i = W$ and $P_i = 1$, $\forall\ i$ using the expression in (1) and average them over 100000 samples. Fig. 3 presents the variation of the ICNR, x with respect to the amount of resources, W. It is observed that the ICNR, x, increases with W. Therefore, increasing the number of auxiliary resources improves the ICNR. A higher ICNR represents lower confusion level. It is also observed that the differential increase is larger for small values of W and smaller for larger values of W. This indicates that beyond a particular threshold, increasing the amount of auxiliary resources is not so effective in improving the ICNR. Intuitively, this is because, the learning capacity of an information consumer is limited and beyond a particular threshold, more training is not so effective.

Let r_i denote the rate at which the i^{th} information consumer receives information from the i^{th} information provider in the absence of confusion. The rate, r_i, can represent the usefulness or relevance of the information projected by the i^{th} information provider . Alternatively, the rate could represent the speed (volume of information per second) at which the i^{th} information provider updates or refines the information it transmits. If the relevance at which information is sent to the consumer, r_i, is already large, then the high relevance can reduce the effectiveness of the auxiliary resources, because the information consumers may tend to lose their focus on the information. Thus, we define a auxiliary gain, $G_i = \alpha \frac{W_i}{r_i}$, which is a factor by which the confusion levels of the consumer can be reduced by education or training. The constant, α, is a scaling factor.

Apart from the confusing information that a consumer receives from other information providers, each consumer has a natural level of uncertainty about the received information due to an individual's dilemma or understanding levels in processing the information. Other factors like media, society, etc, can cause some confusion too. This natural dilemma can be exploited by the other information providers to add to the confusion of the consumer by using their auxiliary resources. Let the natural dilemma or confusion experienced by the i^{th} consumer be N_i. Then, the total confusion level suffered by the i^{th} consumer is $\sum_{j \neq i} P_j h_{ji} \nu_{ji} + N_i W_j$. The effective information received by the i^{th} consumer is $P_i h_{ii}$, as mentioned earlier.

The difference between N_i and r_i is as follows. The relevance, r_i, pertains to how information the information consumer gets from information provider. The term, N_i, is the natural distraction and dilemma of the information consumer. As an example, for a diabetic patient, r_i could represent the extent to which a patient understands the ill effects of a diet with high sugar content, when advised by a doctor, while N_i could represent the natural tendency or temptation of the patient to have food that is high in sugar content.

For the i^{th} information consumer, We define a term called *Information-to-Confusion-Noise Ratio (ICNR)*, x_i, which quantifies the quality of the information absorbed by an information consumer in the presence of confusion. The ICNR, x_i, obtained by the i^{th} consumer who seeks information from the i^{th} information provider can be written as

$$x_i = \frac{P_i h_{ii} G_i}{\sum_{j \neq i} P_j h_{ji} \nu_{ji} + N_i W_j}, \tag{1}$$

where $G_i = \alpha \frac{W_i}{r_i}$ represents the auxiliary gain explained earlier. It is observed that the ICNR of the i^{th} consumer depends on the power or intensity of *all* the information providers. The ICNR is analogous to the signal-to-interference-noise ratio (SINR) in wireless communication systems [20]. Twitalyzer [21] discussed SINR in social networks based on the ratio between the amount of good information and spams or anecdotes shared with other consumers [22]. The criterion for computation does not model the noise due to the presence of other information providers of useful information.

It is observed that a high ICNR *does not indicate correctness of information*. It only indicates that the information consumer is not confused about the information. However, distortion or spread of mis-information can be incorporated by multiplying the trust term, h_{ji} in (1) by a fraction β_{ji}, which is 1 if the information is transferred faithfully without distortion and $0 \leq \beta_{ji} < 1$ if the information is distorted.

Further, the *confusion* discussed here does not include spam messages. Spam can be incorporated in our model as follows. Information consumers can use spam detection techniques, e.g., [23], [24] to detect spam. Once consumer i identifies a particular provider, j, as a spam, then the trust, h_{ji} placed on that provider is zero, in (1). Therefore, the information provider does not add to the confusion level of the consumer. The "confusion" discussed here is for the case when the consumer receives authentic but contradictory information.

B. Problem Definition

The presence of confusion degrades the relevance of the information obtained by each information consumer. The utility, u_i, of the information received by the i^{th} consumer which is also the satisfaction level of the i^{th} information consumer [25], can then be defined as the relevance, r_i, degraded by a factor which depends on the ICNR, x_i. The effective utility of the information, u_i, obtained by the i^{th} consumer is

$$u_i = r_i \pi(x_i), \tag{2}$$

where $\pi(x_i)$ is an increasing function of x_i satisfying $0 = \pi(0) \leq \pi(x_i) \leq 1 = \lim_{x_i \to \infty} \pi(x_i)$. Intuitively, the expression for u_i indicates that consumers prefer values of ICNR, x_i, that results in larger value of u_i, i.e., larger values of x_i according to the Bernoullian utility theory [25]. This also indicates that information is more effectively relevant or more satisfying to consumers when the confusion levels are relatively lower. The utility, u_i is the *effective* *relevance of the information received by the i^{th} information consumer, from the i^{th} information provider.* Therefore, u_i is a parameter that characterizes the satisfaction level of the transmitting information provider as well as the information consumer. Therefore, we term u_i, the utility of the i^{th} *transmit-receive pair*, which indicates the i^{th} transmitting information provider and i^{th} information consumer.

In order to increase u_i, it is essential to increase the ICNR, x_i. For increasing x_i, the i^{th} information provider should increase the power or intensity, P_i. However, this increases the confusion at the j^{th} information consumer $(j \neq i)$, thereby decreasing the ICNR, x_j and hence, u_j. Therefore, the j^{th} information provider should increase P_j in order to compensate for the loss in ICNR. The increase in P_j causes an increase in the confusion of all the other information consumers (including consumer i). Thus, when an information provider increases the power or intensity with which it transmits the information to an information consumer, the other information providers can also increase their respective intensities, adding to the confusion of the consumer. Therefore, the relevance of information absorbed by an information consumer can me maximized by appropriately controlling the power or intensities of *all* the information providers, i.e., by formulating a non-cooperative game between the information providers.

Co-operation between information providers can be included in our model, in three ways. One is a constructive cooperation, where in information providers agree with each other (thus making $\nu_{ji} = 0$ in (1)), thus increasing the ICNR. Another means of cooperation is a malicious cooperation, which, in turn, can be modeled in two ways- either other information providers all decide to contradict the primary information provider (making $\nu_{ji} = 1$, $\forall$ $j \neq i$ in (1)), thereby reducing the ICNR, x_i or when other information providers provide distracting auxiliary information, i.e, by adding links to a web-page or by spam, thereby increasing W_j, $\forall$ $j \neq i$, in (1), thus decreasing the ICNR, x_i.

The objective is to develop a non-cooperative game-theoretic analysis with pricing, to control the power or intensity of information from different information providers in order to provide maximum relevance of information to *all* the information consumers in the network[2]. It is also of interest to determine the confusion levels in the network based on the average aggression level or the average level of passiveness of all the information providers. The desired solutions to the objectives listed above, can be obtained from the Nash equilibrium of the non-cooperative game with pricing. We provide necessary and sufficient conditions for the existence of a Nash equilibrium of the

[2]If a single information provider wishes to maximize the relevance of information to its information consumer, all it has to do is to transmit at its maximum intensity.

proposed non-cooperative game with pricing. The Nash equilibrium solution will also be used to determine the levels of confusion in the network, based on the average levels of aggression of passive behavior of the information providers. The following section provides the game theoretic analysis.

III. Game Theoretic Analysis

We now combine the notions of ICNR presented in Sections II-A and that of the utility described in Section II-B to present a game theoretic analysis to maximize the effective utility of information, u_i, for all the transmit-receive pairs by controlling the power, P_i of all the information providers. Since the effective utility, u_i depends on the ICNR of the i^{th} consumer, x_i, according to (2), which, in turn, depends on the power, P_i of *all* the information providers, from (1), it is possible to model the assignment of appropriate power (called the *Power Control Problem (PCP)*) to maximize the effective utility of the information received by each of the M information consumers, as a non-cooperative game defined in Appendix A. The set of players are the set of information providers. The strategy set is the values of the powers, P_i, $\forall\ i$ and the utility functions are the values of u_i defined in (2). If $\pi(x_i)$ is a non-negative, non-decreasing concave function of x_i, then u_i in (2) satisfies the properties in (23). An example is when $\pi(x_i) = 1 - e^{-x_i}$ so that $u_i = r_i\,(1 - e^{-x_i})$. Fig. 4(a) shows the normalized utility function, u_i as a function of x_i, where the maximum value of u_i is normalized to 1. It is observed that u_i is a non-decreasing concave function of x_i.

Let the vector $\mathbf{p} = [P_i]_{1\leq i\leq M}$ denote the vector of powers for the PCP. The PCP is a problem of obtaining $\mathbf{p}$ that maximizes u_i, in (2), $\forall\ i$, which is the Nash equilibrium of the non-cooperative game. We present a basic formulation of this problem in Section III-A. We then introduce the notion of pricing in Section III-B. The game theoretic formulation is extended to include pricing, in Section III-C. Finally, in Section III-D, we discuss the necessary and sufficient conditions for the existence of the Nash equilibrium to the power control game with pricing.

A. Basic Game formulation

The Nash equilibrium of PCP game is the solution to the optimization problem,

$$\max_{\mathbf{p}} u_i \quad \forall i, \tag{3}$$

subject to the constraints

$$0 \leq P_i \leq P_{\max}^{(i)} \quad \forall i, \tag{4}$$

which follows from the fact that each information provider will have a maximum capacity to be confident or aggressive. Since u_i in (2) satisfies (23), they are increasing functions of x_i, and from (1), x_i is an increasing function of P_i. Hence, u_i is an increasing function of P_i if all other P_j's are fixed. Hence, the maximum value of u_i occurs, $\forall\ i$ at $P_i = P_{\max}^{(i)}$. Therefore, according to Definition A.1 in Section A, the Nash equilibrium for the PCP is $P_i = P_{\max}^{(i)}$, $\forall\ i$. Intuitively this means that for the PCP, all information providers should transmit with maximum intensity or power. A large intensity of transmission indicates large amount of efforts by the information provider to increase the aggression or confidence levels. This could result in high costs for the information providers (in the form of money or energy or design or infrastructure). In order to address this issue, a pricing function can be used which penalizes transmit-receive pairs with information providers that unnecessarily transmit information with larger powers or intensities. The following sub-section presents a discussion on the pricing that can be posed on the transmit-receive pairs.

B. Pricing

The price imposed on transmit-receive pairs could be a monetary price where the the information provider and the information consumer are required to pay for the efforts taken to increase the intensity. As an example, if the intensity is increased by giving references to a book, the price could be the money paid to purchase the book. If the intensity is increased by advertisements of propaganda, then the price could be the cost involved in advertising or the propaganda. Alternatively, the price could be an emotional price, where in, increasing the intensity could result in weakening the relationship between the information provider and the information consumer or in terms of hurting the reputation of the information provider for being overtly aggressive. However, the intensity of the i^{th} information provider, P_i, should not be the only factor in computing the price for the i^{th} transmit-receive pair, explained in detail as follows.

When information providers increase the power or intensity with which they transmit information in order to meet ICNR requirements, they should be penalized lesser than information providers transmitting at larger powers despite perceiving a good ICNR at the respective information consumer. This is because, when the current ICNR is low, it means that the information consumer experiences a lot of noise and hence, the information provider is forced to increase its intensity to maintain acceptable ICNR. However, an information provider that transmits at larger power despite perceiving good ICNR at the respective information consumer, increases intensity for no valid reason and also causes noise or confusion to other information consumers and hence, must be penalized higher.

An alternate means to argue this is to apply the law of diminishing marginal returns [26] of increasing P_i. In other words, the pricing function should be a function of the ICNR and not just the power or intensity of the information.

Let $f_i(x_i)$ denote the price imposed on the i^{th} transmitter when the i^{th} receiver perceives an ICNR of x_i. The "net utility" for the i^{th} transmit-receive pair, $\hat{u}_i$, is then defined as

$$\hat{u}_i = u_i - \lambda f_i(x_i), \tag{5}$$

where λ is the pricing parameter. The parameter λ can be interpreted as an index which determines how high or low the information providers are priced. A higher λ implies that the transmit-receive pairs are priced heavily, while a lower λ indicates lighter pricing. A larger λ can make the network more passive since information providers would be averse to being aggressive (i.e., transmit at maximum intensity), while a smaller λ encourages information providers to be more aggressive (by penalizing them less for their aggression), thereby resulting in an aggressive network. The pricing function, $f_i(x_i)$ should be an increasing function of x_i because, a consumer with larger ICNR pay a larger price.

The pricing function, $f_i(x_i)$ could be a linear function, i.e.,

$$f_i(x_i) = x_i. \tag{6}$$

Alternatively, the pricing function, $f_i(x_i)$, can be a non-linear function of x_i. Apart from being an increasing function of x_i, $f_i(x_i)$ is also desired to satisfy, $f_i'(x_i) = \frac{df_i(x_i)}{dx_i} \leq 1$. *Although this property is not a requirement to carry out the analysis*, it encourages transmit-receive pairs to obtain larger ICNR. This follows from the principle that for a rate of change of price less than one, users buy larger quantities of a commodity because the "per-unit cost" is lower [26]. A particular choice for $f_i(x_i)$ is[3]

$$f_i(x_i) = \frac{x_i^2}{x_i + G_i} \Rightarrow f_i'(x_i) = 1 - \left(\frac{G_i}{x_i + G_i}\right)^2, \tag{7}$$

i.e., $0 \leq f_i'(x_i) \leq 1$. Fig. 4(b) shows the price, $f_i(x_i)$ as a function of x_i for $G_i = 100$.

The non-linear pricing function specified in (7) is motivated as follows. A higher, x_i should result in higher price. Further, for the same ICNR, x_i, if the intensity of the information from the information provider forms a larger fraction of the total received information at an information consumer, i, then the intensity of the corresponding information provider is larger than required and hence, the transmit-receive pair must be priced higher. However,

[3]The function $f_i(x_i)$ in (7) is only a specific choice and is not a unique function that satisfies the desired properties. Discussion on other non-linear pricing functions satisfying the required properties is beyond the scope of this paper.

for the same x_i, if the intensity of the information from the information provider forms a smaller fraction of the total received information at an information consumer, then the corresponding information provider transmits with higher intensity to compensate for the confusion, and hence the transmit-receive pair must be priced lower. With the above considerations, f_i can be written as

$$f_i = x_i \frac{P_i h_{ii}}{\sum_{j=1}^{M} P_j h_{ji} \nu_{ji} + N_i W_j}. \tag{8}$$

From (1) and (8), $f_i(x_i)$ is given by the expression in (7). The function, $f_i(x_i)$ in (7) is not the only choice for non-linear pricing. Any function that satisfies (17) described later, can be used as a pricing function. Discussions on other choices for $f_i(x_i)$ are beyond the scope of this paper.

The PCP with pricing (called the P-PCP) is then achieved by solving the optimization problem,

$$\max_{\mathbf{p}} \hat{u}_i = \max_{\mathbf{p}} \left[u_i - \lambda f_i\right], \forall i, \quad 0 \leq P_i \leq P_{\max}^{(i)}, \forall i, \tag{9}$$

where u_i is the utility function without pricing, given by (2). Note that although the pricing function is written as a function of the ICNR, x_i, it is also a function of $\mathbf{p}$ from (1).

C. Game with Pricing

The optimization problem in (9) can also be modeled as an $M-$person non-cooperative with the payoff function given by $\hat{u}_i$. In order to determine the Nash equilibrium for the game in (9), we proceed as follows. We re-formulate the game as a problem of allocation of optimal ICNR's to the transmit-receive pairs and compute the optimal P_i's from the obtained optimal ICNR's. The optimization problem in (9) can be re-written as

$$\max_{x_i \geq 0} \hat{u}_i(x_i) = \max_{x_i \geq 0} \left[u_i(x_i) - \lambda f_i(x_i)\right], \forall i. \tag{10}$$

Note that the objective function for each transmit-receive pair depends only on the ICNR of that transmit-receive pair and not on the ICNR obtained by the other receivers. Therefore the optimization problems specified in (10) can be solved as M independent optimization problems in each x_i. As an example Figs. 5(a) and 5(b) present the net utility, $\hat{u}_i(x_i)$, as a function of the ICNR, x_i, when deploying the linear pricing function in (6) and the non-linear pricing function in (7), respectively. It is observed that for the behavior of u_i as shown in Fig. 4(a) and pricing functions as in (6) and (7), the net utility, $\hat{u}_i$, is a concave function with a unique maxima. This behavior also agrees with the "inverse-U" behavior of incentives, discussed in [27].

Let $\mathbf{x}^* = \begin{bmatrix} x_1^* & x_2^* & x_3^* & \cdots & x_M^* \end{bmatrix}^T$[4] be the vector of ICNR's that maximize the M objective functions in (10). The corresponding $\mathbf{p}^* = \begin{bmatrix} P_1^* & P_2^* & P_3^* & \cdots & P_M^* \end{bmatrix}^T$ can be obtained by re-writing (1) as

$$\frac{P_i^* h_{ii} G_i}{x_i^*} - \sum_{j \neq i} P_j^* h_{ji} \nu_{ji} = N_i \sum_{j \neq i} W_j \quad \forall i. \tag{11}$$

The equations in (11) (M in total, one for each i) represent a system of simultaneous linear equations in M variables and can be represented by the matrix equation,

$$\mathbf{p}^* = \left(\mathbf{I}_M - \mathbf{D}_1{}^{-1}\mathbf{A}\right)^{-1} \mathbf{D}_1{}^{-1}\mathbf{D}_2(\mathbf{1}\mathbf{1}^T - \mathbf{I}_M)\mathbf{w}, \tag{12}$$

where $\mathbf{1}$ is the column vector of length M with all entries being unity, $\mathbf{I}_M$ is the $M \times M$ identity matrix, $\mathbf{w} = \begin{bmatrix} W_1 & W_2 & W_3 & \cdots W_M \end{bmatrix}^T$ and $\mathbf{A}$, $\mathbf{D}_1$ and $\mathbf{D}_2$ are given by

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{h_{21}\nu_{21}}{h_{11}} & \frac{h_{31}\nu_{31}}{h_{11}} & \cdots & \frac{h_{M1}\nu_{M1}}{h_{11}} \\ \frac{h_{12}\nu_{12}}{h_{22}} & 0 & \frac{h_{32}\nu_{32}}{h_{22}} & \cdots & \frac{h_{M2}\nu_{M2}}{h_{22}} \\ \frac{h_{13}\nu_{13}}{h_{33}} & \frac{h_{23}\nu_{23}}{h_{33}} & 0 & \cdots & \frac{h_{M3}\nu_{M3}}{h_{33}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{h_{1M}\nu_{1M}}{h_{MM}} & \frac{h_{2M}\nu_{2M}}{h_{MM}} & \frac{h_{3M}\nu_{3M}}{h_{MM}} & \cdots & 0 \end{bmatrix}, \tag{13}$$

$$\mathbf{D}_1 = \text{diag}\left(\frac{G_1}{x_1^*}, \frac{G_2}{x_2^*}, \frac{G_3}{x_3^*}, \cdots, \frac{G_M}{x_M^*}\right) \tag{14}$$

$$\text{and } \mathbf{D}_2 = \text{diag}\left(\frac{N_1}{h_{11}}, \frac{N_2}{h_{22}}, \frac{N_3}{h_{33}}, \cdots, \frac{N_M}{h_{MM}}\right), \tag{15}$$

where diag $(y_1, y_2, \cdots y_M)$ is the diagonal matrix with y_1, y_2, $\cdots$, y_M as the entries along the diagonal.

The game described by (9) has a unique Nash equilibrium if and only if one can obtain a unique non-negative x_i^*, $\forall\ i$ that solves (10) and the corresponding power/intensity vector, $\mathbf{p}^*$, obtained from (12) satisfies (4). It is observed that the value of the optimal x_i^* and hence $\mathbf{p}^*$ depends on the pricing parameter, λ. *In the following subsection, we provide necessary and sufficient conditions on* λ*, which result in a unique Nash equilibrium for the game described by (9).*

D. Nash Equilibrium

The necessary condition for the existence of the Nash equilibrium is presented in in Section III-D1. Section III-D2 describes the sufficient condition for the existence of a unique feasible Nash equilibrium.

[4] $(.)^T$ represents the transpose of a vector or a matrix.

1) Necessary Conditions: Applying the first order necessary conditions for maxima, the value of x_i that maximizes the objective function in (10), x_i^*, can be obtained as the value that satisfies

$$u_i'(x_i^*) = \lambda f_i'(x_i^*), \tag{16}$$

i.e., $\frac{d\lambda}{dx_i^*} = \frac{f_i'(x_i^*)u_i''(x_i^*) - u_i'(x_i^*)f_i''(x_i^*)}{[f_i'(x_i^*)]^2}$. Therefore, if u_i and f_i satisfy the property

$$f_i'(x_i^*)u_i''(x_i^*) - u_i'(x_i^*)f_i''(x_i^*) < 0, \tag{17}$$

then x_i^* decreases as λ increases. Although (17) appears restrictive, functions that satisfy (17) can be easily determined. Few examples are listed below.

- If $f_i(x_i) = x_i$ is a linear function as in (6), then $f_i''(x_i) = 0$. Since u_i is a concave function (as specified in (23)) and f_i is an increasing function, (17) is satisfied.
- If $f_i(x_i)$ is chosen to be a non-decreasing convex function, then $f_i'(x_i)$, $f_i''(x_i) > 0$. Since u_i is an increasing concave function according to (23), the condition in (17) is satisfied.

It is observed that for the pricing function, $f_i(x_i)$ specified in (7), $f_i'(x_i) = 1 - \left(\frac{G_i}{x_i+G_i}\right)^2 \geq 0$ and $f_i''(x_i) = \frac{2G_i^2}{(x_i+G_i)^3} \geq 0$, i.e., $f_i(x_i)$ is a non-decreasing convex function of x_i. Therefore, (17) is satisfied.

Choosing u_i and f_i that satisfy (17) provides an upper bound on the pricing parameter, λ, which, in turn, yields a necessary condition for the existence of a unique Nash equilibrium to the power control game with pricing. In order to explain this in detail, let $\lambda_i^{(0)} \triangleq \frac{u_i'(0)}{f_i'(0)}$, be the pricing parameter that results in an optimum SIR, $x_i^* = 0$, for the i^{th} consumer. The following theorem then gives an upper bound on the pricing parameter λ and a necessary condition for the problem in (9) to have a feasible solution.

Theorem 3.1: Let u_i and f_i satisfy (17), $\forall$ i and let $\lambda_{\max} = \min_i \lambda_i^{(0)}$, where $\lambda_0^{(i)} \triangleq \frac{u_i'(0)}{f_i'(0)}$. Then the necessary condition for the optimization problem in (9) to have a feasible solution is $\lambda < \lambda_{\max}$.

Proof: If $\lambda > \lambda_{\max}$, then $\exists j$, $\lambda > \lambda_j^{(0)}$. When deploying utility and pricing functions that satisfy (17), λ is a decreasing function of x_j^*, i.e., x_j^* decreases as λ increases. As mentioned earlier, $\lambda_j^{(0)}$ is the value of λ for which $x_j^* = 0$. Therefore, if $\lambda > \lambda_j^{(0)}$, $x_j^* < 0$. From (1), it is observed that if $P_i \geq 0$, $\forall$ i, then $x_i \geq 0$. Therefore $x_j^* < 0 \Rightarrow \exists$ j', such that $P_{j'} < 0$, which is an infeasible solution according to the constraints specified in (4). Hence, the necessary condition for optimization problem in (9) subject to the constraints in (4) to have a feasible solution is $\lambda < \lambda_{\max}$. ■

Theorem 3.1 can be intuitively interpreted as follows. λ can be interpreted as the extent to which consumers are priced/penalized. $\lambda_{\max}$ can therefore be considered an upper "cut-off" above which transmit-receive pairs suffer so large a penalty even when the information provider transmits at low powers/intensities, that the information providers prefer not to provide any information at all. In a social network, this could represent a scenario where a user providing information could face severe criticism or face emotional stress in the form of loss of relationship, that they refrain from providing any information when sought for or provide information in an extremely passive manner. $\lambda > \lambda_{\max}$ represents a highly passive network because the cost of transmitting information with any intensity is very large and hence, information providers transmit with very low or no aggression.

- For u_i as in (2) and for a linear choice of $f_i(x_i)$ (e.g., $f_i(x_i) = x_i$, $\lambda_{\max} = \min_i r_i h'(0)$.
- For u_i as in (2) and for a non-linear choice of $f_i(x_i)$ as in (7), $\lambda_{\max} = \infty$. Intuitively, this means that even for a very large pricing parameter, information providers can transmit at large intensities. This represents a social network where users tend to be more aggressive in presenting information.

Note that the condition in Theorem 3.1 is not sufficient because it is possible that $\lambda < \lambda_{\max} \Rightarrow x_i^* > 0,\ \forall\ i$ but $\exists\ j$ such that $P_j^* < 0$ or $P_j^* > P_{\max}^{(j)}$. As an example, consider a network with $M = 2$. Let $\lambda < \lambda_{\max}$ so that x_1^* and x_2^* obtained by solving (16) are positive. For this case, (11) reduces to

$$\begin{array}{rcl} \frac{h_{11}G_1}{x_1^*}P_1 - h_{21}\nu_{21}P_2 & = & N_1W_2 \\ -h_{12}\nu_{12}P_1 + \frac{h_{22}G_2}{x_2^*}P_2 & = & N_2W_1 \end{array} \tag{18}$$

The above can be represented by the matrix equation

$$\left[\begin{array}{cc} \frac{h_{11}G_1}{x_1^*} & -h_{21}\nu_{21} \\ -h_{12}\nu_{12} & \frac{h_{22}G_2}{x_2^*} \end{array}\right]\left[\begin{array}{c} P_1 \\ P_2 \end{array}\right] = \left[\begin{array}{c} N_1W_2 \\ N_2W_1 \end{array}\right] \tag{19}$$

which, in turn, can be written as $\mathbf{B}_2\mathbf{p} = \mathbf{n}_2$, where

$$\mathbf{B}_2 = \left[\begin{array}{cc} \frac{h_{11}G_1}{x_1^*} & -h_{21}\nu_{21} \\ -h_{12}\nu_{12} & \frac{h_{22}G_2}{x_2^*} \end{array}\right] \text{and } \mathbf{n}_2 = \left[\begin{array}{c} N_1W_2 \\ N_2W_1 \end{array}\right].$$

The optimum P_1 and P_2 can then be obtained using matrix inversion [28] as $\mathbf{p} = \mathbf{B}_2^{-1}\mathbf{n}_2$, i.e.,

$$\mathbf{p} = \frac{1}{\Delta}\left[\begin{array}{cc} \frac{h_{22}G_1}{x_2^*} & h_{21}\nu_{21} \\ h_{12}\nu_{12} & \frac{h_{11}G_1}{x_1^*} \end{array}\right]\left[\begin{array}{c} N_1W_2 \\ N_2W_1 \end{array}\right], \tag{20}$$

where $\Delta \stackrel{\triangle}{=} \frac{h_{11}h_{22}G_1G_2}{x_1^*x_2^*} - h_{12}h_{21}\nu_{12}\nu_{21}$. It is observed that for sufficiently large values of x_1^* and x_2^*, it is possible that $\Delta < 0$, i.e., P_1^*, $P_2^* < 0$, thus violating (4). Alternatively, when $\Delta > 0$, the values of x_1^* and x_2^* could be so large that it makes $P_1^* > P_{\max}^{(1)}$ or $P_2^* > P_{\max}^{(2)}$, again violating (4).

In the example discussed above, both the circumstances under which (4) is violated, occur when x_1^* and x_2^* are large, which, in turn,occurs when λ is small. This is because, x_i^* is a decreasing function of λ when u_i and f_i satisfy (17). It then raises a fundamental question, *Similar to the upper cutoff for the pricing parameter,* $\lambda_{\max}$ *specified by Theorem 3.1, can we obtain a lower cutoff,* $\lambda_{\min}$ *for* λ *below which the P-PCP becomes infeasible or below which pricing is ineffective?* We answer this question in the following subsection, by applying the theory of $\mathcal{M}$-matrices and also provide a necessary and sufficient condition for the existence of a unique Nash equilibrium for the P-PCP.

2) Sufficient Conditions: From (12), (14) and (15), for positive $\mathbf{x}^*$, $\mathbf{D}_1$ is positive since $G_i > 0, \forall\ i$ and $\mathbf{D}_2$ is positive since the N_i, $h_{ii} > 0$. Also, from (13), (14) and the definition of $\mathcal{Z}-$ matrix provided in Appendix B, $\left(\mathbf{I}_M - {\mathbf{D}_1}^{-1}\mathbf{A}\right)$ is a $\mathcal{Z}$-matrix. Therefore, re-writing (12) as

$$\left(\mathbf{I}_M - {\mathbf{D}_1}^{-1}\mathbf{A}\right)\mathbf{p}^* = {\mathbf{D}_1}^{-1}\mathbf{D}_2(\mathbf{1}\mathbf{1}^T - \mathbf{I}_M)\mathbf{w} \tag{21}$$

and applying 4) in Lemma B.1 in Appendix B, we state Theorem 3.2 below.

Theorem 3.2: A non-negative ICNR vector, $\mathbf{x}^{*}$[5] *results in a non-negative vector,* $\mathbf{p}^*$*, if and only if the* $\mathcal{Z}-$*matrix,* $\left(\mathbf{I}_M - {\mathbf{D}_1}^{-1}\mathbf{A}\right)$ *is an* $\mathcal{M}$*-matrix.*

Proof: If $\mathbf{p}^*$ is non-negative, then according to 4) in Lemma B.1 in Appendix B, the $\mathcal{Z}-$matrix, $\left(\mathbf{I}_M - {\mathbf{D}_1}^{-1}\mathbf{A}\right)$ is an $\mathcal{M}-$matrix. If $\left(\mathbf{I}_M - {\mathbf{D}_1}^{-1}\mathbf{A}\right)$ is an $\mathcal{M}-$matrix, then, from 2) in Lemma B.1 in Appendix B, it is non-singular and $\left(\mathbf{I}_M - {\mathbf{D}_1}^{-1}\mathbf{A}\right)^{-1} > \mathbf{0}$, where $\mathbf{0}$ is the null-vector or the null-matrix. Since $\left(\mathbf{I}_M - {\mathbf{D}_1}^{-1}\mathbf{A}\right)^{-1} > \mathbf{0}$ and $\mathbf{D}_1^{-1}$, $\mathbf{D}_2$, $\left(\mathbf{1}\mathbf{1}^T - \mathbf{I}_M\right)$ and $\mathbf{w}$ are all non-negative (i.e., each element in each of these matrices/vectors are non-negative), $\mathbf{p}^*$ obtained from (12) is non-negative. ■

Theorem 3.2 therefore provides a necessary and sufficient condition to obtain non-negative $\mathbf{p}^*$ from a non-negative ICNR vector, $\mathbf{x}^*$. The vector, $\mathbf{x}^*$, in turn, depends on the pricing parameter, λ, from (16). Thus, one can obtain conditions on λ that results in a non-negative $\mathbf{p}^*$, by following the sequence of steps listed in Algorithm 1.

In order to achieve Step 1) in Algorithm 1, we prove Lemma 3.1 which shows that when all receivers obtain higher ICNR's the powers/intensities of all corresponding information providers increase. The following definition of increasing functions is used in Lemma 3.1.

[5]A positive (non-negative) vector or matrix is one in which all elements are positive (non-negative). A negative (non-positive) vector or matrix is also similarly defined.

Algorithm 1 Sequence of steps to obtain a lower bound on the pricing parameter, λ, which, in turn, provides a sufficient condition for the existence of the Nash equilibrium.

1) First we show that when x_i^* increases, $\forall\ i$, P_i^* also increases, $\forall\ i$.
2) We then show that there is some threshold vector $\hat{\mathbf{x}}$ such that if even one entry of the vector, $\mathbf{x}^*$ is bigger than that in $\hat{\mathbf{x}}$, then it violates the constraints in (4).
3) We combine the results of Steps 1) and 2) with the fact that x_i is a decreasing function of λ, $\forall\ i$ when u_i and f_i satisfy (17), to obtain a lower cut-off, $\lambda_{\min}$ on λ to yield a sufficient condition for the existence of a unique Nash equilibrium of the P-PCP.
4) Finally the result of Step 3) is combined with Theorem 3.1 to obtain a necessary and sufficient condition for the existence of a unique Nash equilibrium.

Definition 3.1: Consider a function $\mathbf{f}: \mathcal{R}^n \rightarrow \mathcal{R}^m$. Let $\mathbf{y}_1, \mathbf{y}_2 \in \mathcal{R}^n$ and let $\mathbf{y}_1 < \mathbf{y}_2$[6]. Then $\mathbf{f}(\mathbf{y})$ is said to be an *increasing function* if $\mathbf{f}(\mathbf{y}_1) < \mathbf{f}(\mathbf{y}_2)$. A *decreasing function* is defined similarly.

Lemma 3.1: Consider two non-negative ICNR vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ such that $\mathbf{x}_1 < \mathbf{x}_2$ Let $\mathbf{p}_1$ and $\mathbf{p}_2$ be the corresponding vectors obtained from (12) and let $\mathbf{p}_2$ be non-negative. Then $\mathbf{p}_1$ is non-negative and $\mathbf{p}_1 < \mathbf{p}_2$. In other words, $\mathbf{p}$ is an increasing function of $\mathbf{x}$.

Proof: Let $\hat{\mathbf{D}}_1^{(1)}$ and $\hat{\mathbf{D}}_1^{(2)}$ be the diagonal matrices as defined in (14) corresponding to $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. It is observed that $\hat{\mathbf{D}}_1^{(1)} > \hat{\mathbf{D}}_1^{(2)}$ since $\mathbf{x}_1 < \mathbf{x}_2$. Any diagonal matrix is a $\mathcal{Z}$-matrix and any positive diagonal matrix is an $\mathcal{M}$-matrix according to the definitions of $\mathcal{Z}$ and $\mathcal{M}$-matrices given in Section B. Therefore, from Lemma B.2 in Appendix B, $\left(\hat{\mathbf{D}}_1^{(1)}\right)^{-1} < \left(\hat{\mathbf{D}}_1^{(2)}\right)^{-1}$. Hence the $\mathcal{Z}$-matrix $\left[\mathbf{I}_M - \left(\hat{\mathbf{D}}_1^{(1)}\right)^{-1}\mathbf{A}\right] > \left[\mathbf{I}_M - \left(\hat{\mathbf{D}}_1^{(2)}\right)^{-1}\mathbf{A}\right]$, another $\mathcal{Z}$-matrix. Since $\mathbf{p}_2$ is non-negative, $\left[\mathbf{I}_M - \left(\hat{\mathbf{D}}_1^{(2)}\right)^{-1}\mathbf{A}\right]$ is an $\mathcal{M}$-matrix by Theorem 3.2. Therefore from Lemma B.2 in Appendix B, $\left[\mathbf{I}_M - \left(\hat{\mathbf{D}}_1^{(1)}\right)^{-1}\mathbf{A}\right]$ is also an $\mathcal{M}$-matrix. Hence, from Theorem 3.2, $\mathbf{p}_1$ is non-negative. Also, from Lemma B.2 in Appendix B $\left[\mathbf{I}_M - \left(\hat{\mathbf{D}}_1^{(1)}\right)^{-1}\mathbf{A}\right]^{-1} < \left[\mathbf{I}_M - \left(\hat{\mathbf{D}}_1^{(2)}\right)^{-1}\mathbf{A}\right]^{-1}$. It then follows from (12) that $\mathbf{p}_1 < \mathbf{p}_2$. ■

The next step to obtain a sufficient condition for the existence of the Nash equilibrium is to show the existence of a threshold ICNR vector, $\hat{\mathbf{x}}$, mentioned in Step 2) in Algorithm 1. The following result from matrix theory will be used to achieve this.

Lemma 3.2: [28] *Let $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ be $n\times n$, $n\times k$, $k\times n$ and $k\times k$ matrices, respectively. Let the $(n+k)\times(n+k)$ block matrix, $\mathcal{A}=\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$. Then, $det(\mathcal{A})= det(\mathbf{A})det(\mathbf{D}-\mathbf{C}\mathbf{A}^{-1}\mathbf{B})$.*

For any k such that $1 \leq k \leq M$, let ${\mathbf{D}_1}^{(k)}$ and $\mathbf{A}^{(k)}$ denote the $k \times k$ leading principal sub-matrices (i.e., the sub-matrices specified by the first k rows and columns) of $\mathbf{D}_1$ and $\mathbf{A}$, respectively. If $\mathbf{Z}^{(1)} = [1]$, then for $k \geq 2$,

[6]The relations, $>$, $\geq$, $<$ and $\leq$ between two vectors or matrices indicate the relation between the corresponding elements of the vectors/matrices.

the $\mathcal{Z}$-matrix $\mathbf{Z}^{(k)} \triangleq \left(\mathbf{I}_k - \left(\mathbf{D}_1{}^{(k)}\right)^{-1} \mathbf{A}^{(k)}\right)$ can be written as $\mathbf{Z}^{(k)} = \begin{bmatrix} \mathbf{Z}^{(k-1)} & \mathbf{f}_k \\ \mathbf{g}_k^T & 1 \end{bmatrix}$ where

$$\mathbf{f}_k \triangleq \begin{bmatrix} -\frac{x_1^* h_{k1} \nu_{k1}}{G_1 h_{11}} \\ -\frac{x_2^* h_{k2} \nu_{k2}}{G_2 h_{22}} \\ \vdots \\ -\frac{x_{k-1}^* h_{kk-1} \nu_{kk-1}}{G_{k-1} h_{k-1k-1}} \end{bmatrix} \text{ and } \mathbf{g}_k \triangleq \begin{bmatrix} -\frac{x_k^* h_{1k} \nu_{1k}}{G_k h_{kk}} \\ -\frac{x_k^* h_{2k} \nu_{2k}}{G_k h_{kk}} \\ \vdots \\ -\frac{x_k^* h_{k-1k} \nu_{k-1k}}{G_k h_{kk}} \end{bmatrix}.$$

The matrix $\left(\mathbf{I}_M - \mathbf{D}_1{}^{-1}\mathbf{A}\right)$ can be formed by a sequence of matrices $\mathbf{Z}^{(1)}$, $\mathbf{Z}^{(2)}$, $\cdots$, $\mathbf{Z}^{(M)}$. The following lemma provides a property about $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k$, which will be used along with Lemma 3.4 in Theorem 3.3 to obtain the threshold vector, $\hat{\mathbf{x}}$, mentioned in Step 2) in Algorithm 1.

Lemma 3.3: The function $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k$ *is an increasing function of* $\mathbf{x}$, $\forall\ k$ *such that* $2 \leq k \leq M$.

Proof: Consider two positive vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ such that $\mathbf{x}_1 < \mathbf{x}_2$. Let $\hat{\mathbf{D}}_1^{(1)}$ and $\hat{\mathbf{D}}_1^{(2)}$ be the diagonal matrices as defined in (14) corresponding to $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. Let $\left(\hat{\mathbf{D}}_1^{(1)}\right)^{(k)}$ and $\left(\hat{\mathbf{D}}_1^{(2)}\right)^{(k)}$ be the $k \times k$ leading principal sub-matrices of $\hat{\mathbf{D}}_1^{(1)}$ and $\hat{\mathbf{D}}_1^{(2)}$, respectively. Let $\mathbf{Z}_1^{(k)} \triangleq \mathbf{I}_k - \left[\left(\hat{\mathbf{D}}_1^{(1)}\right)^{(k)}\right]^{-1} \mathbf{A}_k$ and $\mathbf{Z}_2^{(k)} \triangleq \mathbf{I}_k - \left[\left(\hat{\mathbf{D}}_1^{(2)}\right)^{(k)}\right]^{-1} \mathbf{A}_k$. Following the steps in the proof of Theorem 3.1, $\left(\mathbf{Z}_1^{(k-1)}\right)^{-1} \leq \left(\mathbf{Z}_2^{(k-1)}\right)^{-1}$, $\forall\ k$ such that $2 \leq k \leq M$. $\mathbf{f}_k$ and $\mathbf{g}_k$ are decreasing functions of $\mathbf{x}$. Hence, $-\mathbf{f}_k$ and $-\mathbf{g}_k$ are increasing functions of $\mathbf{x}$. Let $\mathbf{f}_k^{(1)}$ and $\mathbf{g}_k^{(1)}$ be the $\mathbf{f}_k$ and $\mathbf{g}_k$ vectors corresponding to ICNR vector $\mathbf{x}_1$. Similarly, let $\mathbf{f}_k^{(2)}$ and $\mathbf{g}_k^{(2)}$ be the $\mathbf{f}_k$ and $\mathbf{g}_k$ vectors corresponding to $\mathbf{x}_2$. respectively. Since $\mathbf{x}_1 < \mathbf{x}_2$ and $-\mathbf{f}_k$ and $-\mathbf{g}_k$ are increasing functions of $\mathbf{x}$, $-\mathbf{f}_k^{(1)} < -\mathbf{f}_k^{(2)}$ and $-\mathbf{g}_k^{(1)} < -\mathbf{g}_k^{(2)}$. Therefore, $\left(\mathbf{g}_k^{(1)}\right)^T \left(\mathbf{Z}_1^{(k-1)}\right)^{-1} \mathbf{f}_k^{(1)} = \left(-\mathbf{g}_k^{(1)}\right)^T \left(\mathbf{Z}_1^{(k-1)}\right)^{-1} \left(-\mathbf{f}_k^{(1)}\right) < \left(-\mathbf{g}_k^{(2)}\right)^T \left(\mathbf{Z}_2^{(k-1)}\right)^{-1} \left(-\mathbf{f}_k^{(2)}\right) = \left(\mathbf{g}_k^{(2)}\right)^T \left(\mathbf{Z}_2^{(k-1)}\right)^{-1} \mathbf{f}_k^{(2)}$, i.e., $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k$ is an increasing function of $\mathbf{x}$, $\forall\ k$ such that $2 \leq k \leq M$. ■

Lemma 3.2 will be applied to obtain the following lemma, which, in turn, will be used along with Lemma 3.3 in Theorem 3.3, to obtain the threshold ICNR vector, $\hat{\mathbf{x}}$ mentioned in Step 2) in Algorithm 1.

Lemma 3.4: $\left(\mathbf{I}_M - \mathbf{D}_1{}^{-1}\mathbf{A}\right)$ *is an* $\mathcal{M}$*-matrix if and only if* $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k < 1$, $\forall\ k$, $2 \leq k \leq M$.

Proof: From 3) in Lemma B.1 in Appendix B, $\left(\mathbf{I}_M - \mathbf{D}_1{}^{-1}\mathbf{A}\right)$ is an $\mathcal{M}$-matrix if an only if $det\left(\mathbf{Z}^{(k)}\right) > 0$, $\forall\ k$, $2 \leq k \leq M$. From Lemma 3.2, $det\left(\mathbf{Z}^{(k)}\right) = det\left(\mathbf{Z}^{(k-1)}\right)(1 - \mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k)$. Since $\mathbf{Z}^{(1)} = [1]$, by induction on k, $det\left(\mathbf{Z}^{(k)}\right) > 0$, $\forall\ k$, $2 \leq k \leq M$, if an only if $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k < 1$, $\forall\ k$, $2 \leq k \leq M$. ■

The following theorem proves the existence of the threshold vector, $\hat{\mathbf{x}}$, listed in Step 2) in Algorithm 1.

Theorem 3.3: $\exists$ *a positive ICNR vector* $\hat{\mathbf{x}}$ *such that,* $\forall\ \mathbf{x}$ *such that* $\mathbf{x} < \hat{\mathbf{x}}$*, the vector,* $\mathbf{p}^*$*, obtained from (12) satisfies constraints, (4) and for* $\mathbf{x} > \hat{\mathbf{x}}$*, at least one constraint in (4) is violated.*

Proof: From Lemma 3.3, $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k$ is an increasing function of $\mathbf{x}$. Also, when $\mathbf{x} = \mathbf{0}$, $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k = 0$, $\forall\ k$. Hence, for a positive $\mathbf{x}$, $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k > 0$, $\forall\ k$. Further, $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k$ is an unbounded function of $\mathbf{x}$, $\forall\ k$. Hence, $\exists$ a positive ICNR vector $\tilde{\mathbf{x}}$ such that $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k = 1$ for some k. Hence, $\forall\ \mathbf{x}$ such that $\mathbf{x} < \tilde{\mathbf{x}}$, $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k < 1$, $\forall\ k$, i.e., $\left(\mathbf{I}_M - {\mathbf{D}_1}^{-1}\mathbf{A}\right)$ is an $\mathcal{M}$-matrix from Lemma 3.4 and hence, $\mathbf{p}^*$ is non-negative, according to Theorem 3.2. For $\mathbf{x} > \tilde{\mathbf{x}}$, $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k > 1$ for some k and from Lemma 3.4, $\left(\mathbf{I}_M - {\mathbf{D}_1}^{-1}\mathbf{A}\right)$ is not an $\mathcal{M}$-matrix, i.e., $P_k^* < 0$ for some k, according to Theorem 3.2, i.e., $\mathbf{p}^*$ violates at least one constraint in (4).

Consider the case when $\mathbf{x} < \tilde{\mathbf{x}}$. Since $\mathbf{p}^*$ is an increasing function of $\mathbf{x}$ from Theorem 3.2, as $\mathbf{x}$ increases, $\mathbf{p}$ increases and $\exists\ \mathbf{x}'$, such that for $k \in K \subset \{1, 2, \cdots, M\}$, $P_k = P_{\max}^{(k)}$ and $P_j < P_{\max}^{(j)}$, $j \notin K$. For $\mathbf{x} > \mathbf{x}'$, $\exists$ $k \in K$, such that $P_k > P_{\max}^{(k)}$. Let $\hat{\mathbf{x}} = \min(\mathbf{x}', \tilde{\mathbf{x}})$, i.e., the vector in which each element is the smallest of the corresponding elements in $\mathbf{x}'$ and $\tilde{\mathbf{x}}$. For $\mathbf{x} < \hat{\mathbf{x}}$, $0 \le P_k \le P_{\max}^{(k)}$, $\forall\ k$ and for $\mathbf{x} > \hat{\mathbf{x}}$, $\exists\ k$, such that $P_k < 0$ or $P_k > P_{\max}^{(k)}$, i.e., some constraint in (4) is violated. ■

Using the threshold vector, $\hat{\mathbf{x}}$, from Theorem 3.3, it is possible to obtain a lower bound, $\lambda_{\min}$ on λ, below which, the vector $\mathbf{p}$ obtained from (12) is infeasible (as described in Step 3) in Algorithm 1). The following theorem provides the existence of $\lambda_{\min}$ and also provides a necessary and sufficient conditions for the existence of a unique Nash equilibrium for the P-PCP (thus completing Steps 3) and 4) in Algorithm 1).

Theorem 3.4: Let the pricing function $f_i(x_i)$ be a non-decreasing convex function. Then, $\exists\ \lambda_{\min}$ such that the game modeled by the optimization problem in (9) subject to constraints (4) has a unique feasible Nash equilibrium if and only if $\lambda \in (\lambda_{\min}, \lambda_{\max})$, with $\lambda_{\max}$ as specified in Theorem 3.1.

Proof: For $\mathbf{x}^*$ to be positive, λ should be less than $\lambda_{\max}$, according to Theorem 3.1. Consider $\lambda < \lambda_{\max}$, so that $\mathbf{x}^*$ is positive. When the u_i and f_i satisfy (17), x_i^* is a decreasing function of λ. As λ decreases, x_i^* increases $\forall\ i$ and for some $\lambda = \lambda_{\min}$, $\mathbf{x} = \hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$ is as specified in Theorem 3.3. For $\lambda > \lambda_{\min}$, $\mathbf{x} < \hat{\mathbf{x}}$ and $\mathbf{p}$ is feasible (satisfies (4)) according to Theorem 3.3. Similarly, if $\lambda < \lambda_{\min}$, $\mathbf{x} > \hat{\mathbf{x}}$ and $\mathbf{p}$ is infeasible (violates (4)), according to Theorem 3.3. Therefore $\lambda > \lambda_{\min}$ for a positive $\mathbf{x}$ to result in intensities that satisfy (4), i.e., in a feasible Nash equilibrium. In other words, the Nash equilibrium is feasible if and only if $\lambda \in (\lambda_{\min}, \lambda_{\max})$.

When the pricing function, $f_i(x_i)$ is convex, the net utility, $\hat{u}_i = u_i - \lambda f_i(x_i)$ is concave since u_i in (2) is concave. Therefore the optimum x_i^* obtained from (16) is unique [29]. Since $\mathbf{p}^*$ is obtained from (12) by inversion of a non-singular $\mathcal{M}-$matrix, the Nash equilibrium is unique. ■

When $f_i(x_i)$ is not a convex function, Theorem 3.4 can still be applied to prove the existence of a Nash equilibrium. If $f_i(x_i)$ is such that $\hat{u}_i(x_i)$ is concave, then the Nash equilibrium is unique. Otherwise, the Nash equilibrium may not be unique. The linear pricing function in (6) and the non-linear pricing function in (7) are both convex functions and hence, the Nash equilibrium of the P-PCP is unique when deploying these pricing functions.

Similar to how Theorem 3.1 provided an upper "cut-off" for pricing users, Theorem 3.4 intuitively indicates that there is also a lower "cut-off" for pricing users, below which the pricing becomes ineffective. Intuitively, $\lambda < \lambda_{\min}$ represents a scenario where the pricing is so low that transmit-receive pairs suffer no penalty even when the providers of information transmit with large intensities or powers. As more information providers transmit with maximum powers it could result in transmit-receive pairs suffering lower ICNR than their required target ICNR to maximize the utility of the obtained information. A network with $\lambda < \lambda_{\min}$ therefore represents an aggressive network because information providers transmit information with high intensities.

For a social network to be neither extremely aggressive nor extremely passive, transmit-receive pairs should neither be priced too low nor too high. The upper and lower bounds for pricing are provided by Theorems 3.1 and 3.4, respectively. Although Theorem 3.4 proves the existence of λ_{min}, computation of λ_{min} in closed-form is difficult. One can then apply the method described by Fiedler [30] to evaluate λ_{min} numerically. When $\lambda = 0$, it corresponds to the case with no pricing and the the optimal values of P_i is unbounded, $\forall\ i$. But, when an upper bound constraint is posed on the maximum power, then $P_i = P_{\max}^{(i)}$, $\forall\ i$. The following sub-section presents a means to control the pricing parameter, λ, in order to maintain the network (i.e., the information providers in the network) at a desired level of aggression and passiveness.

E. Controlling the Levels of Aggression and Passiveness

Definition 3.2: A network is said to be $\epsilon-$aggressive $(0 \leq \epsilon \leq 1)$, if the average transmit power of all the transmitters is $\geq \epsilon \frac{1}{M} \sum_{i=1}^{M} P_{\max}^{(i)}$. A network is said to be aggressive if it is $\epsilon-$aggressive for $\epsilon = 1$.

An $\epsilon-$ aggressive network for larger values of ϵ represents a network where the transmitters transmit with close to their maximum intensity. Since the intensity represents the authenticity or aggression with which information is presented, an $\epsilon-$aggressive network is representative of an aggressive network for larger values of ϵ.

Definition 3.3: A network is said to be $\delta-$passive $(0 \leq \delta \leq 1)$, if the average transmit power of all the transmitters is $\leq \delta \frac{1}{M} \sum_{i=1}^{M} P_{\max}^{(i)}$. A network is said to be passive if it is $\delta-$passive for $\delta = 0$.

A $\delta-$passive network for smaller values of δ represents a network in which transmitters transmit at less than δ fraction of their maximum intensity, which, in turn, represents a scenario where transmitters are more "passive" as they tend to present information with very low levels of aggression.

We now address the following question. *Is it possible to control the pricing parameter, λ, so that the network can be made $\epsilon-$aggressive or $\delta-$passive for any desired value of δ or ϵ?* Theorems 3.1 and 3.4 imply that it is possible to make the network aggressive (by making $\lambda < \lambda_{\min}$) or make it passive (by making $\lambda > \lambda_{\max}$). The following theorems indicate that is it possible to control the pricing parameter, λ, in order to make the network, $\epsilon-$aggressive or $\delta-$passive, $\forall\ 0 \leq \epsilon, \delta \leq 1$.

Theorem 3.5: For all ϵ, $0 \leq \epsilon \leq 1$, $\exists\ \lambda_{\min}(\epsilon)$ such that the network is $\epsilon-$aggressive for $\lambda < \lambda_{\min}(\epsilon)$.

Proof: The vector, $\mathbf{p}$, is obtained from (12), by the inversion of an $\mathcal{M}-$matrix, and hence, is a continuous function of $\mathbf{x}$ [28]. Also, when u_i and $f_i(x_i)$ satisfy (17), $\mathbf{x}$ is a decreasing function of λ. From Theorem 3.2, $\mathbf{p}$ is a continuous increasing function of $\mathbf{x}$ and hence, a continuous decreasing function of λ. The expression, $\frac{1}{M}\sum_{i=1}^{M} P_i$ is a continuous increasing function of $\mathbf{p}$ and hence, a continuous decreasing function of λ. When $\lambda < \lambda_{\min}$, with $\lambda_{\min}$ as specified in Theorem 3.4, $P_i = P_{\max}^{(i)}$, $\forall\ i$. Therefore, $\forall\ \epsilon \in [0, 1]$, $\exists\ \lambda_{\min}(\epsilon) > \lambda_{\min}$, such that $\frac{1}{M}\sum_{i=1}^{M} P_i \geq \epsilon \frac{1}{M}\sum_{i=1}^{M} P_{\max}^{(i)}$, i.e., the network is $\epsilon-$aggressive. ■

Theorem 3.6: For all δ, $0 \leq \delta \leq 1$, $\exists\ \lambda_{\max}(\delta)$ such that the network is $\delta-$passive for $\lambda > \lambda_{\max}(\delta)$.

Proof: The proof is identical to the proof of Theorem 3.5. ■

Theorems 3.5 and 3.6 therefore imply that it is possible to keep the information providers in the network as aggressive or as passive by controlling the pricing parameter, λ. However, the exact values of $\lambda_{\max}(\delta)$ and $\lambda_{\min}(\epsilon)$ are very complex to obtain for a specified δ or ϵ. Numerical techniques suggested in [30] can be used to obtain $\lambda_{\max}(\delta)$ and $\lambda_{\min}(\epsilon)$.

Unequal number of information providers and information consumers: If the system has M information providers and N information consumers, $M < N$, then some information providers are primary information providers to more than one information consumer. If an information provider is a primary information provider to $\hat{M}$ information consumers, then this information provider can be viewed as $\hat{M}$ virtual information providers. The network then has N virtual information providers and N information consumers and the analysis described in Section III can be applied. As an example, Fig. 6(a) presents a network with $M = 3$ information providers and $N = 4$ information consumers. Information provider 3 is a primary information provider to information consumers 3 and 4. Information provider 3 is then represented as virtual information provider 3 and virtual information provider 4 so that the network has $N = 4$ virtual information providers and $N = 4$ information consumers. If $M > N$, then some information consumers look for multiple primary information providers of information. In this case, these information consumers can be viewed as multiple virtual consumers. The network will then have M information providers and M virtual information consumers and the analysis in Section III can be applied. As an example, Fig. 6(b) shows a network with $M - 4$ information providers and $N = 3$ information consumers. Information consumer 3 is represented as virtual consumer 3 and virtual consumer 4 so that the network now has $M = 4$ information providers and $M = 4$ virtual information consumers. In general, when $M \neq N$, let $\tilde{M} = \max(M, N)$. The number of transmit-receive pairs then becomes $\tilde{M}$.

IV. Applications

We now present additional applications of our analysis presented in Section III-D2. We first discuss a novel admission control scheme to admit new consumers in the network based on their ICNR requirements (Section IV-A). We then present a scenario where our analysis can be applied to maximize the efficiency of users performing tasks assigned by multiple categories of assigners (Section IV-B).

A. Admission Control

Consider a network in which transmit-receive pairs arrive sequentially. This could be in a chat room or a closed group discussing some specialized topic. By arrival of a transmit-receive pair we mean the arrival of a new provider of information or a new information consumer or both. Let there be $k-1$ transmit-receive pairs in the network and let the k^{th} information provider/information consumer arrive. The necessary condition for the feasibility of the powers/intensities of transmission of all information providers is that the matrix $\mathbf{Z}^{(k)}$ in Lemma 3.4 be an $\mathcal{M}-$matrix, i.e., if an only if $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k < 1$. The ICNR for the k^{th} transmit receive pair, x_k^*, affects only the vector $\mathbf{g}_k^T$. Hence using Lemma 3.4, a new transmit receive pair is admitted into the system only if $x_k^* > 0$ and $\mathbf{g}_k^T \left(\mathbf{Z}^{(k-1)}\right)^{-1} \mathbf{f}_k < 1$.

Consider a chaotic discussion forum in which different kind of information flows. An example could be that of a room discussing subjective topics such as politics or spirituality. Here, different users join the discussion one after the other and either seek others' opinion or provide their opinion on the topic. If a particular consumer enters with a specific ICNR requirement, then it signifies the level of clarity the newly entering consumer expects from the forum. A high ICNR requirement represents a requirement for a high level of clarity, which may not be possible in a discussion forum on subjective topics. The admission control policy described in this subsection then enables a network filter out consumers that have unrealistic expectations on the clarity of information in the forum.

B. Prioritizing Tasks to Increase Productivity

Consider an organization or a social structure in which a category of M users (called task assigners) assign tasks to another set of N users called task assignees. The task assigned by the j^{th} assigner carries a priority, P_j. The i^{th} task assignee carries out the task assigned by the j^{th} assigner with efficiency, $0 \leq h_{ji} \leq 1$ ($h_{ji} = 0$ represents 0% efficiency in carrying out the task and $h_{ji} = 1$ represents 100% efficiency. Alternatively, h_{ji} could represent the influence the j^{th} assigner has on the i^{th} assignee, i.e., $h_{ji} = 0$ represents no influence and $h_{ji} = 1$ indicates that the i^{th} assignee carries out the task to the fullest extent. The parameter, h_{ji} represents the relation between the j^{th} assigner and the i^{th} assignee.

As mentioned in Section III-D2, this can be viewed as a system with $\tilde{M} = \max(M, N)$ virtual assigner-assignee pairs. The i^{th} virtual assigner is the primary assigner of tasks. The amount of concentration the i^{th} assignee has on the task assigned by the primary assigner is then $P_i h_{ii}$. The tasks assigned by the other assigners serve as distractions to the i^{th} assignee. The i^{th} assigner can use W_i amount of auxiliary resources to improve the concentration of the i^{th} assignee. These resources could be in the form of rewards, incentives or penalties. Alternatively the auxiliary resources cold be in the form of reminders to complete the task. The i^{th} assignee suffers N_i amount of natural distractions (like entertainment, fatigue, etc). The natural distraction can be scaled by the amount of auxiliary resources used by the other assigners. Therefore, the i^{th} assignee suffers a distraction of $\sum_{j \neq i} P_j h_{ji} + N_i W_j$ due to the other assigners in the network. It is then possible to define a term called *Concentration-to-Distraction-Noise Ratio (CDNR)* similar to the ICNR defined in (1).

Each user has a productivity, r_i (in the absence of distractions) for the task assigned by the primary assigner. Analogous to the gain, G_i defined in Section II-B, it is possible to define a supervisory gain, $G_i = \alpha \frac{W_i}{r_i}$, which provides a factor by which the concentration can be improved by appropriately deploying auxiliary resources. The auxiliary resources are not so effective if the productivity of the user is already large. The CDNR of the i^{th} assignee, x_i, can then be written as

$$x_i = \frac{P_i h_{ii} G_i}{\sum_{j \neq i} P_j h_{ji} + N_i W_j}, \tag{22}$$

which is similar to the ICNR defined in (1). The effective productivity of the i^{th} assignee can then be written as in (2) where r_i is the productivity in the absence of distraction. The function, $\pi(x_i)$ then represents the amount of degradation in the productivity due to distractions.

One can then apply the analysis presented in Section III to determine the optimal priorities assigned to each task so that the productivity on all tasks are maximized. The pricing parameter, λ, then represents the cost incurred to maintain the desired concentration levels. Large λ represents a highly lenient environment and a small λ represents a more strict environment. Theorems similar to Theorems 3.1, 3.4, 3.5 and 3.6 can be obtained in order to control the pricing parameter, λ, so that environment can be made as lenient or as strict as desired. The analysis can be applied to prioritizing tasks in organizations to increase the output of the employees.

V. Numerical Examples

We collect data from Twitter on various topics like "Haiti", "Body Scan", "Indian Premier League (IPL)", from users in different geographical regions, using the architecture we developed in [4]. We treat each region as an information provider. We use this data to measure the contradiction, ν_{ji} between the information obtained from two different information providers, i and j, as explained in Section I.

For any topic, we consider three scenarios. (i) *Distributed Trust:* Scenarios when information consumers trust all information providers almost equally, (ii) *Highly Concentrated Trust:* Information consumers trust one information provider more than all the others (e.g., a consumer may trust information providers from the USA more than all other countries) and (iii) *Moderately Concentrated Trust:* Information consumers trust few information providers more than the others (e.g., a consumer may trust USA and Canada more than others) and among the information providers they trust, they trust all information providers equally. We normalize the maximum intensity, $P_{\max}^{(i)}$ to unity, $\forall\ i$. We perform about 100000 C-based experiments on UBUNTU LINUX platform, and present the averaged results.

We first study the effect of pricing for the various scenarios listed above. We normalize the pricing parameter, λ, so that $\lambda_{\max}$ in Theorem 3.1 is 1. We also normalize the obtained net utility so that the maximum net utility obtained over all scenarios and all pricing parameters is 1. Fig. 7 represents the behavior of the net utility for various values of the pricing parameter under the scenarios (i), (ii) and (iii) described in the previous paragraph, for linear pricing (Fig. 7(a)) as well as non-linear pricing (Fig. 7(b)). The length of the curves are unequal because the minimum value of the pricing parameter, $\lambda_{\min}$ given by Theorem 3.4 is different in different scenarios.

It is observed from Figs. 7(a) and 7(b) that high net utility (i.e., high relevance of information) can be achieved for lower price in the scenario when consumers trust one information provider more than the others. The achieved net utility is either low or the price paid is too high in the scenario when consumers trust all information providers equally. This is because, consumers placing equal trust on all information providers suffer larger confusion because they obtain contradictory information from all the information providers they trust. However, when consumers trust a single information provider or a set of few information providers, the confusion caused is lower. *This does not mean that consumers obtain correct information when they trust a single information provider. This only means that the confusion levels suffered by consumers are lower when they place more trust on few information providers (preferably a single information provider) instead of trusting all the information providers.* This also implies that the information providers can transmit at relatively lower intensities (i.e., spend less resources on advertising or providing auxiliary information to make the information consumer obtain information from them in future) if they build sufficient trust with the information consumer and also build much higher trust compared to other information providers. *No inference can be conclusively drawn about the behavior of linear or non-linear pricing, from Figs. 7(a) and 7(b) because the presented results are for a specific choice of the non-linear pricing function as in (7).*

Fig. 8 presents the aggression level (i.e., average intensity of transmission) of the transmitters for the scenarios of distributed trust, moderately concentrated trust and highly concentrated trust, discussed earlier in this section, with linear (Fig. 8(a)) as well as non-linear pricing (Fig. 8(b)). Results are obtained by normalizing $P^{(i)}_{\max} = 1$, $\forall$ i (the results scale for other values of $P^{(i)}_{\max}$. Also, the pricing parameter, λ, is normalized so that $\lambda_{\max} = 1$. Note that the non-linear pricing function in (7 results in $\lambda_{\max} = \infty$. For our numerical computations we consider values of λ so that $P_i \approx 0$, $\forall$ i. We found that this is satisfied for $\lambda \approx 60$. We study the specific cases, $\epsilon = 0.85$ and $\delta = 0.1$.

It is observed from Fig. 8 that both for linear pricing as well as for non-linear pricing, the scenario, distributed trust, drops more quickly from aggressive to passive, while the scenario where consumers place highly concentrated trust on one transmitter allows a more gradual decay in the aggression level of the network. This is because, in the scenario with distributed trust, the confusion levels suffered by the consumers is larger (as discussed earlier in this Section in Fig. 7). Therefore, even for larger values of pricing parameters, transmitters transmit with intensities close to $P_{\max}$ in order to mitigate confusion.

Such a scenario represents a network with large levels of confusion because, information providers have to be optimally too aggressive (meaning that the noise levels are large) or too passive (i.e., have to suffer low ICNR's *and yet not increase the intensity of transmission due to high pricing parameters).*

For the scenario with moderately concentrated trust and highly concentrated trust, the confusion levels suffered by consumers are lower and hence, the value of λ should be much lower for transmitters to transmit with intensities closer to $P_{\max}$. This also results in smaller values of P_i, for the same ICNR, x_i, in the scenario with highly and moderately concentrated trusts compared to that with distributed trust, i.e., larger values of $\lambda_{\max}$, thus resulting in smoother decay in the aggression level of the network, compared to the scenario with distributed trust. Figs. 8(a) and 8(b) therefore imply that the scenario with moderately and highly concentrated trust gather aggression or passive behavior slowly as compared to that with distributed trust, i.e., is more stable because the aggression exhibits a more smooth decay compared to the scenario with distributed trust. *These scenarios represent a network with smaller levels of confusion.*

Intuitively Figs. 8(a) and 8(b) imply that in scenarios with distributed trust, since the consumer trusts all the providers of information, the information providers oscillate between being overly aggressive and overly passive too quickly, because the information consumers are easily influenced by the other information providers. On the contrary, in the scenarios with moderately and highly concentrated trust, consumers trust few information providers (or one information provider) and hence, the information providers, can be less aggressive and yet provide relevant information to the information consumers because the consumers do not have many other trust-worthy information providers. Distributed trust could represent a network of teenagers who get easily influenced by all providers of information and hence, one should be extremely strict with them at times.

The key insights of our analysis are two fold. For information providers, the understanding of information confusion presents the optimal intensities to present information so that it results in maximum relevance for the intended information seekers. For information seekers, the key insights are the facts that maximum confusion is likely to result when they trust all information providers almost equally and less confusion results when the information seekers place concentrated trust of a few information providers. This means that information seekers should not arrive at a quick decision about the information they receive, in scenarios where they trust all information providers almost equally. When the information seekers place highly concentrated trust on fewer information providers, they experience lesser confusion which enables them to take a quicker decision on the information they obtain.

A generalized scenario: It is noted that the numerical examples shown in this paper are for the specific case when the information providers are Twitter users. As a more general example, consider a customer who wishes to buy men's apparel. The possible choices of stores could be Walmart [31], Kohls [32], Target [33], Sears [34] and J. C. Penney [35]. Let this customer post a query in the Facebook page of J. C. Penney on a specific men's apparel he/she wishes to buy. The primary provider of information is J. C. Penney while the other providers of information could be his/her Facebook friends, Kohls, Walmart, Target, Sears etc. This corresponds to the case of multiple information providers and one receiver discussed at the end of Section III.

Here, the intensities could be a weighted combination of the number of choices (i.e., the variety), a weighted sum of the number and percentage of discount offers, the number of pictures, etc. The trust factor can be a function of the customer's preferences as well as the distance of the nearest store from the customer's premises. The auxiliary information includes viewer's rating of the apparel in the stores and testimonials of previous customers. The analysis presented in this paper can then be used by the various stores to optimize their intensities (the number of advertisements, their discount offers, the number of pictures they post on their web pages).

Computation of P_i's: Since the problem of determining the optimal intensities for the information providers is modeled as a non-cooperative game with complete information, all the information providers know each other's strategy sets and pay offs. Therefore all information providers will be able to use our analysis to determine the P_i's. In a social network context, the strategies are known by the actions taken by the information providers. For example, two competing restaurants can follow each other on Twitter or be friends in Facebook or follow each other's web pages and learn about their P_i's by viewing the pictures of their menus or their pricing strategies, etc. Similarly, in the example discussed in the previous two paragraphs, the various stores (i.e., Kohls, J. C. Penney, Sears, Target, Walmart) know the number of discount offers, the number of pictures posted, the number of links to different types of apparel, etc.

VI. Conclusion

We presented a quantitative analysis to maximize the relevance of information in networks with multiple information providers. A game theoretic approach for controlling intensities of information transmission in social networks was proposed, so that confusion can be minimized. A linear and non-linear pricing function was discussed and necessary and sufficient conditions for the existence of a unique Nash equilibrium were discussed. Applications of our work to admission control for new consumers and task prioritization were also discussed. Some of the key inferences drawn were

- The pricing parameter can be suitably adjusted in order to keep the aggression level of the network as high or as low as desired.

- Networks in which consumers place concentrated trust on fewer information providers achieve more relevant information transfer to the consumers (i.e., suffer from less confusion), compared to those in which consumers trust all the information providers in a similar manner.
- Networks in which consumers distribute their trust almost equally to all the information providers result in instability of aggression, i.e., oscillate between being highly aggressive to highly passive, thus representing a network with high levels of confusion.

Incorporating information flow and diffusion models (e.g., [36], [37] to study the propagation of confusion is a topic under investigation. We point out that the numerical examples in the paper does not cover the generalized scenario when the information providers can be of different types, e.g., a video, a news article and a message from another friend. In future, additional experiments can be performed to collect multi-domain data (e.g., domain blogs, news articles, tweets) and our analysis can be applied to the data.

APPENDIX A
GAME THEORY FUNDAMENTALS

A *game* [38], $\mathcal{G}\left(\mathcal{P},\mathcal{S},\mathcal{U}\right)$, is defined by a set of players, $\mathcal{P}=\{p_1,p_2,\cdots,p_n\}$, a strategy set, $\mathbf{S}_i$, for each player p_i ($\mathcal{S}\triangleq\{\mathbf{S}_1,\mathbf{S}_2,\cdots,\mathbf{S}_n\}$) and a pay off set or a set of utility functions, $\mathcal{U}=\{u_1,u_2,\cdots,u_n\}$, where $u_i:\mathbf{S}_i\rightarrow\mathcal{R}$, is the utility function or pay off function of the player p_i. The objective of a game $\mathcal{G}\left(\mathcal{P},\mathcal{S},\mathcal{U},\right)$ is for each player to choose a strategy $s_i\in\mathbf{S}_i$ to form an optimal strategy vector $\mathbf{s}=\begin{bmatrix} s_1 & s_2 & \cdots & s_n\end{bmatrix}$ such that $u_i(\mathbf{s})$ is maximum for each i. The utility or a pay off function in a game, $u(\chi)$, should be a non-negative, non-decreasing concave function [38], i.e.,

$$u(\chi)\geq 0,\quad\forall\chi,\quad\frac{du}{d\chi}\geq 0,\quad\forall\chi,\quad\frac{d^2u}{d\chi^2}\leq 0,\quad\forall\chi. \tag{23}$$

Definition A.1: [38] A strategy vector, $\mathbf{s}=\begin{bmatrix} s_1 & s_2 & \cdots & s_n\end{bmatrix}$, is said to be a *Nash equilibrium* of the game $\mathcal{G}\left(\mathcal{P},\mathcal{S},\mathcal{U},\right)$ if, $\forall\, i\in\{1,2,\cdots,n\}$, $u_i\left(s_i,\mathbf{s}_{-i}\right)\geq u_i\left(\hat{s}_i,\mathbf{s}_{-i}\right)$, $\forall\,\hat{s}_i\in\mathbf{S}_i$, where $\mathbf{s}_{-i}\triangleq\begin{bmatrix} s_1 & s_2 & \cdots & s_{i-1} & s_{i+1} & \cdots & s_n\end{bmatrix}$. The Nash equilibrium is a strategy vector such that the strategy of each player is the *best response* to the strategies of the other players.

APPENDIX B
ON $\mathcal{Z}-$ AND $\mathcal{M}-$MATRICES

An $n\times n$ matrix $\mathbf{B}=[b_{ij}]_{\substack{1\leq i\leq n\\1\leq j\leq n}}$ is called a $\mathcal{Z}-$matrix if $b_{ij}\leq 0$, $\forall\, i\neq j$. A $\mathcal{Z}$-matrix, $\mathbf{B}$, is called an $\mathcal{M}$-matrix if $\mathbf{B}^{-1}$ is non-negative. A comprehensive study on $\mathcal{M}-$matrices (including the following lemma) can be found in

[39].

Lemma B.1: [39] *The following statements are equivalent for any* $n \times n$ $\mathcal{Z}-$*matrix,* $\mathbf{B}$.

1) $\mathbf{B}$ *is an* $\mathcal{M}$*-matrix.*

2) $\mathbf{B}^{-1}$ *exists and is positive.*

3) *All principal and leading principal minors of* $\mathbf{B}$ *are positive.*

4) $\exists$ *a positive vector* $\mathbf{y}$ *such that* $\mathbf{By}$ *is positive.*

Additional properties on $\mathcal{M}-$matrices including the following lemma, can be found in [40].

Lemma B.2: [40] *If* $\mathbf{B}$ *is an* $n \times n$ $\mathcal{M}$*-matrix and* $\mathbf{C}$ *is an* $n \times n$ $\mathcal{Z}$*-matrix such that* $\mathbf{C} \geq \mathbf{B}$*, then* $\mathbf{C}$ *is an* $\mathcal{M}$*-matrix and* $\mathbf{B}^{-1} \geq \mathbf{C}^{-1}$.

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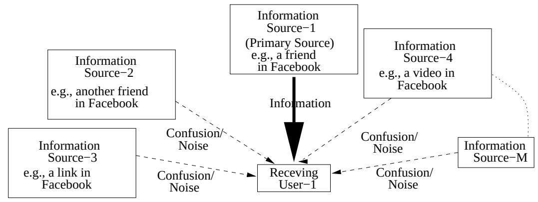


Fig. 1. A scenario where a consumer receives information from an information provider and noise/confusion from other information providers. Here "information source" represents information provider and "receiving user" represents information consumer.

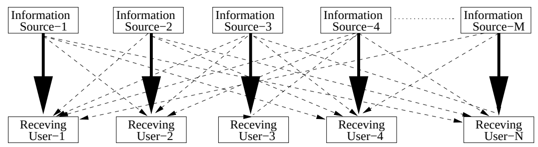


Fig. 2. A network with M information sources and N information consumers. The information from the j^{th} information provider to the i^{th} consumer $(j \neq i)$, is treated as confusion/noise. Here, "information source" represents information provider and "receiving user" represents information consumer.

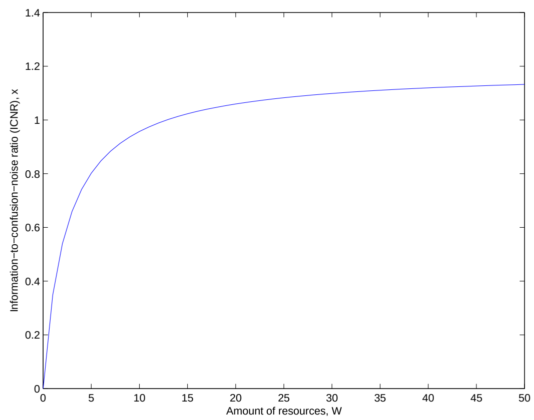
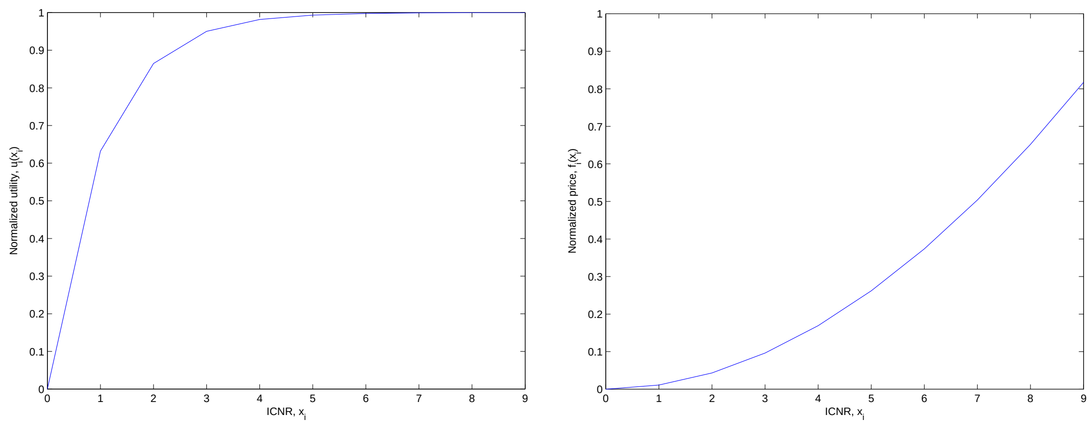


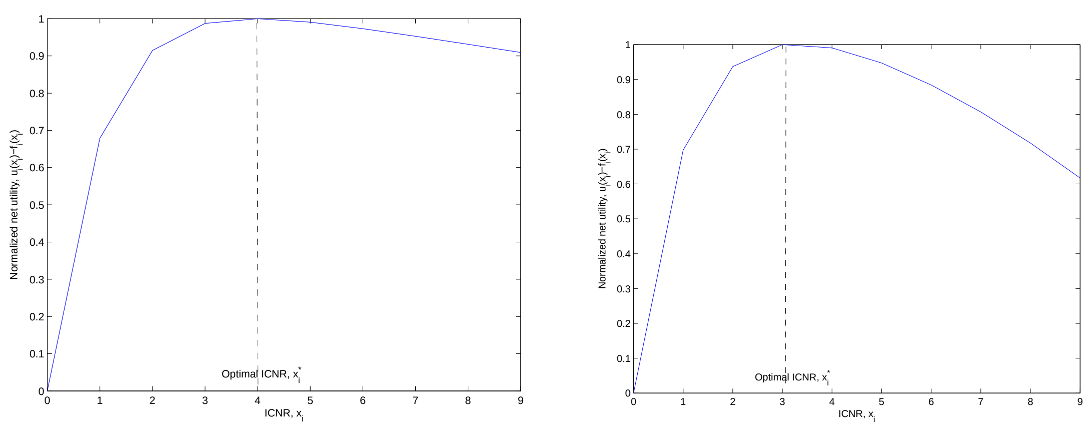
Fig. 3. Behavior of the ICNR with respect to the amount of auxiliary resources, W. More resources improve the ICNR but beyond a threshold, the improvement is negligible.



(a) Normalized utility function, u_i, in (2)

(b) Normalized pricing function, $f_i(x_i)$, in (7)

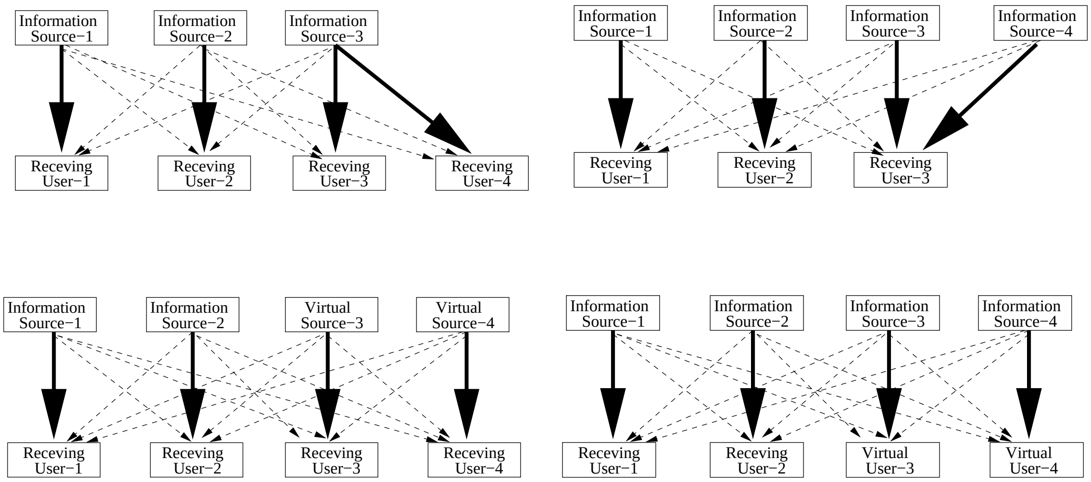
Fig. 4. For u_i, $\pi(x_i)$ is taken to be $\pi(x_i) = 1 - e^{-x_i}$. The largest values of u_i and $f_i(x_i)$ are normalized to 1.



(a) Linear pricing

(b) Non-linear pricing

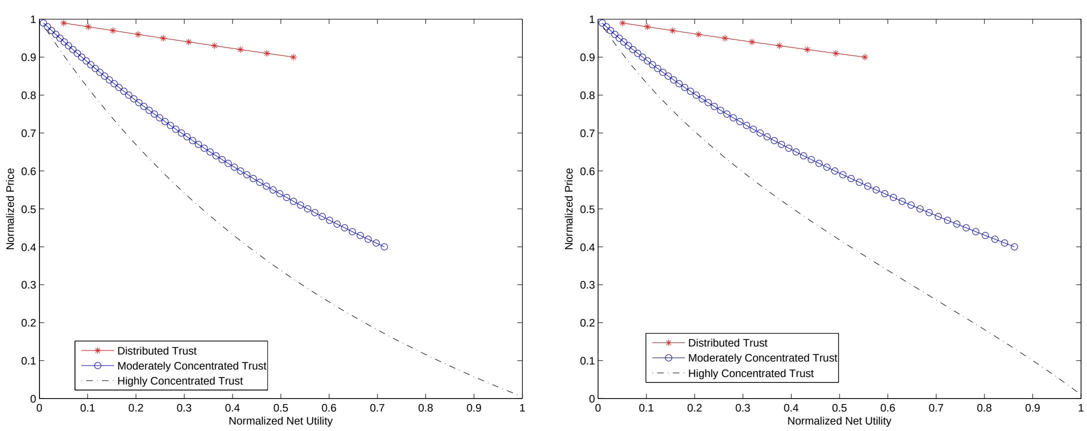
Fig. 5. Normalized net utility, $\hat{u}_i$ in (5) for $\lambda = 1$, as a function of the ICNR, x_i. The largest value of $\hat{u}_i$ is normalized to 1. The utility function, u_i is as shown in Fig. 4(a). Linear pricing corresponds to deploying the pricing function, $f_i(x_i)$ in (6) and non-linear pricing corresponds to deploying the pricing function, $f_i(x_i)$ in (7).



(a) $M = 3$ information providers and $N = 4$ information consumers

(b) $M = 4$ information providers and $N = 3$ information consumers

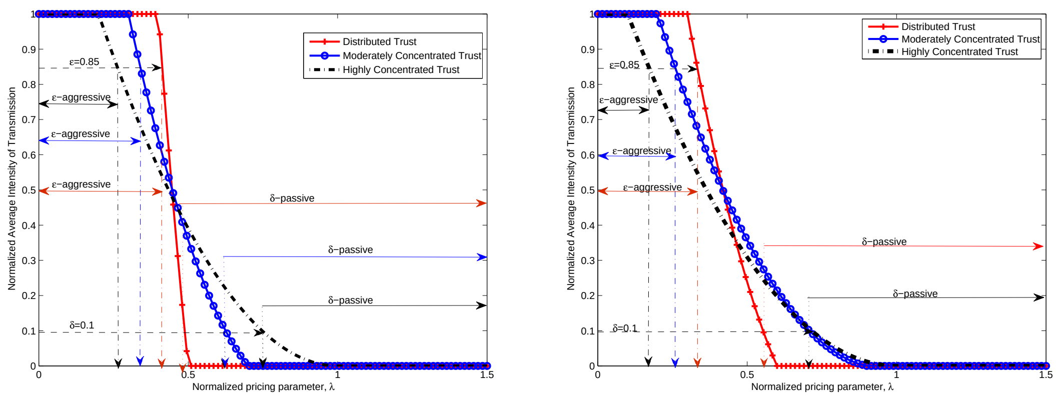
Fig. 6. “Information source” represents information provider and “receiving user” represents information consumer. In Fig. 6(a), information provider 3 is represented as virtual information provider 3 and virtual information provider 4. In Fig. 6(b), information consumer 3 is represented as virtual consumer 3 and virtual consumer 4.



(a) Linear pricing

(b) Non-linear pricing

Fig. 7. Pricing parameter to obtain various values of net utility under different scenarios. The length of the curves are unequal because the minimum value of the pricing parameter, $\lambda_{\min}$ given by Theorem 3.4 is different in different scenarios.



(a) Linear pricing

(b) Non-linear pricing

Fig. 8. Aggressiveness and passiveness level of the network with respect to the pricing parameter.

TABLE I

SNAPSHOT OF TWEETS COLLECTED FROM TWITTER ON FULL BODY SCAN IN AIRPORTS.

Geographical Region	**Total Tweets**	**Supportive Tweets**	**Opposing Tweets**	**Neutral Tweets**
USA	4571	967	753	2852
Europe	304	69	59	176
Asia	168	17	65	86
Oceania	94	17	24	53
Canada	70	14	19	37
South America	34	5	6	23
Middle East	34	10	7	17
Africa	22	3	4	15

TABLE II

PARAMETERS OF RESTAURANTS THAT CAN CONTROL P_i.

Parameter	Value for $R1$	Value for $R2$	Value for $R3$
Number of Links	2	5	3
Health Rating	4	3	5
Numer of Pictures	15	10	5
Discounts (%)	20	25	15

TABLE III

NORMALIZED INTENSITY (NI) CORRESPONDING TO VARIOUS PARAMETERS LISTED IN TABLE II THAT CAN CONTROL P_i FOR THE RESTAURANTS.

Parameter	NI for $R1$	NI for $R2$	NI for $R3$
Number of Links	0.2	0.5	0.3
Health Rating	0.33	0.25	0.42
Numer of Pictures	0.5	0.33	0.17
Discounts (%)	0.33	0.42	0.25

TABLE IV

EVALUATION OF VARIOUS AUXILIARY RESOURCES BASED ON THE SURVEY IN [2].

Auxiliary Resource	**Rating Received**
Professional Education	5.35
MDON Web-site	5.20
Online Handouts	5.13
Online Directories	5.13
Support Groups	4.84
One-on-one Consultation	4.43