A Quantum Algorithm of Constructing Image Histogram

Yi Zhang, Kai Lu, Ying-hui Gao, and Mo Wang

***Abstract*—**Histogram plays an important statistical role in digital image processing. However, the existing quantum image models are deficient to do this kind of image statistical processing because different gray scales are not distinguishable. In this paper, a novel quantum image representation model is proposed firstly in which the pixels with different gray scales can be distinguished and operated simultaneously. Based on the new model, a fast quantum algorithm of constructing histogram for quantum image is designed. Performance comparison reveals that the new quantum algorithm could achieve an approximately quadratic speedup than the classical counterpart. The proposed quantum model and algorithm have significant meanings for the future researches of quantum image processing.

***Keywords*—**Quantum Image Representation, Quantum Algorithm, Image Histogram.

I. Introduction

IN recent years, how to address the high time requirements of the practical image processing applications became a challenge. To cope with this problem, the construction of combining quantum computation with digital image processing has been extensively investigated. Some quantum image representation models have been proposed to utilize the peculiar properties of quantum mechanics to store and process image information, including Qubit Lattice [1], Entangled Image [2], Real Ket [3], and Flexible Representation of Quantum Images (FRQI) [4]. The merging of quantum computation and image processing has proved to be very fruitful. Based on these quantum image models, some quantum image processing algorithms have been researched such as quantum image transformation [5], [6], quantum image compression [3], quantum image watermarking [7], [8], and so on.

However, Qubit Lattice and Entangled Image do not utilize the quantum superposition and so they cannot operate pixels simultaneously; Real Ket and FRQI use the probability amplitude of a qubit to represent the gray scale information, so they cannot distinguish the pixels with different gray scales to operate. Therefore, up to now, some image statistical operations such as image histogram constructing are not supported based on the existing quantum image representations. It becomes a serious limitation to the further development of quantum image processing.

To cope with this problem, in this paper, a novel quantum image representation model is proposed which overcomes all the aforementioned drawbacks of the existing quantum image models. Based on the newly proposed model, a quantum statistical algorithm of constructing image histogram is designed which can achieve an approximately quadratic speedup over the classical constructing method.

II. Novel Quantum Image Representation

As the best and latest quantum image model by now, FRQI [4] utilizes the quantum superposition to store image as (1), therefore all the pixels in the quantum image can be operated simultaneously.

$$\left|\varphi\right\rangle_{FRQI} = \frac{1}{2^n}\sum_{i=0}^{2^{2n}-1}\left(\cos\theta_i\left|0\right\rangle + \sin\theta_i\left|1\right\rangle\right)\left|i\right\rangle \tag{1}$$

However, since FRQI encodes the gray scale information of pixels into the probability amplitude of a single qubit, it is impossible to distinguish the pixels with different gray scales and operate them individually so that FRQI is deficient to do complex image processing, for example image histogram cannot be constructed based on this quantum image model.

In order to improve this quantum image model, in this section, a new quantum image representation is proposed to cope with the drawback of FRQI. It utilizes two entangled qubit sequences to encode the gray scale and position information respectively and the whole image will be stored in the equiprobable superposition of the two qubit sequences. Through the comparison between FRQI and the new model in Fig. 1, the obvious difference is shown that the new quantum image model utilizes the basis state of qubit sequence to represent the gray scale value instead of the probability amplitude of a single qubit in FRQI. The representation expression of quantum image $\left|\varphi\right\rangle$ for a $2^n \times 2^n$ image with gray levels 2^q can be expressed as (2).

Yi Zhang is with the Department of Computer Science, in National University of Defense Technology, Changsha, China, 410073 (e-mail: zhangyinudt@nudt.edu.cn, phone: +86-13787136328).

Kai Lu is with the Department of Computer Science, in National University of Defense Technology, Changsha, China, 410073 (e-mail: kailu@nudt.edu.cn).

Ying-hui Gao is with the Department of Electronic Science and Engineering, in National University of Defense Technology, Changsha, China, 410073 (e-mail: yinghuigao@nudt.edu.cn).

Mo Wang is with the Department of Computer Science, in National University of Defense Technology, Changsha, China, 410073 (e-mail: dlwangmo@nudt.edu.cn).

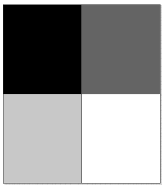
(a)

θ_{0000}	θ_{0001}	θ_{0010}	θ_{0011}
θ_{0100}	θ_{0101}	θ_{0110}	θ_{0111}
θ_{1000}	θ_{1001}	θ_{1010}	θ_{1011}
θ_{1100}	θ_{1101}	θ_{1110}	θ_{1111}

(b)

$\|f(00,00)\rangle$	$\|f(00,01)\rangle$	$\|f(00,10)\rangle$	$\|f(00,11)\rangle$
$\|f(01,00)\rangle$	$\|f(01,01)\rangle$	$\|f(01,10)\rangle$	$\|f(01,11)\rangle$
$\|f(10,00)\rangle$	$\|f(10,01)\rangle$	$\|f(10,10)\rangle$	$\|f(10,11)\rangle$
$\|f(11,00)\rangle$	$\|f(11,01)\rangle$	$\|f(11,10)\rangle$	$\|f(11,11)\rangle$

Fig. 1 (a) FRQI (b) The novel quantum image model



$$\begin{aligned}|\varphi\rangle &= \frac{1}{2}(|0\rangle\otimes|00\rangle+|100\rangle\otimes|01\rangle+|200\rangle\otimes|10\rangle+|255\rangle\otimes|11\rangle)\\ &= \frac{1}{2}(|00000000\rangle\otimes|00\rangle+|01100100\rangle\otimes|01\rangle\\ &\quad+|11001000\rangle\otimes|10\rangle+|11111111\rangle\otimes|11\rangle)\end{aligned}$$

Fig. 2 Image example and its representation expression

$$|\varphi\rangle = \frac{1}{2^n}\sum_{i=0}^{2^{2n}-1} |f(i)\rangle|i\rangle,\ f(i)\in[0,2^q-1] \tag{2}$$

where $f(i)$ is the gray scale value of the corresponding pixel index by position i. Different gray scale values can be distinguished in the novel quantum image model because different basis states of qubit sequence are orthogonal and distinguishable with each other.

Fig. 2 illustrates a 2×2 image with gray range 0-255 and its representation expression of new quantum image model. In this example, $n=1,\ q=8$.

III. Quantum Histogram Constructing Algorithm

Image histogram acts as a graphical representation of the gray scale distribution in a digital image. It takes an indispensable role in many complex image processing algorithms such as image segmentation, image compression and so on. In this section, how to construct quantum image histogram based on the novel quantum image model will be discussed.

In the new quantum image model in Section II, the pixels with different gray scale values are distinguished. It means the quantum image could be rearranged in another superposition of the pixels with different gray scales as (3).

$$|\varphi\rangle = \frac{1}{2^n}\sum_{j=0}^{2^q-1} |j\rangle(\sum_{0\le i\le 2^{2n}-1,\, f(i)=j} |i\rangle) \tag{3}$$

In (3), it is obvious that the whole image is divided into several pixel groups according to the different gray scale values. In order to construct image histogram, we need to calculate all the sizes of pixel group. Then a fast constructing algorithm based on the new quantum image representation will be demonstrated in detail.

The workflow of the new algorithm is divided into 6 steps as follows.

Step1: The whole information of the image is encoded in the new quantum image representation $|\varphi\rangle$ as (4). Two entangled quantum registers are utilized to store the gray scale qubit sequence and the position qubit sequence, respectively.

$$|\varphi\rangle = |\mathrm{Reg}_1\rangle|\mathrm{Reg}_2\rangle = \frac{1}{2^n}\sum_{j=0}^{2^q-1} |j\rangle(\sum |i_j\rangle) \tag{4}$$

where $\sum|i_j\rangle$ denotes the superposition of all the position information of the pixels of which the gray scale values are equal to j.

Step2: An ancillary qubit $|anc\rangle$ is initialized as $|1\rangle$ and the tensor product with $|\varphi\rangle$ is shown as (5).

$$|\varphi\rangle\otimes|anc\rangle = \frac{1}{2^n}\sum_{j=0}^{2^q-1} |j\rangle(\sum |i_j\rangle|1\rangle) \tag{5}$$

Step3: Utilize the quantum accumulation Ω [9] as (6) to add all the values of the ancillary qubit of the pixels in every pixel group. Since $|anc\rangle$ stores the value $|1\rangle$, the result of Ω is just the size of every group. Another quantum register $|\mathrm{Reg}_3\rangle$ is used to store the result of the quantum accumulation Ω. This quantum operation of this step is shown as Fig. 3.

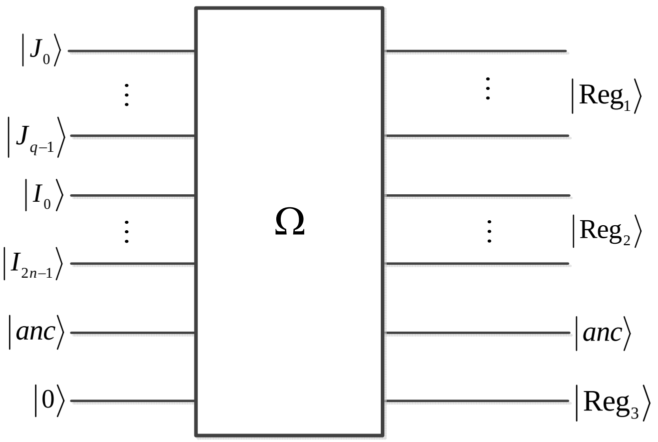


Fig. 3 The quantum accumulation Ω

$$\Omega : \frac{1}{2^n}\sum_{j=0}^{2^q-1}|j\rangle(\sum|i_j\rangle|1\rangle)|\mathrm{Reg}_3\rangle \to \frac{1}{2^n}\sum_{j=0}^{2^q-1}|j\rangle(\sum|i_j\rangle|1\rangle)|sum_j\rangle \tag{6}$$

Step4: In the output quantum state of the quantum accumulation Ω in *Step3*, the information of the image histogram is stored in the entanglement between $|\mathrm{Reg}_1\rangle$ and $|\mathrm{Reg}_3\rangle$ as (7).

$$|\mathrm{Reg}_1\rangle|\mathrm{Reg}_3\rangle = \sum_{j=0}^{2^q-1}\alpha_j|j\rangle|sum_j\rangle, \sum_{j=0}^{2^q-1}|\alpha_j|^2 = 1 \tag{7}$$

Step5: Through the projective measurements for the final quantum state, each sum_j can be observed. If measurement fails, it means there is no pixel with this gray scale and then $sum_j = 0$.

Step6: Repeat *Step1-5* to obtain all the sum_j of the groups to construct the quantum image histogram.

Case Study. Let us utilize the image in Fig. 2 as example to illustrate the whole quantum algorithm.

Firstly, the image will be stored into the new quantum image representation as (8). And (9) represents the tensor product of quantum image $|\varphi\rangle$ and the ancillary qubit $|anc\rangle$.

$$|\varphi\rangle = \frac{1}{2}(|0\rangle\otimes|00\rangle + |100\rangle\otimes|01\rangle + |200\rangle\otimes|10\rangle + |255\rangle\otimes|11\rangle) \tag{8}$$

$$|\varphi\rangle\otimes|anc\rangle = \frac{1}{2}(|0\rangle|00\rangle|1\rangle + |100\rangle|01\rangle|1\rangle + |200\rangle|10\rangle|1\rangle + |255\rangle|11\rangle|1\rangle) \tag{9}$$

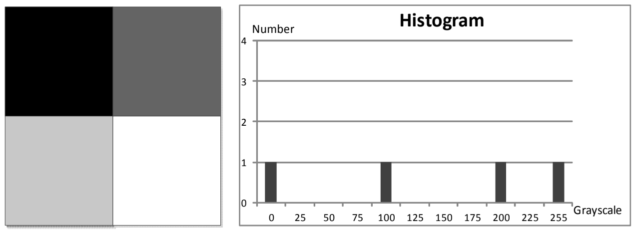


Fig. 4 Example image in Fig. 2 and its image histogram

With the assistance of the ancillary qubit $|anc\rangle$, the quantum accumulation Ω could calculate the sizes of all the pixel groups simultaneously and store the result in another quantum register $|\mathrm{Reg}_3\rangle$ as (10).

$$\Omega(|\varphi\rangle|anc\rangle|\mathrm{Reg}_3\rangle) = \frac{1}{2}(|0\rangle|00\rangle|1\rangle|1\rangle + |100\rangle|01\rangle|1\rangle|1\rangle + |200\rangle|10\rangle|1\rangle|1\rangle + |255\rangle|11\rangle|1\rangle|1\rangle) \tag{10}$$

Therefore, the image histogram is represented in the superposition of $|\mathrm{Reg}_1\rangle$ and $|\mathrm{Reg}_3\rangle$ as (11). Through the observation of projective measurements, the image histogram can be constructed as shown in Fig. 4.

$$|\mathrm{Reg}_1\rangle|\mathrm{Reg}_3\rangle = \frac{1}{2}(|0\rangle|1\rangle + |100\rangle|1\rangle + |200\rangle|1\rangle + |255\rangle|1\rangle) \tag{11}$$

At last, we will focus on the complexity of the new proposed quantum algorithm.

The cost of the whole algorithm boils down to ***Step3*** if we do not consider the preparation procedure of the quantum image. Because the quantum accumulation operation is designed based on Grover's search algorithm [10], the time complexity of obtaining every sum_j is $O(2^n)$ while the gray range is 2^q, the whole algorithm will cost no more than $O(2^{n+q})$ totally. Compared with the classical histogram constructing with time complexity $O(2^{2n})$ for a $2^n \times 2^n$ image, our new quantum algorithm can achieve an approximately quadratic speedup when the image size is large enough, i.e. $q \ll n$. Hence the proposed algorithm will be significant to the practical image applications if quantum computer is available in the near future.

Actually, the newly proposed quantum image representation can be considered as a data set with size 2^{2n} as shown in (2). In the whole field of quantum algorithms, many fast computational algorithms for data set have been studied extensively [11] such as the maximum or minimum element finding algorithm, the mean value determining algorithm, the modal value calculating algorithm and so on. Because all of these researches are based on Grover's search algorithm, the time complexities of these statistical algorithms are no more than $O(2^n)$ for a data set with size 2^{2n}. Therefore, when these existing quantum algorithms are utilized to do the quantum image statistical operations such as constructing histogram for the quantum images, all the time complexities of these quantum algorithms could achieve quadratic speedup over the counterparts in classical images. It is a significant contribution to quantum image analyzing.

IV. Conclusion

In order to achieve high performance image processing, quantum mechanics has been well explored to represent, store and process images. In this paper, in order to design a quantum histogram construction algorithm, a novel quantum image representation is proposed which utilizes two quantum registers to store the color information and the position information respectively. Via quantum counting algorithm, the new proposed quantum histogram construction can achieve an approximate quadratic speedup over the classical counterpart.

Therefore, the novel quantum image model provides more significant diversity inspiration for future quantum image processing and it is more suitable to make image statistical processing than the existing quantum image models in the literature.

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References

[1] Venegas-Andraca, S. E., Ball, J. L.: Storing images in engtangled quantum systems. arXiv:quant-ph/0402085 (2003).
[2] Venegas-Andraca S.E., Ball J.L., K. Burnett, and S. Bose. Processing images in entangled quantum systems. Quantum Information Processing 9:1-11 (2010).
[3] Latorre, J. I.: Image compression and entanglement. arXiv:quant-ph/ 0510031 (2005)
[4] P.Q. Le, F. Dong, K. Hirota, A flexible representation of quantum images for polynomial preparation, image compression, and processing operations, Quantum Information Processing 10(1), 63-84 (2011).
[5] P.Q. Le, A.M. Iliyasu, F. Dong, K. Hirota, Strategies for designing geometric transformations on quantum images, Theoretical Computer Science 412, 1406-1418 (2011).
[6] P.Q. Le, A.M. Iliyasu, F. Dong, K. Hirota, Efficient Color Transformations on Quantum Images, Journal of Advanced Computational Intelligence and Intelligent Informatics, 15(6), (2011).
[7] A. M. Iliyasu, P. Q. Le, F. Dong, K. Hirota. Watermarking and authentication of quantum images based on restricted geometric transformations, Information Sciences 186, 126-149 (2012).
[8] W. Zhang, F. Gao, B. Liu, Q. Wen, H. Chen, A watermark strategy for quantum images based on quantum fourier transform. Quantum Information Processing. DOI 10.1007/s11128-012-0423-6 (2012).
[9] F. Lu and D. C. Marinescu. An RCmax Quantum Scheduling Algorithm. Quantum Information Processing, 6(3), 159-178 (2007).
[10] Grover, L., A fast quantum mechanical algorithm for database search, Proc. of the 28th Ann. ACM Symp. on the Theory of Computing, pp. 212-219 (1996).
[11] M. W. Coffey, Z. Prezkuta. A Quantum Algorithm for Finding the Modal Value. Quantum Information Processing, 7(1), 51-54 (2008).G. O. Young, “Synthetic structure of industrial plastics (Book style with paper title and editor),” in *Plastics*, 2nd ed. vol. 3, J. Peters, Ed. New York: McGraw-Hill, 1964, pp. 15–64.