A quantum pulse gate based on spectrally engineered sum frequency generation

Andreas Eckstein,[1,*] **Benjamin Brecht,**[2] **and Christine Silberhorn**[2]

[1]*Max Planck Institute for the Science of Light, Günther-Scharowsky-Strasse 1, 91054 Erlangen, Germany*
[2]*Applied Physics, University of Paderborn, Warburgerstrasse 100, 33098 Paderborn, Germany*

[*]*andreas.eckstein@mpl.mpg.de*

Abstract: We introduce the concept of a quantum pulse gate (QPG), a method for accessing the intrinsic broadband spectral mode structure of ultrafast quantum states of light. This mode structure can now be harnessed for applications in quantum information processing. We propose an implementation in a PPLN waveguide, based on spectrally engineered sum frequency generation (SFG). It allows us to pick well-defined spectral broadband modes from an ultrafast multi-mode state for interconversion to a broadband mode at another frequency. By pulse-shaping the bright SFG pump beam, different orthogonal broadband modes can be addressed individually and extracted with high fidelity.

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1. Introduction

Ultrashort quantum pulses of light play an ever-increasing role in modern quantum information and communications. They already enable high bit-rate quantum key distribution [1] and high precision quantum clock synchronization protocols [2]. With increasing interest in their rich temporal and spectral structure [3, 4], sum frequency generation has been employed to manipulate it [5–12], but so far with limited modal control. Here, we introduce the concept of the Quantum Pulse Gate (QPG), a SFG-based device using spectral engineering [13–16] to directly target one spectral broadband mode for conversion and separate a single-mode quantum pulse from a multi-mode light state. The QPG acts as a controlled filter for broadband modes, and can also be used to prepare heralded pure single photons in a well defined mode. This opens the possibility to synthesize and analyze quantum pulses, establishing broadband modes

as an interferometrically stable alternative to multiple spatial modes for quantum information processing. Moreover it allows for quantum pulse shaping [17, 18] for engineering efficient single-photon-to-memory coupling [19, 20].

Any optical pulse is decomposable into a complete set of orthogonal basis functions, or broadband modes [3]. Thus it can be considered to be made up of an infinite number of temporally overlapping but independent pulse forms. While for classical light all basis sets are equivalent, for quantum light there may be one special, intrinsic basis choice [21]. For photon pair states this choice determined by a Schmidt decomposition of their bi-photon spectral amplitude into two correlated basis sets of broadband pulse forms, the Schmidt modes [4]. Heralding one of those photons by detecting the other with a single photon detector (SPD), this correlation results in the preparation of a photon in a mixed state of all Schmidt modes present [14]. But with a SPD sensitive to a certain Schmidt mode, it opens up the possibility to prepare pure single photons in the correlated Schmidt mode. Typically though, SPDs and optical detectors in general exhibit very broad spectral response and are not able to discern between different pulse forms.

To compensate for the detectors' shortcomings, one needs to include a filter operation sensitive to broadband modes. It has already been shown that ordinary spectral filters cannot fulfill this role [22,23]: They always transmit part of all impinging broadband modes at once, and thus cannot be matched to a single broadband mode. A sufficiently narrow spectral filter can be used to select a monochromatic mode, however this way, high purity heralded quantum states are impossible [23,24]. Also, most of the original beam's brightness as well as its pulse characteristics are lost.
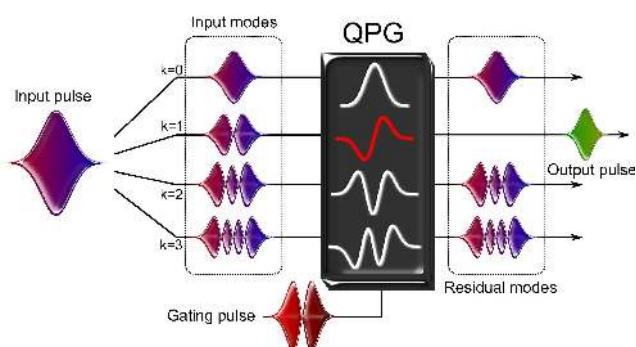

Fig. 1. Quantum Pulse Gate schema: Gating with a pulse in spectral broadband mode u_j converts only the corresponding mode from the input pulse to a Gaussian wave packet at sum frequency.

The idea of using broadband modes as quantum information carriers is especially compelling because of their natural occurrence in ultrafast pulses, and their stability in transmission: Centered around one frequency within a relatively small bandwidth typically, they allow for optical components that are highly optimized for a small spectral range. Since all broadband modes experience the same chromatic dispersion in optical media, they exhibit the same phase modulation and thus stay exactly orthogonal to each other. So a light pulse's broadband mode structure is resilient to the effects of chromatic dispersion, making a multi-channel protocol based on them ideal for optical fiber transmission. Additionally they allow for high transmission rates, as they inherit the ultrashort properties of their 'carrier' pulse, when compared to the 'long' pulses used for classical, narrow-band frequency multiplexing techniques. However, it is extremely challenging to actually access them in a controlled manner: Ordinary spectral filters

and standard optical detectors destroy the mode structure of a beam. A homodyne detector with an ultrafast pulsed local oscillator beam is able to select a single broadband mode by spectral overlap, but only at the cost of consuming the whole input beam [25, 26].

For discrete spatial modes, complete control of a beam's multi-mode structure can be accomplished with linear optics, as combining them to synthesize multi-mode beams and separating constituents without losses is possible [27–30]. In order to exploit the pulse form degree of freedom, we must be able to exact similar control over broadband modes.

An important step towards this goal is to selectively target a single broadband mode for interconversion into a more accessible channel, for instance to shift it to another frequency with SFG. On the single photon level, in the SFG process two single photons "fuse" into one photon at their sum frequency inside a $\chi^{(2)}$-nonlinear material. Well known in classical nonlinear optics, in recent years it has seen increasing adoption in quantum optics for efficient NIR single photon detection [5–8], all-optical fast switching [9], super high resolution time tomography of quantum pulses [10], quantum information erasure [11], and for translating non-classical states of light to different frequencies [12]. Moreover, combined with spectral engineering [13–15], it enables a new type of quantum interference between photons of different color [16].

In this paper we introduce the Quantum Pulse Gate (QPG): A device based on spectrally engineered SFG to extract photons in a well-defined broadband mode from a light beam. We overlap an incoming weak, multi-mode input pulse with a bright, classical gating pulse inside a nonlinear optical material (Fig. 1). Spectral engineering ensures that only the fraction of the input pulse which follows the gating pulse form is converted. The residual pulse, orthogonal to the gating pulse, is ignored. An input quantum light pulse's quantum properties can be preserved in conversion by mode-matching the gating pulse to its intrinsic mode structure.

SFG conversion efficiency can be tuned with gating pulse power, and unit efficiency is possible if the process can be engineered so that input beam and output beam are completely frequency-uncorrelated. Thus the QPG is able to unconditionally filter broadband modes from arbitrary input states, and to convert them into a well-defined Gaussian wave packet at the sum frequency. By pulse-shaping the gating pulse we are able to switch between different target broadband modes during the experiment. By superimposing gating pulses for two different broadband modes, we create interference between those previously orthogonal pulses. In combination with a standard single photon detector we are able to herald pulsed, pure, single-mode single photons from a multi-mode photon pair source.

2. SFG in terms of broadband modes

For a bright classical gating pulse, the effective (i. e. time-integrated) Hamiltonian of collinear SFG that converts a photon in mode 'a' to mode 'c' is given by

$$\mathbf{H} = \int dt\, \hat{\mathbf{H}}(t) = \theta \iint d\omega_i\, d\omega_o\, f(\omega_i, \omega_o)\, \mathbf{a}(\omega_i)\mathbf{c}^\dagger(\omega_o) + \text{h. c.} \tag{1}$$

analogous to the pulse-pumped SPDC Hamiltonian derived in [31]. Here we introduced the coupling constant $\theta \propto \chi^{(2)}\sqrt{P}$ with $\chi^{(2)}$ denoting the second order nonlinear polarization tensor element of the SFG process and P the gating pulse power. The normalized SFG transfer function $f(\omega_i, \omega_o) \propto \alpha(\omega_o - \omega_i) \times \Phi(\omega_o, \omega_i)$ maps the input frequencies ω_i to the sum frequencies ω_o, where α is the spectral amplitude of the classical gating pulse. The phase matching distribution of the SFG process Φ emerges from integrating the spatial part of the fields' phases over the interaction length $\Phi(\omega_o, \omega_i) = \int_0^L dz\, e^{i(k_o(\omega_3) - k_g(\omega_3 - \omega_1) - k_i(\omega_1))z}$ with k_i, k_o, k_g the dispersion relations of the input, output and gating field respectively.

In parametric down-conversion (PDC), the Schmidt decomposition of the joint spectral amplitude of the generated photon pairs reveals their broadband mode structure [4]. Applying the
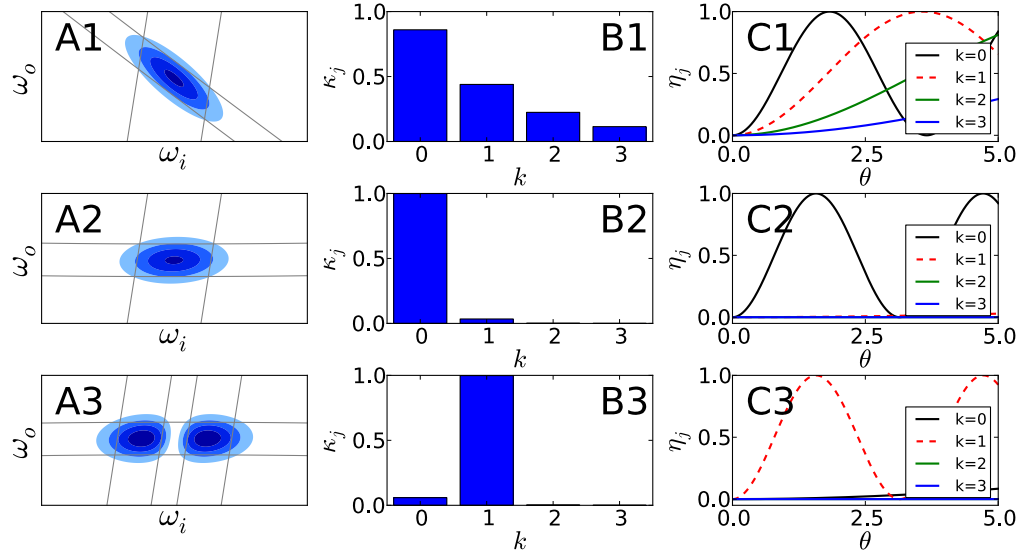
Fig. 2. (A1-A3) SFG transfer function $f(\omega_i, \omega_o)$ with (A1) and without (A2,A3) frequency correlations. (B1-B3) Coefficients κ_j for the first four Schmidt mode pairs of the transfer functions. (C1-C3) SFG efficiencies $\mathbf{A}_j \to \mathbf{C}_j$ for the first four Schmidt modes against gating power dependent SFG coupling constant θ

same approach to SFG [16] to decompose the spectral transfer function we find

$$f(\omega_i, \omega_o) = \sum_j \kappa_j \, \varphi_j(\omega_i) \, \psi_j(\omega_o). \tag{2}$$

The decomposition is well-defined and yields two correlated sets of orthonormal spectral amplitude functions $\{\varphi_j(\omega)\}$ and $\{\psi_j(\omega)\}$ and the real Schmidt coefficients κ_j which satisfy the relation $\sum_j \kappa_j^2 = 1$. If the gating pulse has the form of a weighted Hermite function $u_j(\omega) \propto e^{\frac{(\omega-\omega_0)^2}{2\sigma^2}} H_j\left(\frac{\omega-\omega_0}{\sigma}\right)$ with H_j the Hermite polynomials, the basis functions of both sets are in good approximation Hermite functions as well. In the Schmidt-decomposed form, the transfer function describes a mapping between pairs of broadband modes $\varphi_j(\omega) \to \psi_j(\omega)$.

By defining broadband mode operators $\mathbf{A}_j = \int d\omega \, \varphi_j(\omega) \, \mathbf{a}(\omega)$ and $\mathbf{C}_j = \int d\omega \, \psi_j(\omega) \, \mathbf{c}(\omega)$ corresponding to the Schmidt bases, the effective Hamiltonian from Eq. (1) can be rewritten as

$$\mathbf{H} = \theta \sum_j \kappa_j \left(\mathbf{A}_j \mathbf{C}_j^\dagger + \mathbf{A}_j^\dagger \mathbf{C}_j\right), \tag{3}$$

An optical beam splitter has an effective Hamiltonian of the form $\mathbf{H}_{\mathrm{BS}} = \theta \, \mathbf{a}\mathbf{c}^\dagger + \mathrm{h. c.}$ [32]; so with respect to broadband modes, SFG can be formally interpreted as a set of beam splitters, independently operating on one pair of broadband modes each, such that $\mathbf{A}_j \to \cos(\theta_j)\mathbf{A}_j + \iota\sin(\theta_j)\mathbf{C}_j$. The effective coupling constant $\theta_j = \theta \cdot \kappa_j \propto \sqrt{P}$ takes the role of the beam splitter angle. Its transmission probability – the probability to find a photon in the up-converted mode $\mathbf{C}_j$ if it initially has been in mode $\mathbf{A}_j$ – is $\eta_j = \sin^2(\theta_j)$.

In Fig. 2 A1-C1, we illustrate an example for a non-engineered SFG process, as commonly found in pulsed SFG experiments: The transfer function $f(\omega_i, \omega_o)$(Fig. 2 A1) exhibits spectral correlations, causing more than one non-zero Schmidt coefficient (Fig. 2 B1). This leads to the simultaneous conversion of multiple modes $\mathbf{A}_j$ at once with non-zero coupling constants

$\theta_j \propto \sqrt{P}$ for any given gating pulse power P (Fig. 2 C1). Hence a SFG process in general is not mode-selective.

3. The quantum pulse gate

However, SFG can be made mode-selective with spectral engineering, by eliminating its spectral correlations so that the frequency of an up-converted photon gives no information about its original frequency. Now, Schmidt decomposition yields one predominant parameter $\kappa_j \approx 1$ with all others close to zero and a separable transfer function $f(\omega_i, \omega_o) \approx \kappa_j \varphi_j(\omega_i) \psi_j(\omega_o)$. Also, now the full coupling $\theta_j \approx \theta$ is exploited, allowing for relatively weak gating beams for maximum conversion efficiency. We achieve this by engineering the SFG process such that the input beam group velocity $v_i = k_i^{-1}(\omega_i)$ is matched to the gating pulse group velocity $v_g = k_g^{-1}(\omega_g)$. As a result, the phasematching function Φ is horizontal in Fig. 2 A2 and A3, and Fig. 2 B2 and B3 show that mostly one mode pair is excited. If the phasematching bandwidth is narrow compared to gating pulse width, spectral correlations are negligible, and we can approximate a separable transfer function. The effective SFG Hamiltonian is now a beam splitter-type Hamiltonian:

$$\mathbf{H}_{\text{QPG}} = \theta_j \left(\mathbf{A}_j \mathbf{C}_0^\dagger + \mathbf{A}_j^\dagger \mathbf{C}_0 \right). \tag{4}$$

We note that it commutes with all modes input modes $[\mathbf{A}_k, \mathbf{H}_{\text{QPG}}] = 0$ where $k \neq j$. In other words, the quantum pulse gate is mode-selective and accepts only mode $\mathbf{A}_j$ for up-conversion.

In the Heisenberg picture of quantum state evolution, the action of the QPG on a given quantum light state is described by a Bogoliubov transformation: Linear transformations of its mode operators $\mathbf{A}_{\text{in}} \rightarrow \mathbf{A}_{\text{out}} = \mathbf{U}_{\text{QPG}} \mathbf{A}_{\text{in}} \mathbf{U}_{\text{QPG}}^\dagger$, where $\mathbf{U}_{\text{QPG}} = \mathscr{T} e^{-\iota \int \text{d}t \hat{\mathbf{H}}(t)}$ is the unitary time evolution operator generated by the Hamiltonian operator $\hat{\mathbf{H}}(t)$ that describes traversal of the pulse gate implementation, with $\mathscr{T}$ the time ordering operator. It accurately describes the interplay between the frequency upconversion and its reverse process for an arbitrary pump power and coupling constant θ. In the perturbative case with $\theta \ll 1$, $\mathbf{U}_{\text{QPG}}$ can be developed to first order and time ordering has no effect. For the higher order terms though, time ordering has to be applied to account for interaction between multiple photon conversions at the same time which results in spectral mode distortions in the strongly coupled regime. However, it has been shown that for any frequency conversion process described by a Bogoliubov transformation there exists a Bloch-Messiah reduction into orthogonal, independent processes. The mode structure coincides with the Schmidt decomposition from Eq. (2) in the weak coupling limit, but the spectral modes do not change dramatically for stronger coupling [33]. Numerical analysis of a strongly coupled up-conversion process reveals that the main source of discrepancy is a group velocity mismatch between gating pulse and input pulse [18]. Since we utilize such a group velocity matching to minimize spectral correlation between input and output beam, we neglect time ordering for now and approximate $\mathbf{U}_{\text{QPG}} \approx e^{-\iota \mathbf{H}_{\text{QPG}}}$.

This process implements the QPG, with the bright input pulse used as gate pulse to select a specific broadband mode. By tuning the central wavelength and spectral distribution of the gating pulse, we can control the selected broadband mode's shape, width and central wavelength. We compare the effect of different gating pulse forms: Gating with mode u_0 (i. e. a Gaussian spectrum, Fig. 2 A2-C2) selects input mode $\mathbf{A}_0$, gating with mode u_1 (Fig. 2 A3-C3) selects $\mathbf{A}_1$ from the input pulse for frequency up-conversion. Because of the horizontal phasematching, the target mode is always the Gaussian pulse $\mathbf{C}_0$ regardless which spectral gating mode u_j is used.

Pure heralded single photons are a crucial resource in many quantum optical applications, but the widely used PDC photon pair sources emit mixed heralded photons in general, due to their intrinsic multimode structure [14]. We now consider the application of the QPG to "purify"

those photons by selecting a single broadband mode from the heralding beam. In type-II PDC, a pump photon decays inside a $\chi^{(2)}$-nonlinear medium into one horizontally polarized signal and one vertically polarized idler photon. For a collinear type-II PDC source pumped by ultrafast pulses the general effective Hamiltonian in terms of broadband modes reads

$$\mathbf{H}_{\mathrm{PDC}} = \chi \sum_j c_j \left(\tilde{\mathbf{A}}_j^\dagger \mathbf{B}_j^\dagger + \tilde{\mathbf{A}}_j \mathbf{B}_j \right). \tag{5}$$

We feed the signal photon (containing all broadband modes $\tilde{\mathbf{A}}_j$) from the PDC source into the QPG which is mode-matched such that $\tilde{\mathbf{A}}_0 = \mathbf{A}_0$. We note that for heralding pure single photons or pure Fock states [22], mode-matching is not necessary and an engineered SFG process according to Eq. (4) is sufficient. In that case however, the resulting pulse shape is a coherent superposition of all input modes. Here, only the 0th mode is selected, and the higher modes do not interact with the QPG because the according beam splitter transformations yield the identity $\mathbf{A}_j \to \mathbf{A}_j$ for $j > 0$. We choose the gating pulse power such that $\theta_0 = \frac{\pi}{2}$ for optimal conversion efficiency. Combining the PDC source with a subsequent QPG results transforms the PDC Hamiltonian as $\mathbf{H}_{\mathrm{PDC}} \to \mathbf{H}' = \mathbf{U}_{\mathrm{QPG}} \mathbf{H}_{\mathrm{PDC}} \mathbf{U}_{\mathrm{QPG}}^\dagger$, and we obtain

$$\mathbf{H}' = \iota \chi \mathbf{B}_0^\dagger \mathbf{C}_0^\dagger + \chi \sum_{k=1}^\infty c_j \tilde{\mathbf{A}}_j^\dagger \mathbf{B}_j^\dagger + \mathrm{h.\,c.} \tag{6}$$

Since mode $\mathbf{C}_0$ is centered at the sum frequency of input and gating pulse, it can be split off easily into a separate beam path with a dichroic mirror. Conditioning on single photon events on the path of $\mathbf{C}_0$ provides us with pure heralded single photons in mode $\mathbf{B}_0$. Fig. 3 illustrates this scheme: A photon detection event heralds a pure single photon pulse in broadband mode u_1. This process can be cascaded to successively pick off several modes $\mathbf{A}_j$ from the input beam. Note that if we insert a mode matched QPG into the vertically polarized PDC beam to convert $\mathbf{B}_0$ into $\mathbf{D}_0$, we can unconditionally single out an ultrafast two-mode squeezed vacuum state from a multi-mode squeezer [33, 34].
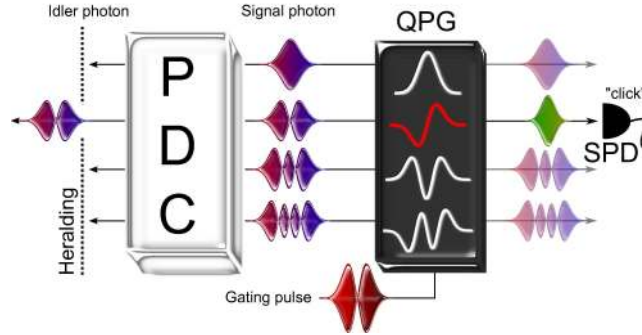

Fig. 3. A QPG application: Generating pure heralded broadband single photons in different modes from a multimode PDC source of broadband-mode-correlated photon pairs

4. Experimental parameters

Finally we describe the numerical methods used to obtain experimental parameters and simulation results. For the SFG process we propose a periodically poled, z-cut Titanium diffused LiNbO$_3$ (Ti:PPLN) waveguide [35] with a cross-section area of $6.5 \mu\mathrm{m} \times 5 \mu\mathrm{m}$ and $L = 50\mathrm{mm}$

length. We employ a standard finite element method to calculate the spatial mode fields inside the waveguide and obtain their corresponding effective refractive indices which evaluate to $n_{\text{eff}}^{(p)} = 2.18$, $n_{\text{eff}}^{(in)} = 2.21$ and $n_{\text{eff}}^{(out)} = 2.32$. The input pulse has a central wavelength of 1550nm and a duration of 2ps. The nonlinear waveguide has a 4.4μm poling period and is heated to 190°C to achieve phasematching for SFG of an input pulse at 1550nm to 557nm. The gating beam is ordinarily polarized, while input and output beam are extraordinarily polarized.

The gating spectrum is centered around 870nm, its corresponding spectral FWHM for mode matching is 0.635nm. The uncorrelated, separable transfer functions in Fig. 2 (A2-A3) are calculated from these parameters, using gating pulses with u_0 and u_1 as spectral amplitude, respectively. Utilizing a completely quantitative model to calculate waveguide dispersion and neglecting the effects of a time-ordered Hamiltonian on the SFG process, we predict a gating pulse energy of only $E_G = 1.36$pJ (or an average power of $P_{\text{av}} = 0.103$mW at 76MHz repetition rate) for unit conversion efficiency $\eta_0 = 1$ of the waveguide's lowest Schmidt mode, i. e. $\theta_0 = \frac{\pi}{2}$. This comparatively low power is due to the careful tailoring of the process which leads to only one pair of modes being excited, such that no power is wasted on higher modes.

Dispersive pulse broadening through group velocity dispersion could in principle affect the re-usability of our pulses in a cascaded setup of QPGs. A Gaussian pulse with central frequency ω_0 and spectral standard deviation σ travelling through a crystal of length L with the propagation constant $k(\omega)$ will in first order approximation elongate by a factor $\left(1 + k''(\omega_0)^2 L^2 \sigma^4\right)^{\frac{1}{2}}$. In our PPLN waveguides, for gating and input pulses as specified above this results in pulse broadening of 0.003% and 0.004% respectively, making this effect neglegible here. The difference of the inverse group velocities of pump and input beam is as small as $k_p'(\omega_{870\text{nm}}) - k_i'(\omega_{1550\text{nm}}) = 3.8 \times 10^{-12} \frac{1}{\frac{m}{s}}$. This group velocity matching results in the horizontal orientation of the phase-matching distribution Φ and a low distortion of the spectral modes at high pump powers [18].
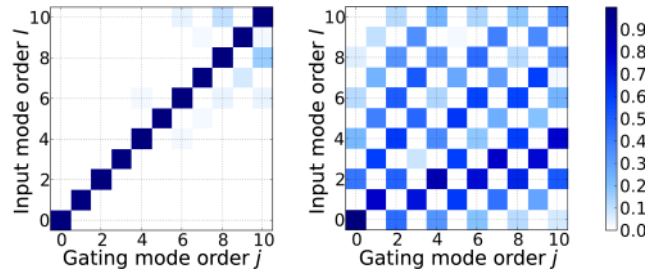

Fig. 4. Overlap between input pulse mode $\tilde{u}_l$ and QPG Schmidt mode φ_j for mode-matched (left) and non-mode-matched (right) case in the weak coupling regime.

In Fig. 4 we illustrate the switching capabilities of our QPG for weak coupling ($\theta \ll 1$), as well as the impact of mode matching. For the given material parameters, we employ gating pulses with pulse form u_0 to u_{10}, determine the Schmidt decomposition of the resulting transfer function $f(\omega_i, \omega_o)$, and plot the fidelity of a certain mode conversion, that is the overlap of the predominant QPG Schmidt function φ_j ($\kappa_j \approx 1$) with a Hermitian input mode $\tilde{u}_l$ from an incident light pulse. On the left, gating and input pulse have equal frequency FWHM, which is essential for good mode matching. Now, by switching the order j of the gating mode (and without changing the physical parameters of the QPG), we select with high fidelity only the input mode j to be converted. For $j \le 10$, the overlap $\left| \int d\omega \tilde{u}_j^*(\omega) \varphi_j(\omega) \right|^2$ exceeds 99%, and the overlap for all other input modes combined therefore is less than 1%: Only a negligible

fraction of modes other than the selected input mode are converted.

In contrast, Fig. 4 (right) has no mode matching, the gating pulse FWHM is twice that of the input pulse. Multiple strong overlaps between SFG Schmidt modes φ_j and input modes $\tilde{u}_l$ appear: A wide range of modes is converted for any given input spectrum. The checkerboard pattern reflects the fact that only modes of the same parity exhibit an overlap, an odd and an even mode are orthogonal regardless of mode-matching.

5. Conclusion and outlook

In conclusion, we have introduced the concept of the QPG, a flexible device to separate well-defined broadband modes from a light pulse based on spectrally engineered SFG. The selected mode can be switched by shaping the gating pulse spectrum and converted with high fidelity. Further, we have given a realistic set of experimental parameters for a QPG realized in a PPLN waveguide and demonstrated the high flexibility of the QPG achieved through shaping the gating pulse form. We proposed as an initial application the preparation of pure heralded single photons from an arbitrary type II PDC source. For pulsed QKD schemes [1], it can act as a de-multiplexer of multiple quantum channels within one physical pulse, and in metrology it may be used to further enhance measurement accuracy beyond the classical limit by replacing multiple squeezed pulses [2] with one multi-mode squeezed pulse of light.

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