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A RAPID BLADE-TO-BLADE SOLUTION FOR USE IN TURBOMACHINERY DESIGN

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ABSTRACT

A rapid technique for solving the blade-to-blade turbomachinery flow problem has been developed. Approximate governing flow equations, which include the effects of compressibility, radius change, rotation, and variable stream sheet thickness are solved using an improved panel method. The development and solution of these equations are described. Sample calculations are presented to illustrate the method's capabilities and accuracy.

NOMENCLATURE

As,Bs,...,Ad	integral equation solution singularity influence coefficients
b	stream sheet thickness
m	meridional blade-to-blade surface coordinate
n	blade shape surface normal
r	radius from turbomachine centerline to blade-to-blade surface
S	blade row pitch or blade spacing in radians
s	blade surface distance
S_1, S_2	blade-to-blade and midchannel surfaces
V	absolute velocity of flow
W	relative velocity of flow
w	disturbance relative velocity
x	transformed meridional coordinate
y	transformed tangential coorddinate
α	flow angle on the blade-to-blade surface
Γ	blade row circulation
φ	tangential blade-to-blade surface coordinate
ρ	fluid density
σ, γ, δ	singularity strengths
θ	orientation of blade surface elements used in the integral equation solution
	angular velocity

Subscripts:

c	known or constant value
est	estimated value
i,j,k	summation indices for singularity strengths
u,i,o,d	upstream, inlet, outlet, and downstream station of the blade row
n,t	surface normal and tangential components
onset	undisturbed flow
rb	integral terms contribution to solution velocity
p	arbitrary point on blade-to-blade surface
m,φ	on blade-to-blade surface of revolution
x,y	on transformed blade-to-blade surface
*	transformed quantity

Overbars:

$(\bar{\ })$	vector quantity
$(\tilde{\ })$	mean quantity

INTRODUCTION

The design and study of turbomachinery rely heavily on the use of inviscid irrotational compressible blade-to-blade flow solutions. As a result many methods and computer codes have been developed to solve this problem. The intent in presenting yet another of these blade-to-blade solutions is to provide the turbomachinery designer with a tool which has a better combination of solution speed, robustness, and versatility than previous methods.

The procedure solves the inviscid irrotational compressible blade-to-blade flow equations on a surface of revolution. One three dimensional effect is modeled by including a variable stream sheet thickness. The governing equations are linearized by approximating the compressibility effects. These approximate equations are solved using an integral equation technique (panel method).

The method does have limitations and is intended primarily for use in preliminary design studies where a variety of blade shapes and flow conditions are to be analyzed. By using this technique the designer can produce blade shapes for analysis by more accurate, but presumably longer running, flow codes.

Specific features of the method are as follows. (1) Computational time for a typical case run on the IBM 3033 computer at NASA Lewis Research Center is less than 4 CPU seconds. After an initial solution, additional solutions for variations in the inlet flow velocity, inlet flow angle, outlet flow angle, rotational speed, or a combination of these effects can be made at an additional expense of less than 0.5 CPU seconds per case. (2) The method can be applied to all types of turbomachinery and flow conditions. However, the quality of the solution does vary with the particular problem being solved. The method loses accuracy as the flow becomes transonic, will not predict shocked flows, and does not accurately determine when a blade row is choked. In addition the method also gives poorer solutions in the uncovered portions of the blade passage for lower solidity blade rows. (3) Since a direct linear equation solver is used, a solution is always produced even if the blade shape is poorly described or extreme flow conditions are given. (4) The method uses an integral equation solution, so that the flow is only calculated on the blade surface. The flow can be calculated at points in the blade passage, but this is inefficient. (5) The solution does not require the use of a computational mesh or grid, so that almost any blade shape or body can be analyzed.

ANALYSIS

The general blade-to-blade surface geometry is shown in figure 1, and an r-z cut through the blade-to-blade surface is shown in figure 2. The surface in figure 1 corresponds to the S_1 surface described by Wu in reference [1]. The governing equations developed by Wu for inviscid, irrotational, compressible flow in the S_1 surface can be written as,

$$\text{Continuity} \quad \frac{\partial}{\partial \varphi}(\rho b V_\varphi) + \frac{\partial}{\partial m}(r \rho b V_m) = 0 \qquad (1)$$

$$\text{Irrotationality} \quad \frac{\partial}{\partial \varphi}(V_m) - \frac{\partial}{\partial m}(r V_\varphi) = 0 \qquad (2)$$

where V_φ and V_m are absolute velocity components, ρ is the fluid density, b is the stream sheet thickness, and r is the local radius of the axisymmetric blade-to-blade surface.

In terms of flow relative to the blades the equations become

$$\text{Continuity} \quad \frac{\partial}{\partial \varphi}(\rho b W_\varphi) + \frac{\partial}{\partial m}(r \rho b W_m) = 0 \qquad (3)$$

$$\text{Irrotationality} \quad \frac{\partial}{\partial \varphi}(W_m) - \frac{\partial}{\partial m}(r W_\varphi) = 2\Omega r \frac{dr}{dm} \qquad (4)$$

where the relative components, W_φ and W_m are given by

$$V_m = W_m$$
$$V_\varphi = W_\varphi + \Omega r \qquad (5)$$

The radius, r, can be transformed out of the governing equation by using the same transformation as Kurmanov, Podvidz, and Stepanov [2].

$$y = \varphi \; ; \; x = \int^m \frac{dm}{r}$$

or

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \varphi} \; ; \; \frac{1}{r}\frac{\partial}{\partial x} = \frac{\partial}{\partial m} \qquad (6)$$

and

$$W_x = r W_m ; \; W_y = r W_\varphi$$

where r is a function of m only. In the present work, the velocity part of this transformation is modified to the following

$$W_x = r \rho b W_m \; ; \; W_y = r \rho b W_\varphi \qquad (7)$$

Note that the transformation preserves the relative flow angle from the m-φ surface to the x-y surface.

$$\tan(\alpha) = \frac{W_\varphi}{W_m} = \frac{W_y}{W_x}$$

Using this transformation the governing equations become

$$\text{Continuity} \quad \frac{\partial}{\partial y}(W_y) + \frac{\partial}{\partial x}(W_x) = 0 \qquad (8)$$

$$\text{Irrotationality} \quad \frac{\partial}{\partial y}\left(\frac{W_x}{\rho b}\right) - \frac{\partial}{\partial x}\left(\frac{W_y}{\rho b}\right) = 2\Omega r \frac{dr}{dx} \qquad (9)$$

Expanding the irrotationality equation (9) gives

$$\frac{\partial}{\partial y}(W_x) - \frac{\partial}{\partial x}(W_y) + \frac{W_y}{\rho b}\frac{\partial}{\partial x}(\rho b) - \frac{W_x}{\rho b}\frac{\partial}{\partial y}(\rho b) = 2\Omega \rho b r \frac{dr}{dx} \qquad (10)$$

The effect of the transformation on the governing equations has been to move the non-linear terms from the continuity equation (8) to the irrotationality equation (10) and to have the radius explicity appear in a single term.

Equation (10) is linearized by replacing (ρb) and W_y in the last three terms by estimated values, which are taken to be functions of x only. With these approximations the governing equations become

$$\text{Continuity} \quad \frac{\partial}{\partial y}(W_y) + \frac{\partial}{\partial x}(W_x) = 0 \qquad (11)$$

$$\text{Irrotationality} \quad \frac{\partial}{\partial y}(W_x) - \frac{\partial}{\partial y}(W_y) = - \frac{W_{y_{est}}}{(\rho b)_{est}}\frac{d}{dx}(\rho b)_{est} + 2\Omega(\rho b)_{est} r \frac{dr}{dx} \qquad (12)$$

<u>Equation solution</u>

Since equations (11) and (12) are linear, superposition of solutions may be used

$$W_x = W_{x_c} + w_x$$
$$W_y = W_{y_c} + w_y \qquad (13)$$

where W_{x_c} and W_{y_c} can be considered as components of a flow which is independent of y, and w_x and w_y are the disturbance to this known flow due to the blade presence. Substituting equations (13) into the governing equations gives

$$W_{x_c} = \text{constant} \qquad (14a)$$

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$$W_{y_c} = \text{constant} + \int^{(\rho b)_{est}} \frac{W_{y\,est}}{(\rho b)_{est}} d(\rho b)_{est}$$

$$- 2\Omega \int^{r} (\rho b)_{est}\, r\, dr \qquad (14b)$$

$$\frac{\partial}{\partial x}(w_x) + \frac{\partial}{\partial y}(w_y) = 0 \qquad (14c)$$

$$\frac{\partial}{\partial x}(w_y) - \frac{\partial}{\partial y}(w_x) = 0 \qquad (14d)$$

In this study a solution to these equations was developed by using the integral equation approach of reference [3]. In the notation of this paper, the onset velocity reference [3] becomes,

$$V_{x_{onset}}\Big|_{ref.\ 3} = W_{onset}\cos(\alpha)$$

$$V_{y_{onset}}\Big|_{ref.\ 3} = W_{onset}\sin(\alpha)$$

$$+ \int^{(\rho b)_{est}} \frac{W_{y\,est}}{(\rho b)_{est}} d(\rho b)_{est} - 2\Omega \int^{r} (\rho b)_{est}\, r\, dr$$

and the distributed singularity expressions of reference [3] are used to satisfy equations (14c) and (14d). The resulting equations for the velocity components tangential and normal to the blade surface can be written as.

$$W_{t_p} = W_{onset}\cos(\theta_p - \alpha) + W_{rb_p}\sin(\theta_p) + \sum_{i=1}^{n} As_{pi}\sigma_i$$

$$+ \sum_{j=1}^{n-1} Av_{pj}\gamma_j + \sum_{k=1}^{m} Ad_{pk}\,\delta_k \qquad (15a)$$

$$W_{n_p} = -W_{onset}\sin(\theta_p - \alpha) - W_{rb_p}\cos(\theta_p) + \sum_{i=1}^{n} Bs_{pi}\,\sigma_i$$

$$+ \sum_{j=1}^{n-1} Bv_{pj}\,\gamma_j + \sum_{k=1}^{m} Bd_{pk}\,\delta_k \qquad (15b)$$

where

$$W_{rb_p} = \int^{(\rho b)_{est}} \frac{W_{y\,est}}{(\rho b)_{est}} d(\rho b)_{est} - 2\Omega \int^{r_p} (\rho b)_{est}\, r\, dr$$

The above equations, (15a) and (15b), correspond to equations (16c) and (16d) in reference [3]. The solution technique of reference [3] may now be applied directly using slightly modified boundary conditions.

<u>Boundary conditions</u>

The boundary conditions for the relative transformed flow problem are

1. On the body surface,

$$\overline{W}.\overline{n} = W_n(s) = 0 \qquad (16a)$$

2. At the upstream boundary,

$$W_x\Big|_u = W_u\cos(\alpha_u) = W_{onset}\cos(\alpha) + w_{x_u}$$

$$(16b)$$

$$W_y\Big|_u = W_u\sin(\alpha_u) = W_{onset}\sin(\alpha) + w_{y_u}$$

3. At the downstream boundary,

$$W_x\Big|_d = W_d\cos(\alpha_d) = W_{onset}\cos(\alpha) + w_{x_d}$$

$$(16c)$$

$$W_y\Big|_d = W_d\sin(\alpha_d) = W_{onset}\sin(\alpha) + w_{y_d}$$

4. Circulation condition,

$$\Gamma = S[(rV)_u - (rV)_d] = \oint r\overline{V}.\overline{ds}$$

or in transformed variables

$$\frac{\Gamma}{S} = \left(\frac{W_y}{\rho b}\right)_u - \left(\frac{W_y}{\rho b}\right)_d + \Omega\,(r_u^2 - r_d^2) \qquad (16d)$$

and

$$\Gamma = \oint \frac{1}{\rho b} \overline{(W + \rho b\,\Omega r^2)} \cdot \overline{(dx^2 + dy^2)^{1/2}} \qquad (16e)$$

5. Far stream limits on w,

$$w_{x_u} = w_{x_d} \longrightarrow 0 \text{ as } x_u, x_d \longrightarrow \pm\infty$$

$$(16f)$$

$$w_{y_u} = -w_{y_d} \text{ as } x_u, x_d \longrightarrow \pm\infty$$

The above conditions, along with the conservation of angular momentum (rV_φ = constant) in areas outside the blade row and equations (15), are used to develop a system of equations similar to equations (17)-(19) in reference [3]. This system of equations is solved by upper and lower triangular matrix decomposition.

Mean flow and stream sheet thickness estimation

The solution of the governing equations requires that the integral terms in equation (14b) be known a priori. These terms can be approximated by using numerical integration and a one dimensional estimate for the flow through the blade passage.

Three techniques have been used to estimate the one dimensional flow in this study. The first is to use the blade shape and passage geometry to calculate approximate flow turning angles, flow areas, and stream sheet thicknesses. These quantities are then used with a one dimensional continuity equation and the isentropic relations to calculate the mean flow velocity and density. The second technique is to specify or assume a mean flow velocity distribution through the blade row and to estimate the stream sheet thickness from the passage geometry (see figure 2). Isentropic relations are used to calculate the density. The simplest mean flow estimate is a linear distribution from blade inlet to exit based on the design velocity diagram. More complex distributions can be made depending on the type of turbomachinery being analyzed and one's knowledge of the flow. The third method is to use a mid-channel stream surface flow solution, which corresponds to an S_2 surface in Wu's notation [1]. Such a solution can provide both the stream sheet thickness and mean velocity distributions.

All three techniques were used successfully in this study. The quality of the overall solution produced by using any one of the mean flow estimates was dependent on the particular blade geometry being analyzed. For highly guided passages like those found in centrifugal machinery, the first technique produced very good results. In lower solidity blading where the flow angle can deviate significantly from the mean blade angle, the second method proved to give the best results. The use of a mid-channel solution also provided good results for a number of cases, but its success depends on how well the mid-channel solution models the flow turning through the blade row. In this work the MERIDL computer code [4] was used to calculate the mid-channel solutions, and did provide good solutions for most cases. For flows where the mean flow is supercritical and the flow turning is controlled by the compressibility of the flow, only the second and third techniques gave reasonable solutions.

Method accuracy

In considering the overall accuracy of this solution, two sources of error are discussed. First is the approximations made in the governing equations. Second is the solution technique used in solving the equations.

By looking at the terms in equation (10) which are neglected or approximated in the final solution, the applicability and accuracy of the solution can be assessed. The only flow for which the solution is exact, in the sense that no approximation has been made to the governing equations, is incompressible flow with constant stream sheet thickness. With the introduction of a variable stream sheet thickness, an approximation of the W_y term is required. This term makes significant contributions to the solution when problems with large or rapid changes in stream sheet thickness are analyzed. The accuracy of the term is controlled by the estimate of W_y. At far stream stations the flow is modeled as being uniform, and the estimate of W_y as being independent of y is consistent. However, near or inside the blade row where blade surface curvatures strongly influence the flow, errors in the solution will occur. Compressible flow calculations require the inclusion of the change in density terms in equation (10). The more compressible the flow the more significant these terms become. In the solution equations used in this study it is assumed that the density only varies in the meridional direction. Wood in reference [5] also makes the same assumption, and it is currently used in the TSONIC code [6] to calculate transonic flows. An estimate of tangential variations in density could have been included in the analysis, but this would require an approximate solution for the blade loading. Calculations indicate that the term is not significant for subcritical flows. However, inaccuracy in the solution will appear when high speed flows with large blade loadings are being analyzed. Such flow occur in supercritical blade designs. The remaining term for change in density in the meridional direction is approximated by using a mean flow estimate to arrive at values of the tangential velocity component and the density. This term tends to dominate the solution as the compressibility effects increase. If the mean flow estimate does not approximate the local flow well, errors in the solution will occur.

Major sources of error in the solution technique are the integral equation solution and the numerical approximation of the integral terms in equations (15a) and (15b). The integral equation solution is second order accurate in terms of the elements used to approximate the blade shape. Its accuracy was discussed in reference [3]. Numerical integration of the terms in equations (15a) and (15b) was accomplished using a simple trapezoidal method. Twenty-two integration points were used between the upstream and downstream boundaries of the flow field with twenty of the points placed between the blade row leading and trailing edges. The accuracy of this integration is consistent with the accuracy of the integral equation solution for a typical design calculation.

Solution modifications

The solution can be modified to produce better results for certain problems. One possible modification is to specify a fictitious mean flow distribution or stream sheet thickness in order to account for viscous flow blockage effects. Another solution modification is to alter the transformation back to physical variables. The velocity transformation can be written as,

$$W = \frac{W_*}{\rho b\, r} = \frac{1}{rb}\left(\frac{1}{\rho}\right)W_*$$

where W is the physical velocity magnitude and W_* is the transformed velocity magnitude. This can be modified to a form similar to the Stockman-Lieblein compressibility correction [7].

$$W = \frac{1}{rb}\left(\frac{1}{\rho}\right)^{W_*/\tilde{W}_*} W_* \tag{17}$$

where $\tilde{W}_*$ is the transformed mean velocity magnitude at the same merdional location as W. This altered transformation models density variations in the tangential direction which were suppressed by the approximations to the governing equations.

Additional solution modifications are possible. The problem with using solution modifications is that they usually produce better solutions only for a particular type of turbomachinery flow.

SAMPLE CALCULATIONS AND DISCUSSION

The sample calculation problems were chosen to demonstrate the capabilities and accuracy of the method. The calculated flows range from incompressible to supercritical. Both centrifugal and axial turbomachinery blade rows are analyzed. All flows are calculated without taking into account viscous effects.

Incompressible flow

Two incompressible flow cases are presented. In both cases a centrifugal compressor blade-to-blade geometry is considered. The transformed surface is shown in figure 3. The blade row is rotating at 68,000 RPM. In the first case a constant stream sheet thickness is assumed. Therefore no approximations are present in the governing equations. The calculated results are shown in figure 4, along with results from the finite difference solution of Katsanis [6]. The calculated surface velocities from the two solutions are in good agreement, including the region of reverse relative flow on the blade pressure surface. The second case differs slightly from the first. A varible stream sheet thickness is assumed, so that the method is solving approximate governing equations. The mean flow is estimated using the blade passage geometry. The assumed stream sheet variation has a ratio of 4.75 to 1 from the blade leading to trailing edge. The comparison of surface velocities with Katsanis [6] is shown in figure 5. Agreement between the two solutions is very good.

Compressible flow, subcritical

Three subcritical flow cases are presented. The density approximation will influence the solutions for these flows. The mean flows used in the solutions were determined from the blade passage geometry in the first and second cases, and was specified in the third case.

The first calculation is for a radial diffuser. The critical velocity ratio at the blade inlet is 0.87 and at the outlet is 0.41. As shown in figure 6, the comparison with Katsanis is excellent except at the trailing edge. The trailing edge velocity spikes calculated by the Katsanis solution are due to the rapid turning of the inviscid flow around the rounded trailing edge of the blade. The real flow would separate forming a viscous wake before such large velocity gradients could appear. In the present solution an extrapolation technique similar to Gostelow's [8] is used to eliminate these spikes.

The second case is a radial inflow turbine design by Klassen using the MERIDL and TSONIC computer codes [4], [6]. The critical velocity ratios across the blade row are 0.48 at the inlet and 0.88 at the exit. The flow incidence with the blade is 60 degrees. Total turning of the flow is 100 degrees. The rotational speed of the rotor is 100,000 RPM. A comparison of the present method with the TSONIC code solution is shown in figure 7. Some differences in the solutions do occur in the uncovered portions of the blade row at the inlet and outlet areas, but overall agreement is good.

The third subcritical case is a high turning low solidity turbine vane. The vane has an axial solidity of 0.63, and a design turning of 75 degrees. The ideal exit critical velocity is 0.896. Since the flow turning deviates significantly from the mean blade angle for this problem, it was necessary to specify the mean flow rather than calculating it using the blade passage geometry. A linear mean flow distribution was assumed based on the vane velocity diagrams. The experimental data of Schwab [9] is used for comparison in figure 8. The agreement is good.

Compressible flow, supercritical

Two supercritical cases are presented. The density approximation will have more of an effect in these cases than it had on the previous solutions. Solution modification techniques were used in calculating these flows.

The first case is a turbine vane which was designed by Detroit Diesel Allison and tested there by Huffman et al. [10]. The mean flow for this calculation was determined by using the blade geometry from the inlet to the throat and a linear distribution from the throat to the blade row outlet flow conditions. The comparison with experimental data is shown in figure 9 for both a supercritical and subcritical exit flow. The agreement for both cases is good. The calculated velocity is a little low on the suction surface near the throat and blade trailing edge.

The final case is a supercritical compressor blade designed by Sanz using the hodograph method of Bauer, Garabedian, and Korn [11]. The critical velocity ratio is 0.74 at the blade inlet and 0.58 at the outlet. The flow is turned through 29 degrees by the cascade. The mean flow for the solution was calculated using the blade passage geometry. This case is a severe test of density approximation, since large blade loadings and high speed flows are present. Solution comparisons are shown in figure 10. The circle symbols show the calculated results when the basic method is applied. The calculation does poorly in the region where the flow is transonic. The calculation in this region can be improved by using equation (17) to modify the solution as shown by the square symbols in the figure. However, the subsonic portion of the solution becomes poorer as a result.

As this case demonstrates, solution modification techniques can be used to match almost any known blade row flow. The difficulty with using solution modifications lies in their general application to the prediction of flows. Hopefully, specific modification techniques could be developed and applied to the prediction of flows in particular classes or types of turbomachinery.

CONCLUSION

The present method rapidly calculates surface velocities for a wide range of turbomachinery blade-to-blade flows. The method produces good results for subcritical flows. Supercritical flows can be calculated, but the solution accuracy is poor. Solution modifications can be made to improve the supercritical flow calculations, however, application of these modifications requires insight into the flow being analyzed and the modifications can not be applied universally to all flows.

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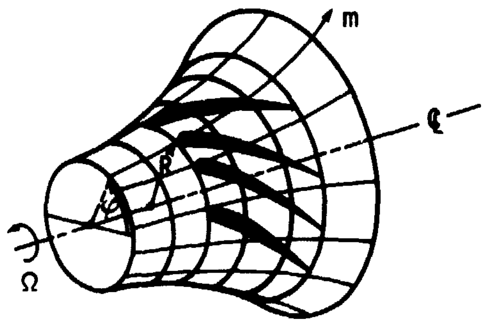

Figure 1. - Blade-to-blade surface of revolution.

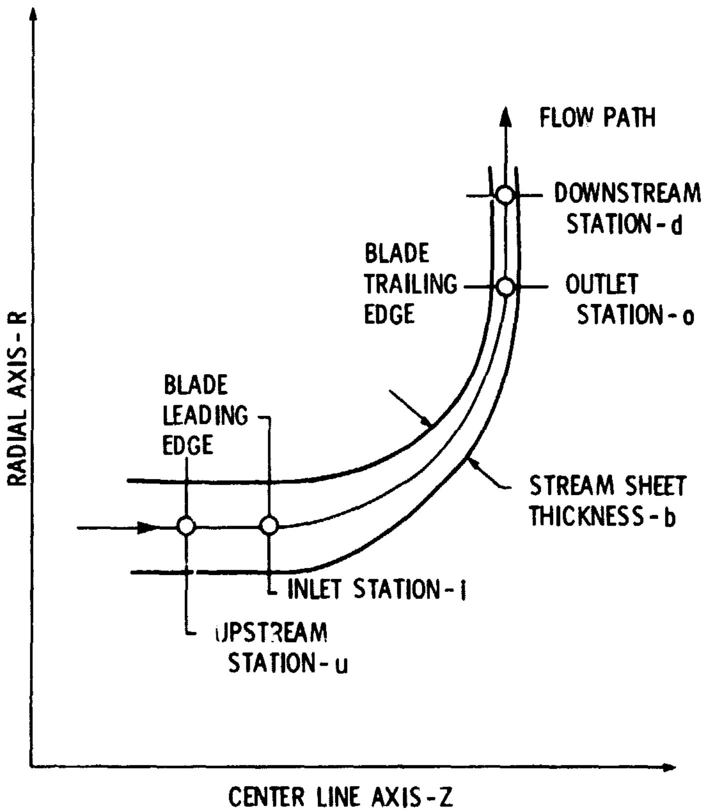

Figure 2. - R-Z cut through blade-to-blade surface.

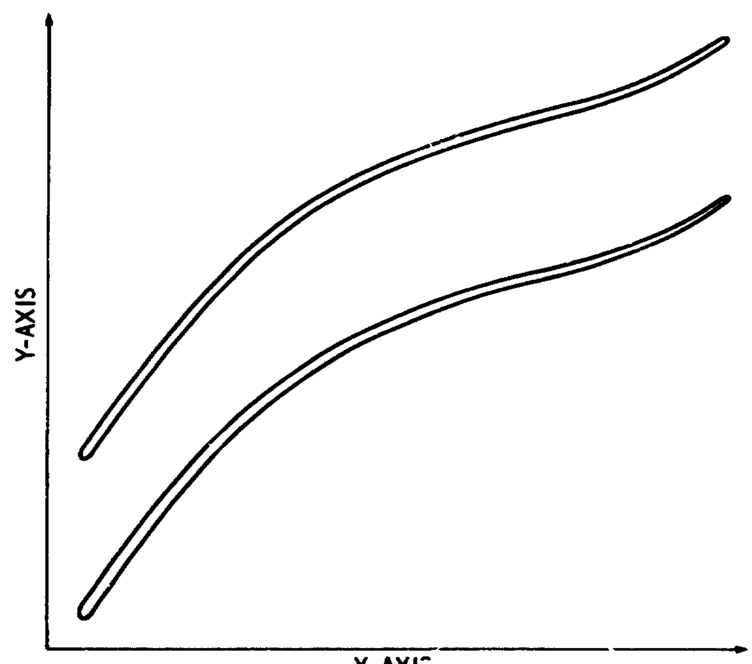

Figure 3. - Transform blade-to-blade surface - centrifugal impeller.

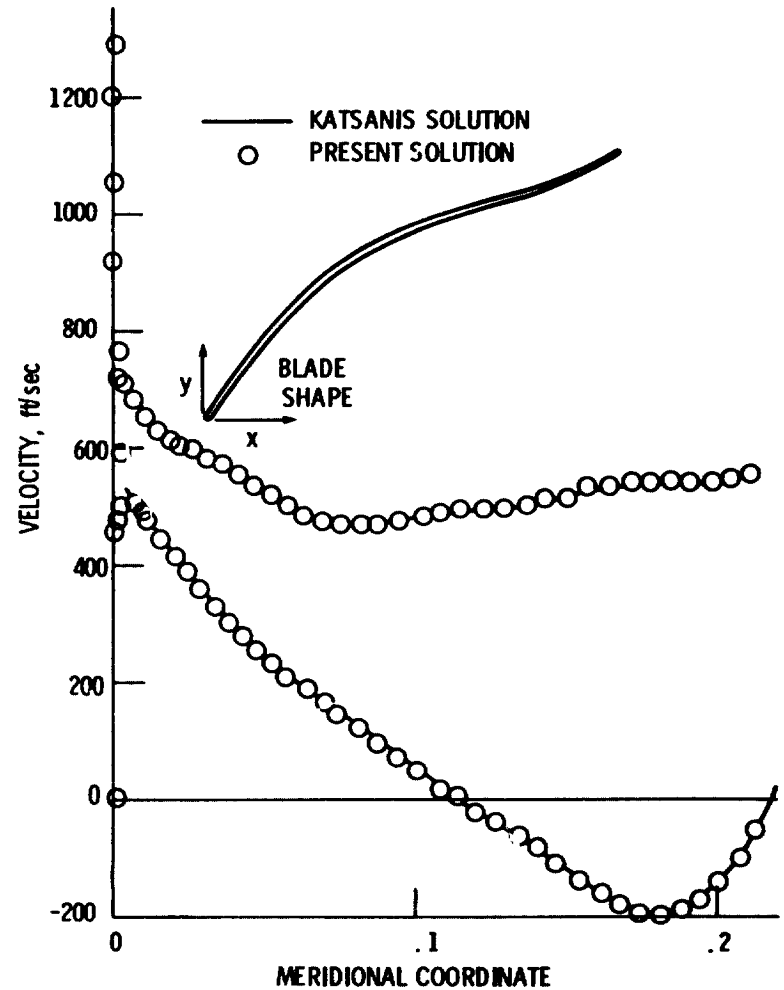

Figure 4. - Incompressible flow - constant stream sheet thickness.

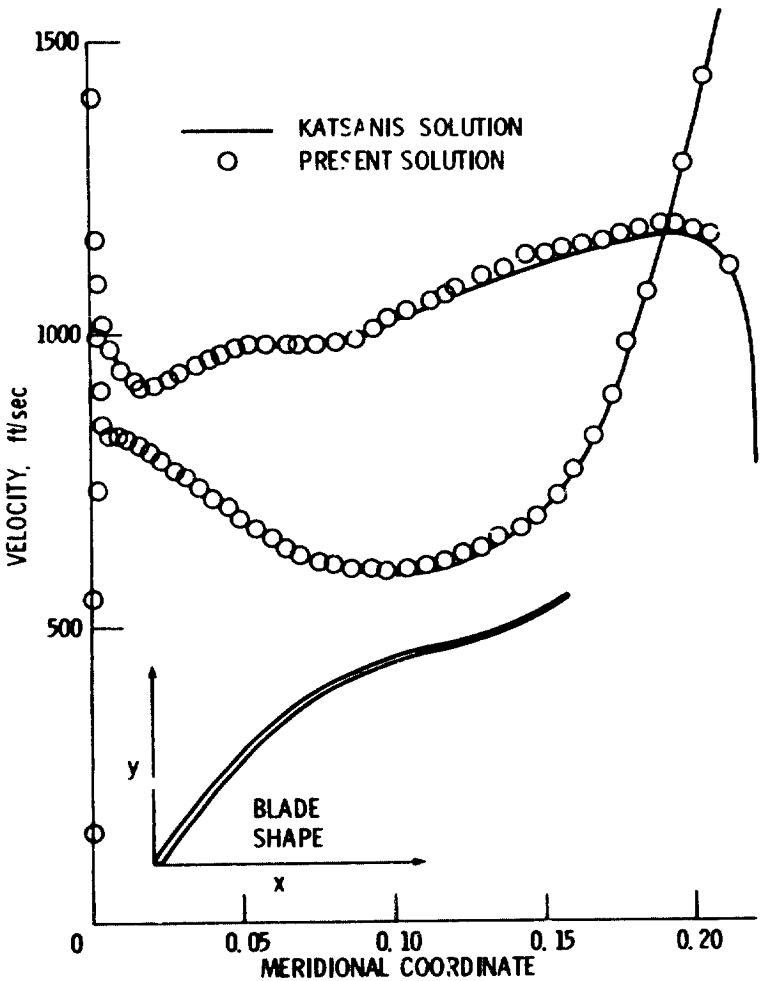

Figure 5 - Incompressible flow-variable stream sheet thickness.

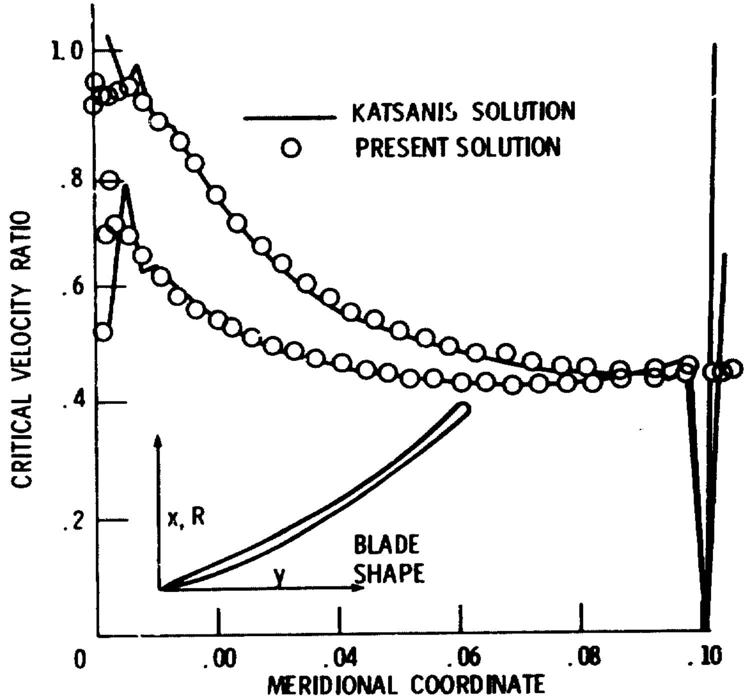

Figure 6. - Radial vane diffuser.

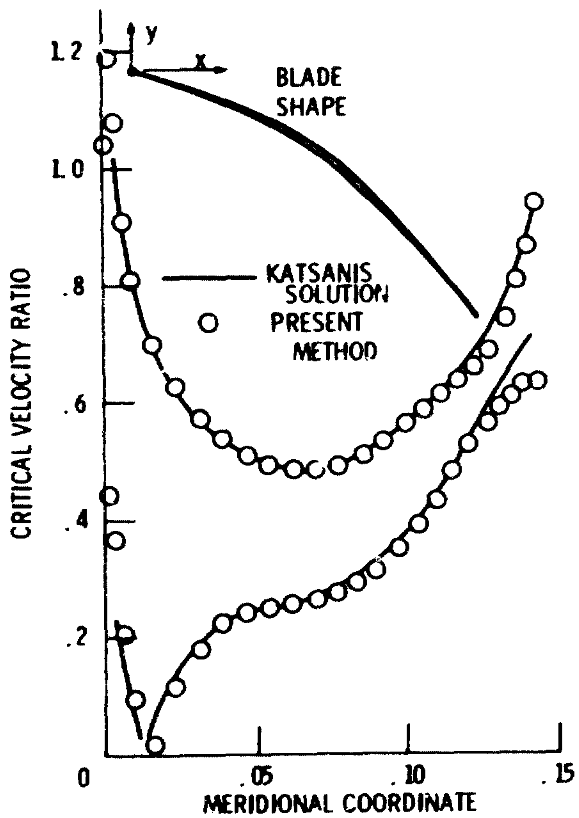

Figure 7. - Radial in-flow turbine.

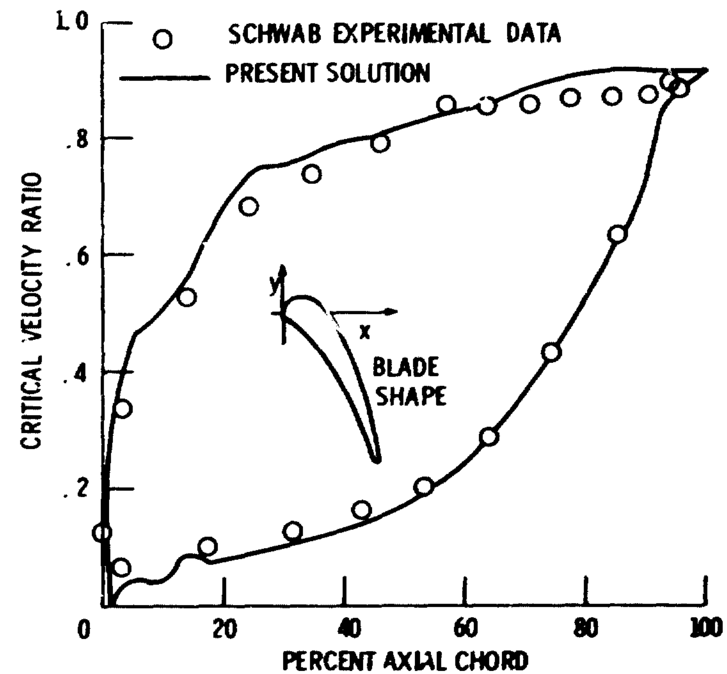

Figure 8. - High turning turbine vane.

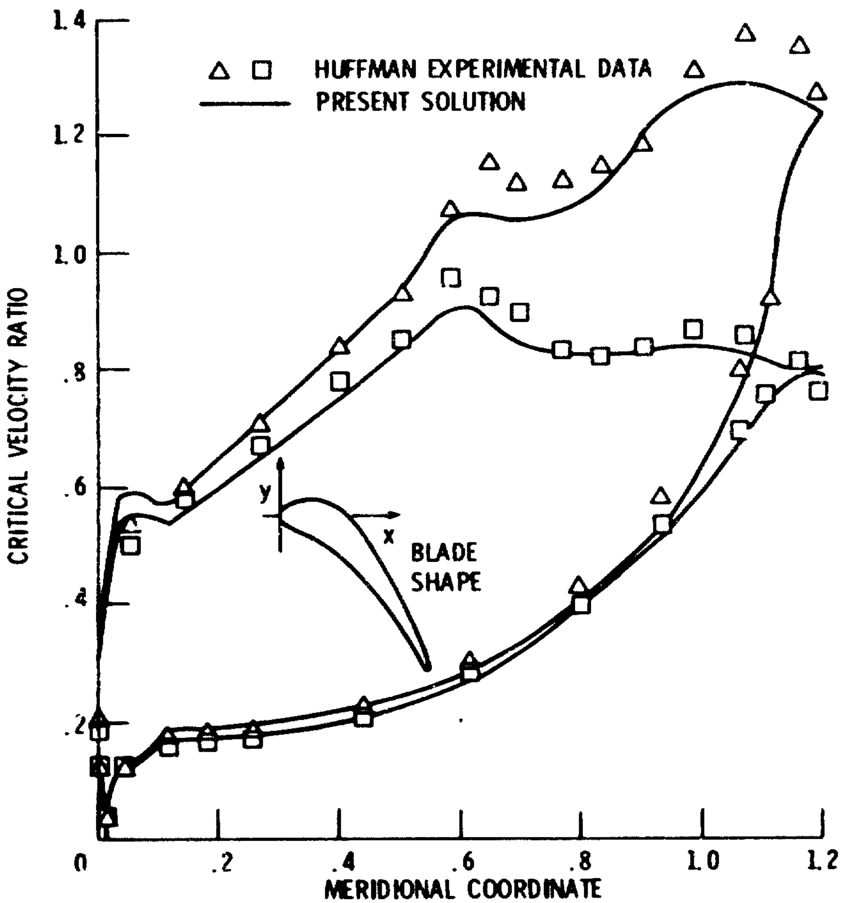

Figure 9. - Supercritical exit turbine vane.

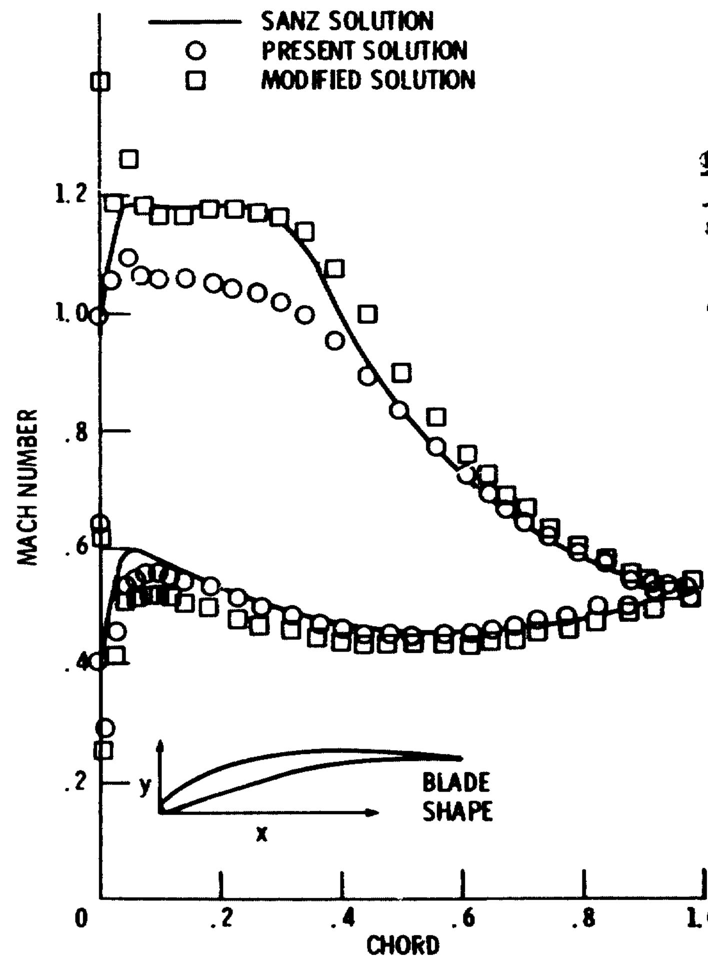

Figure 10. Supercritical compressor blade.