A Rapid Look-up Table Method for Reconstructing MR Images from Arbitrary K-space Trajectories

B. M. Dale, M. Wendt, J. L. Duerk
Departments of Radiology and Biomedical Engineering,
Case Western Reserve University and University, University Hospitals of Cleveland, Cleveland, Ohio 44106

Introduction:

In MRI the problem of reconstructing non-uniformly sampled Fourier data arises when an image is acquired using a non-Cartesian, or non-rectangular, k-space trajectory. Such trajectories may be desirable for a variety of reasons such as rapid acquisition times or good artifact properties.

The methods for reconstructing non-uniformly sampled k-space data can be broadly grouped into those relying on the discrete Fourier transform (DFT) and various gridding algorithms. If these algorithms could be implemented in a computationally efficient manner, more rapid reconstruction would be obtained. This would be a necessary precursor to a truly real time combined acquisition and reconstruction scheme. Such a scheme would be invaluable, for example, for interventional MRI applications.

Look-up tables are a commonly used tool for increasing the computational efficiency of many algorithms, and the purpose of this work was to determine if they could be effectively exploited to achieve real time reconstruction, or to achieve real time gridding.

Material and Methods:

Any linear operation on MRI data can be written as

$$\mathbf{f} = \sum_{i=0}^{M-1} \mathbf{T}(i) \cdot F(k_i) \text{ where}$$

$$\mathbf{f} = \begin{bmatrix} f(0) \\ \vdots \\ f(N-1) \end{bmatrix} \text{ and } \mathbf{T}(i) = \begin{bmatrix} T(i,0) \\ \vdots \\ T(i,N-1) \end{bmatrix}$$

where $F(k_i)$ are the Fourier data points, **T** is the linear operation to be performed (e.g. gridding or DFT) and **f** is the result.

Thus, a gridding operation [1] can be performed by defining **T** as

$$T(i,k) = \left(C(k) * \left(\rho^{-1}(k_i) \cdot \delta(k - k_i)\right)\right) \cdot comb(k) \quad i = 0,1,\cdots,M-1$$

where M is the number of frequency samples, $C(k)$ is a convolution kernel, such as the commonly used Kaiser-Bessel window, and *comb* is a set of δ functions used to represent uniform sampling. The result, **f**, then becomes the gridded k-space data; the image is reconstructed by performing the 2D-FFT.

Alternatively, a DFT can be performed by defining **T** as

$$T(i,n) = \frac{1}{\sqrt{N}} \rho^{-1}(k_i) \cdot e^{j2\pi k_i n/N} \quad n = 0,1,\cdots,N-1 \quad i = 0,1,\cdots,M-1$$

where, N is the number of spatial samples (pixels), the $F(k_i)$ are the Fourier data points, and ρ^{-1} is the density compensation function. Then **f** becomes the reconstructed image. This follows directly from the DFT synthesis equation

$$f(n) = \frac{1}{\sqrt{N}} \sum_{i=0}^{M-1} \rho^{-1}(k_i) \cdot F(k_i) \cdot e^{j2\pi k_i n/N} \quad n = 0,1,\cdots,N-1$$

where the usual requirement for equidistant frequency samples has been relaxed.

Whenever all the k_i are known beforehand, as is the case in MRI, **T** can be calculated in advance and can be stored in memory as a look-up table. Then either the DFT reconstruction or the gridding can be performed by multiplying each $\mathbf{T}(i)$, or each entry in the table, by each $F(k_i)$, or each acquired data point, and accumulating the results into **f**, the final result.

The principal advantage of the gridding table is that, for a small convolution window, the vast majority of the points in the table, **T**, are equal to zero. This allows the use of a sparse matrix representation (see Fig. 1) with the zeroes neither being stored nor calculated at run time.

Both the DFT and the gridding methods were implemented and compared using such tables. The convolution kernel (gridding tables only), was 4 by 4 array of grid points, which is larger than is typically used elsewhere in situations where time is critical [2]. Several numerical phantoms were reconstructed in order to simulate a variety of trajectories and to test the method. In-vivo trials were also performed during an IMRI procedure on a porcine neck using a 180 view, 256 points/view (echo at center), radial k-space true-FISP acquisition on a Siemens 0.2T Magnetom Open MRI system. The sequence had a TE of 9.4 ms, TR of 20.8 ms, acquisition time of 3.8 s and BW/pixel of 78 Hz.

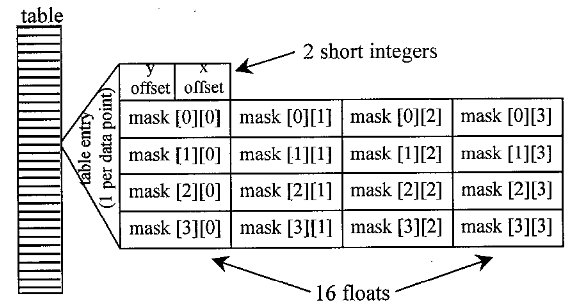

FIG. 1. Structure of the gridding table for a 4 by 4 convolution window. The x and y offset values accomplish the sparse matrix representation. The mask elements are the only non-zero elements.

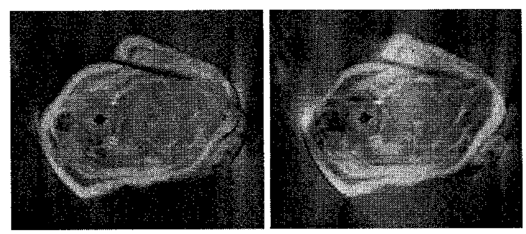

FIG. 2. Reconstructed magnitude image of pig neck (a). Gridded in 60 ms using the table-based method described above. Filtered back-projection (b) of same data, included for comparison.

Results:

The reconstructed images for the pig experiment are shown in Fig. 2. The gridded and FFT reconstructed image shows reduced artifacts relative to the filtered back-projection reconstructed image. The reconstructed phantom images showed peak deviations (relative to analytical reconstruction) of less than 2% of the total gray-level range.

The gridding table required 3 MB of memory. Due to memory constraints the DFT tables were kept small (32^2 points in reconstructed image) and only tested with the phantoms.

The table-based gridding took 60 ms on a 350 MHz Pentium II. For comparison a 512^2 point FFT took 390 ms on the same system, the standard gridding [1] took 4.78 s, and the filtered back-projection took 1 min. The DFT tables would take 7.5 min (extrapolated from small tables).

Conclusions:

A table-based gridding operation can be executed in much less time than the acquisition, despite the large size of the tables (relative to the size of the data cache), even using modest computational resources. Further, the method can be more computationally efficient than conventional gridding since many calculations need only be performed once and stored in a look-up table. Currently, the DFT tables are too large to use on a PC (2.8 GB for the radial sequence, extrapolated from small tables). The DFT tables remain interesting because they directly yield the reconstructed image without recourse to the FFT. With sufficient computer power, this should allow the image to be updated after acquiring each data point. Even with modest computational power the limiting step is the FFT, not the gridding. This represents a significant step towards a truly real time, non-Cartesian, acquisition/reconstruction scheme by demonstrating a more temporally efficient method for arbitrary K-space trajectory reconstruction.

References:

1. O'Sullivan, J.D. *IEEE Trans Med Imaging*, MI-4, 200, 1985.
2. Irarrazabal, P, et al. *Magn Reson Med*, 30, 207, 1993.

Acknowledgments:

This work is supported by the Whitaker Foundation and Siemens Medical Systems