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A rate-sensitive analysis of R/C beams subjected to blast loads

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Abstract

The behavior and response of structural reinforced concrete elements under severe blast loads are investigated numerically. The analytical approach utilizes the Timoshenko beam theory for the analysis of reinforced concrete beams. A formally proposed rate-sensitive nonlinear material models are used to predict the dynamic property of structural steel and concrete under the blast loads. Comparison between the rate-sensitive analysis and rate-insensitive analysis in which the static strength of material is enhanced by a factor to roughly reflect the effect of strain-rate is given. The numerical results are also compared with the limited experimental data. It is demonstrated that the present consideration of the influence of strain-rate on the structural responses in design manuals leads to over conservative.

1 Introduction

Recent advances in computational methods have provided a tremendous opportunity for the realistic assessment of the response of structures subjected to severe dynamic loads. Although it is well established that the rate of straining can affect significantly the material response, it is still common practice in designing blast-resistant structures to use material models which do not account for the strain-rate effect or to take into account of enhancement of static strength of materials when assessing the dynamic responses of the structures. A recent study [3,4] demonstrated that the strain-rate effect can have considerable influence on the response of steel and reinforced concrete structures subjected to explosion loading, since this type of loading is usually associated with localized high strain-rates. Therefore, the behavior of structural components under the various dynamic loading conditions can be predicted realistically only if the effect of strain-rate on the material response is accounted for.

The rate-sensitive response of structural steel and concrete under dynamic loading has been studied extensively over past three decades. Based on experimental evidence, the most notable outcome of such studies was the observation that the yield stress increases with the rate of straining, and hence a distinction was made between static and dynamic yielding. Several material models were proposed to reflect the effect of the strain-rate on the elasto-plastic material response [3,4,7,8]. Amongst these efforts, the elasto/viscoplastic model incorporating the so-called overstress concept, which was developed by Malvern[7] and Perzyna [8], received much attention mainly due to its simplicity. The formally proposed rate-sensitive steel and concrete models [3,4] have demonstrated successfully in predicting steady-state and transient responses of the materials and are used in this investigation.

Previous analytical and experimental work performed on soil-structure interaction phenomena has pointed out the need to model not only the material nonlinearity but also the shear effects on the structural responses, particularly in high strain-rate environments. To account for the shear effects, a layered Timoshenko beam element is developed in this paper. Material and geometric nonlinearity, using the Lagrangian formulation, are included.

The blast loaded reinforced concrete beams are analyzed by the proposed layered Timoshenko beam element formulation. Several numerical examples are given, and comparison between the results by the rate-sensitive analysis and the rate-insensitive analyses is presented. The numerical results are also compared with the limited experimental data.

2 Rate-sensitive material model

The rate-sensitive material models used in this paper is based on the general elasto/viscoplastic theory proposed by Malvern[7] and Perzyna[8], According to the elasto/viscoplastic theory , the response of an elasto/viscoplastic material consists of an elastic part, which develops instantaneously, and a time-dependent viscoplastic part, which is related to the overstress. For an uniaxial stress state, the viscoplastic rate-sensitive response is hence described by the following equations:

$$\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_p \tag{1}$$

$$\dot{\varepsilon}_e = \frac{\dot{\sigma}}{E} \tag{2}$$

$$\dot{\varepsilon}_p = \frac{f\langle X \rangle}{E} \tag{3}$$

$$X = \sigma - g(\varepsilon_p) \tag{4}$$

where ε is the total strain, ε_e is the elastic strain, ε_p is the plastic strain, σ is the stress, E is Young's modulus, X is the overstress, the dot $(\cdot)$ denotes the rate of variation with respect of time and $g(\varepsilon_p)$ represents the static stress-strain relationship in the plastic range. The function $f\langle X\rangle$ is the rate function, which reflects the rate-sensitivity of the material. In this function, the bracket $\langle\ \rangle$ implies that the expression is activated only when $X > 0$, that is when the stress point is outside the plastic limit curve.

For analytical purposes, it is more convenient to combine (1)-(4) into the following governing equation:

$$\dot{X} + f\langle X\rangle = E(1-\mu)\dot{\varepsilon} \tag{5}$$

where

$$f\langle X\rangle = E(1-\mu)\dot{\varepsilon}_*\left(e^{\frac{X}{SN}} - 1\right)^N \tag{6}$$

The material constants for rate-sensitivity S, N and $\dot{\varepsilon}_*$ may be determined based on constant strain-rate experimental data. For a given $\dot{\varepsilon}$, the governing equation can be analyzed analytically only when $N = 1$ or numerically when $N \neq 1$ [3].

In this investigation, the bilinear kinematic strain-hardening and the trilinear strain-softening models are utilized for the static stress-strain relationship of the structural steel and concrete respectively, which are represented diagrammatically in Fig. 1 and Fig. 2. The detail of the models can be found in [3,4].

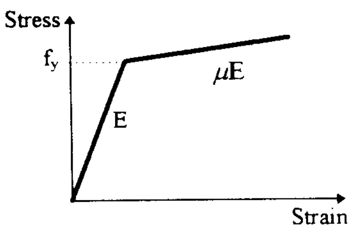

Figure 1. Bilinear steel model

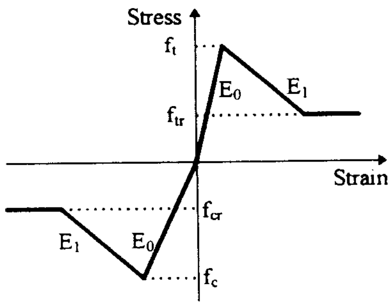

Figure 2. Trilinear concrete model

3 Layered Timoshenko beam finite element formulation

3.1 Assumed displacement field

The layered Timoshenko beam which is developed herein is based on the assumption that plane sections originally normal to the neutral axis remain plane after deformation but not necessarily normal to the neutral axis. This implies that the axial displacement $\bar{u}(x,z)$ may be expressed directly in term of $\theta(x)$ the rotation of the normal so that

$$\bar{u}(x,z) = u_0(x) - z\theta(x) \tag{7}$$

in which $u_0(x)$ is the axial displacement at the neutral axis. Note that the normal rotation $\theta(x)$ is equal to the slope of the neutral axis $d\bar{w}/dx$ minus a rotation β which is due to the transverse shear deformation. Thus we have

$$\theta(x) = d\bar{w}/dx - \beta \tag{8}$$

Notice also that the lateral displacement $\bar{w}(x,z)$ is given by the lateral displacement at the neutral axis so that

$$\bar{w}(x,z) = w(x) \tag{9}$$

3.2 Strain-displacement relationship

The axial and shear strains, ε_x and γ_{xz}, are related to the axial and lateral displacements, $\bar{u}$ and $\bar{w}$, and shear deformation, β, by

$$\varepsilon_x = \frac{\partial \bar{u}}{\partial x} - z\left(\frac{\partial^2 \bar{w}}{\partial u^2} - \frac{\partial \beta}{\partial x}\right) + \frac{1}{2}\left(\frac{\partial \bar{w}}{\partial x}\right)^2 \tag{10}$$

$$\gamma_{xz} = \beta + \left(\beta - \frac{\partial \bar{w}}{\partial x}\right)\frac{\partial \bar{u}}{\partial x} \tag{11}$$

3.3 Stress-strain relationship

As the interaction between the shear and moment in the structural elements is a complicated phenomenon and is not well understood, the layered Timoshenko beam model proposed in this paper does not included the shear and moment interaction. Therefore, the axial and shear may be calculated separately as follows:

$$\sigma_x = E_t \varepsilon_x \tag{12}$$

$$\sigma_{xz} = G_t \gamma_{xz} \tag{13}$$

where E_t and G_t are tangent young's modulus and shear modulus, respectively. The bilinear model shown in Fig. 1 is utilized to describe axial and transverse deformations of steel as well as the transverse deformation of concrete. The

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trilinear model is used to represent the axial stress-strain relationship of concrete.

3.4 Dynamic equilibrium equation
The dynamic equilibrium equation of the Timoshenko beam may have the form based on the virtual work principle

$$M\ddot{y} + p_a + p_f + p_s = f \tag{14}$$

where M is mass matrix , f is the external applied force vector, p_a is the internal axial resistance vector, p_f is the internal flexural resistance vector and p_s is the shear resistance vector. They may have the forms

$$p_a = \int_0^l [B_a]^T \overline{N} dx \tag{15}$$

$$p_f = \int_o^l [B_f]^T \overline{M} dx \tag{16}$$

$$p_s = \int_o^l [B_s]^T \overline{Q} dx \tag{17}$$

in which B_a , B_f and B_s are strain matrixes of axial, flexural and shear deformations respectively. The stress resultants $\overline{N}$, $\overline{M}$ and $\overline{Q}$ are contributed by the axial and shear stresses in each layer, that is

$$\overline{N} = \sum_l b_l \sigma_{xl} t_l \tag{18}$$

$$\overline{M} = \sum_l b_l \sigma_{xl} z_l t_l \tag{19}$$

$$\overline{Q} = \sum_l b_l \tau_{xzl} z_l t_l \tag{20}$$

where b_l , t_l and z_l is the breath, thickness and z-coordinate at the middle of the layer l respectively. If the beam is divided into two nodal elements and linear shape function is utilized, the internal resistance of an element may have the forms

$$p_a^{(e)} = \left[\overline{N}, 0, 0, -\overline{N}, 0, 0\right]^T \tag{21}$$

$$p_f^{(e)} = \left[0, 0, \overline{M}^{(e)}, 0, 0, -\overline{M}^{(e)}\right]^T \tag{22}$$

$$p_s^{(e)} = \left[0, -\overline{Q}^{(e)}, -(\overline{Q}l)^{(e)}/2, 0, \overline{Q}^{(e)}, -(\overline{Q}l)^{(e)}/2\right]^T \tag{23}$$

and consistent nodal force vector of element is given as

$$f^{(e)} = \left[0, (ql)^{(e)}/2, 0, 0, (ql)^{(e)}/2, 0\right]^T \tag{24}$$

The dynamic equilibrium equation (14) may be solved by explicit predictor-corrector Newmark scheme, and the critical time step is given [6] by

$$\Delta t_{cr} = \frac{2}{\omega_{\max}(\delta + 0.5)} \tag{25}$$

where δ is the integration parameter of Newmark scheme and

$$\omega_{\max} = \max\left(\omega^{f}_{\max}, \omega^{s}_{\max}, \omega^{a}_{\max}\right) \tag{26}$$

For elastic system,

$$\omega^{f}_{\max} = \frac{12c\sqrt{I/A}}{L^2} \tag{27}$$

$$\omega^{s}_{\max} = \frac{c}{\sqrt{I/A}}\left(\frac{k}{2(1+\nu)}\right) \tag{28}$$

$$\omega^{a}_{\max} = \frac{2c}{L} \tag{29}$$

where $\omega^{f}_{\max}$, $\omega^{s}_{\max}$ and $\omega^{a}_{\max}$ are the maximum natural frequencies of the beam element for flexure, shear and axial deformations respectively, c is elastic wave speed, L is the element length, k is shear area corrector factor, ν is Poission's ratio, I and A are the inertial moment and area of the section respectively.

4 Numerical examples and discussions

4.1 Basic input parameters

In order to illustrate strain-rate effect on the response of reinforced concrete structures, a simply-supported beam as shown in Fig. 3 is investigated in this

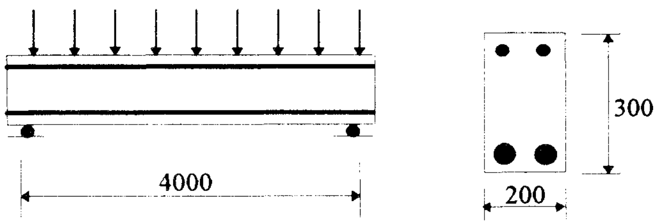

Figure 3. Configuration of reinforced concrete beam (units in mm)

paper. The numerical analysis is divided into two categories: one is the rate-sensitive analysis and the other is the rate-insensitive analysis in which the static strength is enhanced by a factor to reflect the influence of strain-rate. The material properties of the steel and concrete assumed for these analyses are summarized as follows: (1) for steel, the Young's modulus $E = 210000 MPa$,

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shear modulus $G = 125000MPa$, static yielding strength $f_y = 310MPa$, static shear yielding strength $\tau_y = 155MPa$, hardening parameter $\mu = 0.005$, mass density $\rho = 7.8 \times 10^{-9} N \sec^2 mm^{-4}$ and the rate-sensitive material parameters are $S = 6MPa$, $N = 1$, $\dot{\varepsilon}_* = 5 \times 10^{-5} \sec^{-1}$; (2) for concrete in transverse deformation, shear modulus $G = 9400MPa$, static shear yielding strength $\tau_s = 1.365MPa$, hardening parameter $\mu = 0$, and the rate-sensitive parameters are $S = 1.15MPa$, $N = 2$, $\dot{\varepsilon}_* = 4 \times 10^{-4} \sec^{-1}$.; (3) for concrete in axial tensile deformation, secant modulus $E_o = 20000MPa$, static peak strength $f_t = 1.8MPa$, softening modulus $E_1 = -10000MPa$, residual strength $f_{tr} = 0.72MPa$, and the rate-sensitive parameters are $S = 0.378MPa$, $N = 2$, $\dot{\varepsilon}_* = 1.5 \times 10^{-4} \sec^{-1}$; (4) for concrete in axial compressive deformation, secant modulus $E_o = 15000MPa$, static peak strength $f_c = -19.5MPa$, softening modulus $E_1 = -7500MPa$, residual strength $f_{cr} = -7.8MPa$, and the rate-sensitive parameters are $S = 1.93MPa$, $N = 2$, $\dot{\varepsilon}_* = 6.66 \times 10^{-4} \sec^{-1}$. It is noted that the rate-sensitive parameters of the steel used in the rate-sensitive analysis are fitted by least-square method based on CEB's recommendation[11] and the rate-sensitive parameters of concrete in tension and compression are also fitted by least-square method based on data given by Souroushian et al[10] and John & Shah[1], respectively. The strength enhancement factors of steel and concrete due to the strain-rate effect used in the rate-insensitive analysis are based on [2] and the values of 1.3 for steel and 1.2 for concrete are taken. The mass density of concrete are assumed as $\rho = 2.3 \times 10^{-9} N \sec^2 mm^{-4}$ in all analyses. The beam is divided into 12 elements. The Newmark integration parameters are $\delta = 0.5$ and $\beta = 0.25$ and time integration step of $\Delta t = 10^{-5}$ sec is used in all analyses.

The simplified equivalent SDOF method [2] which is frequently used in the present design manuals of blast-resistant structures is utilized to give applied blast loads which correspond to the resistance of the SDOF elastic/ideally-plastic system with the ductility ratio of 3. Three different steel ratios in tensile region ($\rho = 0.5\%$, 1% and 1.5%) are assumed and the steel ratios in compressive region are taken as half of the steel ratios in tensile region.

4.2 Triangle load

In order to reflect the effect of different duration of the triangle blast load as shown in Fig. 4a, the dynamic responses of the reinforced concrete beams under the loads of four different duration are predicted by the proposed models.

The maximum displacements at the center of the beam are summarized in Table 1. It is demonstrated that the difference of the maximum displacements at the center of the beams between the rate-sensitive and the rate-insensitive predictions is significant, from 15 to 76%. It is shown that the displacements by the rate-insensitive analysis are always larger than those by the rate-sensitive analysis, which implies that the present consideration in design manuals for the effect of strain-rate is over conservative.

Fig. 5 shows the typical displacement-time histories.

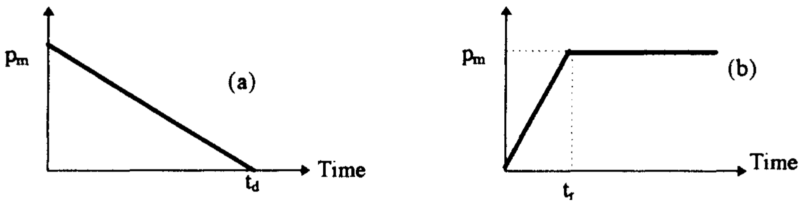

Figure 4. (a) Triangle load, (b) Ramp load

Table 1. Maximum displacemental responses under the triangle loads

$\rho(\%)$	t_d/T	$P_m(N/mm)$	$y^{rd}_{max}(mm)$	$y^{ri}_{max}(mm)$	$\delta=(y^{ri}_{max}-y^{rd}_{max})/y^{ri}_{max}(\%)$
0.5	0.25	49.1	7.0	12.5	44
0.5	1	19.6	8.0	20.0	60
0.5	4	15.1	10.0	42.5	76
0.5	16	14.1	11.0	75.0	85
1.0	0.25	97.7	14.0	20.0	30
1.0	1	39.1	17.0	30.0	43
1.0	4	30.1	22.0	58.0	63
1.0	16	28.1	24.0	100.0	76
1.5	0.25	144.8	22.0	26.0	15
1.5	1	57.9	27.0	37.0	27
1.5	4	49.7	34.0	68.0	50
1.5	16	41.6	39.5	108.0	63

(ρ = steel ratio in tensile region, T = period, P_m = peak applied load, y^{ri}_{max} and y^{rd}_{max} = maximum displacement by the rate-insensitive and rate-sensitive analyses, respectively)

4.3 Ramp load

The R/C beam under the ramp load as shown in Fig. 4b is investigated. The results are presented in Table 2. It is demonstrated again that the difference between the rate-sensitive and the rate-insensitive predictions is significant, about 55%.

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Table 2. Maximum displacemental responses under the ramp loads

$\rho(\%)$	t_r/T	$p_m(N/mm)$	$y^{rd}_{max}(mm)$	$y^{ri}_{max}(mm)$	$\delta = (y^{ri}_{max} - y^{rd}_{max})/y^{ri}_{max}(\%)$
1.0	0.25	19.4	12.0	27.0	56
1.0	1	22.3	10.5	22.5	53

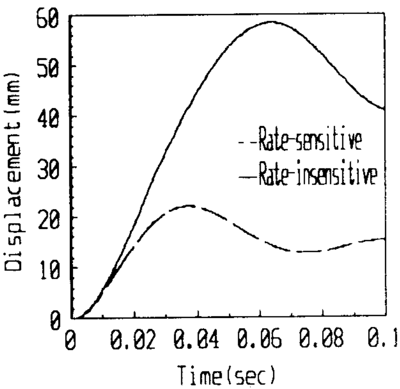

Figure 5. Displacement-time history under the triangle load

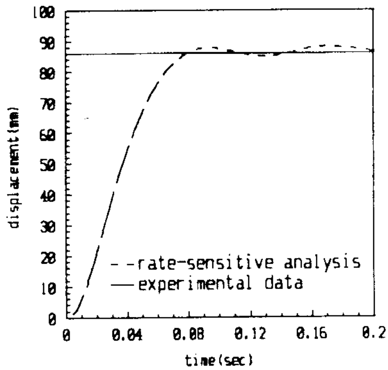

Figure 6. Comparison between the calculation and experiment

4.4 Comparison with experiment

The response of the simply supported beam subjected to uniform blast pressure was studied experimentally by Allgood and Swihart[12]and the result is used to compare the proposed numerical analyses. In the experiment, a step pressure was applied, having an amplitude equal to the static ultimate load($p_u = 0.156MPa$) of the beam. Based on [12], the length of the beam $l = 4320mm$, the static yield strength of steel $f_y = 340MPa$ and the static peak compressive strength of concrete $f_c = 28.5MPa$, the tensile steel area $A_t = 924mm^2$ and compressive steel area $A_c = 279mm^2$. The other parameters are assumed as described previously.

Fig. 6 depicts the comparison between the results of the rate-sensitive analysis and the experiment. The solid line represents the maximum displacement in the experiment. A good agreement is observed. However, it is worth pointing out that the rate-insensitive analysis, in which the static strengths of the materials are enhanced by a factor as described before, predicts the beam will collapse under the same blast load as in the rate-sensitive analysis.

5 Conclusions

This paper presents an investigation into the influence of material rate-sensitivity on dynamic responses of reinforced concrete beams subjected to blast loads.

The paper first outlined the rate-sensitive steel and concrete models and then formulated Timoshenko beam element. Finally, the simply-supported reinforced concrete beams under the triangle and ramp loads are analyzed and several numerical examples are given. The comparison between the maximum displacemental responses of the rate-sensitive and the rate-insensitive predictions is presented. The comparison of the proposed rate-sensitive analyses with the limited experimental data is also given, and good agreement is obtained.

It is demonstrated that the rate-sensitivity on the response of reinforced concrete beam subjected to blast loads is significant, and the consideration of enhancement of static strength to account for the effect of strain-rate on the structural response in the manuals is inadequate and leads to over conservative in designing reinforced concrete structures.

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