

A Rational Expectations Equilibrium with Informative Trading Volume

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Abstract

Grossman (1976) shows how market prices aggregate private information. In this paper I show how trading volume helps investors to interpret the aggregate information in the price. I construct a model where investors trade for two reasons: private information and risk sharing. When trading volume is high, investors know that private signals are dispersed. They therefore weight the market price heavily relative to their own signals. Conversely, when trading volume is low, investors weight their private signals more heavily. This model offers a closed form solution of a rational expectations equilibrium where all investors learn from (1) private signals, (2) market price and (3) aggregate trading volume.

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1 Introduction

In a financial market every investor is interested in the private information that other investors might possess. There are several ways to learn what other market participants know. For example, starting with Grossman (1976) a large number of authors have shown how investors can extract information from the price.¹ In this paper, I provide the first closed form solution for a rational expectations equilibrium where all investors infer information about the state of the economy from (1) private signals, (2) the market price and (3) aggregate trading volume.²

Several empirical studies support the idea that trading volume contains information about future returns. For example, Llorente, Michaely, Saar, and Wang (LMSW, 2002) solve and test a model where trading volume predicts changes in the autocorrelation of returns.³ However, the investors in LMSW's model rationally ignore trading volume when they update their beliefs, since volume does not contain any information beyond their own private signals and the market price. Therefore, in LMSW, trading volume provides information only to an outside observer of the economy, but not to investors within the economy.

My model shows how investors within the economy can learn from trading volume, and how volume information differs from information contained in the price.

¹Authors who examine how investors can learn from prices include Grossman (1976), Grossman and Stiglitz (1980), Hellwing (1980,1982), Verrecchia (1982), Admati (1985), Brown and Jennings (1988), Grundy and McNichols (1989), and Wang (1993), and also Admati and Pfleiderer (1988), Foster and Viswanathan (1990, 1993, 1996), Back, Cao, and Willard (2000). Glosten and Milgrom (1985) show how investors can learn from the bid-ask spread. Glosten (1994) shows how investors can learn from the limit order book.

²Authors who develop trading volume models include Pfleiderer (1984), Kim and Verrecchia (1991), Easley and O'Hara (1992), Campbell, Grossman, and Wang (1993), Harris and Raviv (1993), Shalen (1993), Blume, Easley, and O'Hara (1994), Wang (1994), He and Wang (1995), Lo and Wang (2000), Gallmeyer, Hollifield, and Seppi (2005).

³For other evidence see also Gallant, Rossi, and Tauchen (1992), Campbell, Grossman, and Wang (1993), Conrad, Hameed, and Niden (1994), Lee and Swaminathan (2000), Gervais, Kaniel, and Mingelgrin (2001), and Connolly and Stivers (2003).

The main result of this model is that trading volume reveals the relative quality of the aggregate private information in the economy. Under high trading volume, the aggregate information is more precise compared to private signals than under low trading volume. Investors therefore use volume to decide how they should weight the market price relative to their own private signals when they update their beliefs. When trading volume is high, investors weight the market price more heavily. Conversely, when volume is low, investors weight their private signals more heavily.

In order to show how investors infer information from trading volume, I develop a model where a large number of small investors observe private noisy signals about a future dividend. In addition to their endowment of information, the investors are also endowed with private claims to a risky future labor income. The dividend income and the labor income are correlated, so that investors have two motives for trading: private information and risk sharing.

Private signals and labor endowments are identically distributed for all investors, so that all investors observe information of identical quality. Therefore, investors weight their signals equally when they update their beliefs. As a result, the equilibrium price depends on the average signal and the average exposure to the labor risk. Since investors are uncertain about the average labor risk, they are not able to fully infer the average dividend signal from the price.

In addition to the uncertainty about the aggregate dividend information and aggregate labor risk, the investors are also uncertain about the cross-investor correlation of the individual errors in their private dividend signals. The correlation of the individual signal errors is important, since this correlation determines the quality of the aggregate information relative to the private information. For example, if the signals are perfectly correlated, then the average signal contains the same information as the individual signals. However, if investor specific signal errors are uncorrelated, then investors know more on aggregate than they know individually. Investors therefore need to know the correlation of the individual signals, in order to assess the precision of the average signal in the price.

In a symmetric economy where private information and labor risks are identically distributed for all investors, trading volume reveals the correlation of signals in the following way: since investors weight their signals and endowments identically when they calculate their demands, the number of shares that a given investor buys or sells depends only on the differences between his private signal and endowment and the signal and endowment of the average investor in the economy. Therefore, the individual trades are functions only of the investor specific components of signals and endowments. Hence, if these components are independent across investors, and if the number of investors in the economy is large, the per capita trading volume depends only on the distribution and not on the realization of these components. As a result, investors can infer the distribution of signals from trading volume. In particular, if investors are uncertain about the correlation of their private signals, then trading volume reveals this correlation.

In the equilibrium, the number of shares that investors trade increases with the dispersion of their signals. Therefore, if trading volume is high, investors realize that the correlation of investor specific signal errors is low and that the quality of the aggregate information in the price is high. In that way, trading volume helps investors to disentangle two dimensions of uncertainty: the realization of the aggregate signal and the precision of this signal. Since volume does not reveal the realization of the aggregate signal, and since the price alone does not reveal the precision of the signal, all investors fully rationally condition their demands on both price and trading volume.

In traditional models of heterogeneous information, such as Grossman and Stiglitz (1980), investors form a weighted average of their private signals and the market price when they update their beliefs. In these models, all investors know the optimal weights for the price and the signals, since these weights are independent of the state of the economy. In my model, investors are uncertain how they should weight the market price relative to their own private information. Observing trading volume removes this uncertainty. Under low trading volume, the quality of the aggregate

signal in the price does not exceed the quality of their private signals. Since the price contains additional noise from the aggregate labor income shock, investors weight their own signals more heavily than the price when volume is low. However, under high trading volume, the quality of the aggregate information exceeds the quality of the individual signals. Therefore, investors weight the price more heavily when trading volume is high.

The idea that investors have a risk sharing and a private information motive for trading has been previously employed for example by Wang (1994) and LMSW (2002). However, in these models, there are only two agents that trade with each other. Therefore, trading volume does not provide any information for the investors beyond the information that they can infer from their own private signals and the market price. The technical difficulty that arises if investors are allowed to observe trading volume is that volume is a sum of absolute values and therefore not normally distributed. Asset pricing models with heterogeneously informed investors usually rely on the properties of the normal distribution in order to be tractable.

My model solves the problem that trading volume is not normally distributed by transforming a non-linear optimization problem into a problem that is linear conditional on the observation of trading volume. Several other authors have examined alternative approaches. For example, Bernardo and Judd (1996) show how to numerically solve a model where investors learn private information from trading volume. Their numerical approach has the advantage that it covers a large set of possible assumptions, however, a numerical approach does not provide the same clean economic intuition as an analytical solution.

Blume, Easley, and O'Hara (BEH, 1994) provide a closed-form solution for a model where investors learn from past prices and past trading volume. Similar to my model, the investors in BEH face two dimensions of uncertainty: the realization and the quality of other investors' signals. However, in order to solve their model, BEH have to assume that investors are not fully rational: even though investors know the price at which they trade, they employ this price in order to update

their beliefs only after they have completed their trade. BEH (page 160) comment on the difficulty of solving a rational expectations equilibrium where investors learn from trading volume: “Alternatively, there could be nonrevealing equilibria in which traders condition on price and volume. However, as volume is a sum of absolute values it cannot be normally distributed. So although such an equilibrium might exist there seems to be no hope of constructing it, and hence no hope of using a contemporaneous data approach to study volume.” As I show in this paper, the case for a non-revealing equilibrium where investors condition their demands on prices and on volume is not completely hopeless.

The remainder of this paper is organized as follows. In section 2 I describe the setup of the model. In section 3, I show that an equilibrium exists. In section 4 I describe the properties of this equilibrium. Section 5 concludes.

2 Setup of the model

The economy is populated by a countable set of investors. I will refer to an individual investor as investor i , $i = 1, 2, \dots$. There are two time periods, $t = 0$ and $t = 1$. Figure 1 shows a picture of the time line. At time $t = 0$, investor i is endowed with

$$N_i = N + n_i$$

units of a non-traded asset. At time $t = 1$ the investors receive a payoff of Y for each unit N_i they are endowed with at time $t = 0$. The total non-traded income of investor i at time $t = 1$ is therefore given by $N_i Y$. I will refer to $N_i Y$ as the labor income of investor i , even though other interpretations are possible.

In addition to the labor income, the investors also receive income from their investments in the financial market. The financial market consists of two assets: a risk free bond and a risky firm. One dollar invested in the bond at time $t = 0$ pays one dollar at time $t = 1$. Investors can buy or sell an unlimited amount of the bond. Investors can trade shares of the firm at time $t = 0$ at the equilibrium price P . At

time $t = 1$ the firm pays a liquidating dividend D for each share the investors hold at time $t = 0$ after trading. At time $t = 0$ the investors observe private noisy signals about this dividend. The signal of investor i is given by

$$\hat{D}_i = D + \epsilon_i$$

where ϵ_i is an error term. The correlation of the ϵ_i across investors determines how much the investors disagree about the future payoff. There are two possible states of the world regarding this correlation. In state H , the dispersion of beliefs is high. In this state, the ϵ_i are independent across investors. In state L the dispersion of beliefs is low. In this state, all individual signals contain the same error term $\epsilon_i = \epsilon$, so that the errors are perfectly correlated across investors and all investors observe the same signal $D + \epsilon$. The investors might have some information about this dispersion of beliefs, however they do not know the realization of the correlation with certainty. For any given realization of the correlation state the random variables D , N , Y , $\{\epsilon_i\}_{i=0}^\infty$, $\{n_i\}_{i=0}^\infty$ are jointly normally distributed with mean zero and variances σ_D^2 , σ_N^2 , σ_Y^2 , σ_ϵ^2 and σ_n^2 .⁴ All variables are uncorrelated except for the correlation of the ϵ_i in state L , and except for D and Y , which are correlated with $\text{Cov}[D, Y] = \sigma_{DY} > 0$.

Let X_i be the demand of investor i . Let

$$X = \lim_{h \rightarrow \infty} \frac{1}{h} \sum_{i=1}^h X_i$$

be the per capita demand, provided that this limit exists. An equilibrium is given by a price P that satisfies

$$X = \text{supply per capita} \tag{1}$$

⁴Since all variables are normally distributed, dividends and labor income can be negative. It is possible to choose means and variances so that the probability for negative payoffs will be arbitrarily small. However, since all investors know these distributions, the choice of the mean does not affect the trading volume. In order to simplify the notation I set therefore all the means equal to zero.

with probability one. To simplify the notation I will set the supply equal to zero, and I assume that all investors own zero shares prior to the trading date $t = 0$. The assumption of zero supply means that all dividends that investors who hold long positions of the firm receive are paid by investors who hold short positions. This assumption will remove a constant from the equilibrium price, but it will not affect any of the results.

Since investors hold zero shares before trading, the number of shares that investor i trades is given by $|X_i|$. Let

$$V = \lim_{h \rightarrow \infty} \frac{1}{h} \sum_{i=1}^h |X_i|$$

be the (double counted) per capita trading volume, provided that this limit exists.

Investor i chooses his demand X_i by maximizing

$$\mathbb{E} \left[-e^{-\rho W_i} \middle| \mathcal{F}_i \right]$$

where ρ is the coefficient of absolute risk aversion, W_i is the future wealth, and \mathcal{F}_i is the information set of investor i . This information set is given by

$$\mathcal{F}_i = \{\hat{D}_i, N_i, P, V\}.$$

Investors can therefore condition their demand on their private signals, their private labor endowments, the equilibrium price, and the equilibrium trading volume. Note that investors know N_i , their own exposure to the labor risk, but they do not observe N , the economy wide exposure to this risk. This assumption will prevent the equilibrium price from completely revealing all private information.

3 The equilibrium

Let

$$\hat{D} = \lim_{h \rightarrow \infty} \frac{1}{h} \sum_{i=1}^h \hat{D}_i$$

be the aggregate information about the future dividend. Then we have

$$\hat{D} = \begin{cases} D + \epsilon & \text{in state } L \\ D & \text{in state } H \end{cases}$$

In state L , all investors observe the aggregate signal $\hat{D} = D + \epsilon$ directly. In state H , the individual signal errors are uncorrelated, so that they cancel each other out in the average signal. In this case, the quality of the aggregate signal exceeds the quality of the individual signals.

Theorem 1 (Equilibrium). *Let $s \in \{L, H\}$ be an indicator variable, such that $s = L$ if the dispersion of signals is low, and $s = H$ if the dispersion of signals is high. If $\rho\sigma_\epsilon\sigma_n\sigma_{DY} < \sigma_D^2$, then there exists an equilibrium with the following properties:*

(a) *the price is given by*

$$P = \Phi_D^s \hat{D} - \Phi_N^s N,$$

where

$$\Phi_D^L = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2}, \quad \Phi_N^L = \frac{\rho\sigma_{DY}\sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2},$$

and

$$\Phi_D^L < \Phi_D^H < 1, \quad \Phi_N^L < \Phi_N^H$$

(b) *the equilibrium demands are given by*

$$X_i^L = -\frac{\sigma_{DY}}{\sigma_D^2} n_i \quad \text{and} \quad X_i^H = \frac{1}{\rho\sigma_\epsilon^2} \epsilon_i - \Psi n_i$$

where $0 < \Psi < \frac{\sigma_{DY}}{\sigma_D^2}$

(c) *the trading volume is given by*

$$V^L = \sqrt{\frac{2}{\pi} \frac{\sigma_{DY}\sigma_n}{\sigma_D^2}} \quad \text{and} \quad V^H = \sqrt{\frac{2}{\pi} \left(\frac{1}{\rho^2\sigma_\epsilon^2} + \Psi^2\sigma_n^2 \right)}$$

and we have

$$V_L < V_H.$$

For a proof see appendix A. To understand the nature of the equilibrium, note first that part (c) of the theorem shows that trading volume is low when investors observe homogeneous signals, and trading volume is high when signals are dispersed. Conditional on the state of dispersion, volume is constant. As a result, investors can infer the dispersion from volume, but volume does not provide any information in addition to the information about the dispersion.

Since, in the equilibrium, all investors know the dispersion of signals, and since, conditional on the dispersion, all exogenously determined random variables are jointly normally distributed, there exist a conditional linear equilibrium. Part (a) of the theorem shows that, conditional on the dispersion of signals, the price P is a linear function of the aggregate signal \hat{D} and the exposure to the aggregate labor risk N . The price decreases with N , since the labor payoff Y and the dividend D are positively correlated.

The coefficients of \hat{D} and N depend on the information state of the world. The dividend coefficient Φ_D^H is larger than the coefficient Φ_D^L , since the investors know more in aggregate in state H , then they know in state L . The coefficient Φ_N^H is larger than the coefficient Φ_N^L , since the investors cannot distinguish between \hat{D} and N in state H . Therefore investors interpret a low realization of N partly as good news about D , and hence amplify the impact of N on the price.

Part (b) of the theorem shows the equilibrium demands of the investors. Since all investors hold zero shares before they start trading, the demand X_i is equivalent to the number of shares that investor i buys or sells. Note that these equilibrium trades can also be written as

$$X_i^L = -\frac{\sigma_{DY}}{\sigma_D^2} (N_i - N) \quad \text{and} \quad X_i^H = \frac{1}{\rho\sigma_\epsilon^2} (\hat{D}_i - D) - \Psi (N_i - N).$$

In state L , investor i sells shares if his labor risk exposure N_i is higher than the average risk exposure N . In this state, the demands are independent of the dividend signals \hat{D}_i since all investors observe the same signal $\hat{D}_i = D + \epsilon$. In state H , investors do not only trade to share risk, but also because they are heteroge-

neously informed. In this case, the equilibrium demands depend on the differences between the individual signals \hat{D}_i and the average signal D and the individual labor endowments N_i and the average endowment N .

Note that, since $0 < \Psi < \frac{\sigma_D \Psi}{\sigma_D^2}$, the amount of trading due to risk-sharing is lower in state H than in state L . Investors share less risk in state H , since investors know more about the future payoff D , if signals are dispersed. Figure 2 shows a geometric interpretation of the aggregate trading volume if signals are dispersed.

The demands X_i depend only on the differences between individual and average signals and endowments, since information and labor risks are identically distributed across investors. If signals and endowments are identically distributed, then investors weight their signals and endowments identically when they form their demands. As a result, the price depends on the averages whereas the equilibrium demands depend on the differences of signals and endowments. Therefore, the price is a function only of the common components \hat{D} and N , and the demands are functions only of the investor specific components ϵ_i (in state H) and n_i . As a result, for a given state of signal dispersion, equilibrium demands and the price are independent.

Since the equilibrium demands depend only on investor specific components ϵ_i and n_i , and since these components are independent across investors, the per capita trading volume is given by the expected trade of investor i . As a result, volume depends only on the distribution of signals and endowments across investors. Conditional on the dispersion of signals, trading volume is independent of the realization of signals and endowments. Since volume depends only on the distribution of signals and endowments, investors can learn this distribution from observing volume. The investors can therefore use trading volume to learn the dispersion of beliefs in the economy. Given this information, the investors use their private signals and the price to estimate the future dividend. In that way, observing trading volume helps the investors to separate two sources of uncertainty: uncertainty about the realization of the aggregate information, and uncertainty about the quality of the aggregate information.

I have shown so far how trading volume depends on two extreme cases: the error terms ϵ_i are perfectly correlated or they are not correlated at all. In order to examine how trading volume depends on the correlation of error terms in general, assume the dividend signal of investor i is given by

$$\hat{D}_i = D + \sqrt{1 - \omega} \eta + \sqrt{\omega} \epsilon_i, \quad \sigma_\eta^2 = \sigma_\epsilon^2 \quad (2)$$

where $0 \leq \omega \leq 1$, η is a common error term and ϵ_i are investor specific error terms, which are independent and identically distributed across investors. The case $\omega = 0$ corresponds to state L , and the case $\omega = 1$ corresponds to state H in the theorem. Since $\sigma_\eta^2 = \sigma_\epsilon^2$ the total variance of the signal does not depend on ω . Hence, the specification of the dividend signal in (2) allows to examine the effect of a change in the correlation of signal errors, independent of the effect of a change in the total error variance. Figure 3 shows the equilibrium trading volume, assuming that all investor know ω . Calculating prices and demands for the case $\omega \in (0, 1)$ is a simple extension to the proof in Appendix A. As Figure 3 shows, trading volume increases with the dispersion of signals. For any given level of ω , trading volume increases if the total error variance decreases. This last result, which has previously been shown for example by Pflleiderer (1984), is due to the fact that investors trade more aggressively, if they are more certain about the future value of the asset.

4 Properties of the equilibrium

4.1 Updating of beliefs

Corollary 1. *Under the assumptions of the theorem the expected future dividend conditional on the information of investor i is given by*

$$E[D|\mathcal{F}_i] = \psi_D^L \hat{D}_i$$

if trading volume is low, and

$$E[D|\mathcal{F}_i] = \psi_D^H \hat{D}_i + \psi_N N_i + \psi_P P,$$

if trading volume is high, and we have $\psi_P > 0$, $\psi_N > 0$, and $0 < \psi_D^H < \psi_D^L$.

Corollary 1 shows that investors use the price to update their beliefs only when trading volume is high. Under low trading volume, the aggregate signal is identical to the individual signals. Since the uncertain aggregate labor risk N prevents the price from revealing the aggregate signal, investors completely ignore the price when they update their beliefs. However, under high trading volume, the quality of the aggregate information exceeds the quality of the private information. Therefore, if trading volume is high, investors reduce the weights on their private signals and weight the price more heavily. In this case, the investors also use their own labor risk N_i in order to estimate the aggregate risk N in the price.

4.2 Trading strategies

Corollary 2. *Under the assumptions of the theorem the demand of investor i is given by*

$$X_i^L = \Psi_D^L \hat{D}_i - \Psi_N^L N_i - \Psi_P^L P$$

$$X_i^H = \Psi_D^H \hat{D}_i - \Psi_N^H N_i - \Psi_P^H P$$

where all coefficients are greater than zero and we have $\Psi_P^H < \Psi_P^L$.

Corollary 2 shows that, after controlling for the signals \hat{D}_i and the labor endowment N_i , investors always trade against the price. However, investors trade less aggressively against the price when trading volume is high than when trading volume is low. The reason for this behavior is that, under high trading volume, the price does not fully adjust to the aggregate information in the economy. Investors interpret therefore high prices partially as good news and low prices partially as bad news.

4.3 Volume predicts the future risk premium

I have so far assumed that the risky asset has a zero net supply. This assumption removes a risk premium from the return, since the average investor does not require a risk premium if he does not hold any shares. The following result shows the relation between volume and the risk premium, if the asset has a positive supply.

Corollary 3. *Assume the per capita supply of the risky asset is given by $S > 0$, and assume that all investors hold S shares prior to trading. Let $P_{S=0}^s$, $s \in \{L, H\}$ be the equilibrium price in the theorem. Then we have*

$$P^L = P_{S=0}^L - \Phi_S^L S \quad \text{and} \quad P^H = P_{S=0}^H - \Phi_S^H S,$$

and

$$E[D - P|V^s] = \Phi_S^s S$$

and

$$E[D - P|V^L] > E[D - P|V^H] > 0.$$

Corollary 3 shows that the price decreases with the supply of the risky asset. The expected future return is given by the risk premium that investors require in order to hold the per capita supply S . Since the quality of the aggregate information increases with trading volume, investors require a lower risk premium when trading volume is high. Hence, the expected future return decreases as current trading volume increases.

4.4 Volume and the autocorrelation of returns

Many practitioners seem to believe that price movements on high volume are more likely to continue than price movements on low volume.⁵ I will show in the following that these beliefs are a rational outcome of this model. I define the autocorrelation

⁵Here are two pieces of anecdotal evidence:

“Downturns that come on heavy volume often are considered more likely to continue,

of returns γ , conditional on the realized volume V , as the relation between the unexpected part of the price $P - E[P]$ and the future return $D - P$:

$$\gamma = E[(P - E[P])(D - P)|V]. \quad (3)$$

For example, if $\gamma > 0$ then the future return will be high, if the price is unexpectedly high today.

Corollary 4. *Under the assumptions of the theorem, we have*

$$\gamma = -(\Phi_N^L)^2 \sigma_N^2$$

if trading volume is low, and

$$\gamma = \Phi_D^H(1 - \Phi_D^H)\sigma_D^2 - (\Phi_N^H)^2\sigma_N^2$$

if trading volume is high, where $\Phi_D^H(1 - \Phi_D^H) > 0$.

Under low trading volume, returns are negatively autocorrelated, since shocks to the aggregate labor risk affect the current price without affecting future payoffs. The aggregate dividend signal $\hat{D} = D + \epsilon$ does not induce autocorrelation, since the price fully incorporates this information. Under high trading volume, there are two effects on the autocorrelation of returns. The average labor risk N has a negative effect and the average dividend signal $\hat{D} = D$ has a positive effect. The effect of the average signal \hat{D} on the autocorrelation is positive, since this information is not fully incorporated into the price at time 0. The direction of the total effect depends on the parameter values. Figure 4 shows how the autocorrelation increases monotonically with the dispersion of signals (and therefore also with trading volume). If investors are sufficiently informed (σ_ϵ^2 is small), returns are positively autocorrelated when trading volume is high.

rather than those on more limited volume, according to some traders.” (Wall Street Journal, July 8, 2003, page C1)

“Those that say this latest rally may last at least until Christmas point to a pickup in trading volume.” (Wall Street Journal, November 14, 2004, page C16)

5 Summary

In a financial market, every investor is interested in the information that other market participants might possess. There are several ways how investors can learn what other investors know. In this paper, I provide a closed form solution of a rational expectations equilibrium where investors infer information about the state of the economy from (1) private signals, (2) the market price and (3) aggregate trading volume.

In this model investors receive private signals of a future dividend as well as individual labor income shocks that create a risk-sharing motive for trading. Investors are uncertain about the correlation of the individual errors in their private signals. Knowing this correlation is important, since it determines the quality of the aggregate information in the economy. The aggregate information is more precise relative to private signals when individual signals errors are uncorrelated, than when signal errors are correlated.

In the equilibrium, trading volume increases with the dispersion of signals. Hence, if trading volume is high, investors know that signals are dispersed, and that the precision of the aggregate information in the price is high. Investors therefore weight the market price heavily relative to their own private signals. Conversely, if trading volume is low, investors weight their own signals heavily relative to the market price.

Appendix A: Existence of the equilibrium

Assume trading volume is given by $V = V_L$ if the dispersion of signals is low, and $V = V_H$ if the dispersion of signals is high, where $V_L < V_H$, and, conditional on the state of dispersion of signals, V_L and V_H are constants. Then all investors know whether the economy is in state L or in state H . Assume the economy is in state L , so that all investors observe the same signal $\hat{D} = D + \epsilon$, and assume the price is given by

$$P = \Phi_D^L \hat{D} - \Phi_N^L N \tag{4}$$

Let $\mathcal{F}_i = \{\hat{D}, N_i, P, V\}$ be the information set of investor i . Then the demand of investor i is given by

$$X_i = \frac{\mathbb{E}[D - P|\mathcal{F}_i] - \rho \text{Cov}[D - P, N_i Y|\mathcal{F}_i]}{\rho \text{Var}[D - P|\mathcal{F}_i]}, \quad (5)$$

where

$$\mathbb{E}[D - P|\mathcal{F}_i] = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} \hat{D} - P \quad (6a)$$

$$\text{Var}[D - P|\mathcal{F}_i] = \frac{\sigma_D^2 \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} \quad (6b)$$

$$\text{Cov}[D - P, N_i Y|\mathcal{F}_i] = \frac{\sigma_{DY} \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} N_i. \quad (6c)$$

Plugging the demand into the equilibrium condition in (1) and solving for the price and comparing coefficients with (4), we get Φ_D^L and Φ_N^L in part (a) of the Theorem. Plugging these coefficients into (4), and (4) into (5), we get the equilibrium demand

$$X_i = -\frac{\sigma_{DY}}{\sigma_D^2} n_i.$$

Since the n_i are independent across investors, we have

$$V^L = \lim_{h \rightarrow \infty} \frac{1}{h} \sum_{i=1}^h |X_i| = \mathbb{E}|X_i| = \sqrt{\frac{2}{\pi}} \frac{\sigma_{DY} \sigma_n}{\sigma_D^2} \quad (7)$$

Next, assume the economy is in state H , so that the signal errors ϵ_i are uncorrelated. Assume the price is given by

$$P = \Phi_D^H D - \Phi_N^H N \quad (8)$$

Then the demand of investor i is given by (5). Let $\tilde{\mathcal{F}}_i = \{\hat{D}_i, N_i, V\}$ and $\mathcal{F}_i = \{\hat{D}_i, N_i, P, V\}$. To simplify the notation, I will write Φ_D for Φ_D^H and Φ_N for Φ_N^H in the following. Then

we have

$$\mathbb{E}[D|\tilde{\mathcal{F}}_i] = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} \hat{D}_i \quad (9a)$$

$$\text{Var}[D|\tilde{\mathcal{F}}_i] = \frac{\sigma_D^2 \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} \quad (9b)$$

$$\text{Cov}[D, N_i Y|\tilde{\mathcal{F}}_i] = \frac{\sigma_{DY} \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} N_i \quad (9c)$$

$$\mathbb{E}[P|\tilde{\mathcal{F}}_i] = \Phi_D \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} \hat{D}_i - \Phi_N \frac{\sigma_N^2}{\sigma_N^2 + \sigma_n^2} N_i \quad (9d)$$

$$\text{Var}[P|\tilde{\mathcal{F}}_i] = \Phi_D^2 \frac{\sigma_\epsilon^2 \sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} + \Phi_N^2 \frac{\sigma_n^2 \sigma_N^2}{\sigma_N^2 + \sigma_n^2} \quad (9e)$$

$$\text{Cov}[D, P|\tilde{\mathcal{F}}_i] = \Phi_D \frac{\sigma_D^2 \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} \quad (9f)$$

Let

$$Z = \frac{\text{Cov}[D, P|\tilde{\mathcal{F}}_i]}{\text{Var}[P|\tilde{\mathcal{F}}_i]} \quad (10)$$

Then we have

$$\mathbb{E}[D - P|\mathcal{F}_i] = \mathbb{E}[D|\tilde{\mathcal{F}}_i] + Z \left(P - \mathbb{E}[P|\tilde{\mathcal{F}}_i] \right) - P \quad (11a)$$

$$\text{Var}[D|\mathcal{F}_i] = \text{Var}[D|\tilde{\mathcal{F}}_i] - Z \text{Cov}[D, P|\tilde{\mathcal{F}}_i] \quad (11b)$$

$$\text{Cov}[D, Y|\mathcal{F}_i] = \frac{\sigma_{DY} \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} \left(1 - Z \Phi_D \right) \quad (11c)$$

Plugging (11) into the demand of investor i in (5) and plugging these demands into the equilibrium condition in (1) and solving for the price we get

$$P = \frac{1}{1 - Z} \left[\frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} (1 - Z \Phi_D) D + \left(Z \Phi_N \frac{\sigma_N^2}{\sigma_N^2 + \sigma_n^2} - \rho \frac{\sigma_{DY} \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} (1 - Z \Phi_D) \right) N \right]$$

Comparing coefficients with (8), we get

$$\Phi_D = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2(1-Z)} \quad (12a)$$

$$\Phi_N = \rho \frac{\sigma_{DY} \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} \frac{\sigma_N^2 + \sigma_n^2}{\sigma_N^2 + \sigma_n^2(1-Z)} (1 - \Phi_D Z) \quad (12b)$$

From (9f) and (12a) we have

$$\text{Cov}[D, P|\tilde{\mathcal{F}}_i] = \Phi_D^2 \sigma_\epsilon^2 \left(1 - \frac{\sigma_\epsilon^2 Z}{\sigma_D^2 + \sigma_\epsilon^2} \right).$$

Using (10) we get

$$\text{Cov}[D, P|\tilde{\mathcal{F}}_i] \left(1 + \frac{\Phi_D^2}{\text{Var}[P|\tilde{\mathcal{F}}_i]} \frac{\sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} \right) = \Phi_D^2 \sigma_\epsilon^2.$$

Using (9e) and (10) again we get

$$Z = \frac{\Phi_D^2 \sigma_\epsilon^2}{\Phi_D^2 \sigma_\epsilon^2 + \Phi_N^2 \frac{\sigma_n^2 \sigma_N^2}{\sigma_N^2 + \sigma_n^2}}, \quad (13)$$

so $0 < Z < 1$. So we have $\frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} < \Phi_D < 1$ and $0 < \Phi_N < \rho \frac{\sigma_{DY} \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} \frac{\sigma_n^2 + \sigma_N^2}{\sigma_N^2}$. Define $f(Z) = Z - K$, where K is the right hand side of (13), and Φ_D and Φ_N are given by the right hand sides of (12). Then we have $f(0) < 0$ and $f(1) > 0$. Hence, since $f(Z)$ is continuous, the existence of the equilibrium follows from the intermediate value theorem.

For the demand we have from (5) and (11)

$$X_i = \frac{1}{\rho \text{Var}[D|\mathcal{F}_i]} \left(\Psi_D \hat{D}_i - \Psi_N N_i - \Psi_P P \right), \quad (14)$$

where

$$\Psi_D = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} (1 - Z \Phi_D) \quad (15a)$$

$$\Psi_N = \rho \frac{\sigma_{DY} \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} (1 - Z \Phi_D) - Z \Phi_N \frac{\sigma_N^2}{\sigma_N^2 + \sigma_n^2} \quad (15b)$$

$$\Psi_P = 1 - Z \quad (15c)$$

Plugging (8) into (14) we get

$$X_i = \frac{1}{\rho \text{Var}[D|\mathcal{F}_i]} \left[(\Psi_D - \Psi_P \Phi_D) D + \Psi_D \epsilon_i - (\Psi_N - \Psi_P \Phi_N) N - \Psi_N n_i \right] \quad (16)$$

From (12a) and (15) we have

$$\begin{aligned} \Psi_D - \Psi_P \Phi_D &= \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} (1 - Z \Phi_D) - (1 - Z) \Phi_D \\ &= \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} \left(1 - \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2 (1 - Z)} Z \right) - (1 - Z) \Phi_D \\ &= (1 - Z) \Phi_D - (1 - Z) \Phi_D \\ &= 0 \end{aligned} \quad (17)$$

Similarly, from (12b) and (15) we have

$$\Psi_N - \Psi_P \Phi_N = \rho \frac{\sigma_{DY} \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} (1 - Z \Phi_D) - Z \Phi_N \frac{\sigma_N^2}{\sigma_N^2 + \sigma_n^2} - (1 - Z) \Phi_N = 0 \quad (18)$$

From (11b) and (15a) we get

$$\frac{\Psi_D}{\rho \text{Var}[D|\mathcal{F}_i]} = \frac{1}{\rho \sigma_\epsilon^2} \quad (19)$$

From (11b), (12b), and (18) we get

$$\frac{\Psi_N}{\rho \text{Var}[D|\mathcal{F}_i]} = \frac{\sigma_{DY}}{\sigma_D^2} \frac{\sigma_N^2 + \sigma_n^2}{\sigma_N^2 + \sigma_n^2 (1 - Z)} (1 - Z)$$

So we get from (16), (17), (18), and (19)

$$X_i = \frac{1}{\rho \sigma_\epsilon^2} \epsilon_i - \frac{\sigma_{DY}}{\sigma_D^2} \frac{\sigma_N^2 + \sigma_n^2}{\sigma_N^2 \frac{1}{1-Z} + \sigma_n^2} n_i$$

Since ϵ_i and n_i are independent across investors we have

$$V^H = \sqrt{\frac{2}{\pi} \left(\frac{1}{\rho^2 \sigma_\epsilon^2} + \left(\frac{\sigma_{DY}}{\sigma_D^2} \frac{\sigma_N^2 + \sigma_n^2}{\sigma_N^2 \frac{1}{1-Z} + \sigma_n^2} \right)^2 \sigma_n^2 \right)}. \quad (20)$$

Hence, comparing (7) and (20), we have

$$V^H > \sqrt{\frac{2}{\pi}} \frac{1}{\rho \sigma_\epsilon} > \sqrt{\frac{2}{\pi}} \frac{\sigma_{DY} \sigma_n}{\sigma_D^2} = V^L,$$

if

$$\rho \sigma_\epsilon \sigma_n \sigma_{DY} < \sigma_D^2.$$

Appendix B: proof of Corollary 1

If the dispersion of signals is low, we have $E[D|\mathcal{F}_i] = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} \hat{D}_i$ by (6a). Investors do not use the price to update their beliefs since they cannot directly observe the aggregate labor risk shock N . If the dispersion of signals is high, we have

$$\begin{aligned} E[D|\mathcal{F}_i] &= E[D|\tilde{\mathcal{F}}_i] + Z \left(P - E[P|\tilde{\mathcal{F}}_i] \right) \\ &= \frac{\sigma_D^2}{\sigma_D^2 + \sigma_\epsilon^2} (1 - Z\Phi_D^H) \hat{D}_i + Z\Phi_N^H \frac{\sigma_N^2}{\sigma_N^2 + \sigma_n^2} N_i + ZP \end{aligned}$$

by (11a). So since $\Phi_N^H > 0$, $\Phi_D^H \in (0, 1)$, and $Z \in (0, 1)$, we have $\psi_N^H > 0$, $\psi_P^h > 0$, and $0 < \psi_D^H < \psi_D^L$.

Appendix C: proof of Corollary 2

Let $\text{Var}_L = \text{Var}[D|\mathcal{F}_i]$, if the dispersion of signals is low, and let Var_H be the corresponding variance if the dispersion of signals is high. Then we have $\Psi_P^L = \frac{1}{\rho \text{Var}_L}$ by (5). From (6b) and (11b) we have $\text{Var}_H = \text{Var}_L(1 - \Phi_D^H Z)$. So $\psi_P^H = \frac{1-Z}{1-\Phi_D^H Z} \psi_P^L$. So, since $\Phi_D^H \in (0, 1)$ and $Z \in (0, 1)$, we have $\psi_P^H < \psi_P^L$.

Appendix D: proof of Corollary 3

Assume the price is given by

$$P = \Phi_D^L(D + \epsilon) - \Phi_N^L N - \Phi_S^L S$$

when the dispersion of signals is low, and by

$$P = \Phi_D^H D - \Phi_N^H N - \Phi_S^H S$$

when the dispersion of signals is high. Then it follows directly from Appendix A that

$$\begin{aligned} \Phi_S^L &= \rho \frac{\sigma_D^2 \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} \\ \Phi_S^H &= \rho \frac{\sigma_D^2 \sigma_\epsilon^2}{\sigma_D^2 + \sigma_\epsilon^2} (1 - Z\Phi_D) \end{aligned}$$

hence $0 < \Phi_S^H < \Phi_S^L$.

Appendix E: proof of Corollary 4

Assume the dispersion of signals is low. Then we have

$$\begin{aligned} \mathbb{E}[(P - \mathbb{E}[P])(D - P)|V] &= \mathbb{E}\left[\left(\Phi_D^L(D + \epsilon) - \Phi_N^L N\right)\left((1 - \Phi_D^L)D - \Phi_D^L \epsilon - \Phi_N^L N\right)\right] \\ &= \Phi_D^L\left(\sigma_D^2 - \Phi_D^L(\sigma_D^2 + \sigma_\epsilon^2)\right) - (\Phi_N^L)^2 \sigma_N^2 \\ &= -(\Phi_N^L)^2 \sigma_N^2 \end{aligned}$$

Similarly, we have for high dispersion

$$\mathbb{E}[(P - \mathbb{E}[P])(D - P)|V] = \Phi_D^H(1 - \Phi_D^H)\sigma_D^2 - (\Phi_N^L)^2 \sigma_N^2$$

where $0 < \Phi_D^H < 1$ by appendix A.

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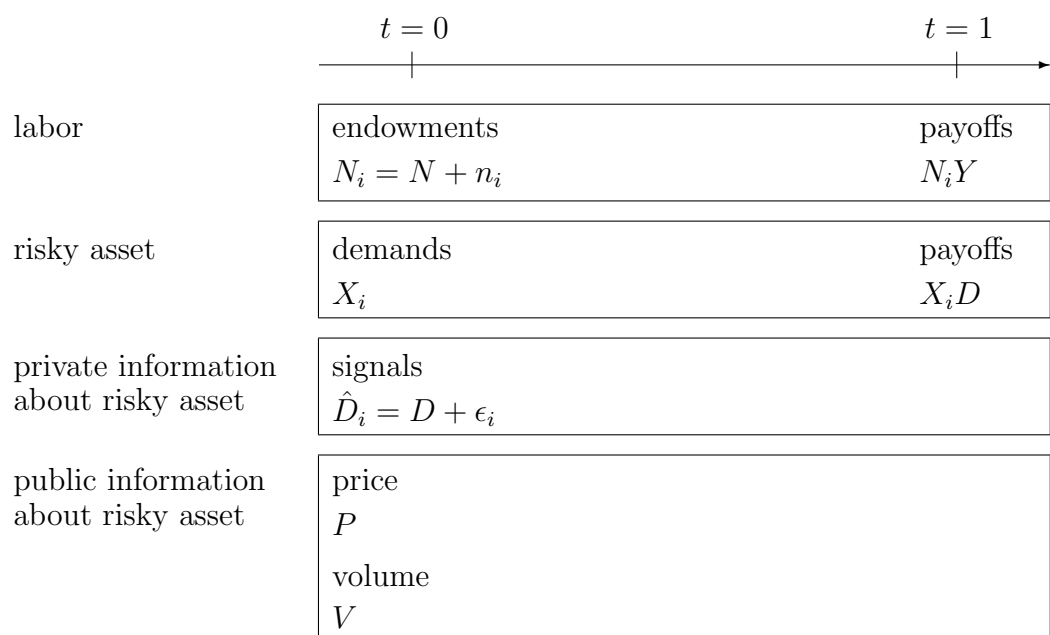


Figure 1: **Time line.** This figure shows endowments, demands, payoffs, private information and public information.

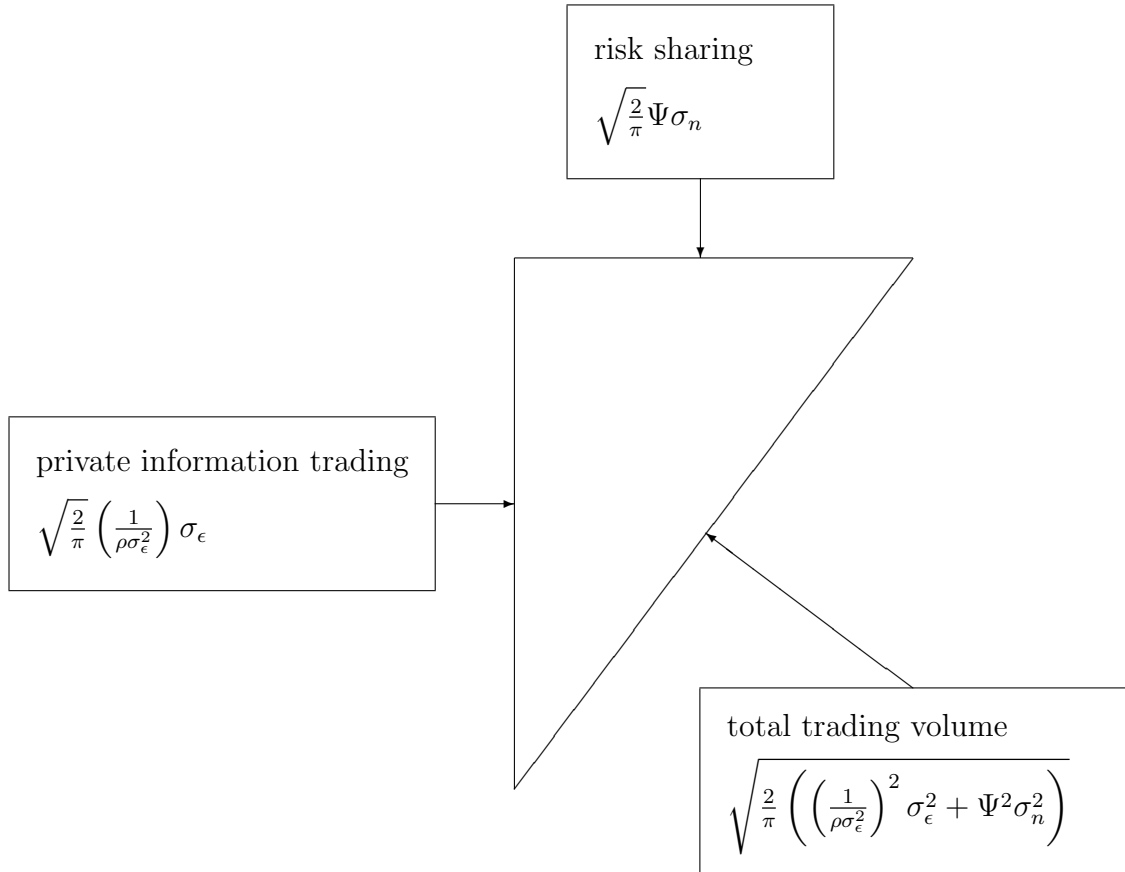


Figure 2: **Composition of trading volume.** This figure shows how aggregate trading volume relates to the amount of volume generated by information based trading and the amount generated by risk sharing. The information based trading is the volume that would occur in a state of the world where $n_i = 0$ for all i , so that investors do not trade to share risk. The risk sharing trading is the volume that would occur in a state of the world where $\epsilon_i = 0$ for all i , so that investors do not trade based on their private information. The total volume is less than the sum of its two components, since risk sharing and information trading partially offset each other for the average investor in the economy.

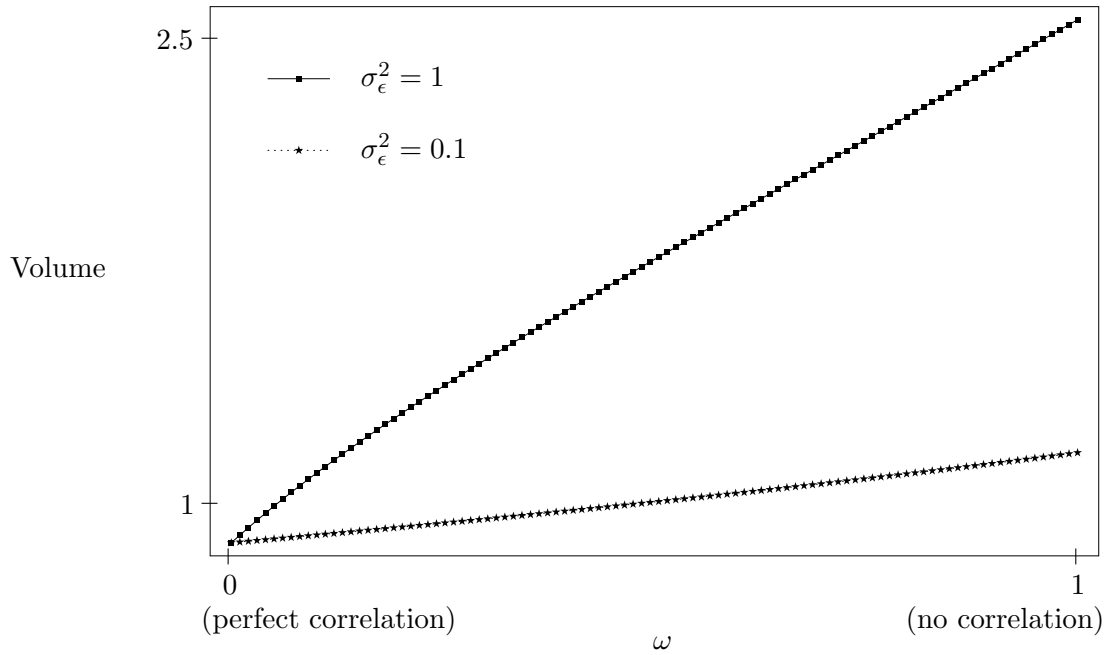


Figure 3: **Trading Volume and the Dispersion of Signals.** This figure shows how trading volume in a symmetric economy with a large number of small investors depends on the correlation of the investor specific signal errors, holding the total variance of the error terms constant. The private signal of investor i is given by $\hat{D}_i = D + \sqrt{1-\omega}\eta + \sqrt{\omega}\epsilon_i$, where η is a common error term and ϵ_i is an investor specific error term, and $\sigma_\eta^2 = \sigma_\epsilon^2$. The total variance of the signal errors is given by $(1-\omega)\sigma_\eta^2 + \omega\sigma_\epsilon^2 = \sigma_\epsilon^2$. The remaining parameters are given by $\sigma_D^2 = \sigma_{DY} = \sigma_N^2 = \sigma_n^2 = 1$.

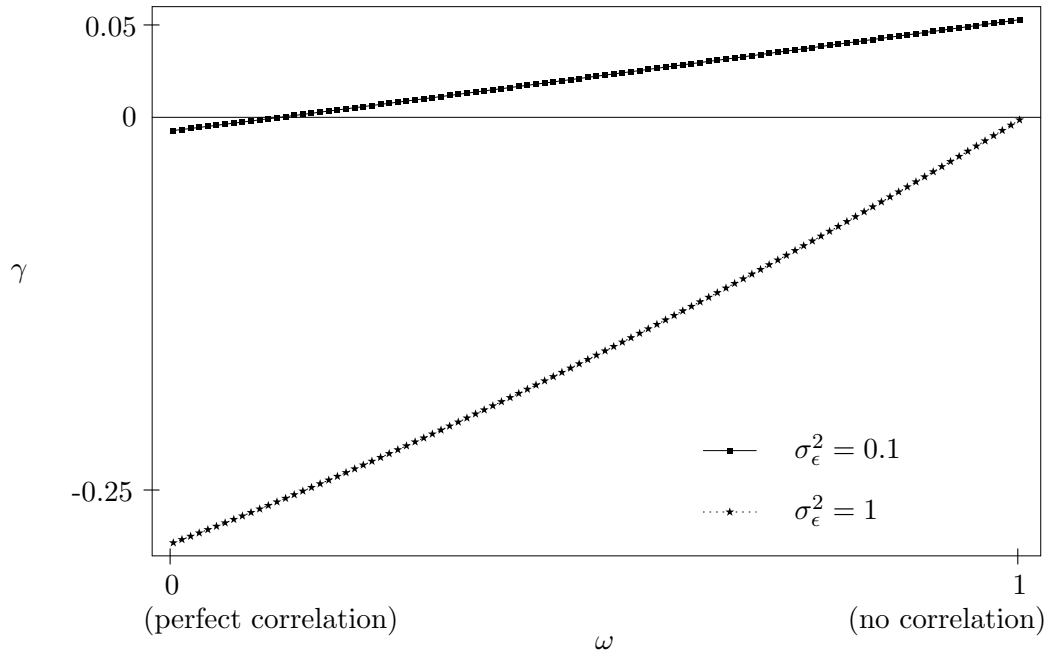


Figure 4: **Autocorrelation and the Dispersion of Signals.** This figure shows how the autocorrelation γ , as defined in (3) depends on the correlation of the investor specific signal errors, holding the total variance of the error terms constant. The private signal of investor i is given by $\hat{D}_i = D + \sqrt{1-\omega}\eta + \sqrt{\omega}\epsilon_i$, where η is a common error term and ϵ_i is an investor specific error term, and $\sigma_\eta^2 = \sigma_\epsilon^2$. The total variance of the signal errors is given by $(1-\omega)\sigma_\eta^2 + \omega\sigma_\epsilon^2 = \sigma_\epsilon^2$. The remaining parameters are given by $\sigma_D^2 = \sigma_{DY} = \sigma_N^2 = \sigma_n^2 = 1$.