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# Thermodynamic theory of stress distribution in epitaxial Pb(Zr,Ti)O<sub>3</sub> thin films

H. Joon Kim, S. Hoon Oh, and Hyun M. Jang<sup>a)</sup>

*Department of Materials Science and Engineering, and National Research Laboratory (NRL) for Ferroelectric Phase Transitions, Pohang University of Science and Technology (POSTECH), Pohang 790-784, Korea*

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A phenomenological thermodynamic model has been developed to account for the effects of the film thickness on various properties of ferroelectric thin films. To this end, we have suitably incorporated a position-dependent stress distribution function into the elastic Gibbs function. Various physical properties can be predicted as a function of the film thickness using this modified thermodynamic formalism. A comparison of the theoretical predictions with experimental values of the average strain and the para-ferro transition temperature indicates that the tensile stress caused by the cubic-tetragonal displacive phase transition dominates over the compressive thermal stress in the epitaxially oriented tetragonal Pb(Zr, Ti)O<sub>3</sub> thin films. © 1999 American Institute of Physics. [S0003-6951(99)05546-1]

It is known that structure and dielectric/ferroelectric properties of ferroelectric thin films are greatly influenced by the film thickness.<sup>1-4</sup> This thickness dependence is more pronounced in epitaxially grown thin films. Funakubo and co-workers<sup>3</sup> experimentally demonstrated that the lattice parameters and dielectric susceptibilities of epitaxially grown PbTiO<sub>3</sub> thin films were increasingly dependent on the film thickness with decreasing total thickness.

Yano and co-workers<sup>4</sup> examined dielectric properties of epitaxially grown BaTiO<sub>3</sub>[100]||Pt[100]||MgO[100] thin films having various thicknesses. Using phenomenological thermodynamic approach, they further showed that the thickness dependence of dielectric properties was mainly caused by a two-dimensional stress originated from the stress at the film/substrate interface. However, they assumed that a thin film having a certain thickness has its own characteristic value of average stress. Moreover, they simplified the stress distribution over the thickness direction by an average two-dimensional stress, which is valid for an infinitesimally thin film. In reality, however, the stress originated from the interface gradually relaxes with the distance from the interface.

In view of this, the main purpose of the present study is to develop a quantitative thermodynamic model that can account for the effects of the film thickness on various properties of epitaxial thin films. To this end, we have suitably incorporated a position-dependent stress distribution function into the elastic Gibbs function. The thin-film system that we have chosen is an epitaxially grown tetragonal Pb(Zr, Ti)O<sub>3</sub> (PZT) because it is the most promising candidate for modern ferroelectric random access memory (FRAM) devices and for numerous thin-film microsensors/actuators used in microelectromechanical system (MEMS) devices.

Considering a thin film having a certain thickness as a piling-up of infinitesimally thin hypothetical two-dimensional layers, one can reasonably assume that the rate of stress relaxation over the thickness direction is propor-

tional to the local stress at this particular position. Then, the variation of a stress over the thickness direction in a given thin film can be represented by the following first-order differential equation:

$$\frac{d\sigma(z)}{dz} = -\alpha_t \sigma(z), \quad (1)$$

where  $\sigma(z)$  is the local stress at a position  $z$  from the film/substrate interface, and  $\alpha_t$  is a decline parameter and is a measure of stress relaxation with the distance. Then, the stress distribution function,  $\sigma(z)$ , comes out to be

$$\sigma(z) = \sigma_0 e^{-\alpha_t z}, \quad (2)$$

where  $\sigma_0$  is the stress at the film/substrate interface and is independent of the film thickness.

We will develop a modified phenomenological thermodynamic formalism to determine the two unknown parameters,  $\sigma_0$  and  $\alpha_t$ , from measured values of average strains and average para-ferro transition temperatures for PZT thin films having various thicknesses. Based on the Landau-Devonshire phenomenological theory, one can expand the elastic Gibbs function for a proper ferroelectric derived from a phase of  $P_{m3m}$  symmetry using a Taylor series in powers of polarization and stress,<sup>5,6</sup> namely,

$$\begin{aligned} \Delta G = & \alpha_1(P_1^2 + P_2^2 + P_3^2) + \alpha_{11}(P_1^4 + P_2^4 + P_3^4) \\ & + \alpha_{12}(P_1^2 P_2^2 + P_2^2 P_3^2 + P_3^2 P_1^2) + \alpha_{111}(P_1^6 + P_2^6 + P_3^6) \\ & + \alpha_{112}[P_1^4(P_2^2 + P_3^2) + P_2^4(P_3^2 + P_1^2) + P_3^4(P_1^2 + P_2^2)] \\ & + \alpha_{123}P_1^2 P_2^2 P_3^2 - 1/2s_{11}(X_1^2 + X_2^2 + X_3^2) \\ & - s_{12}(X_1 X_2 + X_2 X_3 + X_3 X_1) - 1/2s_{44}(X_4^2 + X_5^2 + X_6^2) \\ & - Q_{11}(X_1 P_1^2 + X_2 P_2^2 + X_3 P_3^2) - Q_{12}[X_1(P_2^2 + P_3^2) \\ & + X_2(P_3^2 + P_1^2) + X_3(P_1^2 + P_2^2)] - Q_{44}(X_4 P_2 P_3 \\ & + X_5 P_3 P_1 + X_6 P_1 P_2), \end{aligned} \quad (3)$$

<sup>a)</sup>Electronic mail: hmjang@postech.ac.kr

where  $P_i$  and  $X_i$ , respectively, are the magnitude of the polarization vector along the direction,  $i$ , and the  $i$ -component stress, respectively. In the reduced notation,  $X_1, X_2, X_3$  denote the tensile stresses and  $X_4, X_5, X_6$  the shear components.  $\alpha_1$  is the dielectric stiffness and is assumed to be a linear function of temperature near the Curie temperature (Curie–Weiss law).  $\alpha_{ij}$  and  $\alpha_{ijk}$  are the high-order stiffness coefficients at a constant stress,  $s_{ij}$  are the elastic compliances measured at a constant polarization, and  $Q_{ij}$  are the electrostrictive coefficients written in polarization notation. All the coefficients except for  $\alpha_1$  are assumed to be independent of temperature, and their optimized values are given in a series of papers reported by Haun and co-workers.<sup>7,8</sup>

We now introduce the stress distribution function into the elastic Gibbs function for a tetragonal phase. The following relations hold under a two-dimensional stress  $\sigma(z)$  at a given position  $z$ :

$$X_1 = X_2 = \sigma(z), \quad X_3 = X_4 = X_5 = X_6 = 0. \quad (4)$$

The spontaneous polarization ( $P_s = P_3$ ) can be derived from  $\Delta G$  for a tetragonal phase using the stability criterion, i.e.,  $\partial\Delta G/\partial P_i = E_i = 0$ :

$$P_1^2 = P_2^2 = 0,$$

$$P_3^2(z) = P_s^2(z) = \frac{-\alpha_{11} + [\alpha_{11}^2 - 3\alpha_{111}\{\alpha_1 - 2Q_{12}\sigma(z)\}]^{1/2}}{3\alpha_{111}}. \quad (5)$$

Similarly, the strain can be derived using the relation,  $\partial\Delta G/\partial X_i = -x_i$ , and  $x_1$  for a tetragonal phase is given by

$$x_1(z) = \frac{Q_{12}\{-\alpha_{11} + [\alpha_{11}^2 - 3\alpha_{111}\{\alpha_1 - 2Q_{12}\sigma(z)\}]^{1/2}\}}{3\alpha_{111}} + (s_{11} + s_{12})\sigma(z). \quad (6)$$

On the other hand, the strain developed along the  $a$  axis can be estimated from the experimentally determined lattice parameters using the following relation:

$$x_1 = (a_T - a'_c)/a'_c, \quad (7)$$

where  $a_T$  is the  $a$ -axis lattice constant for a tetragonal phase, and  $a'_c$  is that for a hypothetical cubic phase extrapolated to room temperature.

To correlate the above theoretical prediction with experimentally available values, we have further computed average values of the strain as a function of the film thickness. To do this, we have used the mean value theorem. Using Eq. (6) one can eventually obtain the following expression of  $\bar{x}_1$  after a tedious integration procedure:

$$\begin{aligned} \bar{x}_1 &= \frac{1}{t} \int_0^t x_1(z) dz \\ &= -\frac{Q_{12}\alpha_{11}}{3\alpha_{111}} - \frac{Q_{12}}{3\alpha_{111}\alpha_1 t} \left[ 2\{\sqrt{ae^{-\alpha_1 t} + b} - \sqrt{a+b}\} \right. \\ &\quad \left. + \sqrt{b} \left\{ \ln \left( \frac{\sqrt{ae^{-\alpha_1 t} + b} - \sqrt{b}}{\sqrt{ae^{-\alpha_1 t} + b} + \sqrt{b}} \right) - \ln \left( \frac{\sqrt{a+b} - \sqrt{b}}{\sqrt{a+b} + \sqrt{b}} \right) \right\} \right] \\ &\quad - (s_{11} + s_{12}) \frac{\sigma_0}{t\alpha_1} [e^{-\alpha_1 t} - 1], \end{aligned} \quad (8)$$

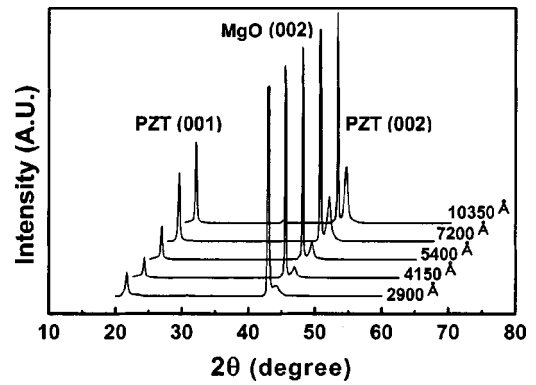


FIG. 1. X-ray  $\theta$ - $2\theta$  scan profiles of  $\text{PbZr}_{0.4}\text{Ti}_{0.6}\text{O}_3$  thin films with various thicknesses.

where  $a = 6\alpha_{111}Q_{12}\sigma_0$  and  $b = \alpha_{11}^2 - 3\alpha_{111}\alpha_1$ .

We now consider the para-ferro transition temperature ( $T_c$ ) as a function of  $\sigma(z)$ . Using the expression of the elastic Gibbs function at the Curie point and its partial derivative with respect to  $P_c$  in the absence of any external field (i.e.,  $\partial\Delta G/\partial P_c = E_c = 0$ ), one can derive an expression for the spontaneous polarization at the Curie point.<sup>6</sup> Substituting this expression into the stability criterion,  $\partial\Delta G/\partial P_c = E_c = 0$ , and using the mean value theorem again, we have eventually obtained the following relation for the average para-ferro transition temperature in the tetragonal phase field:

$$\begin{aligned} \bar{T}_c &= \frac{1}{t} \int_0^t T_c(z) dz \\ &= T_0 + \frac{\alpha_{11}^2}{4\beta\alpha_{111}} - \frac{2Q_{12}\sigma_0}{\beta\alpha_1 t} [e^{-\alpha_1 t} - 1] \\ &\quad + \frac{2(s_{11} + s_{12})\sigma_0^2}{2\beta\alpha_1 t} [e^{-2\alpha_1 t} - 1] \\ &\quad + \frac{12\alpha_{111}^4(s_{11} + s_{12})\sigma_0^4}{4\beta\alpha_1 t\alpha_{11}^4} [e^{-4\alpha_1 t} - 1], \end{aligned} \quad (9)$$

where  $T_0$  is the Curie–Weiss temperature,  $\beta \equiv 1/2\epsilon_0 C$ , and  $C$  is the Curie constant.

To examine our theoretical treatments, we have prepared epitaxially oriented tetragonal PZT thin films (with Zr/Ti = 0.4/0.6) having various film thicknesses on MgO(001) using a pulsed laser deposition (PLD) method. The deposition conditions used in the present study are as follows: (i) LASER source; KrF eximer with a pulse width of 30 ns at 10 Hz, (ii) fluence; 60 mJ/cm<sup>2</sup>, (iii) oxygen pressure; 0.05 Torr, (iv) substrate temperature; 650 °C.

The  $c$ -axis lattice constant was estimated using conventional x-ray  $\theta$ - $2\theta$  scans (with  $\text{Cu } K\alpha$  radiation) while the  $a$ -axis lattice constant was determined using {303} rocking curves. The x-ray diffraction (XRD)  $\theta$ - $2\theta$  patterns of the PZT thin films are shown in Fig. 1. Only peaks originating from the (00 $l$ )-type diffraction were found. This indicates that the films have a tetragonal perovskite structure and are highly  $c$ -axis oriented. Using  $2\theta_{(001)}$  values obtained from the  $\theta$ - $2\theta$  scans we have calculated the  $c$ -axis lattice parameters. To evaluate the lattice parameters in the direction parallel to the film/substrate interface (i.e.,  $a$  axis), the asymmetric {303} rocking experiment was performed (see Fig. 2). The

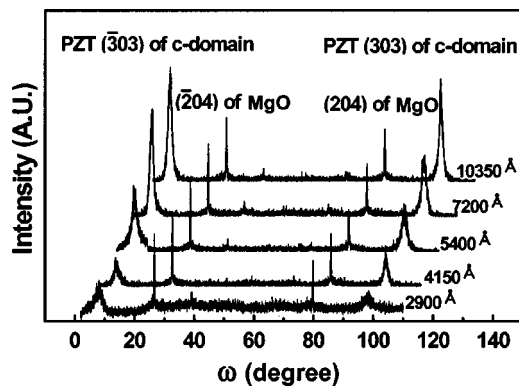


FIG. 2. Asymmetric {303} rocking curves of  $\text{PbZr}_{0.4}\text{Ti}_{0.6}\text{O}_3$  thin films with various thicknesses.

specimen is rotated to obtain the rocking curves of (303) peaks while the reflected beam is collected in a counter fixed at the  $2\theta_{(303)}$  corresponding to the Bragg angle for (303) planes. The  $a$ -axis lattice parameter is determined using the following relation:

$$a_T = \frac{c}{\tan(\Delta\omega_{303}/2)}, \quad (10)$$

where  $\Delta\omega_{303}$  is the difference between  $\omega_{303}$  for (303) planes and  $\omega_{\bar{3}03}$  for ( $\bar{3}03$ ) planes in the rocking curves. One can readily obtain the average strain values ( $\bar{\epsilon}$ ) of epitaxial  $\text{PbZr}_{0.4}\text{Ti}_{0.6}\text{O}_3$  thin films at room temperature using these experimental results and Eq. (7).

The average para-ferro transition temperatures of epitaxial  $\text{PbZr}_{0.4}\text{Ti}_{0.6}\text{O}_3$  thin films with various thicknesses were measured using a differential thermal analyzer (DTA7, Perkin Elmer). The film thickness was measured using an  $\alpha$  stepper (Tencor Instruments).

We are now in a position to estimate the two unknown parameters,  $\sigma_0$  and  $\alpha_t$  in Eq. (2). For temperatures above  $T_c$ ,  $\sigma_0$  has only a thermal stress. Thus, we can directly obtain  $\sigma_0$  for  $T > T_c$  from the difference in the thermal expansion coefficient between PZT and MgO substrate. Using these values of  $\sigma_0$  and the experimental values of the average transition temperature, one can numerically estimate  $\alpha_t$  as a function of the film thickness from Eq. (9). We found that  $\alpha_t$  was essentially independent of temperature with a good linear correlation between  $\alpha_t$  and the film thickness,  $t$ . The next step is to estimate  $\sigma_0$  for  $T < T_c$ . Using the estimated value of  $\alpha_t$  and the experimental value of the average strain for a given film thickness, one can subsequently estimate  $\sigma_0$  for room temperature from Eq. (8). Consequently, we have obtained the following stress distribution function for the epitaxially oriented tetragonal PZT thin films:

$$\sigma(z, t) = \sigma_0 e^{-\alpha_t z}, \quad (11)$$

where  $\sigma_0 = +1.16$  GPa at room temperature, and  $\alpha_t = +3.925 \times 10^{-4} - 2.325 \times 10^{-8}t$  in an  $\text{\AA}$  unit. A positive sign of  $\sigma_0$  signifies a tensile stress at the interface.  $\alpha_t$  is independent of temperature and has a dimension of  $(\text{length})^{-1}$ .

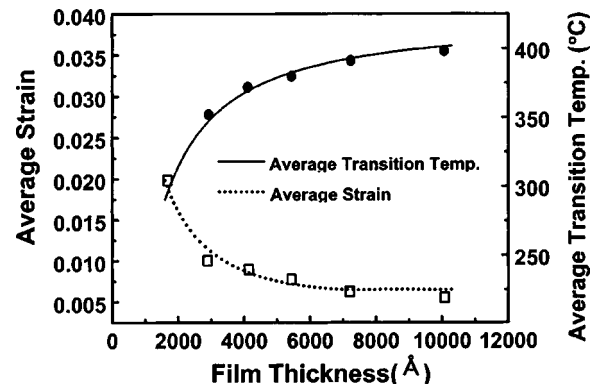


FIG. 3. Comparison of the theoretical predictions with the experimental data of average strain and transition temperature of  $\text{PbZr}_{0.4}\text{Ti}_{0.6}\text{O}_3$  thin films as a function of film thickness.

Figure 3 compares the theoretical computations of the average strain and the transition temperature with experimental values as a function of the film thickness,  $t$ . As shown in the figure, the theoretical prediction reasonably describes the experimental values for a wide range of the film thickness. One important feature of Fig. 3 is that the average strain of the PZT thin film decreases while the average transition temperature increases rapidly with increasing film thickness. This indicates that the thin film is under a high tensile stress. Our theoretical computation clearly predicts a reverse trend under a compressive stress. This further indicates that, for epitaxially grown tetragonal PZT thin films, the tensile stress caused by the cubic-tetragonal displacive phase transition (GPa order) dominates over the compressive thermal stress (MPa order).

In conclusion, a phenomenological thermodynamic model has been developed to account for the effects of the film thickness on various properties of epitaxial ferroelectric thin films. By correlating the present theoretical predictions with the experimental data, it is shown that the tensile stress caused by the cubic-tetragonal displacive phase transition dominates over the compressive thermal stress in the epitaxially grown tetragonal PZT thin films on MgO(001) substrates.

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