A Reformulation-Linearization Technique for Solving Discrete and Continuous Nonconvex Problems

by

Hanif D. Sherali

Department of Industrial and Systems Engineering,
Virginia Polytechnic Institute and State University,
Blacksburg, Virginia, U.S.A.

and

Warren P. Adams

Department of Mathematical Sciences,
Clemson University,
Clemson, North Carolina, U.S.A.

KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

CONTENTS

PREFACE xiv

ACKNOWLEDGMENTS xx

COPYRIGHT PERMISSIONS xxi

1 INTRODUCTION 1

1.1 Discrete Linear and Nonlinear Mixed-Integer Problems 2

1.2 Continuous Nonconvex Polynomial Programming Problems 14

1.3 Coping with Large-Scale Representations 17

PART I DISCRETE NONCONVEX PROGRAMS 21

2 RLT HIERARCHY FOR MIXED-INTEGER ZERO-ONE PROBLEMS 23

2.1 Basic RLT Process for Linear Mixed-Integer 0-1 Problems 25

2.2 Validity of the Hierarchy of Relaxations and the Convex Hull Representation 34

2.3 Further Insights Into the Structure of the Relaxations 43

2.4 Characterization of the Facets of the Convex Hull of Feasible Solutions 49

2.5 Extension to Multilinear Polynomial Programs and Specializations for Equality Constraints 52

3 GENERALIZED HIERARCHY FOR EXPLOITING SPECIAL STRUCTURES IN MIXED-INTEGER ZERO-ONE PROBLEMS 61

3.1 Generalized RLT for Exploiting Special Structures (SSRLT) 63

3.2 Validation of the Hierarchy for SSRLT 75

3.3 Composing *S* and *S*-Factors for Some Special Structures 78

3.3.1 Generalized Upper Bounding (GUB) or Multiple Choice Constraints 78

3.3.2 Variable Upper Bounding Constraints 87

3.3.3 Sparse Constraints 90

3.4 Using Conditional Logic to Strengthen RLT/SSRLT Constraints 91

3.4.1 Examining Sequential Lifting as an RLT Process 93

3.4.2 Numerical Example to Illustrate Conditional Logic Based Enhancement of SSRLT 95

3.4.3 Application to the Traveling Salesman Problem 98

4 RLT HIERARCHY FOR GENERAL DISCRETE MIXED-INTEGER PROBLEMS 103

4.1 The Discrete Problem and its Equivalent Zero-One Representation 104

4.2 Hierarchy of Relaxations in the Transformed Zero-One Space 106

4.3 Structure of a Parallel Hierarchy in the Original Variable Space 110

4.4 Equivalence of the Hierarchies in the Transformed and Original Spaces 112

4.5 Illustrative Example 125

4.6 Translating Valid Inequalities from Zero-One to General Discrete Spaces 127

5 GENERATING VALID INEQUALITIES AND FACETS USING RLT 131

5.1 Convex Hull Characterization and Facets for the Quadric Boolean Polytope 133

5.2 Convex Hull Characterization and Facets for GUB Constrained Knapsack Polytopes 146

5.3 Tight Representations and Strong Valid Inequalities for Set Partitioning Problems 160

5.3.1 Notation 161

5.3.2 A Specialized Hierarchy of Relaxations for the Set Partitioning Polytope 163

5.3.3 Insights into Deleting Fractional Vertices and Generating Manageable Relaxations 177

6 PERSISTENCY IN DISCRETE OPTIMIZATION 185

6.1 RLT-Based Persistency for Unconstrained 0-1 Polynomial Programs 188

6.2 RLT-Based Persistency for Constrained 0-1 Polynomial Programs 212

6.3 A Modified RLT Approach 228

6.3.1 Persistency for unconstrained 0-1 Polynomial Programs 237

6.3.2 Persistency for Constrained 0-1 Polynomial Programs 247

6.4 Relationships to Published Results 257

PART II CONTINUOUS NONCONVEX PROGRAMS 261

7 RLT-BASED GLOBAL OPTIMIZATION ALGORITHMS FOR NONCONVEX POLYNOMIAL PROGRAMMING PROBLEMS 263

7.1 Polynomial Programs Having Integral Exponents 265

7.1.1 A Branch-and-Bound Algorithm 272

7.1.2 An Illustrative Example 279

7.2 Polynomial Programs Having Rational Exponents 281

7.2.1 A Branch-and-Bound Algorithm 289

7.2.2 An Illustrative Example 293

8 REFORMULATION-CONVEXIFICATION TECHNIQUE FOR QUADRATIC PROGRAMS AND SOME CONVEX ENVELOPE CHARACTERIZATIONS 297

8.1 Reformulation by Generating Quadratic Constraints (RLT-LP) 300

8.2 An Illustrative Example: Higher Order Constraints 302

8.3 Fundamental Insights and Results for the Level-One RLT Relaxation 306

8.4 Construction of Convex Hulls and Convex Envelopes: General Results and Some Special Cases 315

8.5 Enhancements of the RLT Relaxation: RLT-NLP 335

8.5.1 Eigen-Transformation (RLT-NLPE) and Identification of Selected Constraints (SC) 336

8.5.2 Reformulation-Convexification Approach: Inclusion of Suitable Nonlinear Constraints in RLT-LP to Derive RLT-NLP 338

8.6 A Lagrangian Dual Approach for Solving RLT Relaxations 340

8.7 A Preliminary Computational Comparison of the Bounding Problems 343

8.8 Design of a Branch-and-Bound Algorithm 347

8.8.1 Scaling 347

8.8.2 Branch-and-Bound Search Strategy 347

8.8.3 Optimality Criterion 348

8.8.4 Range Reductions 348

8.8.5 Branching Variable Selection 352

8.8.6 Finding Good Quality Feasible Solutions 355

8.8.7 Summary of the Algorithm 357

8.9 Computational Results 359

9 REFORMULATION-CONVEXIFICATION TECHNIQUE FOR POLYNOMIAL PROGRAMS: DESIGN AND IMPLEMENTATION 369

9.1 Application of RLT to a Class of Quadrified Versus the Original Polynomial Program 372

9.1.1 Quadrification Process and the Application of RLT to the Quadrified Polynomial Program 373

9.1.2 Dominance of LP(PP) over LP($\overline{\text{QPP}}$) 378

9.2 A Computational Comparison: Evaluation of the Quadrification Process 383

9.3 Relaxations for Univariate Polynomial Programs 385

9.3.1 Squared Grid-Factor Constraints (SGF) and Associated Problem SGF-LB 387

9.4 Squared Lagrangian Interpolation Polynomial Constraints (SLIP) and Associated Problem SLIP-LB 388

9.5 Computational Evaluation of Relaxations for Univariate Problems 389

9.6 Relaxations and an Algorithm for Multivariate Problems..... 390

9.6.1 Regular RLT and Convex Variable Bounding Constraints 391

9.6.2 Constraint-Factor Based Restrictions 392

9.6.3 Constraint Filtering Scheme and Lower Bounding Problem..... 394

9.6.4 Range-Reduction Strategies..... 397

9.6.5 Branch-and-Bound Algorithm 398

9.7 Computational Results..... 399

PART III SPECIAL APPLICATIONS TO DISCRETE AND CONTINUOUS NONCONVEX PROGRAMS 403

10 APPLICATIONS TO DISCRETE PROBLEMS..... 405

10.1 Mixed-Integer Bilinear Programming Problems..... 407

10.1.1 An Equivalent Linear Reformulation..... 409

10.1.2 Design of an Algorithm..... 412

10.1.3 Computational Experience..... 419

10.2 Zero-One Quadratic Programming Problems 423

10.2.1 An Equivalent Linear Reformulation..... 425

10.2.2 Design of an Algorithm..... 427

10.2.3 Computational Experience..... 432

10.3 Miscellaneous Applications 434

11 APPLICATIONS TO CONTINUOUS PROBLEMS..... 441

11.1 Squared Euclidean Distance Location-Allocation Problem.....443

11.1.1 RLT-Based Relaxation 447

11.1.2 A Branch-and-Bound Enumeration Algorithm 454

11.1.3 Computational Results.....459

11.2 Linear Complementarity Problem 465

11.2.1 A Reformulation of the LCP.....466

11.2.2 Proposed Algorithm 472

11.2.3 Computational Results.....479

11.3 Miscellaneous Applications 486

REFERENCES 493