A Reinforcement Procedure Leading to Correlated Equilibrium: A Correction

Sergiu Hart* Andreu Mas-Colell†

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In Hart and Mas-Colell [2001], one needs to replace $1/(m^i - 1)$ by $1/m^i$ in Formula (7); that is,

$$\begin{aligned} p^i_{t+1}(k) &:= \left(1 - \frac{\delta}{t^\gamma}\right) \min\left\{\frac{1}{\mu} Q^i_t(j,k), \frac{1}{m^i}\right\} + \frac{\delta}{t^\gamma}\frac{1}{m^i}, \text{ for } k \neq j; \text{ and} \\ p^i_{t+1}(j) &:= 1 - \sum_{k \in S^i : k \neq j} p^i_{t+1}(k). \end{aligned} \tag{7}$$

Also, μ should satisfy $\mu \geq 2Mm^i$ (instead of $\mu > 2M(m^i - 1)$). With these changes,[1] Theorem 2 is true as stated.

The reason for this correction is that in the Proof of Step 8 we use the estimate of $\mathbf{O}(1/\sqrt{w})$ obtained in the Lemma of Step M7 in Hart and Mas-Colell [2000]. However, this estimate depends on the lower bound β on the diagonal elements of the matrix[2] Π. The correction we have made to (7) guarantees that $\Pi_t(j,j) \geq 1/m^i$ for all $j \in S^i$ and all $t \geq 1$.

*Center for Rationality and Interactive Decision Theory; Department of Mathematics; and Department of Economics, The Hebrew University of Jerusalem, Feldman Building, Givat-Ram, 91904 Jerusalem, Israel. *E-mail*: `hart@huji.ac.il` *URL*: `http://www.ma.huji.ac.il/~hart`

†Department of Economics and Business; and CREI, Universitat Pompeu Fabra, Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. *E-mail:* `mcolell@upf.es`

[1]In fact, one may replace $m^i - 1$ by $m^i - 1 + \varepsilon$ for any $\varepsilon > 0$, both in (7) and in the lower bound on μ; we chose $\varepsilon = 1$ for simplicity.

[2]It is easy to see that the bound obtained in that Lemma is $c(\beta)w^{-1/2}$, and $c(\beta) \to \infty$ as $\beta \to 0$. In Hart and Mas-Colell [2000], as well as in Theorem 1 of this paper, the diagonal elements of all the matrices Π_t are indeed uniformly bounded away from 0 (for instance: in Theorem 1

The only place in the proof which is affected by this change is the Proof of Step 4: The first case is now $(1/\mu)\,C_t^+(j,k) \leq 1/m$, and its proof remains the same. In the second case, when $(1/\mu)\,C_t^+(j,k) \geq 1/m$, we have

$$\Pi_t(j,k) = \frac{1-\delta_t}{m} + \frac{\delta_t}{m} = \frac{1}{m} \leq \frac{1}{\mu}C_t^+(j,k),$$

and, for the opposite inequality,

$$\begin{aligned}\frac{1}{\mu}C_t^+(j,k) &\leq \ldots \leq \frac{2M}{\mu} + \frac{M}{\mu}\overline{Z}_t(j,k) \leq \frac{1}{m} + \frac{M}{\mu}\overline{Z}_t(j,k) \\ &= \Pi_t(j,k) + \frac{M}{\mu}\overline{Z}_t(j,k).\end{aligned}$$

References

Hart, S. and A. Mas-Colell [2000], "A Simple Adaptive Procedure Leading to Correlated Equilibrium," *Econometrica* 68, 1127–1150.

Hart, S. and A. Mas-Colell [2001], "A Reinforcement Procedure Leading to Correlated Equilibrium," in *Economic Essays*, edited by G. Debreu, W. Neuefeind and W. Trockel, Springer, 181–200.

here, $\Pi_t(j,j) \geq \delta/m$). However, in the original Formula (7), we had $\Pi_t(j,j) \geq \delta t^{-\gamma}/m$, which goes to 0 as $t \to \infty$.