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A Relation between Peculiar Velocity and Density Parameter in the One-Dimensional Inhomogeneous Universe

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We derive the relation between the peculiar velocity and the density parameter in a one-dimensional inhomogeneous universe for the case in which the density distribution is described by the Zeldovich solution. We find that the simple relation obtained in the linear perturbation theory holds without any correction in spite of a fully non-linear nature of the problem. We also find that the inclusion of the cosmological constant in the present context does not change the above result.

Our Universe is believed to be homogeneous and isotropic in a scale larger than at least 100 Mpc. In such a scale the averaged spacetime is a very useful concept and is described by the Robertson-Walker (RW) geometry which is characterized by a few parameters.[1] One of them is the so-called density parameter Ω which is the ratio between the mean density and the critical density. Efforts have been made to determine the density parameter by the observation of the deviation from the expected homogeneous Hubble flow. This is because the deviation (the peculiar velocity) is induced by the gravitational attraction of the overdensity whose evolution is governed by the averaged expansion rate and hence by the mean density. For example, our local group of galaxies is falling towards the center of the Virgo cluster because of the overdensity of Virgo. For the analysis of the Virgo infall people mostly use the relation between the peculiar velocity and density contrast δ derived in the linear perturbation theory,[2),3)] or sometimes take the spherical model with non-linear effect taken into account.[4),5)] In reality, however, the Virgo centric infall is not quite a linear phenomena, nor described by a spheical symmetric flow. Therefore, it is highly desirable to see whether the non-linear correction as seen in the spherical model has a feature universal to other cases. As a first step to answer this question, we consider the case where the evolution of the density contrast is given by an exact solution, so-called Zeldovich solution in a one-dimensional universe. It is not realistic, but situations of arbitrary large density contrast may be treated. Moreover it is an exact solution and thus it allows an analytic treatment.

The basic equations for the density perturbation in the Newtonian cosmology in the one-dimension with zero pressure becomes as follows:[3)]

$$\frac{\partial \rho}{\partial t}+3\frac{\dot{a}}{a}\rho+\frac{1}{a}\partial_x(\rho v)=0\,, \tag{1}$$

$$\frac{\partial v}{\partial t}+\frac{1}{a}v\partial_x v+\frac{\dot{a}}{a}v=g\,, \tag{2}$$

$$\partial_x g=-4\pi G\rho_b a\delta\,, \tag{3}$$

where a is the scale factor of the universe given by a prescribed function depending on the spatial curvature of the RW geometry and g is the gravitational force due to the density contrast δ defined by $(\rho-\rho_b)/\rho_b$ in the region we are interested in. ρ_b is the average background density. All variables are assumed to be only functions of t and x. Let us define new variables for ρ, ρ_b, v and t as

$$\bar{\rho}\equiv\rho a^3\,,\quad \bar{\rho}_b\equiv\rho_b a^3\,,\quad \bar{v}\equiv va\,,\quad dt\equiv a^2 d\tau\,.$$

Then the above equations take the form[1]

$$\frac{\partial\bar{\rho}}{\partial\tau}+\partial_x(\bar{\rho}\,\bar{v})=0\,, \tag{4}$$

$$\frac{\partial\bar{v}}{\partial\tau}+\bar{v}\partial_x\bar{v}=a^3 g\,, \tag{5}$$

$$\partial_x g=-4\pi G\rho_b a\delta\,. \tag{6}$$

An exact solution to the above set of equations is known by Zeldovich,[6]

$$x=q+B(\tau)S(q)\,, \tag{7}$$

$$\bar{v}=\frac{dB}{d\tau}S\,, \tag{8}$$

$$\bar{\rho}=\frac{\bar{\rho}_b}{1+B\dfrac{dS}{dq}}\,. \tag{9}$$

We find from Eqs. (6) and (9)

$$1+\delta=\frac{1}{1+B\dfrac{dS}{dq}}\,, \tag{10}$$

$$g=4\pi G\rho_b aBS\,. \tag{11}$$

Using these equations it is easy to derive the formal expression for the peculiar velocity

$$v=\frac{2g}{3H\Omega}\frac{dB}{da}\,. \tag{12}$$

Remarkably this is an exact expression and is the same expression as that one obtains in the linear perturbation theory for the peculiar velocity, except that there appears B instead of the temporal part of the growing solution D in the linear perturbation theory.[3] We now examine B. The equation for B is obtained from Eq. (5),

$$\frac{d^2B}{d\tau^2}=4\pi G\rho_b a^4 B\,. \tag{13}$$

Using the variable t this leads

$$\frac{d^2B}{dt^2}+2\frac{\dot{a}}{a}\frac{dB}{dt}=4\pi G\rho_b B\,. \tag{14}$$

This equation is again exactly the same as that for the temporal part of the density perturbation D. The solution of this equation coincides with the linear theory if $\delta \ll 1$, since $B \propto \delta$ from Eq. (10) in such a case. Thus we have $B \propto D$ and

$$\frac{a}{B}\frac{dB}{da}=\frac{a}{D}\frac{dD}{da}\,. \tag{15}$$

Consequently, we obtain in the one-dimensional case the v-Ω relation which takes the form identical with that in the linear perturbation theory.

Now we consider the case in which a non-vanishing cosmological constant Λ exists. The gravitational force g is generated by the density contrast and Λ makes no effect in that. Therefore, the basic equations given by Eqs. (1), (2), (3) and (14) are not modified even if a non-vanishing Λ exists. The only change due to the existence of Λ is the Ω dependence of the coordinate time t and hence of B. It is shown that the Ω dependence of the solution of Eq. (14) does not depend much on whether Λ term in the Einstein equation exists or not.[7] Hence we conclude that the v-Ω relation does not change in the one-dimensional non-linear situation considered here irrespective of the existence of the cosmological constant.

It would be instructive to compare our findings with the case for the spherical flow model. In the latter case, the v-Ω relation has a substantial non-linear correction. It is approximately represented by $(1+\bar{\delta})^{-0.25}$, which appears on the left-hand side of Eq. (12) and this form is often used in a realistic analysis for the Virgo centric flow.[5] Our analysis explicitly demonstrates that the non-linear correction depends on the morphology of the density enhancement. This implies that the non-linear correction is model-dependent and the results that rest on a particular correction form are not reliable.

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