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A remote tutoring system for algebra enhanced by Mathematica™

H. Nishizawa, K. Nakayama, and T. Kanaya
Department of Electrical Engineering, Toyota National College of Technology, Eisei-cho 2-1, Toyota 471, Japan
E-mail: nisizawa@tctcc.cc.toyota-ct.ac.jp

Abstract

A remote tutoring system of algebra, especially for the factorization of polynomials, is described. The system is aimed at improving the calculating ability of students aged between fifteen and seventeen in Kosen (Colleges of Technology in Japan), and serves each student as an effective training opportunity in the field of factorizing polynomials through the well-organized Kosen computer network. The system receives string data sent by the students, determines if the final expression is fully factorized, finds errors in intermediate expressions, and tells the students of the completeness of the factorization and positions of the errors in the given expressions, if they exist.

The system consists of a WWW (World Wide Web) server that collaborates with the symbolic computational program Mathematica™, both running on a Power Macintosh™ personal computer, and WWW browsers running on personal computers connected to the server through a local area computer network. In this system, a custom function written in the Mathematica™ language evaluates the input data sent from the students. The function is called by the WWW server through a CGI (Common Gateway Interface) program. It has rules and methods for evaluating the mathematical expressions. With this system, students get more valuable information about their calculating process than from most CAI programs which request the students to fill in the blanks of the prepared expressions or request a selection from several candidates.

1 Background

Kosen (Colleges of Technology in Japan) accepts a wide age-range of students between fifteen and twenty-two on a campus ,[1] which is the same age-range for freshmen in high schools to seniors in universities. Thus the professors and the lecturers of Kosen have many different types of classes at the same time; for example, a class of basic algebra for younger students and also a class of vector analysis for older students.

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Under these circumstances, adequate exercise necessary for the younger students tends to be sacrificed, because classes for younger students require less technical knowledge but more time, mostly in exercises, compared to classes for the older students. Most professors prefer lectures for the older students because of the reduced time consumed by exercises. The introduction of teaching assistants could improve the situation, but financial support for T.A.s is not yet authorized for Kosen. Lack of exercise in fundamental mathematics appears to be one of the serious drawbacks of Kosen.[2]

To improve the situation, the authors took advantage of the well-organized computer network on campus and started developing a remote tutoring system for fundamental mathematics.[3,4] The system utilizes the symbolic computational program Mathematica™ for the flexible tutoring which is characterized by informative messages to the students and accepts flexible inputs.

2 Concept of the System

Most CAI programs, which request filling in the blanks of a given expression or selecting an answer from several candidates, do not give enough information for improving the

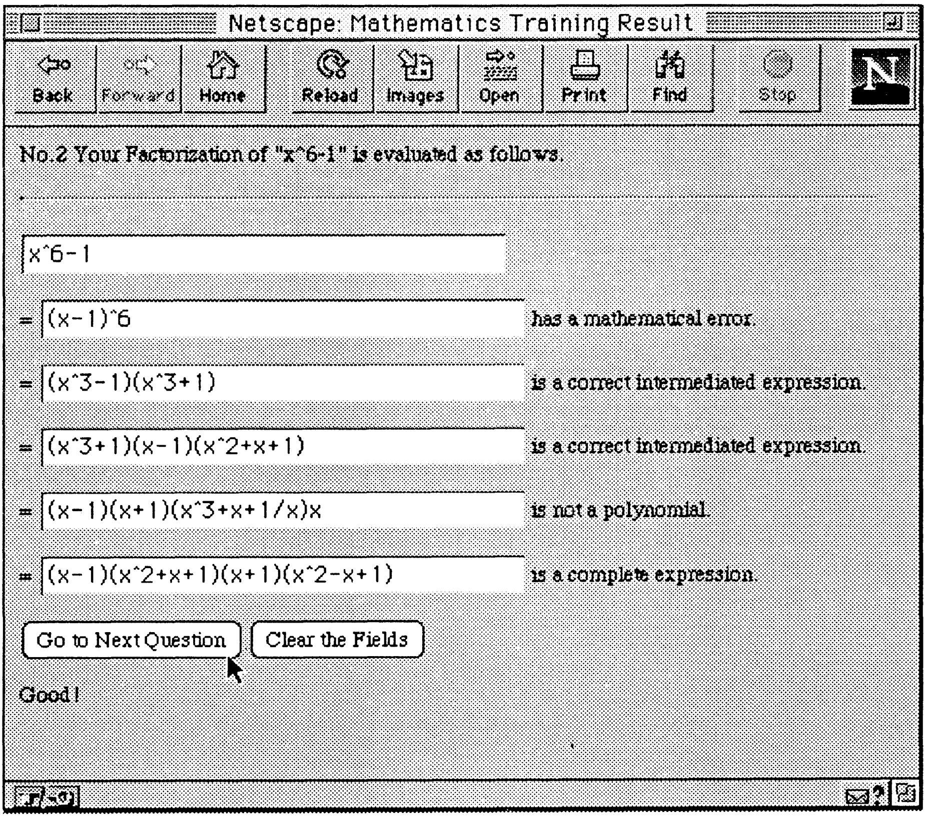

Figure 1 : A screen shot of a remote user after the evaluation by the tutoring system.

calculating ability of students. They only tell if their selections are correct or not, and do not tell anything about the intermediate calculating process nor anything about an error in the expression of the students. Another deficit of these programs is not allowing any variability in the expressions. Only one correct answer is allowed and other expressions are neglected from the start even if they are mathematically equivalent to the answer. Although these programs are sufficient to measure the achievement of students, they are not appropriate for effective exercise because of the poor information returned from the programs and the strict limitation of the expressions. The main purpose of the tutoring system described in this paper is to give students more useful information about their calculating processes and allow varied expressions in the inputs.

Figure 1 shows an example of the evaluated result. The expression of second line $(x-1)^6$ is not mathematically equal to the first line x^6-1. The third through fifth lines have equality but they are not sufficient for the condition of factorization. Only the sixth line is evaluated as a fully factorized expression. The students get the comment concerning each line of their expressions.

The tutoring system returns to students two valuable pieces of information. First, information on the completeness of the calculation. When the students are informed of the incompleteness of their final expressions, they can begin further analysis in order to complete the calculation. Second, information on the position of an error. A valuable clue for the students is to find by themselves the reason for the error.

Flexibility of the input data is another characteristic of the system (Table 1). Although the input data type are limited to the Mathematica™ input form, students are allowed to input their expressions using their calculating tactics, and variation in detail has no limitation as long as the expressions have mathematical equivalence. The length of an expression line or the number of lines are not limited. All input expression lines are evaluated by the system. To achieve these characteristics in a short time, the many built-in functions and flexible programming capability of Mathematica™ are essential.

The openness of the system is the last point to consider. To allow access from many terminals around the campus, the system utilizes WWW technology. By building the host system with a WWW server program, selection of the user-side hardware becomes flexible. Any computers which run WWW browsers can be used as user terminals.

Table 1 All expressions listed below are equivalent to the factorized expression $(x+1)^2(x-1)^2$. They are not only mathematically equal to it, but also are factorized completely.

$(x+1)(1-x)(x+1)(1-x)$	$-(x+1)^2(x-1)(1-x)$
$\frac{1}{4}(2x+2)^2(x-1)^2$	$9\left(\frac{1}{3}x+\frac{1}{3}\right)^2(x-1)^2$

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3 System Layout

The tutoring system is composed of a WWW server and WWW clients connected to each other with the local area network on campus (Figure 1). The WWW server is a Power Macintosh™ computer, on which a WWW server program, WebSTAR™, a symbolic computational program, Mathematica™ (version 2.2.2), and two custom CGI programs are running. One of the CGI program selects a factorization question from a list of problems in the text file for a student. WebSTAR™ receives the student's input sent from his/her WWW browser and passes it to the other CGI program, which calls a custom Mathematica™ function written in the Mathematica™ language through the MathLink, a built-in communication protocol between the Mathematica™ kernel and other programs. The custom Mathematica™ function evaluates the input in the form of a Mathematica™ input according to the evaluation criteria to be described later, which includes completeness of the input expressions and the position of an error if it exists. The CGI program receives the result returned from the function and transmits it through WebSTAR™ to the student's WWW browser.

On the returned page displayed by the browser, the student knows not only the completeness of his/her calculation, but also the position of an error. He/She is allowed a wide variety of expressions as input data to the system, and all of his/her mathematical expressions are immediately examined.

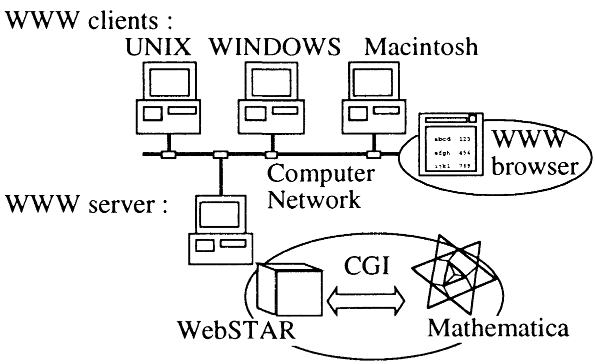

Figure 2: The layout of the remote tutoring system.

4 Criteria of Evaluation

During a factorization exercise, a student produces several lines of expressions until they reach his/her final expression. The tutoring system separately evaluates each line of the expressions as follows:

First, the system evaluates the equality of every line of the expressions sent from the student with the standard expression, in another words, the answer. It expands a line of expression t sent from the student and the answer s , and compares it using the Mathematica™ built-in functions; **Expand[*t*] == Expand[*s*].** If the result is **True**,

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the line of expression t has equality with the answer and line t is evaluated as an appropriate intermediate expression. Otherwise, the comment added to the line tells that the line has a mathematical error.

After the evaluation of mathematical equality, a custom function **factorizedQ[*tl*, *s*, {*x*}]** evaluates if they meet the condition as the final expression. The function receives an expression *tl*, the answer *s*, and a list of independent valuables $\{x\}$ used in them, and returns the result. The function examines the Head and the other elements of the expressions *tl*. A factorized expression must have a Head of "Times" or "Power" when expressed in the FullForm of the Mathematica™ input, and the elements must be polynomials of variables $\{x\}$ or constants. Rational expressions should not be included in the elements. A built-in function **PolynomialQ[ ]** finds invalid expressions in the elements. The function **factorizedQ[ ]** then counts the number of multiplied polynomials in the elements. If the order of powers of an element is greater than one, then the function **factorizedQ[ ]** is recursively called with the element as argument *tl*, and the number of multiplied polynomials in the element is counted. The total number of multiplied polynomials in the final expression *tl* and the answer *s* are compared. If the number of polynomials in the expression *tl* is equal with the number in the expression *s*, *tl* is evaluated to be fully factorized. Incomplete factorization is characterized by a final expression *tl* of a lower number of multiplied polynomials. An incomplete final expression includes polynomials of higher order of powers in the elements, which must be further factorized by a multiplication of polynomials of lower order of powers.

5 Operation of the System

The custom function **factorizedQ[ ]** was tested with about 500 lines of expression collected from manuscript answers for a class of students. All the decisions made by the tutoring system were reasonable, although it judged some lengthy expressions like $(x+1+1)(x+2)$ as appropriate final expressions.

Thirteen questions of fairly complicated factorization (Table 2) are given to the students, which are composed of up to three variables and order of powers between two to six. Both right and wrong answers are given to the function as inputs, and its judgements are observed.

For example, the fully factorized expression of x^6-1 is $(x-1)(x^2+x+1)(x+1)(x^2-x+1)$ in the restriction of real coefficients (Figure 1). Although $(x^3-1)(x^3+1)$ is a correct intermediate expression, it is not completely factorized, because (x^3+1) and (x^3-1) could be factorized to $(x+1)(x^2-x+1)$ and $(x-1)(x^2+x+1)$, respectively. $(x^3-1)(x^3+1)$ has only two multiplied polynomials compared to four for $(x-1)(x^2+x+1)(x+1)(x^2-x+1)$. $(x+1)(x-1)(x^4+x^2+1)$ and $(x^3+1)(x-1)(x^2+x+1)$ are not completed either, which have three multiplied polynomials. The completeness of factorization is determined by counting the number of multiplied polynomials.

One of the tricky expressions was $(x-1)(x+1)(x^3+x+1/x)x$, which has four multiplied elements but is not completely factorized. In this case, the function **PolynomialQ[ ]** finds the invalidity of the third element, which is not a polynomial but

Table 2: The questions tested the custom function factorizedQ[ ].

expanded expression	factorized expression
$(a-b)^2-10(a-b)+25$	$(a-b-5)^2$
$a^2+2ab+b^2-1$	$(a+b+1)(a+b-1)$
$x^2+xy-6y^2+x+13y-6$	$(x-2y+3)(x+3y-2)$
$2x^2-2xy-4y^2+5x-7y+2$	$(2x-4y+1)(x+y+2)$
$a^2-b^2+4bc-4c^2$	$(a+b-2c)(a-b+2c)$
$x^3+2x^2y-9x-18y$	$(x+2y)(x+3)(x-3)$
$(x+y+z)^3-x^3-y^3-z^3$	$3(x+y)(y+z)(z+x)$
$a^3(b-c)+b^3(c-a)+c^3(a-b)$	$(a-b)(b-c)(a-c)(a+b+c)$
$(x-1)(x+2)(x-3)(x+4)+24$	$(x^2+x-8)(x-2)(x+3)$
$(x^2-5x+3)(x^2-5x+5)+1$	$(x-1)^2(x-4)^2$
$a^4+b^4+c^4-2b^2c^2-2c^2a^2-2a^2b^2$	$(a+b+c)(a-b-c)(a+b-c)(a-b+c)$
x^6-1	$(x-1)(x^2+x+1)(x+1)(x^2-x+1)$

a rational expression.

If we allow to use complex coefficients, the fully factorized expression of x^6-1 becomes $(x-1)\left(x+\frac{1+\sqrt{3}i}{2}\right)\left(x+\frac{1-\sqrt{3}i}{2}\right)(x+1)\left(x-\frac{1+\sqrt{3}i}{2}\right)\left(x-\frac{1-\sqrt{3}i}{2}\right)$ instead of $(x-1)(x^2+x+1)(x+1)(x^2-x+1)$. But the expression is commented as "it includes complex coefficient, which is not allowed to be used here." When the coefficients are restricted to real numbers.

The system has been tested under remote access through the campus network, where the user interface described in HTML(Hyper Text Markup Language) and response time are examined. The current page displayed on a WWW browser (Figure 1) has instructions and a description of the expression to be factorized on the top, and a number of fields for inputs below. Each field is to be filled with one expression. Each line of the expression is independently evaluated regardless of the order of the line. The lower left button on the screen tells "Go to Next Question" if one of the lines holds a fully factorized expression, but "Evaluate Again" if there exist no fully factorized expressions.

Figure 3 shows the response time of the tutoring system when one through four users requested the evaluation at the same time. The consumed time is measured from the beginning of the evaluation to the completion of displaying the result. The response time of the system , 1.1 to 5.3 seconds, is about one-fifth of another CAI system we have developed ,[5] which consists of a WWW server collaborating with a relational database manager. The quicker response of this system mainly depends upon the

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Input expressions used at the response measurement :

△
$$\begin{aligned}&(x-1)(x+2)(x-3)(x+4)+24\\&=\left(x^2+x-2\right)\left(x^2+x-12\right)+24\\&=\left(x^2+x\right)^2-14\left(x^2+x\right)+24+24\\&=\left(x^2+x\right)^2-14\left(x^2+x\right)+48\\&=\left(x^2+x-6\right)\left(x^2+x-8\right)\\&=(x-2)(x+3)\left(x^2+x-8\right)\end{aligned}$$

○
$$\begin{aligned}&a^6-1\\&=\left(a^3\right)^2-1\\&=\left(a^3+1\right)\left(a^3-1\right)\\&=(a+1)\left(a^2-a+1\right)(a-1)\left(a^2+a+1\right)\end{aligned}$$

□
$$\begin{aligned}&x^2+2x+1\\&=(x+1)^2\end{aligned}$$

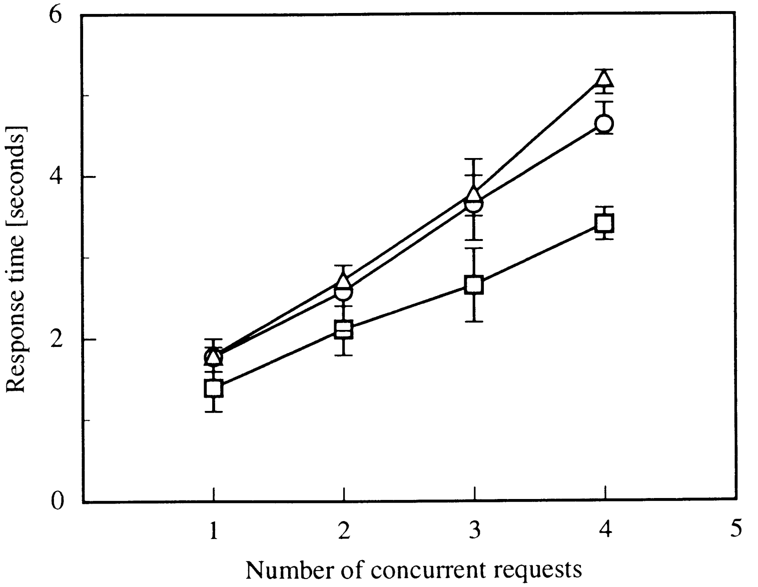

Figure 3 : Response time of the system when 1 to 4 evaluations are requested at the same time.

processing speed of Mathematica™. The response time has a small dependency on the number of concurrent requests and the complexity of the input expressions. Under this condition, the response time mainly determined by the network speed including the performance of the WWW server.

Weak points of the system are the complexity of the input method using a keyboard and confusion of the independent variables. Students, who are not familiar with the Mathematica™'s input form, tend to input a single variable "*xy*" instead of multiplied two variables "*x y*", which includes a blank between "*x*" and "*y*". Improvement of the interface must be done.

6 Extension of Applied Field

The evaluation method adapted to the tutoring system is applicable to other fields of algebra and extension of the applied field is now underway. These fields include solving quadratic equations or inequalities, expanding polynomials, simplifying rational expressions, etc. in algebra. The applied field is also extended to other fields of mathematics and also to the fields of engineering. For example, mathematical expressions deduced from linguistic description in physics or engineering problems, could be evaluated by applying the method of this tutoring system.

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