A RESERVOIR ANALYSIS OF THE DENVER EARTHQUAKES: A CASE OF INDUCED SEISMICITY

by

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ABSTRACT

Injection of fluid waste into Precambrian crystalline rocks at the Rocky Mountain Arsenal triggered earthquakes in the 1960s. Available data suggest that the waste fluid was injected into a reservoir composed of connected vertical fractures. Earthquakes are believed to be results of lateral sliding motions along fracture planes.

A mathematical model is constructed to simulate fluid pressure build-up caused by injection. Computed pressure build-up is related to the spatial distribution of earthquake epicenters. The results show that the earthquakes are confined to that part of the reservoir where the pressure build-up exceeds 32 bars. This critical value is interpreted as the pressure build-up above which earthquakes occur. The existence of this critical pressure is consistent with the Hubbert-Rubey theory on the role of fluid pressure in fault movement. The migration of earthquake epicenters away from the injection well, a phenomenon noted by previous investigators, can be accounted for by the outward propagation of the critical pressure build-up.

The analysis is extended to examining the effects of fracture widening under high injection pressure. The results show that the effect is confined to a small region within one kilometer of the injection well.

INTRODUCTION

Historical Background

During 1961 a deep injection well was drilled by the U.S. Army Corps of Engineers at the Rocky Mountain Arsenal (RMA), located northeast of Denver, Colorado, for the purpose of disposing waste water. The well completely penetrated the sedimentary rocks of the Denver Basin and was drilled to a depth of 3671 meters into crystalline Precambrian bedrock. Injection took place into the bottom 21 meters of open hole, which was completed in a highly fractured Precambrian gneiss.

Routine waste disposal operations began on March 8, 1962. Pressure injection was accomplished by using one or more of four constant-displacement pumps (approximately 380 l/min each). During injection, the pressure at wellhead varied from zero (gravity flow) to a maximum of about 72 bars.

The injection history from 1962 to 1966 can be divided into four characteristic periods. From March 1962 to September 1963, waste fluid was injected under pressure into the well. Between October 1963 and September 1964, no injection took place. From October 1964 to March 1965, injection was accomplished by gravity flow. Pressure injection resumed in April 1965 but was discontinued in February 1966. A total of 625 million liters of waste fluid was disposed of in the well during the 4-year period.

Shortly after the start of the injection program, minor earthquakes were detected in the Denver area. Between April 1962 and

August 1967, over 1500 "Denver earthquakes" (also known as "Derby earthquakes") were recorded at the seismograph station at Bergen Park (Major and Simon, 1968). Some of the earthquakes exceeded Richter magnitudes of 3 and 4.

In November 1965, David Evans (1966), a Denver geologist, publicly suggested a direct relationship between fluid injection at the RMA well and earthquakes in the Denver area. He based his hypothesis on (1) an apparent correlation between the volume of fluid injected into the well and the frequency of the earthquakes and (2) a study by Wang (1965), which showed that the majority of the earthquakes had epicenters within 8 km of the well. Because of Evans' suggested injection–earthquake relationship, the waste disposal operation at the RMA was discontinued. This was followed by a number of more detailed investigations conducted by the Colorado School of Mines, the U.S. Geological Survey, and the U.S. Army Corps of Engineers.

Although no fluid has been injected into the well since February 1966, the earthquake activity continued. In 1967, three major earthquakes, each with a Richter magnitude greater than 5, shook the Denver area and caused minor structural damage. After 1967, however, the number of earthquakes began to decline (Fig. 1). The present indication is that the swarm of activity that occurred between 1962 and 1967 has virtually disappeared (Major, 1978).

Purpose of the Study

The purpose of this study is to examine the relationship between earthquakes and fluid injection at the Rocky Mountain Arsenal. Although the injection–earthquake hypothesis has been proposed many times in the

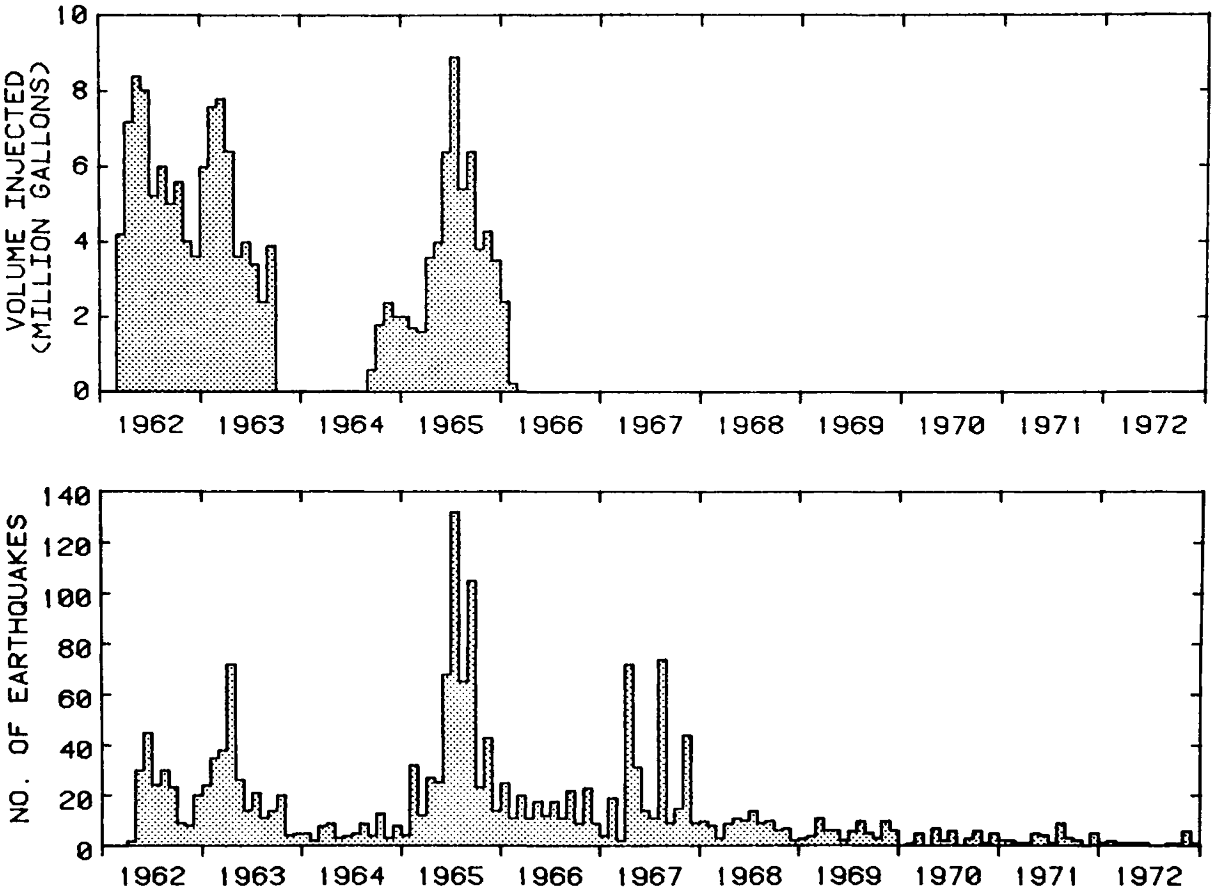

Fig. 1. Comparison of fluid injected and the frequency of earthquakes at the Rocky Mountain Arsenal. -- Upper graph shows monthly volume of fluid waste injected in the disposal well. Lower graph shows number of earthquakes per month. The apparent correlation for the period 1962-1966 was first noted by Evans (1966).

past, further examination is needed because various investigators have pointed out that the three major earthquakes of 1967 reduced the quality of Evans' original correlation between injected volume and the number of earthquakes (Major and Simon, 1968). In later studies, fluid pressure in the RMA well and the injection energy were examined. None of these studies, however, could adequately account for the earthquake activities after 1966.

This study examines the injection–earthquake relationship through the use of a mathematical model, which simulates pressure build-up in the Precambrian reservoir. The first portion of the research is directed to determine an appropriate mathematical model that describes fluid flow in the reservoir. The pressure build-up due to fluid injection is next calculated. Finally, a comparison is made between the spatial distribution of fluid pressure in the reservoir and the spatial distribution of earthquakes. From this comparison, the mechanism through which fluid injection and earthquakes are related can be determined.

THE RESERVOIR

Evidence for a Fractured Zone

It has been established that the waste fluid from the RMA was injected into a fracture zone in the Precambrian rocks beneath the Denver Basin. Examination of cores from the RMA well confirmed the presence of fractures in the Precambrian interval (Scopel, 1964). It is believed that the reservoir permeability is confined primarily to these fractures; the reservoir rock itself is much less permeable. Evans (1966) found that the Precambrian core was split apart along a vertical fracture plane. He theorized that this might have been an open fracture. Sheriden, Wrucke, and Wilcox (1966) studied the petrography of the recovered core and further found that the fractures and microbreccias in the cores were very similar to fracture zones in the Front Range granites. They suggested that a fracture zone may occur in the general vicinity of the RMA well. In a later study, Snow (1968) also suggested that the fractured Precambrian rocks beneath the Denver Basin are of common origin with the fractured Precambrian rocks of the Front Range.

There is other evidence that indicates the presence of a fracture zone at the RMA well. Van Poollen (1966) noted a linear relationship between reservoir pressure and the square root of the elapsed time since the well was shut in. Three and a half years later, van Poollen and Hoover (1970) found the shut-in pressure continued to follow this linear relationship. They interpreted this to mean that fluid flow in the reservoir was essentially linear. This type of transient pressure behavior is a strong indication that the waste fluid was injected into a linear fracture zone.

Perhaps the best evidence suggesting the existence of a linear fracture zone is the location of earthquake epicenters that were recorded in the vicinity of the RMA. Between 1966 and 1968 various seismic arrays were installed by the U.S. Geological Survey at the RMA. Although these devices were in oepration intermittently, sufficient data were collected so that a zone of earthquakes could be clearly outlined. The results of this survey indicated that the earthquake epicenters were consistently located in an area that is elliptical in shape, approximately 10 km long and 3 km wide, and contains the RMA well (Fig. 2). The trend of the major axis of this seismic zone was approximately N. 60° W. The analysis of these earthquakes suggested that they occurred as a result of right-lateral strike-slip motions along vertical planes having the same trend as the seismic zone (Healy, Jackson, and Van Schaack, 1966; Healy and others, 1968; Hoover and Dietrich, 1969).

Although there is no surface indication of faulting, the evidence, taken collectively, leaves little doubt that a series of vertical faults existed in the Precambrian rocks prior to the injection of fluid and that the recorded earthquakes represented lateral shear motion along these fault planes. Given the number of fracture and fault trends in the Front Range west of Denver, it is likely that some of these fault zones extend into the Precambrian rocks beneath the Denver Basin. The RMA well seems to have penetrated such a fault zone.

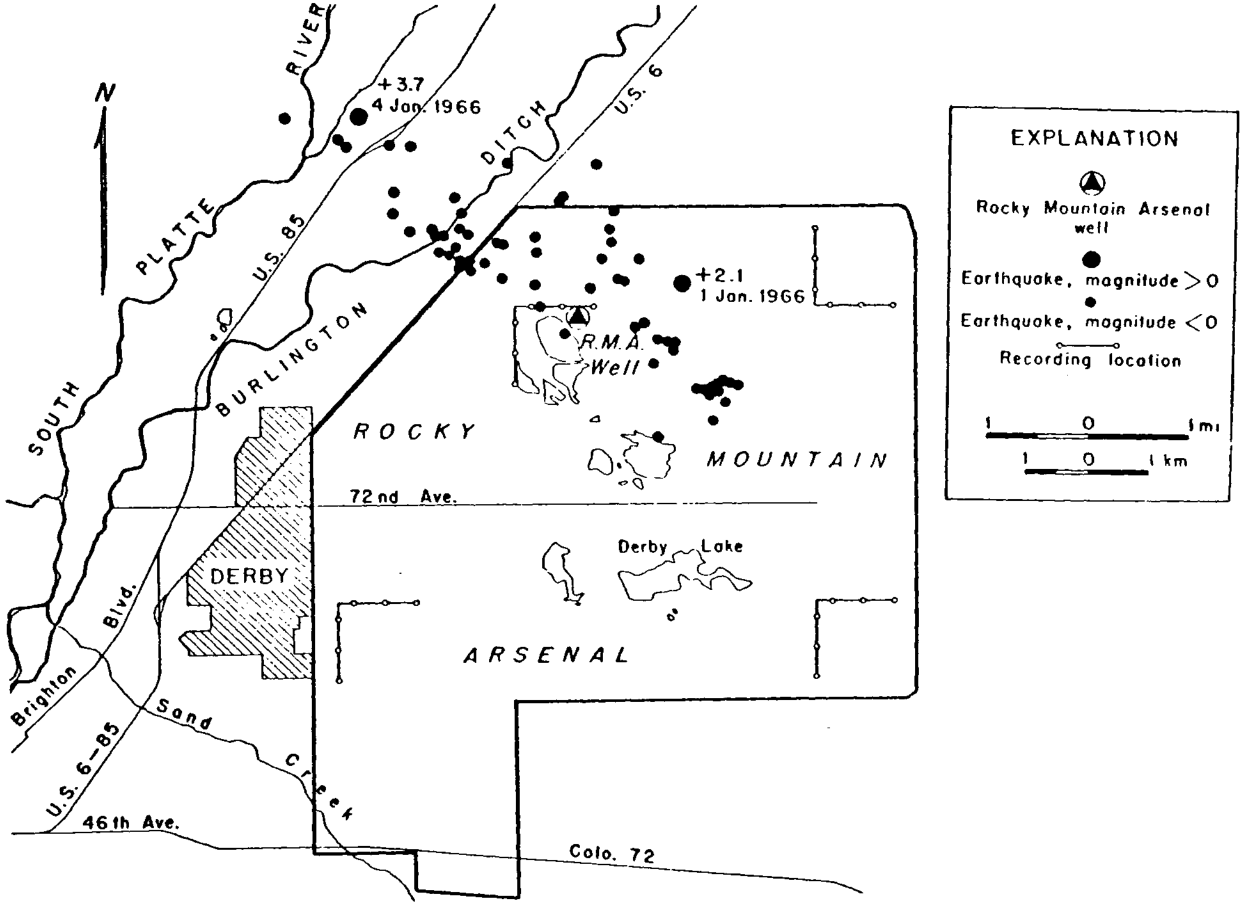

Fig. 2. Location of earthquakes recorded from mobile microseismic stations during January and February, 1966.--From Healy and others (1966, Fig. 3)

Vertical Extent of the Reservoir

The vertical extent of the Precambrian reservoir can be inferred from the depth range of the Denver earthquakes. Depth of earthquake hypocenters have been investigated by Wang (1965), Healy and others (1966), and Hoover and Dietrich (1969). Wang reported that a number of earthquakes were located at depths greater than 30 km. Healy and others (1966) inspected Wang's data and noted that most of the earthquakes reported in Wang's study were located with less than four stations. These four stations were not optimally located to detect earthquakes in the RMA vicinity. Healy and others (1966) concluded that Wang's location data were subject to errors of 10 km or more.

Using early data collected by the U.S. Geological seismic array installed on the RMA, Healy and others found that earthquake hypocenters clustered much more closely around the RMA well than was indicated in Wang's study. All the earthquakes studied by Healy and others were located at depths between 4.5 and 5.5 km.

A comprehensive list of earthquake hypocenters recorded by the U.S. Geological Survey seismic array during 1967 and 1968 was given by Hoover and Dietrich (1969). In Figure 3 the hypocenters of these earthquakes are plotted on a northwest-southeast cross section taken through the trend of the epicenters. This plot suggests that the earthquake zone extends approximately 3.3 km in depth from 3.7km to 7.0 km below land surface. The vertical extent of the reservoir in the Precambrian rocks is expected to be confined to this range.

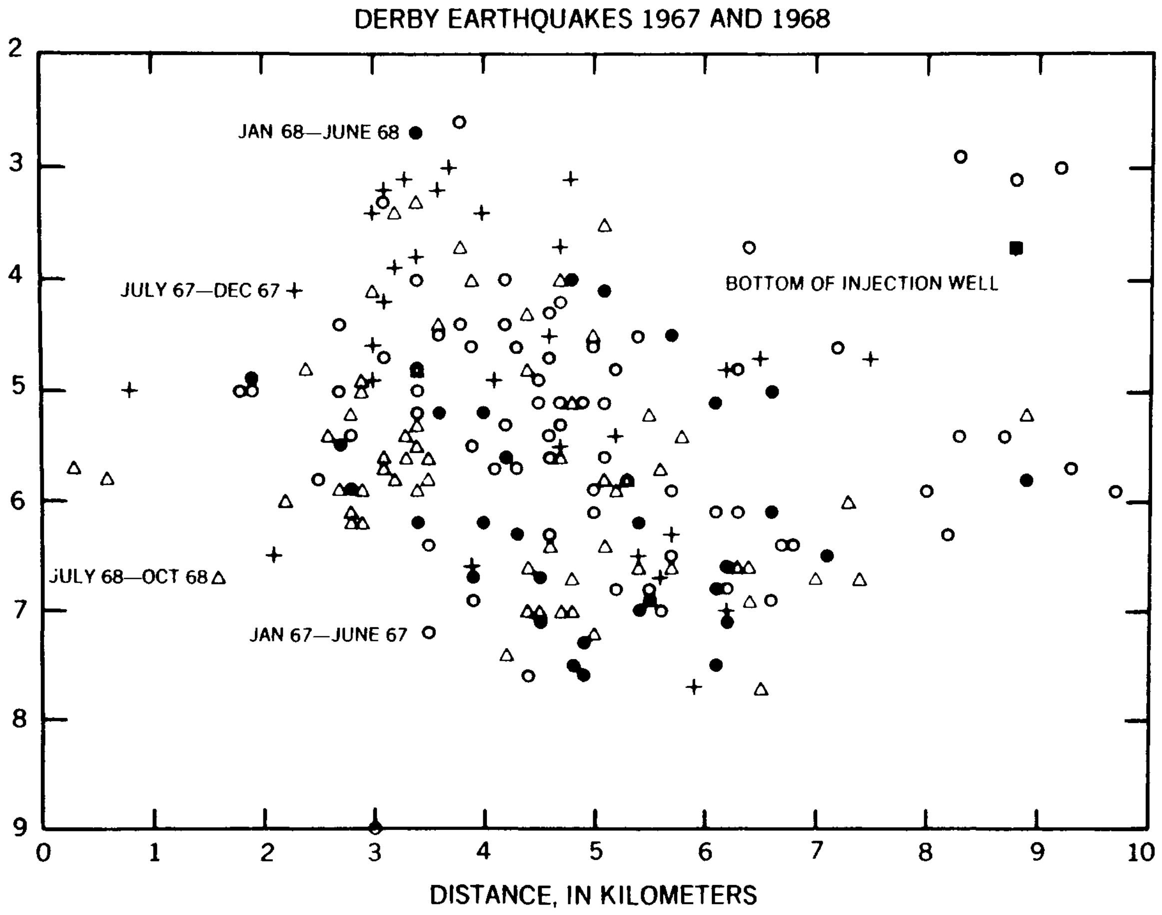

Fig. 3. Northwest-southeast cross section on which are plotted the earthquakes reported by Hoover and Dietrich (1969) for 1967 and 1968

Transmissivity

Numerous pressure measurements have been made in the RMA well for the purpose of estimating the transmissivity of the Precambrian reservoir. These data can be divided into three categories: (1) injection tests conducted prior to the start of the waste disposal operations, (2) continuous pressure recordings during the 4-year disposal operation period, and (3) pumping tests conducted in the fall of 1968.

Prior to the waste injection opration, a total of nine injection tests were performed for the open interval between 3650 m and 3671 m. Five of these injection tests were conducted during September 1961 and the remaining four during January 1962. For all the injection tests pressure measurements were made only during the shut-in period following injection. Pressure data are given by Rowland (1962) and Ball, Sells, and Downs (1966).

The September 1961 tests were conducted through the drill pipe, using the drill rig equipment. Except for the last two runs, during which Amerada subsurface gages were used, pressure readings were taken with a surface recorder. In general the pressure data were of poor quality. Using the Horner method of analysis, van Poollen (1966) calculated transmissivity values ranging from 1.11×10^{-6} m^2/s to 4.05×10^{-5} m^2/s, with a probable average of 2.36×10^{-5} m^2/s.

In January 1962, after the well was completed, four additional injections tests were performed. Pressure recordings from a subsurface Amerada gage were available for the last three tests, and the data obtained were generally of better quality than those from earlier tests. The calculated transmissivities, however, ranged from 2.50×10^{-5} m^2/s to

9.13×10^{-5} m^2/s (van Poollen, 1966). These values are somewhat higher than those computed from the September 1961 tests. Van Poollen suggested that the high values could be explained by a cleaning of the fractures caused by the long period of fluid withdrawal prior to the January injection tests.

In addition to data from the injection tests, transient wellhead pressure, which was continuously recorded during the actual waste disposal operation, may also be used to estimate reservoir transmissivity. Continuous daily wellhead pressure charts are available for the periods from May 1962 to September 1963 and from April 1965 to February 1966. Although the injection rate changed frequently, there were several occasions during which a long period of constant injection was followed by another long period of either constant injection at a different injection rate or by shutdown. Pressure data for these periods are particularly suitable for estimating reservoir transmissivity. Using 15 such periods, van Poollen (1966) calculated transmissivity values, which ranged from a low of 8.78×10^{-6} m^2/s to a high of 3.13×10^{-5} m^2/s. In general, however, most of the calculated transmissivities were close to the average value of 1.63×10^{-5} m^2/s.

In the fall of 1968 a series of pumping tests were conducted at the RMA well. Drawdown data from these tests can be used as additional estimates of the reservoir transmissivity. From the results of these pumping tests, van Poollen (1969) noted that transmissivity of the Precambrian reservoir appeared to be a function of pumping rate. The calculated transmissivities were 1.05×10^{-5}, 6.17×10^{-6}, and 3.61×10^{-6} m^2/s for pumping rates of 7.89×10^{-4}, 1.28×10^{-3}, and 1.58×10^{-3} m^2/s, respectively.

A summary of the calculated reservoir transmissivities is given in Table 1. The wide range of values, spanning two orders of magnitude, is not unexpected considering the quality of the data, mechanical difficulties, and the many factors (such as variable pumping rates, wellbore damage, fluid composition, and temperature) that were not taken into account. It was decided that the average value of 1.63×10^{-5} m^2/s, computed from the 15 periods of rate change during the waste injection operation, was probably a good estimate of reservoir transmissivity determined from the short-term data. While the pressure data from the September 1961 tests were poor and the calculated transmissivity values from January 1962 tests were much higher than the rest, the 15 periods analyzed span the entire 4-year operation of the RMA well; calculated transmissivities were constantly close to the average value.

Initial Fluid Pressue

During the final stages of well construction, considerable lost circulation was encountered while drilling the Precambrian interval. Because loss of circulation may alter the natural fluid pressure in the vicinity of the well, the initial downhole pressure in the Precambrian reservoir was not known.

In 1966, after injection was discontinued at the RMA well, Ball and Downs (1966) estimated the initial downhole pressure to be 328 bars. At about the same time, van Poollen (1966) computed a value of 339 bars. Assuming fresh water at 20°C in the well tubing, Ball and Downs' value would put the initial fluid level at 325 meters below land surface, while van Poollen's estimate would give an initial fluid level of 208 meters below land surface.

Table 1. Range of calculated transmissivity values

Dates	Type of Test	Number of Tests	Range of Calculated Transmissivity (m^2/s)		
			Low	High	Average
Sept 19-20, 1961[a]	Injection	5	1.11×10^{-6}	4.05×10^{-5}	2.36×10^{-5}
Jan 1-3, 1962[a]	Injection	4	2.50×10^{-5}	9.13×10^{-5}	7.05×10^{-5}
Mar 8, 1962 Feb 20, 1966[a]	Change of Injection Rate	15	8.78×10^{-6}	3.13×10^{-5}	1.63×10^{-5}
Sept 2 to Oct 26, 1968[b]	Pumping	3	3.61×10^{-6}	1.05×10^{-5}	6.76×10^{-6}

a. Data from van Poollen (1966). b. Data from van Poollen (1969).

By the end of 1967, however, it became apparent that the earlier estimates of initial reservoir pressure were incorrect. On December 22, 1967, fluid level in the RMA well had already dropped to 350 meters below land surface (Hurr, 1977) and was continuing to fall off at a rate of 0.3 meters per day. In a later calculation, van Poollen (1968) revised the pressure estimate to 269 bars (fluid level at 923 meters below land surface). Water-level measurements since the beginning of 1968 indicate that this value is a more reasonable estimate of the initial downhole pressure in the Precambrian reservoir (Healy and others, 1968).

MATHEMATICAL MODEL OF THE RESERVOIR

Preliminary Model

To compute pressure build-up and the subsequent fall-off caused by fluid injection, a conceptualized reservoir must be constructed. In the present study, the reservoir is assumed to be composed of a series of connected vertical fractures, which are more or less parallel to one another and are generally aligned in the direction of the zone of earthquakes, N. 60° W. The reservoir is taken to extend in depth over the depth range of the earthquake hypocenters, i.e., from 3.7 to 7.0 km. As a further simplification, it is also assumed that fluid flow in the fractured reservoir can be approximated by flow in a porous medium so that a continuum model may be used. An oblique view of the idealized reservoir is shown in Figure 4.

A two-dimensional flow model was used to analyze the transient pressure in the reservoir during and after fluid injection. Although two-dimensional flow is a somewhat restrictive assumption it is acceptable for three reasons:

1. Pressure build-up computed from the two-dimensional model may be interpreted as the average pressure build-up over the depth of the reservoir (see Appendix A). A comparison between the horizontal distribution of earthquake epicenters and the depth-averaged pressure build-up is a reasonable approach to examining earthquake–pore pressure relationship.

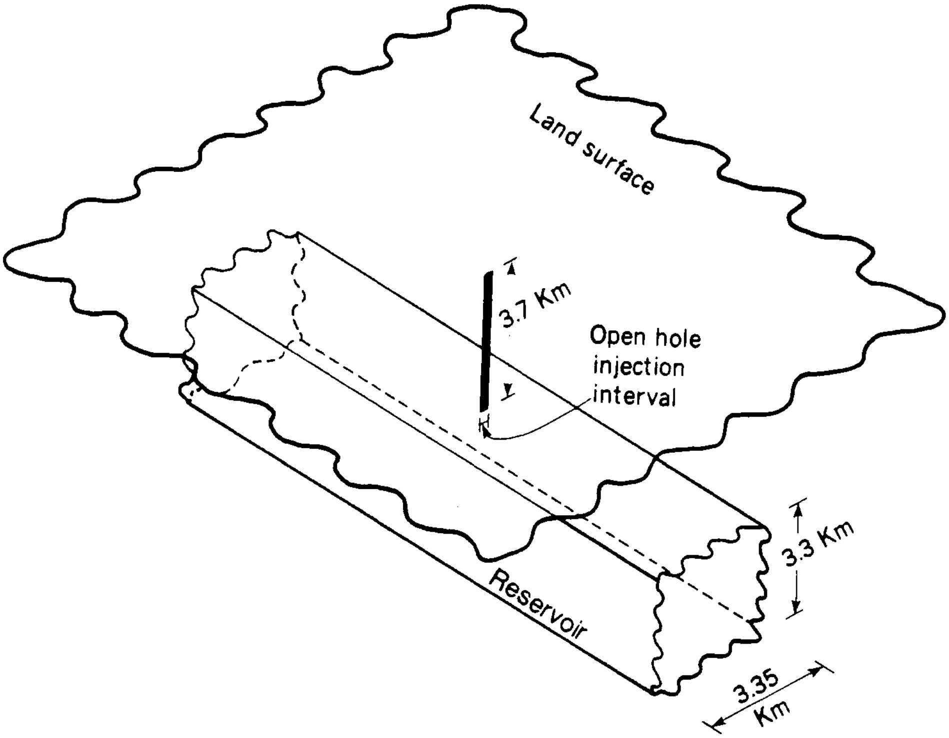

Fig. 4. Oblique view of idealized reservoir

2. The long-term seismic data (1962-1967) give only the horizontal spread of earthquake epicenters. Good depth data are only available for 1967 and 1968. Thus, even if the three-dimensional pressure field can be calculated, the earthquake–pore pressure comparison can only be made on a two-dimensional basis for most of the 11-year period.
3. Present available information is insufficient to warrant a three-dimensional analysis. For example, Snow (1968) studied the hydraulic character of fractured metamorphic rocks of the Front Range and found that fracture permeability decreased with depth due to increase in fracture spacing and decrease in fracture aperture. Unfortunately, the manner in which permeability varies with depth in the reservoir below the RMA is unknown.

Development of the partial differential equation governing depth-averaged hydraulic head build-up in the reservoir is given in Appendix A. The final form of the equation is

$$T\left(\frac{\partial^2 \bar{h}}{\partial x^2} + \frac{\partial^2 \bar{h}}{\partial x^2}\right) = \frac{\partial \bar{h}}{\partial t} - Q(t)\,\delta(x - x_0)\,\delta(y - y_0) \qquad (1)$$

where $\bar{h}$ is the depth-averaged build-up of hydraulic head above the initial water level, S and T are the storage coefficient and transmissivity of the reservoir, respectively, Q(t) is the variable injection rate, and x_0 and y_0 are the coordinates of the injection well. The depth-averaged pressure increase, Δp, can be computed directly from $\bar{h}$ by

$$\Delta p = \gamma \bar{h} \qquad (2)$$

where γ is the specific weight of the fluid.

The preliminary model of the reservoir in the Precambrian interval was a two-dimensional infinite strip. The injection well was represented by a point source located at the point halfway between the two impermeable boundaries. The infinite strip was aligned in the N. 60° W. direction, and the well was taken to be the origin of the x-y axis systems (Fig. 5).

Assuming that hydrostatic conditions existed initially throughout the reservoir, the analytical solution of equation (1) can be obtained by using the solution for a point source in an infinite two-dimensional reservoir and applying image well theory (Ferris and others, 1962). For a constant injection rate Q, the hydraulic head build-up is given by

$$\bar{h}(x,y,t) = \frac{Q}{4\pi T} \sum_{m=-\infty}^{\infty} W\left(\frac{[x^2 + (y + mw)^2]S}{4Tt}\right) \tag{3}$$

where t is the time from the start of injection, w is the width of the strip, and W is the well function. If the injection rate varies with time in a step-like fashion (e.g., using monthly averages), then the superposition theorem can be applied and the build-up after n different injection rates is given by

$$\bar{h}(x,y,t) = \frac{1}{4\pi T} \sum_{i=1}^{n} (Q_i - Q_{i-1}) \sum_{m=-\infty}^{\infty} W\left(\frac{[x^2 + (y + mw)^2]S}{4T(t - t_{i-1})}\right) \tag{4}$$

where t_i is the starting time of period i and Q_i is the injection rate for that period. (Note that $t_0 = 0$ and $Q_0 = 0$.) The hydraulic head after shut-in can be computed by setting the last injection rate to zero.

For this model, the parameters to be estimated are transmissivity (T), storage coefficient (S), and the width of the infinite strip (w). In

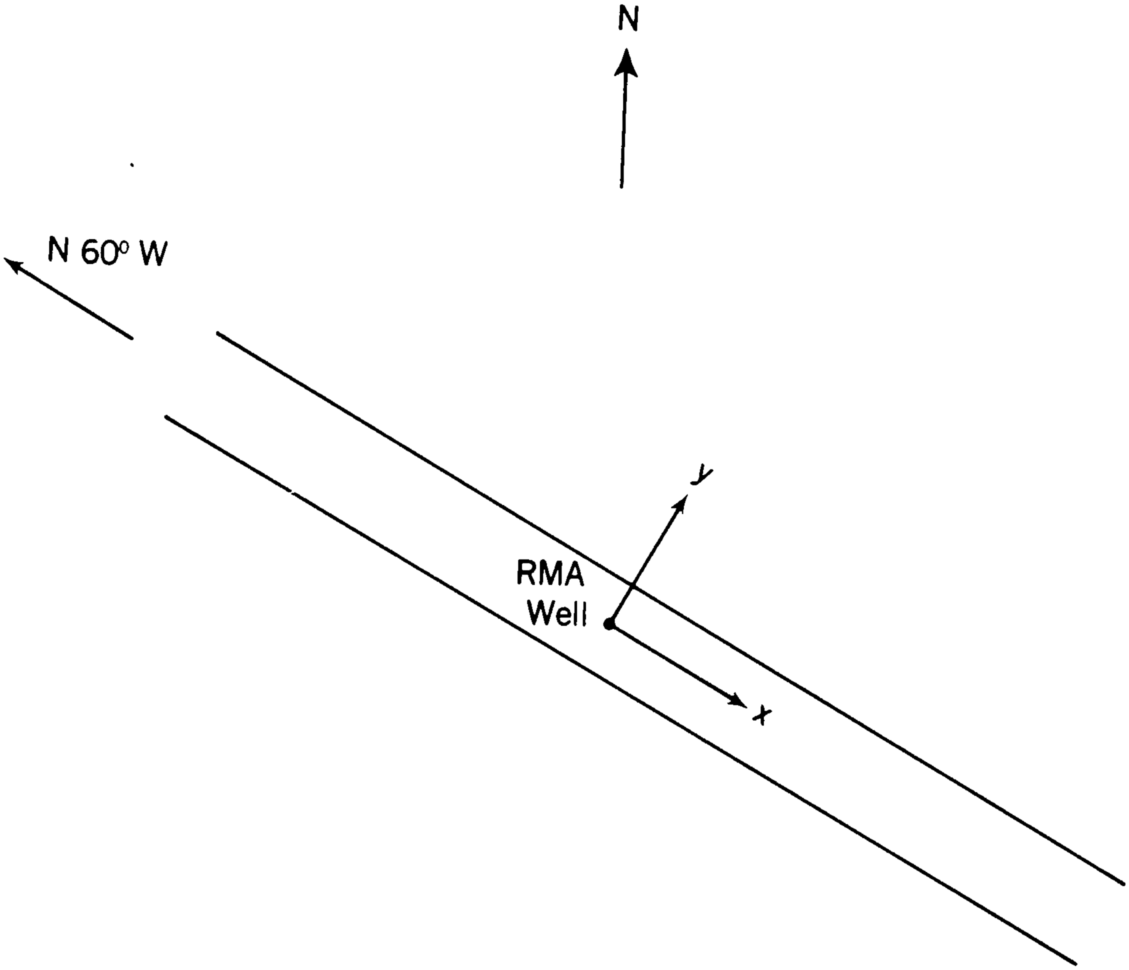

Fig. 5. Plan view of the infinite strip reservoir used for analysis

addition, the initial depth of the water level (d_0) must also be determined to compute depth to water.

Calibration of the Preliminary Model

The preliminary model was calibrated using the history of observed water levels in the disposal well after waste injection was discontinued (Fig. 6). These measurements were made by the U.S. Geological Survey as part of a continuous well monitoring program. The fall-off data (Hurr, 1977) were taken over a period of 9 years (February 1966 to March 1975) following final shut-in. Only the earlier data were available to previous investigators. The later measurements added significant information to the present analysis.

The purpose of the model calibration was to find appropriate values for T, S, w, and d_0 so that, given the injection history, the analytical solution would produce a fall-off curve that would closely match the fall-off data observed in the RMA well. It was expected from the outset that there would probably be no unique solution to this calibration problem. The purpose, rather, was to determine parameters consistent with values calculated in previous studies.

The calibration method was basically one of trial and error. As a first guess values obtained from well tests were used, i.e., $T = 1.63 \times 10^{-5}$ m^2/s and $d_0 = 923$ m. The storage coefficient (S) was arbitrarily set to 1.0×10^{-5}, and the width of the infinite strip was set to 3 km. The hydraulic head build-up in the well was computed by equation (4), using a distance of 0.086 m (radius of the open hole) from the point source. (The exact distance is unimportant because the hydraulic head distribution near the well was relatively uniform during fall-off.) The

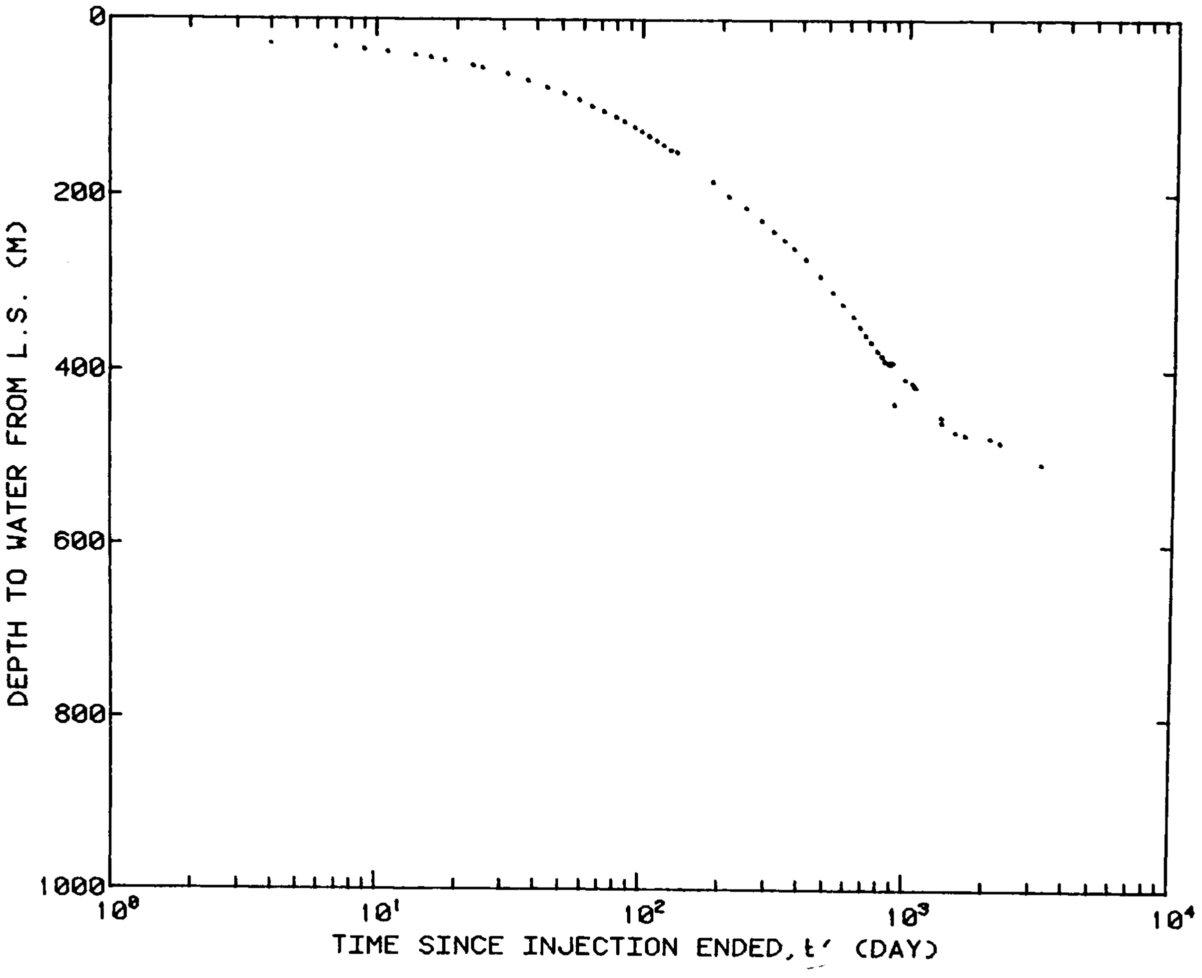

Fig. 6. Observed water levels in the RMA disposal well since injection ended

computed fall-off curve is shown in Figure 7. A comparison between the computed curve and the observed data shows immediately that the original estimate of d_0 is too large. Notice, however, that if the initial depth were decreased by 160 meters to 763 meters below land surface, the computed curve would match the observed data remarkably well (Fig. 7).

From here on the calibration process consisted of changing one or more of the parameters, computing the fall-off curve, and comparing it with the observed data. This procedure was repeated systematically until a set of parameters that generated a fall-off curve that fitted the observed data to a satisfactory degree was found. The parameters that gave the best fit were found to be: $T = 1.08 \times 10^{-5}\ m^2/s$, $S = 1.0 \times 10^{-5}$, $w = 3.35$ km, and $d_0 = 813$ m.

A comparison of the fall-off curve computed using the best-fit parameters with the observed data is shown in Figure 8. Although the fit is not perfect and the residuals are correlated, such imperfections are not unexpected, considering the fact that a greatly simplified model was used to simulate the reservoir in what must be a highly complex system of fractures beneath the Denver Basin.

It should be noted, however, that a significant difference in the shape of the computed and observed fall-off curves can be seen for the later times. After approximately 1,000 days from shut-in, the observed data exhibit a sharp decrease in the rate of fall-off. Such a feature was not found in any of the fall-off curves generated during the calibration process. It was decided that the infinite strip reservoir model must be modified to incorporate this feature of the observed fall-off curve.

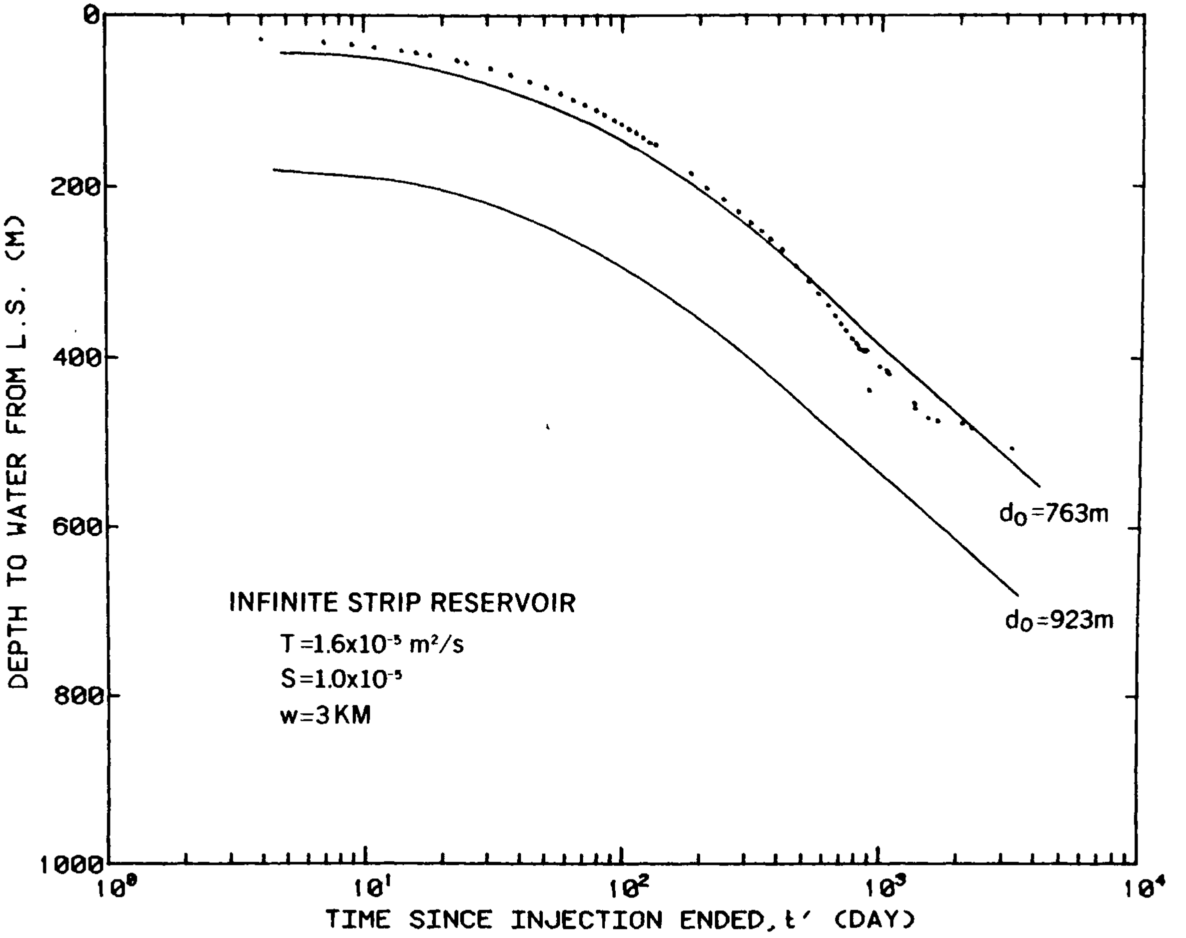

Fig. 7. Comparison of computed versus observed water levels following injection in the RMA disposal well: first trial for infinite strip reservoir model

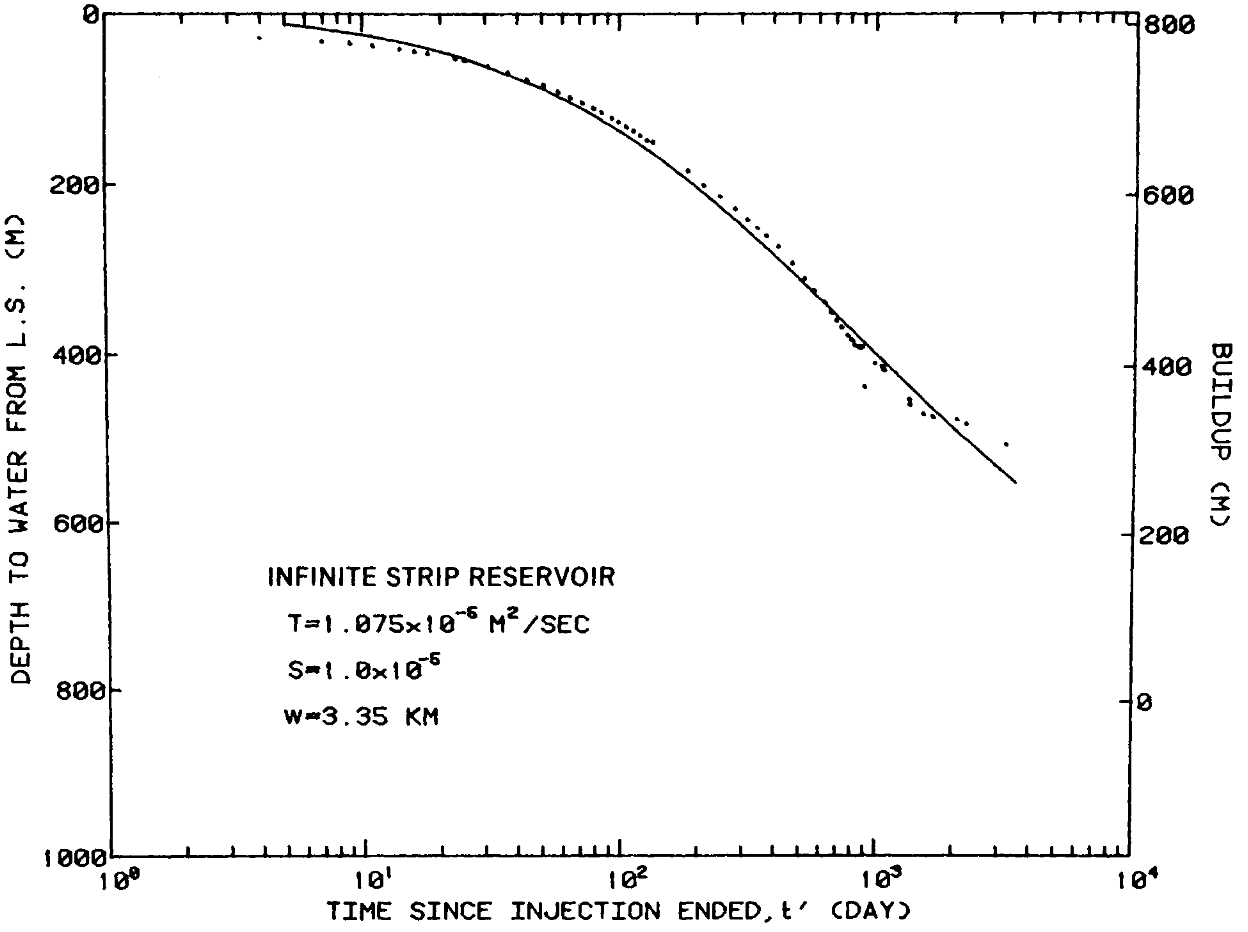

Fig. 8. Comparison of computed and observed water levels: best fit for infinite strip reservoir model

Modification of the Preliminary Model

One method of producing a sudden change in fall-off rate is to replace the infinite strip model by a semi-infinite strip model with an impermeable boundary at one end. If the impermeable end were located sufficiently far from the well, then the fall-off curves for early times computed by both models would be the same. For later time, however, the effect of the impermeable end would no longer be negligible and the fall-off rate in the semi-infinite strip would be slower than in the infinite strip.

The analytical solution of equation (1) for a semi-infinite strip reservoir is given by

$$h(x,y,t) = \frac{1}{4\pi T} \sum_{i=1}^{n} (Q_i - Q_{i-1}) \sum_{m=-\infty}^{\infty} \{ W\left(\frac{[x^2 + (y + mw)^2]S}{4T(t - t_{i-1})}\right) + W\left(\frac{[(x + 2\ell)^2 + (y + mw)^2]S}{4T(t - t_{i-1})}\right)\} \tag{5}$$

Note that an additional model parameter, the distance (ℓ) from the point source to the impermeable end, has now been introduced.

Calibration of the Modified Model

The calibration procedure for the modified model was essentially the same trial-and-error method as before. The values for T, S, w, and d_0 were kept the same as in the infinite strip model. Only the value of ℓ was varied. This procedure would yield a fall-off curve identical to that of the infinite strip model for early time. The best-fit value for ℓ was found to be 30.5 km. A comparison between the computed and observed fall-off curves is shown in Figure 9.

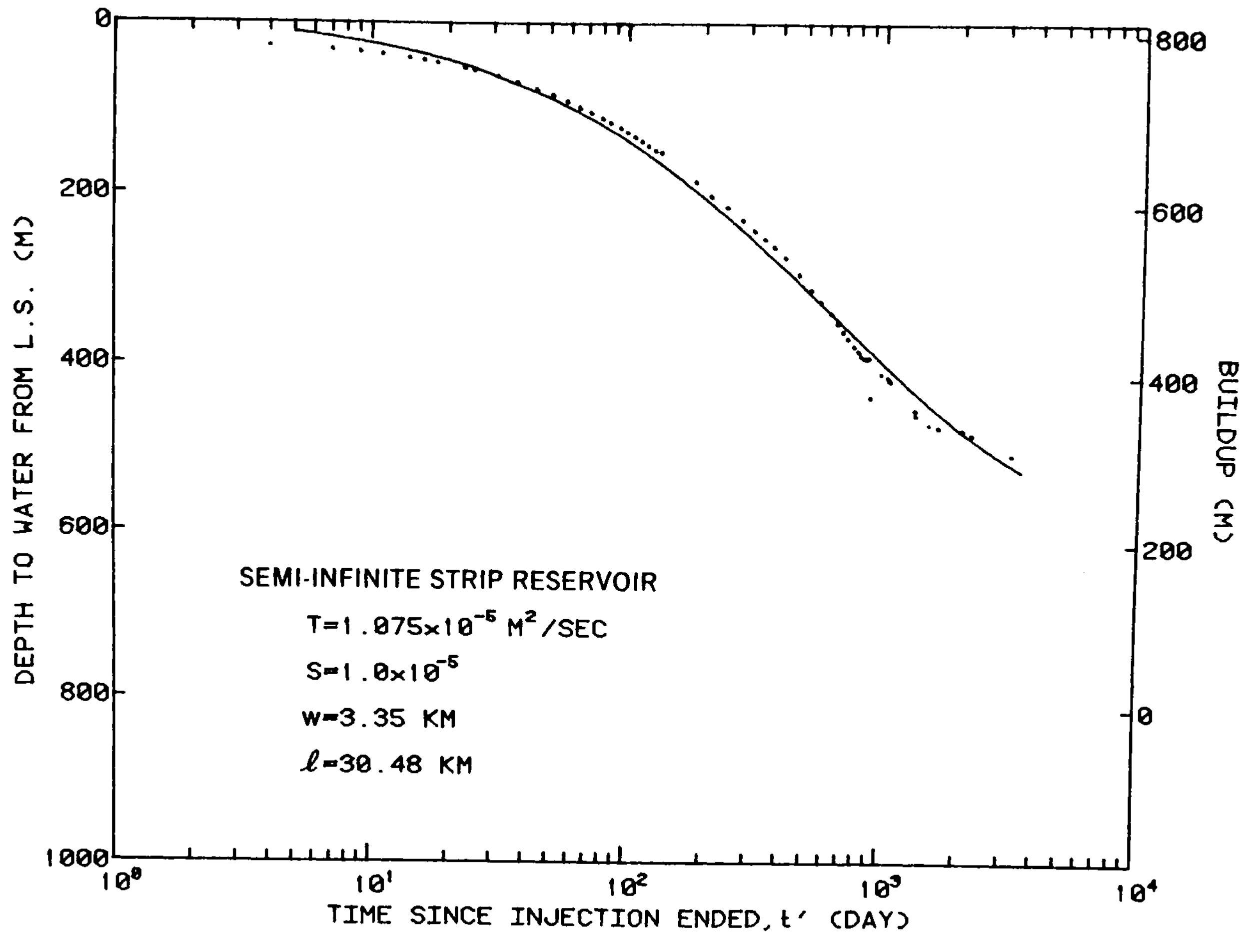

Fig. 9. Comparison of computed and observed water levels: best fit for the semi-infinite strip reservoir model

To investigate whether the impermeable end was due to the discontinuity in the Precambrian rocks at the Front Range of the Rocky Mountains, a Precambrian structural map of the Denver Basin (Haun, 1968) was examined. This map showed that the mountain front was considerably farther than 30 km from the RMA well, but the best-fit value of ℓ placed the impermeable end in an area in which the reservoir is intersected by a set of vertical faults trending northeasterly (Fig. 10). Displacements along these faults were found to be vertical (Haun, 1968). The linear reservoir in the Precambrian rocks may thus have been rendered discontinuous by vertical displacements along these northeasterly trending faults.

The semi-infinite strip reservoir model was considered to be the best model to explain the fall-off data observed in the RMA well, because this model is supported by hydrologic, geophysical, and geologic evidence. In particular,

1. The transmissivity of the semi-infinite strip model was calibrated using data recorded over a period of 9 years. In contrast, transmissivity estimated in previous studies was calculated by the Horner method, which assumes an infinite reservoir, using pressure data recorded over periods of days or hours. Both the long-term and short-term data lead to similar estimates of reservoir transmissivity.
2. The width of the semi-infinite strip reservoir determined by the reservoir analysis closely approximates the width of the observed seismic zone. This correlation further supports the hypothesis that a wide fault or fracture zone exists in the Precambrian rocks beneath the RMA.

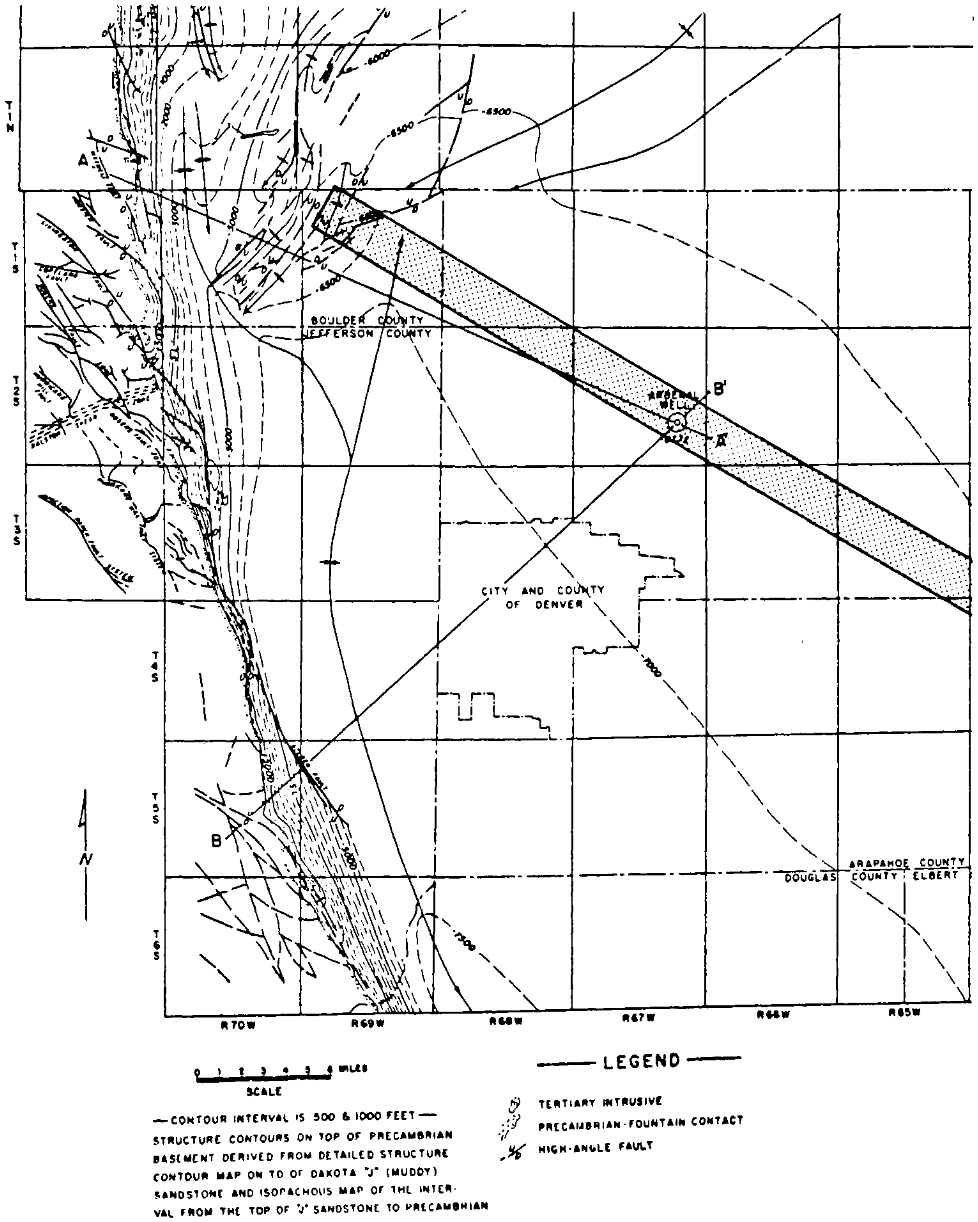

Fig. 10. Precambrian structural contour map of the Denver Basin with the semi-infinite strip reservoir indicated. --Modified from Haun (1968)

3. The position of the impermeable end of the semi-infinite strip as determined by hydrologic modeling of the reservoir is supported by geologic evidence of vertical faulting.
4. The calibrated initial fluid pressure falls within the range estimated in previous studies.

SEISMIC DATA

Two seismic recording stations were in operation when the first Denver earthquake occurred in April 1962. One station was located 34 km west of Denver at Bergen Park and was operated by the Colorado School of Mines. The other station was located at Regis College in Denver. Between 1966 and 1968, additional seismic recording networks were installed by the U.S. Geological Survey and the Colorado School of Mines in the Denver area and at the RMA.

Due to high background noise from the Denver area, the seismograph at Regis College was operated at low magnification. The Regis record may therefore be incomplete because earthquakes of small magnitudes were undetected. The U.S. Geological Survey network was installed after 1966 and was in operation for a few years. Seismograms from the Bergen Park station provide the only continuous reliable record of earthquake activity in the Denver area since 1962.

Major and Simon (1968) presented a seismic study of the Denver earthquakes using seismograms from the Bergen Park observatory for the period from April 1962 to August 1967. By measuring the time interval between P and S wave arrivals and calculating the ratio of amplitudes on the north-south and east-west seismograms (defined as the N-E ratio), they were able to determine the approximate distance between an earthquake epicenter and the observatory and the apparent direction of the epicenter from the observatory. The study was limited to those earthquakes that produced first motions large enough to be measured accurately but small enough not to be off scale.

Results from the seismic study showed that the time interval between P and S wave arrivals from the Denver earthquakes were nearly the same but there were significant variations in the direction from the observatory to the epicenters as determined by the N/E value. From these observations, Major and simon concluded that most of the earthquakes occurred about 44 km from the observatory and that the width of the active zone was probably less than 6.4 km. The area extent of this zone is shown in Figure 11.

Figure 12, taken from the same study, shows the azimuthal distributions of the Denver earthquakes observed at Bergen Park from 1962 to 1967. The distributions indicate that most of the Denver earthquakes occurred between N/E values of 0.3 and 0.6. A comparison of this zone of concentrated earthquake activity with the epicentral zone determined by Healy and others (1966) shows that the Bergen Park data are consistent with the data recorded by seismic arrays at the RMA (Fig. 11).

Major and Simon have noted an interesting phenomenon in the azimuthal distributions of the Denver earthquakes. There seemed to be a slow migration of the center of maximum activity to the northwest (in the direction of the higher N/E values). This phenomenon was also observed in a later seismic investigation by Hoover and Dietrich (1969).

To extend the azimuthal study for the period after August 1967, seismograms from the Bergen Park observatory for the period from September 1967 to December 1972 were obtained. A catalog of Denver earthquakes during this period was provided by Presgrave (1978).

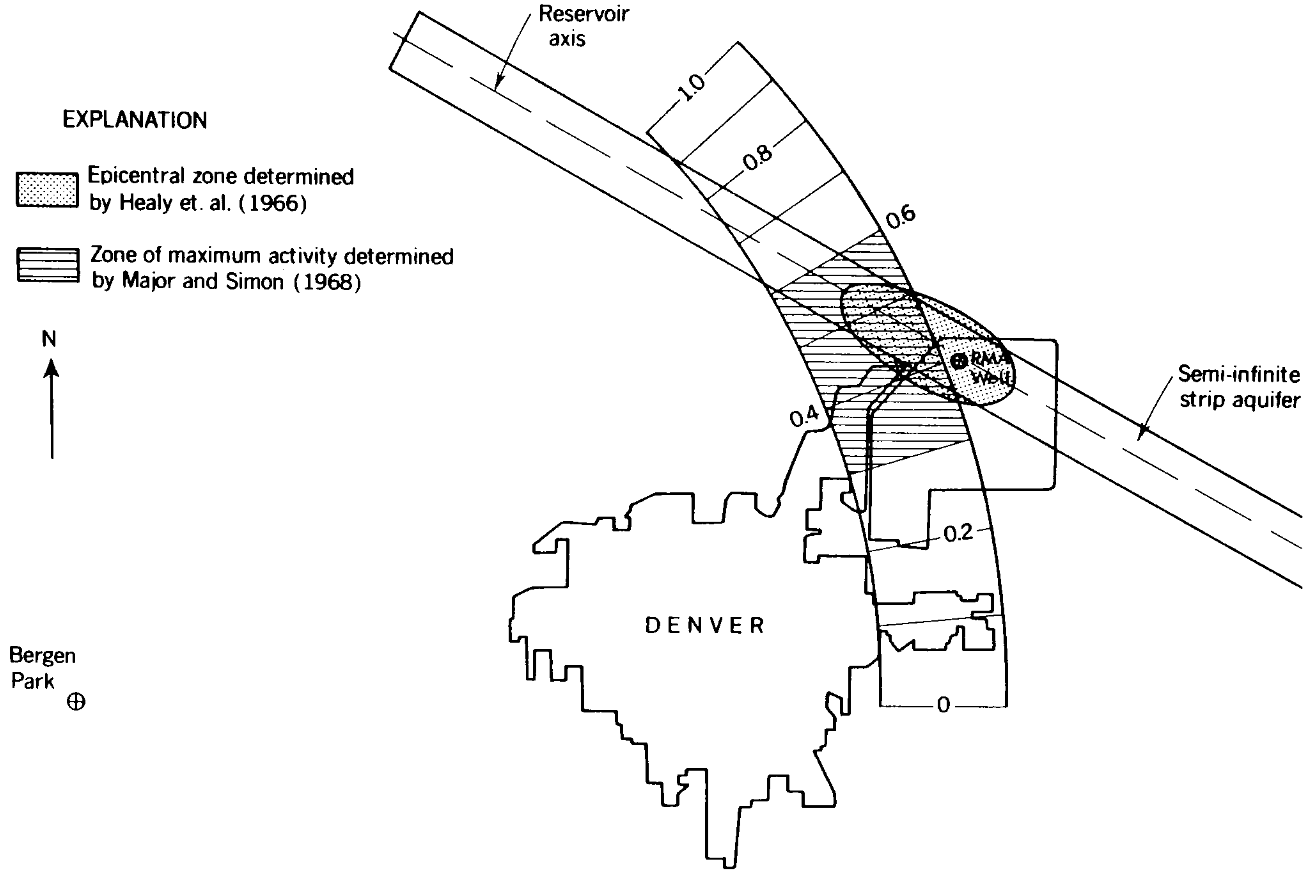

Fig. 11. Map showing the semi-infinite reservoir and its relationship to epicentral zone of earthquakes mapped by Healy and others (1966) and epicentral zone determined by Major and Simon (1968). -- Numbers are N/E values.

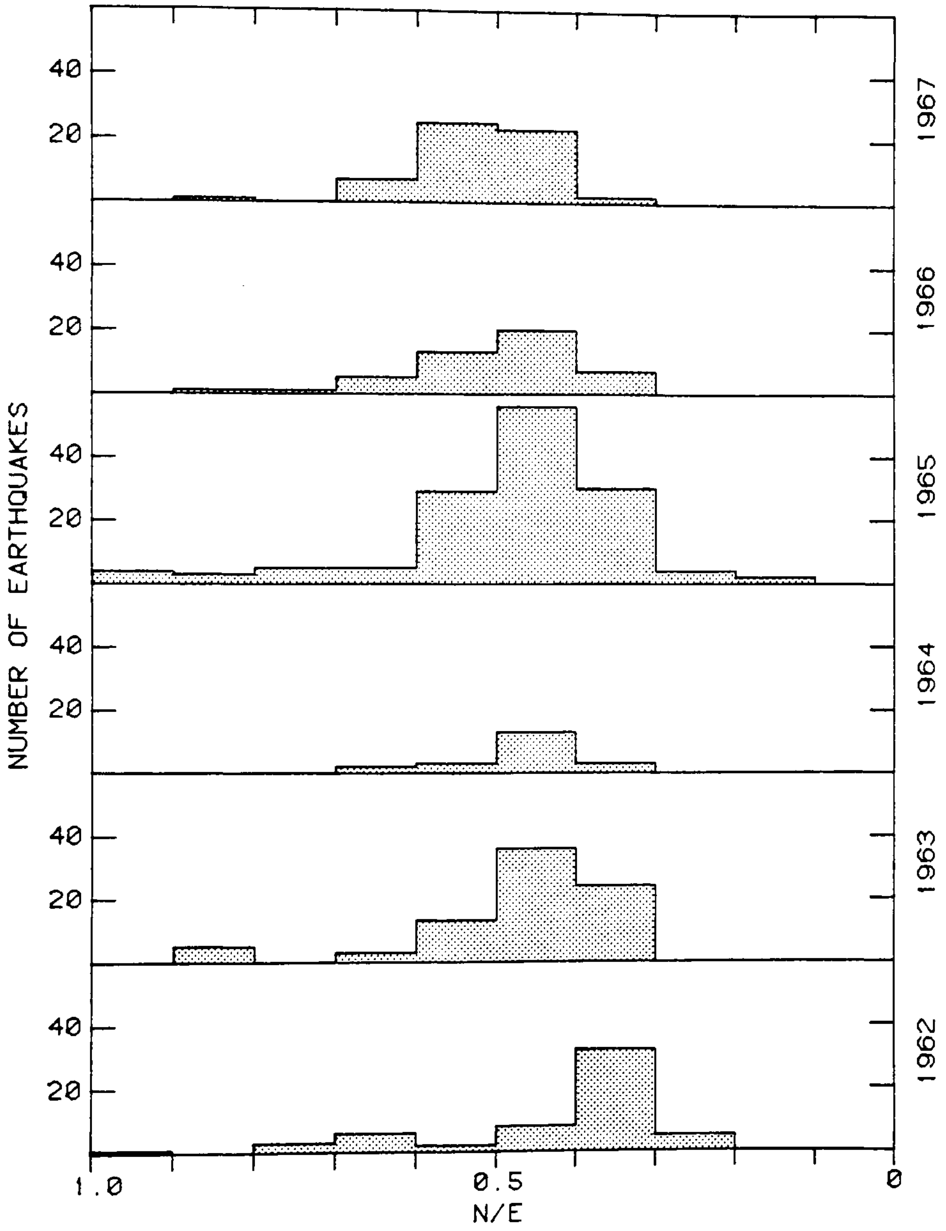

Fig. 12. Azimuthal distributions of Denver earthquakes observed at Bergen Park from April 1962 to August 1967. -- Major and Simon (1968).

Following the same method used by Major and Simon, a similar azimuthal study of the Denver earthquakes was conducted. The azimuthal distributions from 1968 to 1972 are shown in Figure 13. Due to the significant decrease in the number of earthquakes, the northwestward migration noted by Major and Simon is no longer observable. In general, however, the maximum activities are still centered about zones of higher N/E values.

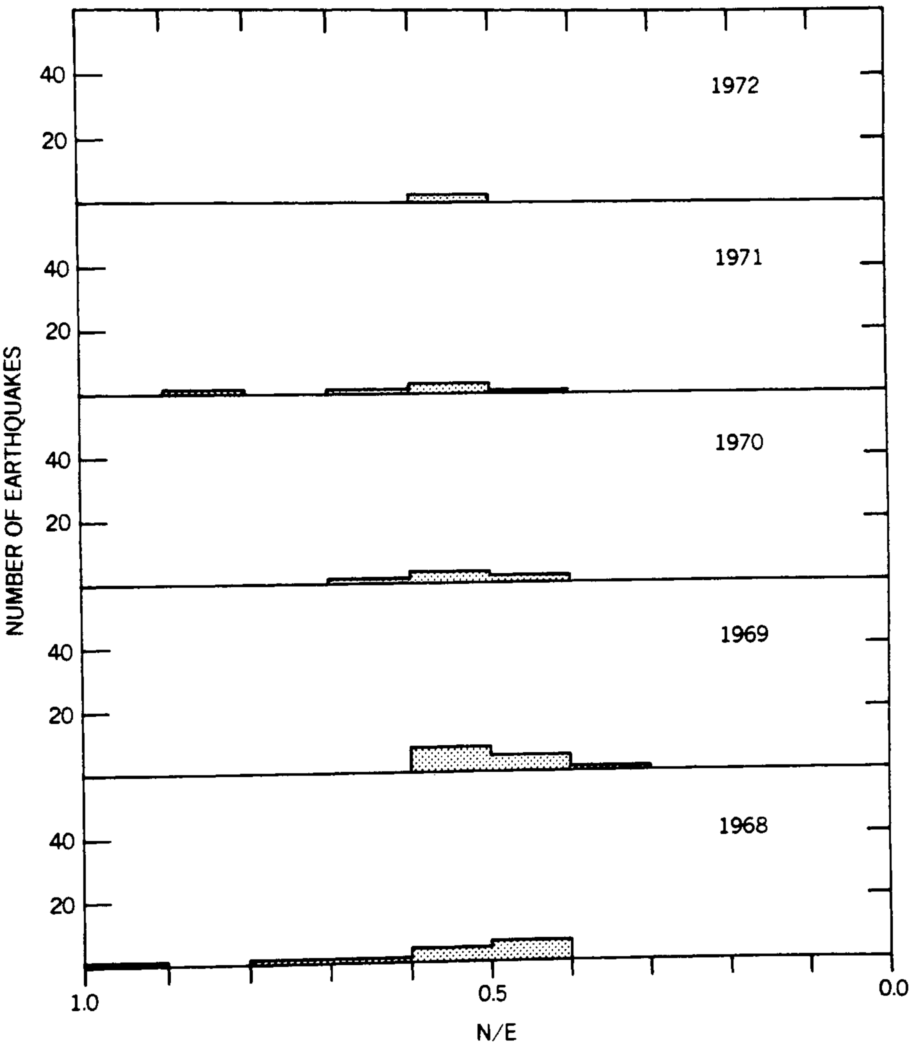

Fig. 13. Azimuthal distributions of Denver earthquakes observed at Bergen Park from 1968 through 1972

RELATIONSHIP BETWEEN EARTHQUAKES AND RESERVOIR PRESSURE

Having determined a likely model for the Precambrian reservoir, the pressure build-up caused by fluid injection can be computed. If the Denver earthquakes were related to the waste injection program at the RMA, a relationship will most likely be found between pressure build-up in the reservoir and earthquake epicenters. In this section the relationship between the horizontal distribution of the computed reservoir pressure build-up and that of the earthquake epicenters for the period 1962 to 1972 will be examined.

There is one problem that makes the direct comparison of reservoir pressure build-up with earthquake location more difficult. While the pressure build-up is computed on a semi-infinite strip, the earthquake distributions (from the azimuthal study described earlier) are given in the active zone defined by two concentric arcs and two lines emanating from Bergen Park. To facilitate a meaningful comparison between pressure and earthquake distributions, it is necessary to choose a common ground on which the distributions can be compared.

The pressure–earthquake comparison was made along the axis of the reservoir. Because flow in the reservoir is essentially linear, pressure variations across the width will be small, except near the well. Thus, the pressure profile along the reservoir axis will be a good indication of the overall pressure distribution in the reservoir.

To construct the spatial distribution of earthquake frequencies along the reservoir axis, the axis line was divided into 10 segments, using the 11 points of intersection between the reservoir axis and the 11 lines, with N/E values of 0 through 1.0, emanating from Bergen Park. The number of earthquakes in each section of the active zone is then lumped into the corresponding segment of the reservoir axis. Constructed in this manner, the bar graphs for spatial distribution of earthquake frequencies will have a horizontal scale in terms of distance along the reservoir axis and the divisions between N/E values will be progressively smaller as the ratio increases from 0 to 1.0.

Figures 14 through 22 permit comparisons of the reservoir pressure build-up and spatial distribution of earthquake frequencies along the reservoir axis for the nine characteristic periods from 1962 to 1972 given in Table 2. For each period two graphs are given. The upper graph shows the distribution of earthquakes for a period, and the lower graph shows the computed reservoir pressure build-up along the reservoir axis for the first, middle, and last months of the same period.

If the Denver earthquakes were caused by excessive pressure build-up in the reservoir, there should be a critical or threshold value above which earthquakes will occur. (The earthquake mechanisms will be discussed later.) This critical value can be estimated in the following way. Because the earthquake activity essentially ceased by the end of 1972, the pressure build-up everywhere in the reservoir must have dropped below the critical value by that time. The maximum pressure build-up during January 1973 was computed to be 32 bars. This value was taken to be the critical value for the pressure build-up in the reservoir.

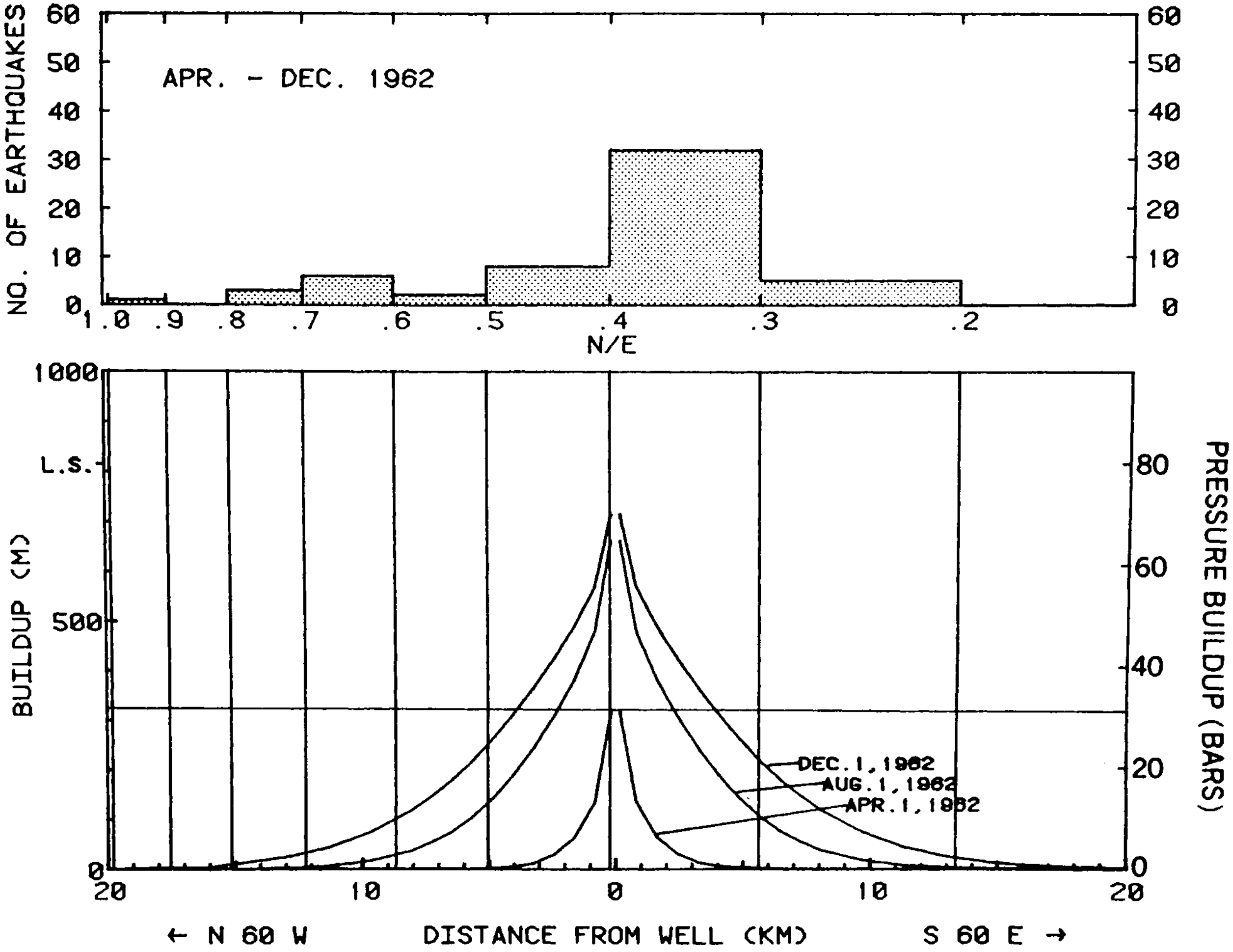

Fig. 14. Spatial distribution of earthquake frequencies and computed pressure build-up along reservoir axis, April-December, 1962

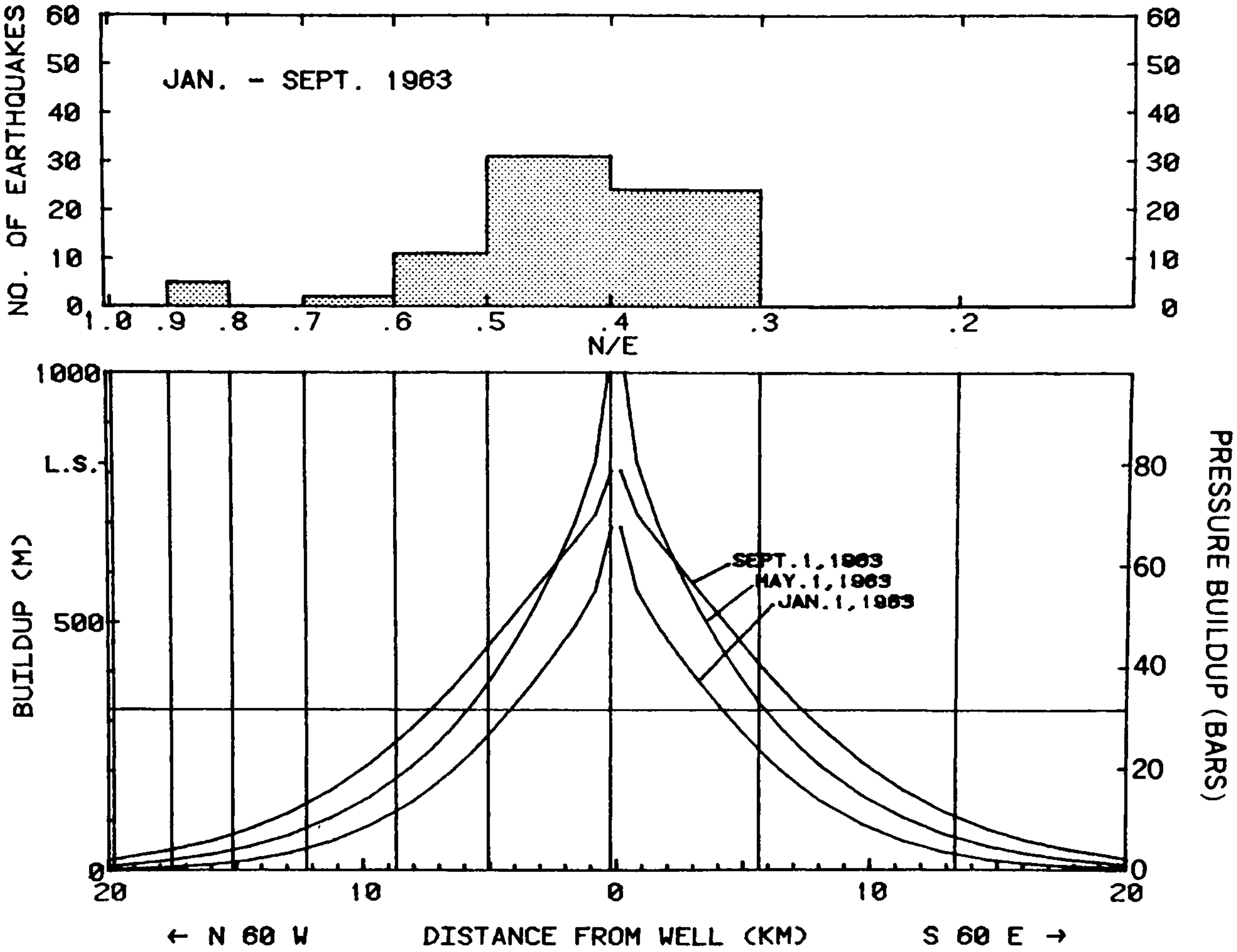

Fig. 15. Spatial distribution of earthquake frequencies and computed pressure build-up along reservoir axis, January-September, 1963

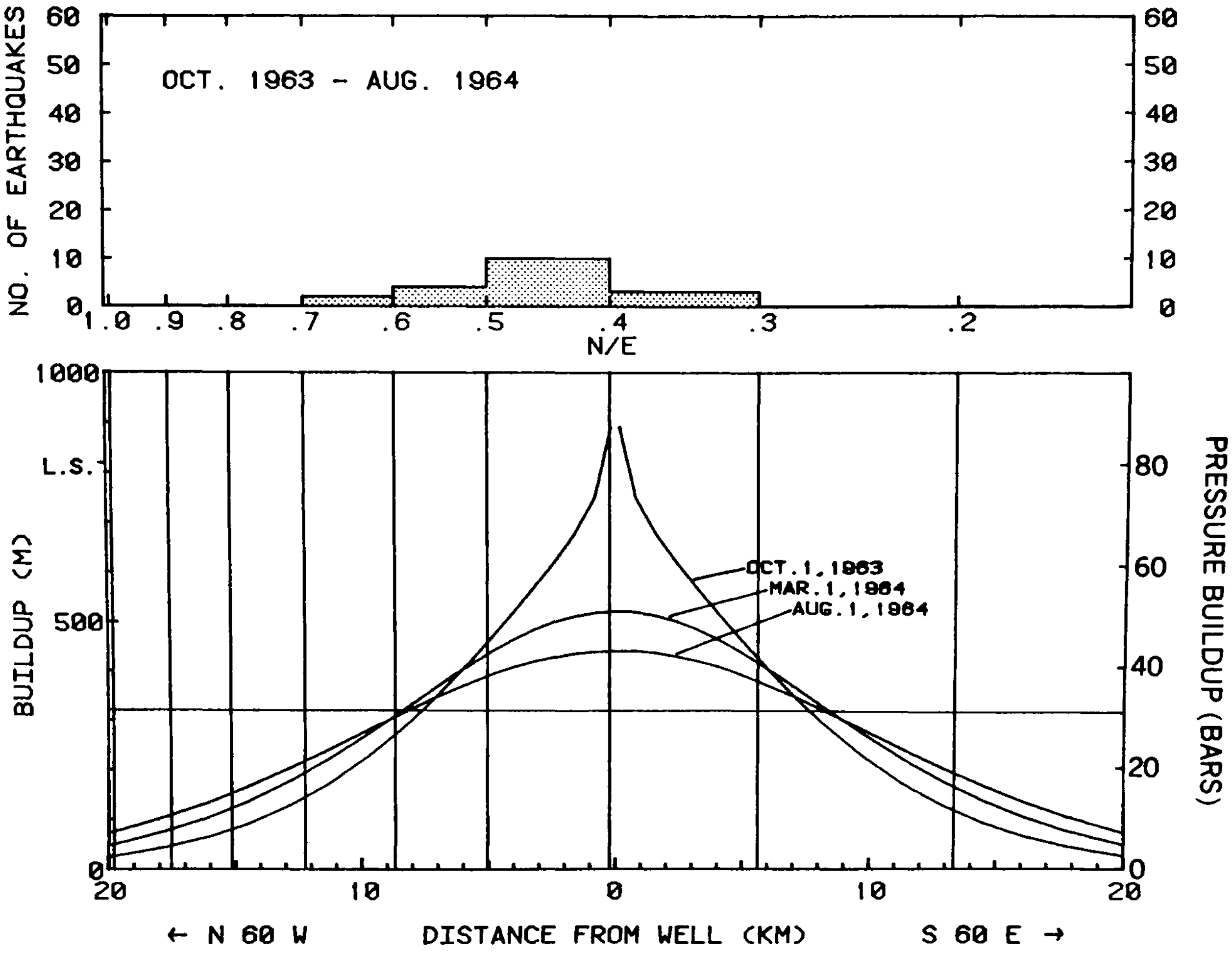

Fig. 16. Spatial distribution of earthquake frequencies and computed pressure build-up along reservoir axis, October 1963-August 1964

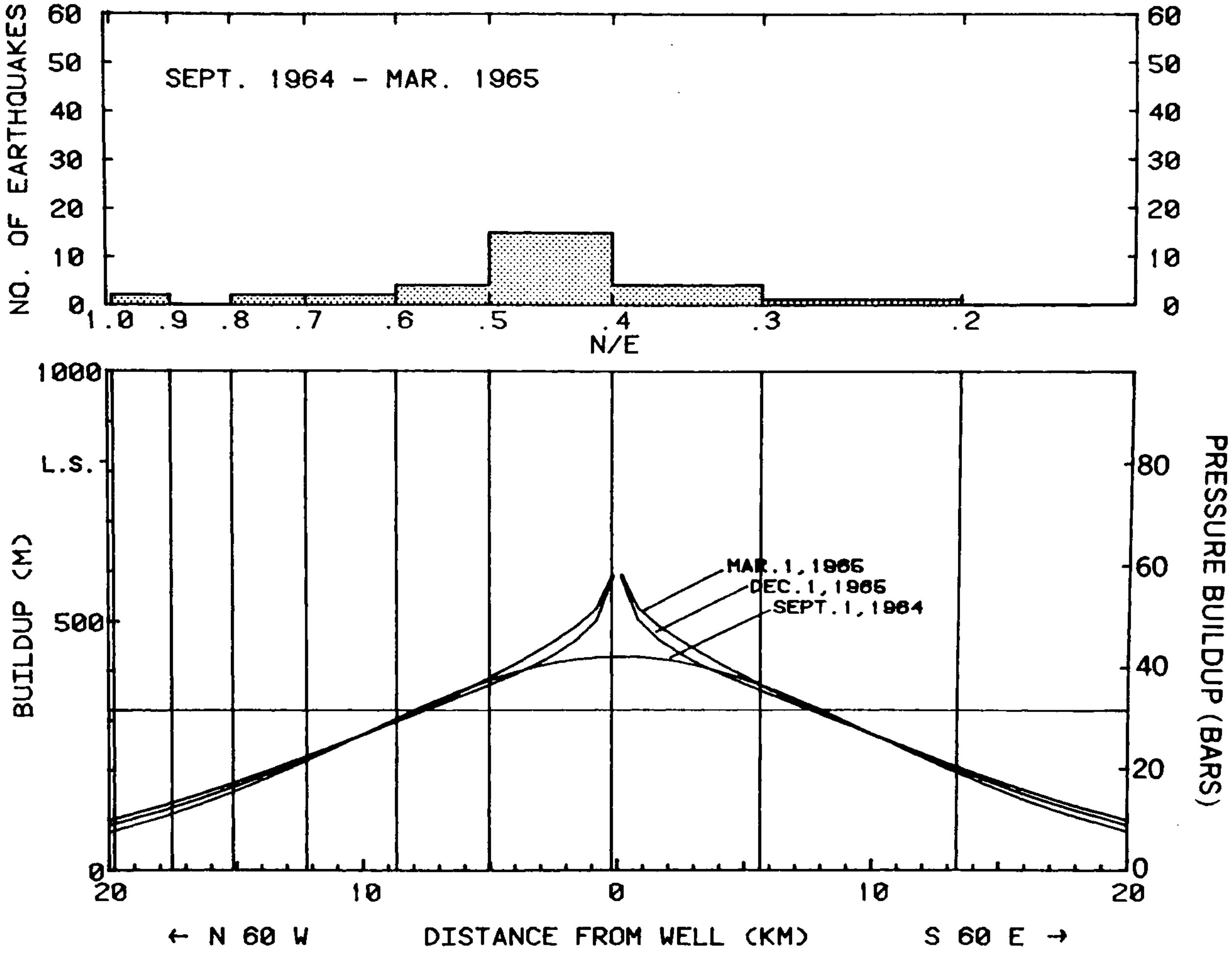

Fig. 17. Spatial distribution of earthquake frequencies and computed pressure build-up along reservoir axis, September 1964-March 1965

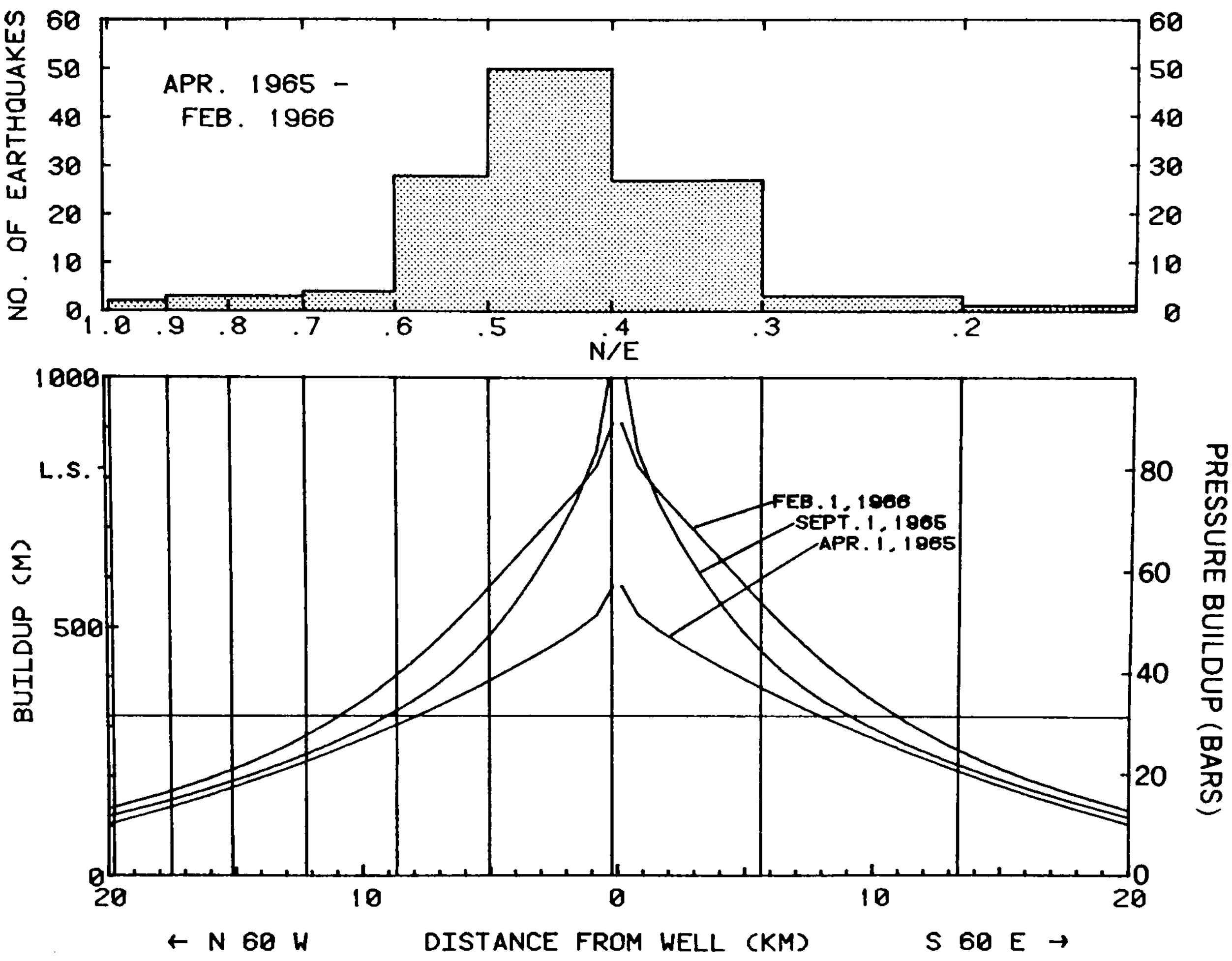

Fig. 18. Spatial distribution of earthquake frequencies and computed pressure build-up along reservoir axis, April 1965-February 1966

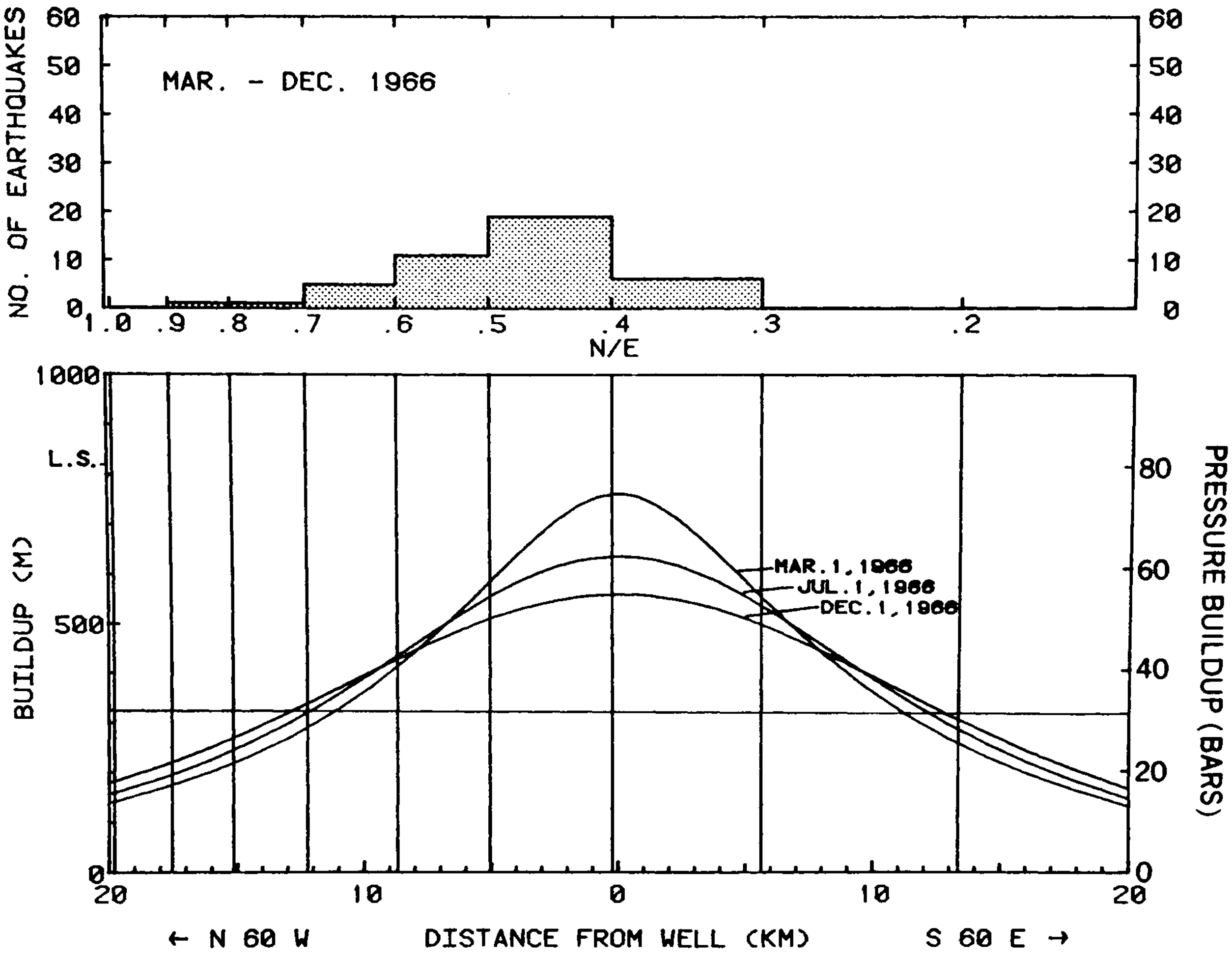

Fig. 19. Spatial distribution of earthquake frequencies and computed pressure build-up along reservoir axis, March-December, 1966

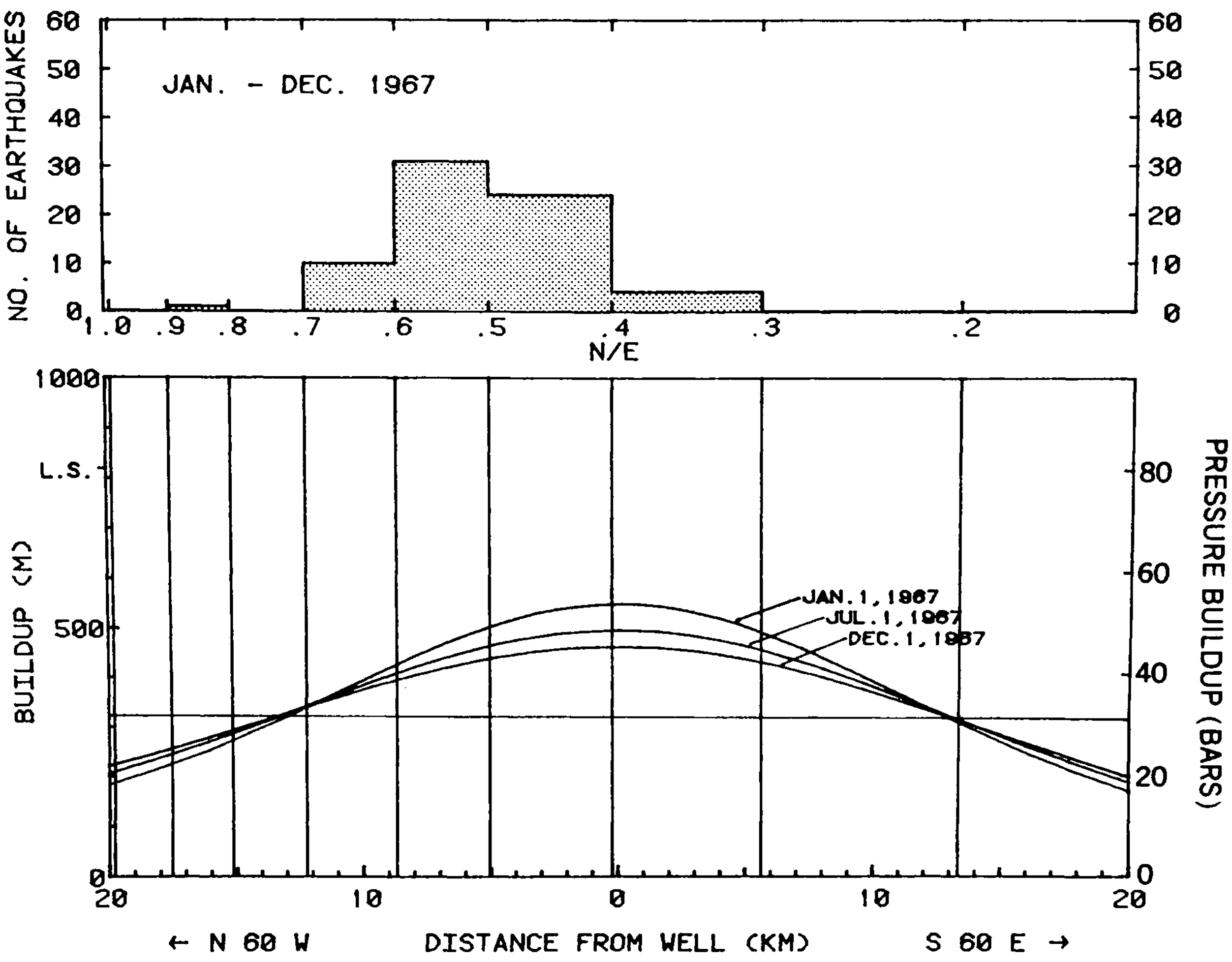

Fig. 20. Spatial distribution of earthquake frequencies and computed pressure build-up along reservoir axis, January-December, 1967

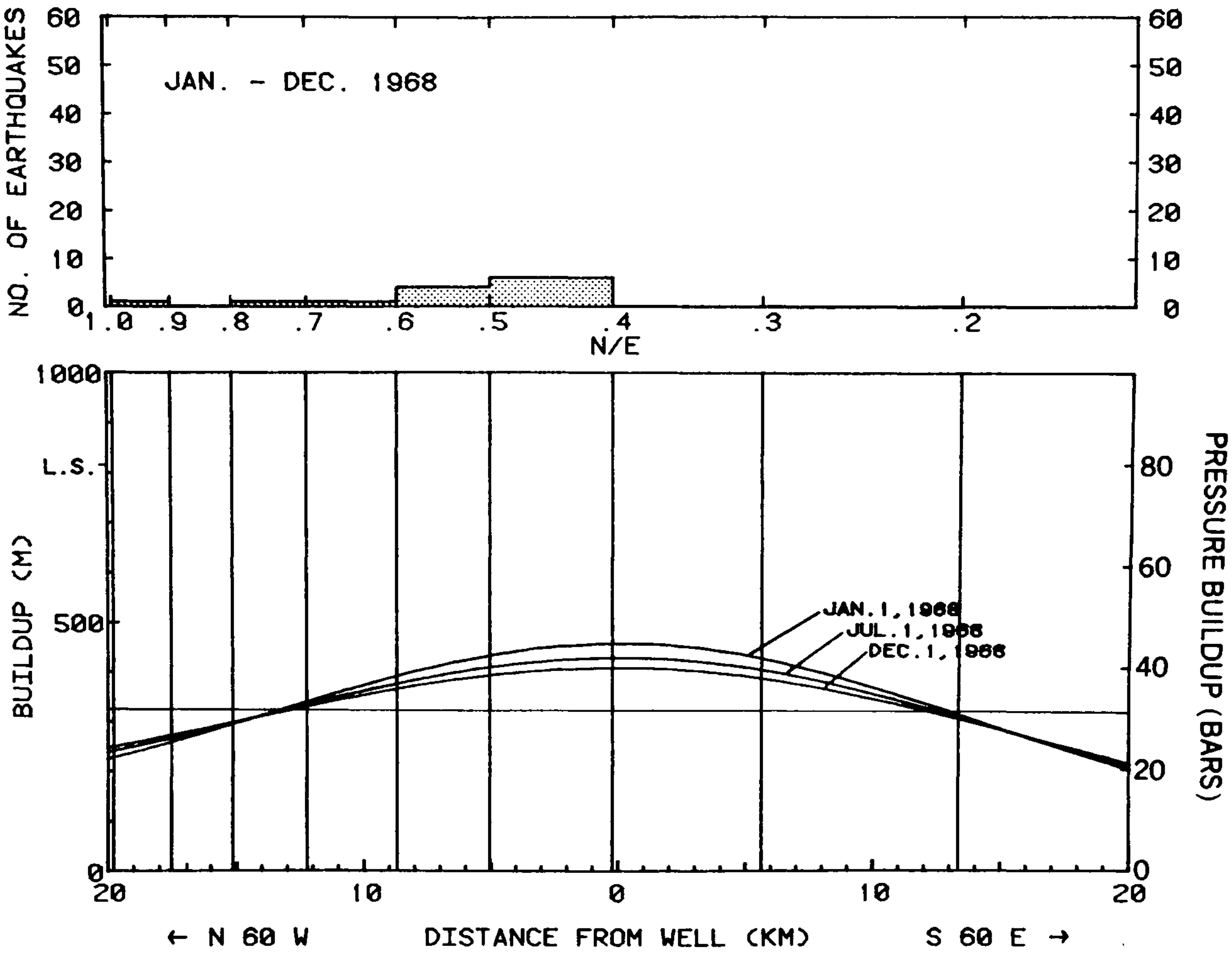

Fig. 21. Spatial distribution of earthquake frequencies and computed pressure build-up along reservoir axis, January-December, 1968

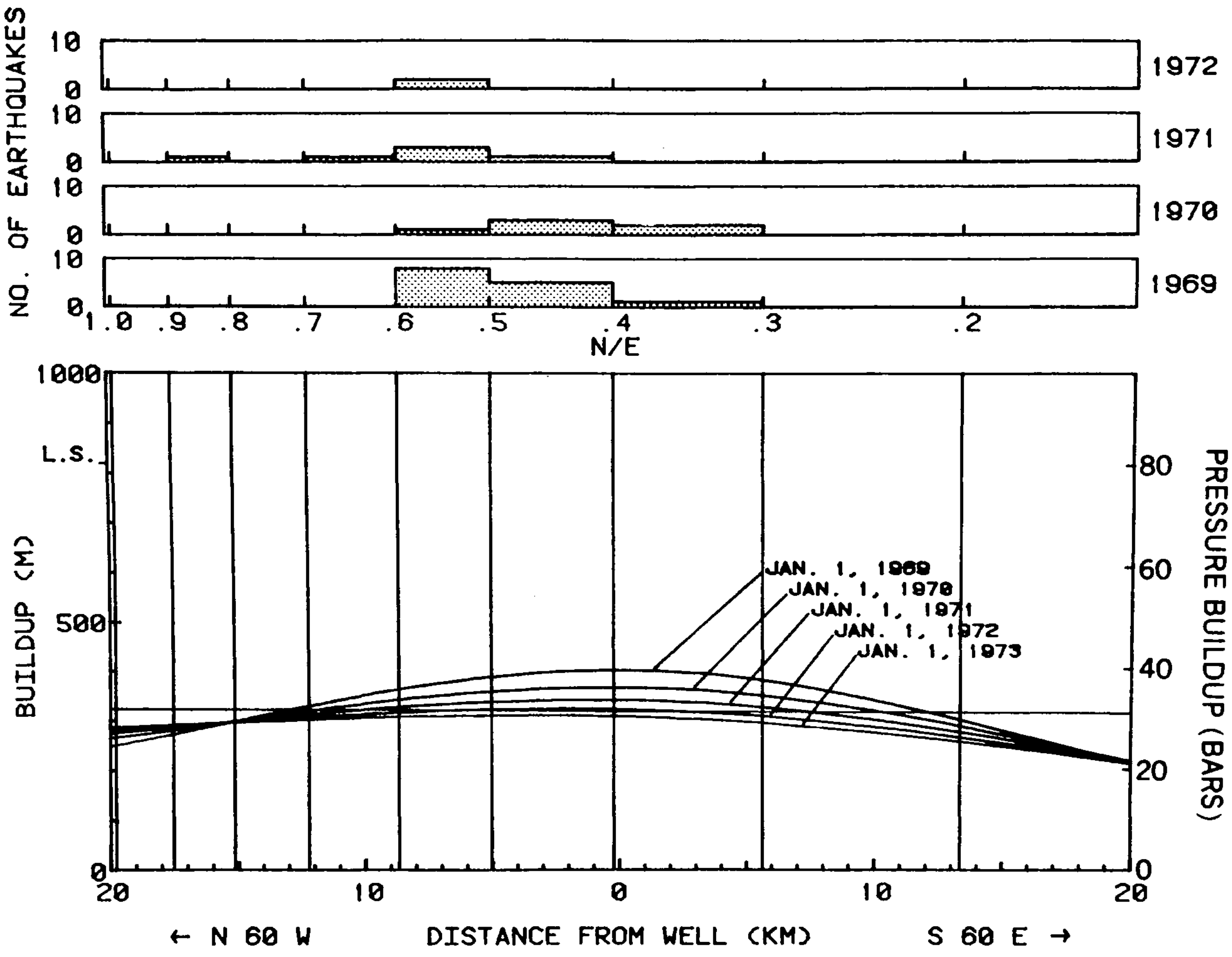

Fig. 22. Spatial distributions of earthquake frequencies and computed pressure build-ups along reservoir axis, 1969, 1970, 1971, 1972

Table 2. Characteristics of the nine time periods for which spatial distribution of earthquake frequencies and computed pressure build-up are compared

Period	Operation
April-December 1962	Pressure injection
January-August 1963	Pressure injection
October 1963-August 1964	Shut-in
September 1964-March 1965	Gravity injection
April 1965-February 1966	Pressure injection
March-December 1966	Shut-in
January-December 1967	Shut-in
January-December 1968	Shut-in
January 1969-January 1973	Shut-in

Examination of Figures 14 through 22 reveals that the spatial distribution of earthquake frequencies is indeed governed by the pressure build-up according to the critical pressure build-up hypothesis. Horizontal lines were drawn corresponding to the 32 bars on each graph of pressure build-up. The figures show that earthquakes are largely confined to that part of the reservoir where the pressure build-up is above the critical value. In addition, northwestward migration of the earthquake activity, as noted by Major and Simon (1968), can now be explained by the outward propagation of the critical pressure build-up from the injection well. This feature is best illustrated in Figures 18, 19, and 20. As the critical pressure build-up propagates from N/E value of 0.6 to 0.7, the number of earthquakes in this section also increases significantly.

It should be noted that there is a consistent lack of seismic activity in the section southeast of the well between N/E values of 0.2 and 0.3, even when the pressure build-up in this section exceeds the critical value. The present model cannot explain this observation. Such a lack of activity may be attributed to several possible factors among which are changes in the regional stress field or changes in the reservoir transmissivity.

EARTHQUAKE MECHANISM

Most seismologists now agree that the Denver earthquakes were of tectonic origin, i.e., they resulted from sudden releases of tectonic strain energy stored in the Precambrian rocks beneath the Denver Basin. Seismic studies by Major and Simon (1968) and Healy and others (1968) showed that the Denver earthquakes exhibit a frequency-versus-magnitude relationship similar to that in other tectonically active areas such as southern California. Energy calculations by Carder (1966) and Rubey (1966) also showed that the total energy released by the earthquakes cannot be accounted for by the work done in injecting the waste fluid in the reservoir. Ball and Downs (1966) have also argued that the geologic setting of the Denver area was conducive to stress build-up within the rock. Consequently, most investigators who believe in the injection–earthquake relationship are of the opinion that fluid injection "triggered" the release of strain energy that was stored in the basement rock by natural processes of deformation.

Many triggering mechanisms have been proposed in previous studies. For example, thermal stress caused by the injection of cold fluids (20°C) into an initially hot reservoir (150°C) was suspected to be a major triggering force. Chemical reactions between the waste fluid and the reservoir rock may also have weakened the strength of the rock, thus allowing slippage to occur along fracture planes.

The most widely accepted mechanism, however, attributes the occurrence of the earthquakes directly to the increase in fluid pressure in the reservoir. This hypothesis states that the increase in fluid pressure serves to reduce the frictional resistance against the shearing stress along a fracture plane. If the fluid pressure is increased to a point where the frictional resistance becomes less than the shearing stress on a fracture plane, slippage will occur and the result is an earthquake. This mechanism has been generally referred to as the Hubbert-Rubey mechanism.

The Hubbert-Rubey Mechanism

The original work of Hubbert and Rubey (1959) actually concerns the role of pore pressure in the mechanics of overthrust faulting. They introduced the concept of rock movements caused by a Mohr-Coulomb-type failure in a fluid-filled rock environment. This concept was first cited by Evans (1966) in his paper on injection–earthquake relationship and subsequently gained wide acceptance as the mechanism through which injection has caused the earthquakes.

Given a rock in which the pore spaces are filled with fluid, the total stress, S_T, on the rock is supported jointly by the effective stress, σ, of the rock itself and the pore pressure, p, of the fluid in the rock, i.e.,

$$S_T = \sigma + p. \tag{6}$$

If S_1 and S_3 are the maximum and minimum (total) principal stresses on the rock, the corresponding effective principal stresses are

$$\sigma_1 = S_1 - p \tag{7}$$

and

$$\sigma_3 = S_3 - p \tag{8}$$

In regions where strike-slip faulting occurs along a vertical plane, the maximum and minimum principal stresses are both in the horizontal direction (Anderson, 1951). If the angle between the fault plane and the minimum principal stress is θ, the effective normal, σ_n and shearing, τ, stresses along the fault plane are respectively given by

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta \qquad (9)$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \qquad (10)$$

where σ_1 and σ_3 are the maximum and minimum effective stresses as defined by equations (7) and (8).

According to the Mohr-Coulomb theory, failure (or slippage) along a fault plane will occur if the following relationship between τ and σ_n holds:

$$\tau \geq \tau_0 + \mu \sigma_n \qquad (11)$$

where τ_0 and μ are the cohesion and coefficient of friction of the fault plane, respectively.

The Hubbert-Rubey theory can now be explained as follows: as pore pressure increases, σ_1 and σ_3 will decrease according to equations (7) and (8). As a result, σ_n will also decrease, as indicated by equation (9). Note, however, that according to equation (10), τ will remain the same no matter how p varies. This results from the fact that fluid cannot support any shearing stress. When the pore pressure increases to the point where equation (11) becomes valid, slippage will occur and an earthquake is generated.

The above analysis has been applied to the Denver earthquakes by Healy and others (1968). Although they did not perform any reservoir simulation (only observed downhole pressures were used), they showed that the occurrence of the Denver earthquakes was consistent with the Hubbert-Rubey theory. The present study further shows a correlation between spatial distribution of earthquake epicenters and fluid pressures build-up above a critical value. The existence of this critical pressure build-up is an additional feature that suggests that the Hubbert-Rubey mechanism is the dominant mechanism through which fluid injection has triggered earthquakes.

Possibility of Spontaneous Earthquake Activity

An important result of the present study is that the Denver earthquakes were triggered by a relatively small increase in reservoir pressure (32 bars). Such a small value of critical pressure build-up suggests that the basement rock at the RMA was already very close to failure prior to injection. This observation opens up the possibility that Denver earthquakes may also occur spontaneously.

Prior to 1962, the only useful seismic data were from the seismograph station at the University of Colorado in Boulder. This station was in operation between 1954 and 1959, and a study of the seismograms for this period by Krivoy and Lane (1966) revealed 13 events that might have been earthquakes in the Denver area. Because all of these events occurred during normal working hours, Krivoy and Lane attributed them to the result of artificial explosions. Reexaminations of the seismograms by Leet (1966) and Carder (1966), however, cast doubts as to whether all

13 events were from artificial sources. Both Leet and Carder are of the opinion that some of the events were natural earthquakes. Hadsell (1968) made a search of newspaper accounts of earthquakes in Colorado and found reports of a major earthquake on November 7, 1882. Using reports from 25 newspapers, he determined that the earthquake came from the Denver area and that the Richter magnitude was over 5. This earthquake is not unlike the three major earthquakes of 1967, which suggests that the Denver area may not have been totally immune to earthquake activities prior to 1962. If this hypothesis is true, the role of the waste disposal operation was to greatly increase the number of earthquakes during the injection period and during the subsequent few years after shut-in.

EFFECTS OF HIGH FLUID PRESSURE ON FRACTURE APERTURE

Evidence of Hydraulic Fracturing

Van Poollen (1966, 1969) and Ball and others (1966) have noted that the transmissivity of the Precambrian reservoir appeared to be much greater during injection than during shut-in or fluid withdrawal. In addition, the transmissivity seemed to increase as injection pressure was increased. They interpreted this observation to mean that the fractures in the reservoir were forced open as fluid was injected under pressure. These fractues may then close again when fluid is withdrawn and the pressure in the reservoir lowered.

Such changes in transmissivity with fluid pressure are not uncommon in fractured reservoirs and may indicate that hydraulic fracturing may occur during injection under high pressure. From the theory of hydraulic fracturing (Hubbert and Willis, 1957), it is known that if the well bore is connected to a preexisting fracture in a reservoir and if the reservoir pressure beyond the influence of stress disturbance caused by the borehole exceeds the original regional stress normal to the plane of the fracture, the fracture will be held open to allow more rapid flow. For the RMA well, Healy and others (1968) noticed a large discontinuity in injection rate with fluid pressure. They took this to be evidence of Hydraulic fracturing.

To further investigate the occurrence of hydraulic fracturing at the RMA well, daily pressures recorded at the arsenal during the injection operations were examined. It was found that at the start of most shut-in periods the wellhead fluid pressure dropped abruptly to approximately 17.2 bars, after which the fall-off proceeded at a much slower rate. Such a sudden drop in fluid pressure to a particular level followed by slow decay is another indication that hydraulic fracturing took place during injection. The "instantaneous shut-in pressure" of 17.2 bars at the well head (377 bars downhole) must be the pressure that is just sufficient to hold the fractues open and should be equal to the region stress normal to the fault plane (Kehle, 1964).

Simulation of Rapid Flow in Open Fractures

During the periods of fluid injection at the RMA, downhole fluid pressure was sometimes increased to 430 bars. Because this value exceeds the pressure needed to hold the fractures open, rapid fluid flow in open fractures must play an important role in determining the reservoir pressure near the well.

To examine the areal extent of open-fracture flow, the semi-infinite strip model was further modified so that transmissivity was made a function of hydraulic head. The transmissivity at any point in the reservoir was set at 1.08×10^{-5} m^2/s when the build-up in hydraulic head at that point was below 989 m. At any point in the reservoir where the build-up in hydraulic head was above 989 m, the transmissivity was abruptly increased to a much higher value. Such a transmissivity–hydraulic head relationship was formulated to simulate rapid flow in fractures opened by high fluid pressure.

This formulation makes the model nonlinear, and solution must be obtained by numerical techniques. The nonlinear solution technique used a simple iteration procedure whereby the transmissivity was lagged as new estimates of hydraulic head were computed. The transmissivity was then updated, and the entire procedure was repeated until convergence was achieved.

A trial run was made assuming the open-fracture transmissivity to be 100 times the normal transmissivity value. As expected, the computed hydraulic head near the well during periods of high injection was found to be much lower than the hydraulic head computed using a constant transmissivity model. A comparison of the hydraulic head profiles of September 1965 computed by the two models is shown in Figure 23. These two profiles are shown here because they exhibit the greatest difference in hydraulic head. As shown in the figure, the effects of open-fracture flow are restricted to near the well; the two profiles differ by less than 10 percent at distances greater than 1 km from the well. In fact, for most of the injection period, the pressure profiles computed by the two models are almost identical.

It should be noted that the downhole pressure computed from this model, which simulates rapid flow in open fractures, never reached 430 bars at any time during the trial simulation. This suggests that the transmissivity–hydraulic head relationship used in the above trial run represents an extreme case of fracture widening near the well. Even for such an extreme case, the computed pressure profile is essentially the same as that computed using a constant transmissivity model, except for a small region near the well. Thus, the quality of the above relationship between earthquake and pressure distributions remains unchanged.

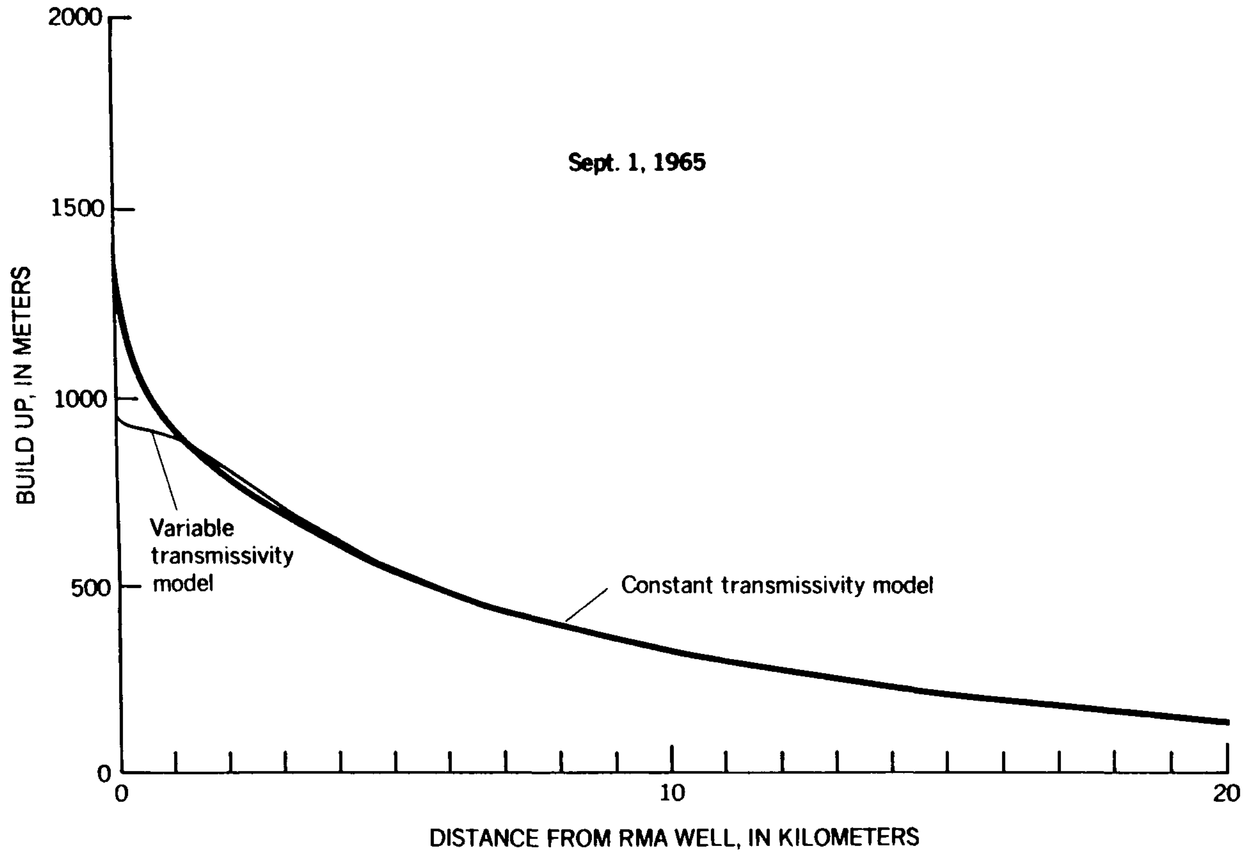

Fig. 23. Computed reservoir pressure build-up illustrating effect of fracture opening under high pressure. -- The variable transmissivity model simulates the effect of rapid flow in open fractures when the build-up in hydraulic head exceeds 989 m (a downhole pressure of 377 bars).

CONCLUSIONS

Waste fluids were injected into the fractured Precambrian bedrock below the Rocky Mountain Arsenal between 1962 and 1966. Soon after injection began, minor earthquakes were detected in the vicinity of the RMA. These earthquakes were found to occur along a long, narrow, seismic zone aligned in a N. 60° W. direction. Many investigators have suggested that a reservoir composed of connected vertical fractures aligned in the same N. 60° W. direction exists in the Precambrian bedrock. Earthquakes were believed to be results of lateral movements along the fault zone and triggered by the increase in pore pressure due to injection. This pore pressure–earthquake hypothesis is examined in this thesis by analyzing the pressure history in the Precambrian reservoir.

The configuration and hydrologic properties of the reservoir were determined from seismic and water-level data. Seismic arrays installed at the RMA in 1966 provided detailed locations of earthquake hypocenters from 1966 to 1968. Observed decline in water level at the injection well since injection ceased provided information on reservoir parameters and boundaries. The two sets of data together suggest that the reservoir is 3.35 km wide, extends 30.5 km to the northwest and infinitely to the southeast, and spans a depth from 3.7 to approximately 7.0 km below land surface. The reservoir has a transmissivity of 1.08×10^{-5} m^2/s and a storage coefficient of 1×10^{-5}. It is assumed that fluid flow in fractured rocks can be approximated by fluid flow in a porous medium.

Comparison of horizontal distribution of pressure build-up and earthquake epicenters for the period from 1962 and 1972 indicates that earthquakes are confined to that part of the reservoir where pressure build-up exceeds 32 bars. This critical value is interpreted as pressure build-up above which earthquakes occur. This result is consistent with the results found at Rangely where earthquakes were controlled by controlling fluid pressures (Raleigh, Healy, and Bredehoeft, 1976). The earthquakes at RMA and the experiment at Rangely indicate that the Hubbert-Rubey hypothesis on the role of fluid pressures in faulting is the dominant process at work.

The reservoir analysis was extended to an examination of the effects of fracture widening due to injection under high pressure. The results show that the pressure distribution computed with the effects of fracture widening differs from the pressure distribution computed without the effect only in a small region within one kilometer of the injection well. The quality of the relationship between earthquake and pressure distribution remains unchanged.

At this point the evidence seems rather conclusive that the increase in fluid pressure triggered the swarm of earthquakes at the RMA. This thought is not original; as pointed out, a number of investigators, starting with Evans (1966), have made this point. By considering the ground-water reservoir in the analysis of the injection-earthquake relationship, it is possible to tie up many of the loose ends left by earlier investigations.

APPENDIX A

DEVELOPMENT OF EQUATION (1)

Referring to Figure 4, which shows the configuration of the reservoir, note that the dimension of the open-hole injection interval is much smaller than the dimension of the reservoir. Consequently, the open hole may be modeled as a point source located at the top boundary of the reservoir.

If the reservoir is assumed to be homogeneous, then the transient three-dimensional distribution of hydraulic head, $h(x,y,z,t)$, is governed by the partial differential equation

$$K\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) = S_s \frac{\partial h}{\partial t} - Q(t)\,\delta(x - x_0)\,\delta(y - y_0)\,\delta(z - x_0) \qquad \text{(A-1)}$$

where K and S_s are the hydraulic conductivity and the specific storage, respectively, of the porous medium and x_0, y_0, z_0 denote the position of the point source.

Let the top and bottom boundaries of the reservoir be located at $z = b$ and $z = 0$, respectively. The boundary conditions at the top and bottom boundaries can be written as

$$\frac{\partial h}{\partial z} = 0 \text{ at } z = 0 \text{ and } z = b \qquad \text{(A-2)}$$

The average hydraulic head over the depth of the reservoir can be defined as

$$\bar{h}(x,y,t) = \frac{1}{b}\int_0^b h(x,y,z,t)\,dz \qquad \text{(A-3)}$$

The depth-averaged hydraulic head can be computed in two ways:

1. Solving equation (A-1) and then depth-averaging the solution according to equation (A-3), or
2. Depth averaging equation (A-1) and then solving the resulting two-dimensional equation.

The two methods yield identical solutions.

Proceeding with the second method, the depth-averaged equation takes the form:

$$\frac{1}{b}\int_0^b [K(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2})]dz = \frac{1}{b}\int_0^b [S_s\frac{\partial h}{\partial t} - Q(t)\,\delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)]dz \tag{A-4}$$

Splitting up the integrals and noting that the integration is with respect to z only, equation (A-4) becomes

$$K[\frac{\partial^2}{\partial x^2}(\frac{1}{b}\int_0^b h dz) + \frac{\partial^2}{\partial y^2}(\frac{1}{b}\int_0^b h dz) + \frac{1}{b}\int_0^b \frac{\partial^2 h}{\partial z^2}dz]$$
$$= S_s\frac{\partial}{\partial t}(\frac{1}{b}\int_0^b h\,dz) - Q(t)\,\delta(x-x_0)\,\delta(y-y_0)\,\frac{1}{b}\int_0^b d(z-z_0)dz \tag{A-5}$$

The third expression on the left-hand side can be rewritten as

$$\int_0^b \frac{\partial^2 h}{\partial z^2}dz = [\frac{\partial h}{\partial z}]_{z=b} - [\frac{\partial h}{\partial z}]_{z=0} = 0$$

according to the boundary condition (A-2). Also

$$\int_0^b \delta(z-z_0)\,dz = 1$$

Now using the definition of depth-averaged hydraulic head given in equation (A-3), equation (A-5) becomes

$$K\left(\frac{\partial^2\bar{h}}{\partial x^2}+\frac{\partial^2\bar{h}}{\partial y^2}\right)=S_s\frac{\partial\bar{h}}{\partial t}-\frac{Q(t)}{b}\,\delta(x-x_0)\,\delta(y-y_0) \qquad \text{(A-6)}$$

Multiplying through by b and letting $T = Kb$ and $S = S_s b$,

$$T\left(\frac{\partial^2\bar{h}}{\partial x^2}+\frac{\partial^2\bar{h}}{\partial y^2}\right)=S\frac{\partial\bar{h}}{\partial t}-Q(t)\,\delta(x-x_0)\,\delta(y-y_0) \qquad \text{(A-7)}$$

Equation (A-7) is thus the equation governing depth-averaged hydraulic head.

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