A Review of Block Designs for Test Treatments – Control(s) Comparisons

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ABSTRACT

In practice there may arise experimental situations where it is desired to compare several treatments called the test treatments to a standard treatment called control. The main interest here lies in making test treatment-control comparison with as much precision as possible and comparison within the test treatments are of less importance. For example in agricultural experiments, the aim of the experimenter is to test a set of new varieties of a crop with an already existing variety and to determine which of the varieties perform better in comparison to the existing variety. Balanced Treatment Incomplete Block (BTIB) designs have been defined for this situation. The designs are balanced with respect to test treatment-control comparisons. The concept of BTIB is further extended to define Balanced Two Disjoint Sets of Treatments (BTDT) designs when there are more than one control. Some methods of constructing these designs are presented here. Some class of row-column designs, which are balanced for test treatments vs. control comparisons, referred to as the Balanced Treatment vs. Control Row-Column (BTCRC) designs are also described when heterogeneity is to be eliminated in two directions.

Key words: Balanced Treatment Incomplete Block (BTIB) design, Balanced Two Disjoint Sets of Treatments (BTDT) design, Balanced Treatment vs. Control Row-Column (BTCRC) design.

INTRODUCTION

Designs are usually characterized by the nature of grouping of experimental units and the procedure of random allocation of treatments to the experimental units. Block (Row-Column) designs are useful in experiments requiring elimination of heterogeneity in one (two) direction. These designs are useful in agricultural experiments for situation where the experimenter is interested in making comparison of all possible treatment pairs. The design adopted should be such that it allows these comparisons to be made with as high a precision as possible. However, in practice there may arise experimental situations where it is desired to compare several treatments called test treatments to standard treatment called control. For example, in screening experiments or in the beginning of a long-term experimental investigation, where it is initially desired to determine the relative performance of new test treatments with respect to the control. Let there be v treatments (such as new types of hybrid varieties, method of cultivation, pesticides, herbicides etc.) and an existing (old) one is to be replaced by one of these newer kinds. In such situations, the experimenter is not interested in making comparisons among all the treatments, but the main interest is to compare the new (test) treatments with the old (control) treatment and thus a higher precision is desired for these estimates.

The earliest work on this problem was carried out by Dunnett (1955). He posed (but did not solve) the problem of optimally allocating experimental units to control and test treatment so as to maximize the probability associated with the joint confidence statement concerning the many-to-one comparison between the mean of the control treatment and the mean of the test treatments. This optimal allocation problem was solved by Bachhofer and his coworkers (1969, 1970, 1981).

In all the above papers, it was assumed that a Completely Randomized (CR) design has been used. However, many practical situations may require the blocking of experimental units in order to cut down on bias and improve the precision of the experiment. If the block size is large enough to accommodate one replication of all the test treatments and additional control

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treatments as well, then the design of experiment can be carried out using the optimal allocations described in Bechhofer (1969) and Bechhofer and Nocturne (1970) with only the usual modifications.

2. Balanced Incomplete Block Designs for Comparing Test Treatments with Control

The situation that commonly occurs in practice is when the block size is less than the total number of treatments. Bechhofer and Tamhane (1981) introduced a general class of incomplete block designs appropriate for the problem called Balanced Test Treatment Incomplete Block (BTIB) designs. The designs are balanced with respect to test treatment-control comparisons.

Let the treatments be denoted by 0,1,…,v with 0 denoting the control treatment and 1,2,…,v denoting the $v \geq 2$ test treatments. Let $k < v + 1$ be the size of each block and b the number of blocks for experimentation. If treatment i is assigned to the h^{th} plot of j^{th} block ($0 \leq i \leq v$, $1 \leq h \leq k$, $1 \leq j \leq b$), then the usual additive linear model is,

$$y_{ijh} = \mu + \alpha_i + \beta_j + e_{ijh},$$

y_{ijh} denote the corresponding response variable, μ is the grand mean, α_i is i^{th} treatment effect and β_j is j^{th} block effect. The e_{ijh} are assumed to be iid $N(0, \sigma^2)$ random variables.

Definition 2.1: [Bechhofer and Tamhane (1981)]. For given (v, b, k), consider a design with the incidence matrix $N = ((n_{ij}))$, where n_{ij} is the number of replications of the i^{th} treatment in the j^{th} block. Let $\lambda_{ii'} = \Sigma n_{ij}n_{i'j}$ denote the total number of times the i^{th} treatment appears with the i'^{th} treatment in the same block over the whole design ($i \neq i'$; $0 \leq i, i' \geq v$). Then the necessary and sufficient conditions for a design to be BTIB are

$$\lambda_{01} = \lambda_{02} = ,\ldots, = \lambda_{0v} = \lambda_0 \text{ (say)}$$
$$\lambda_{12} = \lambda_{13} = ,\ldots, = \lambda_{v-1,v} = \lambda_1 \text{ (say)}$$

In other words, each test treatment must appear with (i.e. in the same block) the control treatment the same number of times (λ_0) over the design, and each test treatment must appear with every other test treatment same number of times (λ_1) over the design. As a consequence of this definition of BTIB design, **N** is the (v+1) x b incidence matrix, given as

$$\mathbf{N} = \begin{bmatrix} \mathbf{N_1} \\ \mathbf{n} \end{bmatrix},$$

where $\mathbf{N_1}$ is a (v x b) incidence matrix pertaining to v test treatments and **n** is a (1 x b) incidence matrix pertaining a control treatment. The vector of replication is $\left(r\mathbf{1}'_{v_1} \quad r_0\right)'$ and vector of block sizes is $(k_1,\ldots,k_b)'$, r and r_0 being the replication number of test treatments and control treatment respectively. Therefore the information matrix is given by

$$\mathbf{C} = \begin{bmatrix} r\mathbf{I_v} & \mathbf{0} \\ \mathbf{0} & r_0 \end{bmatrix} - \begin{bmatrix} \mathbf{N_1} \\ \mathbf{n} \end{bmatrix} \mathbf{K}^{-1} \begin{bmatrix} \mathbf{N_1'} & \mathbf{n}' \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} r\mathbf{I_v} - \mathbf{N_1K^{-1}N_1'} & -\mathbf{N_1K^{-1}n}' \\ -\mathbf{nK^{-1}N_1'} & r_0 - \mathbf{nK^{-1}n}' \end{bmatrix}.$$

2.1 Construction of (BTIB) designs

In this section, some methods of constructing BTIB designs for comparing a set of test treatment to a control treatment are given. The designs are balanced with respect to test treatments-control treatment comparisons according to Definition 2.1.

Method 2.1.1: Consider a BIB design with parameters (v, b, r, k, λ), then adding one control in each block of this design results in a reinforced BIB design or a BTIB design with parameters v, b, r, r_0 = k, k, λ_0 = 1 and λ_1 = 1.

Example 2.1.1: Consider a BIB with parameters v = 7, b = 7, r = 3, k = 3, λ = 1. By adding the control treatment (0) to each block, following BTIB design is obtained:

$$\begin{Bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 4 & 5 & 6 & 7 & 1 \\ 4 & 5 & 6 & 7 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{Bmatrix}$$

Method 2.1.2: Start with a BIB design with t > v treatments in b blocks. Replace the treatments v + 1, v + 2, …, t to zeros. A BTIB design with parameters v, b, r, $r_0 = r(t-v)$, k, $\lambda_0 = (t-v)\lambda$ and $\lambda_1 = \lambda$ is obtained.

Example 2.1.2: Consider the following BIB design with parameters (7, 3, 1):

$$\begin{Bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 & 1 & 2 \end{Bmatrix}$$

Replacing the 6's and 7's by zeros, the following BTIB design with parameters v=5, b=7, r=3, r_0=6, k=3, $\lambda_0 = 2$ and $\lambda_1 = 1$ is obtained:

$$\begin{Bmatrix} 1 & 2 & 3 & 4 & 5 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 3 & 4 & 5 & 0 & 0 & 1 & 2 \end{Bmatrix}$$

If treatment 5 is also replaced then the following BTIB design would be obtained with parameters 4, 7, 3, 9, 3, $\lambda_0 = 3$, $\lambda_1 = 1$.

$$\begin{Bmatrix} 1 & 2 & 3 & 4 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 1 & 2 \end{Bmatrix}$$

Method 2.1.3: Suppose that for given (v, k) there is a design D_1 with $\lambda_0 > 0$. Then new design D_2 for the same (v, k) can be obtained by taking a "complement" of D_1 in the following way. Separate the blocks of D_1 in different sets so that each block in given set has zero assigned in an equal number of plots (0 times, 1 time etc.). For example, consider the above design the blocks of which can be separated into three sets as follows:

$$D_1 = \begin{Bmatrix} 1 & . & 0 & 0 & 0 & . & 0 & 0 & 0 \\ 2 & . & 1 & 2 & 3 & . & 0 & 0 & 0 \\ 4 & . & 3 & 3 & 4 & . & 1 & 2 & 4 \end{Bmatrix}$$

For each set of D_1 write its "complementary" set (with zero assign in the same number of plots) which will result in a BTIB design. These complementary sets for above are

$$\begin{Bmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 3 & 4 & 4 \end{Bmatrix}, \quad \begin{Bmatrix} 0 & 0 & 0 \\ 1 & 1 & 2 \\ 2 & 4 & 4 \end{Bmatrix}, \quad \begin{Bmatrix} 0 \\ 0 \\ 3 \end{Bmatrix};$$

This is a BTIB design with parameters 4, 7, 4, 5, 3, $\lambda 0 = 2$, $\lambda 1 = 2$.

3. Balanced Block Designs for Comparing Two disjoint Sets of Treatments

Section 2 gives the concept of BTIB design for comparing a set of treatments to single control treatment. But there are situations in which more than one control treatment is required. The problem is that there are two sets of treatments [Majumdar (1988), Jaggi (1992), Jaggi *et al*. (1996)]. One set T of cardinality v_1, containing test treatments denoted by $1,2,\ldots,v_1$ and the second set S of cardinality v_2, containing standard or control treatments denoted by $(v_1+1),\ldots,v$, where $v = v_1 + v_2 \geq 4$ and $T \cap S = \phi$. The primary interest of experimenter is to estimate the contrast $(\tau_t - \tau_s)$ with as much precision as possible, where $t \in T$ and $s \in S$. The comparisons between the treatments within the set are not of interest.

The concept of BTIB design is extended to compare a set of test treatments to a set of control treatments. The designs for comparing two disjoint sets of treatment are called as Balanced Two Disjoint Sets of Treatments (BTDT) designs. The two sets of treatments are disjoint in the sense that there are no common treatments between the two sets.

These designs are balanced with respect to test treatments-control treatments comparisons. Let N be a $(v_1 + v_2)$ x b incidence matrix, given as

$$\mathbf{N} = \begin{bmatrix} \mathbf{N_1} \\ \mathbf{n} \end{bmatrix}$$

where N_1 is a $(v_1 \times b)$ incidence matrix pertaining to v1 test treatments and n is a $(v_2 \times b)$ incidence matrix pertaining to v2 control treatments. Also $\mathbf{r} = \left(r_t \mathbf{1}'_{v_1} \quad r_c \mathbf{1}'_{v_2}\right)'$ and $k = (k_1,\ldots,k_b)'$, r_t and r_c being the replication number of test treatments and control treatments respectively and k is the vector of block sizes. Therefore, the information matrix is given by

$$\mathbf{C} = \begin{bmatrix} r_t \mathbf{I}_{v_1} & \mathbf{0} \\ \mathbf{0} & r_c \mathbf{I}_{v_2} \end{bmatrix} - \begin{bmatrix} \mathbf{N_1} \\ \mathbf{n} \end{bmatrix} \mathbf{K}^{-1} \begin{bmatrix} \mathbf{N_1'} & \mathbf{n}' \end{bmatrix}$$

$$= \begin{bmatrix} r_t \mathbf{I}_{v_1} - \mathbf{N_1 K^{-1} N_1'} & -\mathbf{N_1 K^{-1} n'} \\ -\mathbf{n K^{-1} N_1'} & r_c \mathbf{I}_{v_2} - \mathbf{n K^{-1} n'} \end{bmatrix}$$

3.1 Construction of BTDT Design

Method 3.1.1: This method is developed by the Jaggi in 1992. Let N_1 be the incidence matrix of a partially balanced incomplete block (PBIB) design with two associate classes and with parameters v_1, b_1, r_1, k_1, λ_{11}, λ_{12}, m, n, and N_2 be the incidence matrix of another PBIB design with the same association scheme (m,n) and parameter v_1, b_2, r_2, k_2, λ_{21}, λ_{22}, m, n.

Case (A):

If $\frac{\lambda_{11}}{k_1+v_2} + \frac{\lambda_{21}}{k_2} = \frac{\lambda_{12}}{k_1+v_2} + \frac{\lambda_{22}}{k_2} = \lambda$, then

$$\mathbf{N} = \begin{bmatrix} \mathbf{N_1} & \mathbf{N_2} \\ \mathbf{1_{v_2} 1'_{b_1}} & \mathbf{0} \end{bmatrix}$$

is the incidence matrix of a BTDT design for comparing v1 test treatment to v_2 control treatments with parameters $v=v_1+v_2$, $b=b_1+b_2$, $\mathbf{r}' = \begin{bmatrix} (r_1+r_2)\mathbf{1}'_{v_1} & b_1 \mathbf{1}'_{v_2} \end{bmatrix}$
$\mathbf{k}' = \begin{bmatrix} (k_1+v_2)\mathbf{1}'_{b_1} & k_2 \mathbf{1}'_{b_2} \end{bmatrix}$.

Case (B):

If $\frac{\lambda_{11}}{k_1+v_2} + \frac{\lambda_{21}}{k_2+v_2} = \frac{\lambda_{12}}{k_1+v_2} + \frac{\lambda_{22}}{k_2+v_2} = \lambda$,

then $\mathbf{N} = \begin{bmatrix} \mathbf{N_1} & \mathbf{N_2} \\ \mathbf{1_{v_2} 1'_{b_1}} & \mathbf{1_{v_2} 1'_{b_2}} \end{bmatrix}$

is the incidence matrix of a BTDT design for comparing a set of v_1 test treatments to a set of v_2 control treatments with parameters $v=v_1+v_2$, $b=b_1+b_2$,
$\mathbf{r}' = \begin{bmatrix} (r_1+r_2)\mathbf{1}'_{v_1} & (b_1+b_2)\mathbf{1}'_{v_2} \end{bmatrix}$,
$\mathbf{k}' = \begin{bmatrix} (k_1+v_2)\mathbf{1}'_{b_1} & (k_2+v_2)\mathbf{1}'_{b_2} \end{bmatrix}$.

Example 3.1.1: Consider a semi-regular GD design (SR6) with parameters $v_1 = 6$, $b_1 = 9$, $r_1 = 3$, $k_1 = 2$, $m = 2$, $n = 3$, $\lambda_{11} = 0$, $\lambda_{12} = 1$ and another regular GD design (R94) with $v_1 = 6$, $b_2 = 6$, $r_2 = 4$, $k_2 = 4$, $m = 2$, $n = 3$, $\lambda_{21} = 3$, $\lambda_{22} = 2$, both the designs having the same association scheme. The design obtained by taking the blocks of these two designs together and augmenting two new treatments ($v_2 = 2$) in each block of first design, gives following BTDT design with the parameters v = 8, b = 15, r = 7, $r_0 = 9$, k = 4:

(1, 2, 7, 8); (3, 4, 7, 8); (5, 6, 7, 8);
(1, 6, 7, 8); (3, 2, 7, 8);
(5, 4, 7, 8); (1, 4, 7, 8); (3, 6, 7, 8);
(5, 2, 7, 8); (1, 2, 4, 6);
(2, 3, 5, 1); (3, 4, 6, 2); (4, 5, 1,3);
(5, 6, 2, 4); (6, 1, 3, 5);

Method 3.1.2: This method of construction of design for making comparisons between two sets of treatments is derived from the use of variance balanced block designs with equal and unequal replications and equal and unequal block sizes. Using this method, design with equal as well as unequal block sizes, and equal and unequal replications can be constructed.

Consider any variance balanced block design with unequal replication numbers and unequal block sizes. The design obtained by deleting one block from this design is a BTDT design. Then the information matrix C is

$$\mathbf{C} = \begin{bmatrix} (\theta-1)\mathbf{I}_k - \left[\frac{\theta}{v} - \frac{1}{k_1}\right]\mathbf{1}_{k_1}\mathbf{1}'_{k_1} & -\frac{\theta}{v}\mathbf{1}_{k_1}\mathbf{1}'_{v-k_1} \\ -\frac{\theta}{v}\mathbf{1}_{v-k_1}\mathbf{1}'_{k_1} & \theta\mathbf{I}_{v-k_1} - \frac{\theta}{v}\mathbf{1}_{v-k_1}\mathbf{1}'_{v-k_1} \end{bmatrix}$$

Example 3.1.2: Consider the following variance balanced design with unequal replication and unequal block sizes and with parameters v = 6, b = 11, $\mathbf{r} = \begin{bmatrix} 4\mathbf{1}'_4 & 5\mathbf{1}'_2 \end{bmatrix}$, $k = \begin{bmatrix} 4\mathbf{1}'_2 & 2\mathbf{1}'_9 \end{bmatrix}$

1	1	1	1	2	2	3	3	4	4	5
2	2	5	6	5	6	5	6	5	6	6
3	3									
4	4									

The design obtained by deleting the first block i.e. (1, 2, 3, 4) is a BTDT design with unequal replication numbers, $v_1 = k_1 = 4$, $v_2 = v - k_1 = 2$, $b = 10$, $\mathbf{r} = \left[3\mathbf{1}_4' \quad 5\mathbf{1}_2'\right]$, $\mathbf{k} = \left[4 \quad 2\mathbf{1}_9'\right]$.

4. Row-Column Designs for Comparing Test Treatments with a Control

This section considers the designs for comparing several treatments with a control when heterogeneity is to be eliminated in two directions. Row-column designs, which are balanced for test treatments vs. control comparisons referred to as the Balanced Treatment vs. Control Row-Column (BTCRC) designs has been proposed by Majumdar and Tamhane (1996).

Suppose $v \geq 2$ treatments, labeled 1,2,…,v are to be compared with a control, labeled 0, in a row-column design with $a \geq 2$ rows and $b \geq 2$ columns. Assume that only one treatment is applied in each of the a b plots. Let y_{ijk} be the observation on the i^{th} treatment applied in the j^{th} row and k^{th} column ($0 \leq i \leq v$, $1 \leq j \leq a$, $1 \leq k \leq b$). Fixed-effects additive linear model assumed is as follows:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk},$$

where μ is the grand mean, α_i is the i^{th} treatment effect, β_j is the j^{th} row effect, γ_k is the k^{th} column effect and ε_{ijk} are uncorrelated random error with zero mean and constant variance σ^2.

Here again the interest is to estimate the treatments vs. control contrasts $\alpha_0 - \alpha_i$, i = 1,…,v. Let the row-column design have m_{ij} incidences of the i^{th} treatment in the j^{th} row and n_{ik} incidences of the i^{th} treatment in the k^{th} column ($0 \leq i \leq v$, $1 \leq j \leq a$, $1 \leq k \leq b$). Let $M = \{m_{ij}\}$ and $N = \{n_{ij}\}$ denote the row and column incidence matrices, respectively. Further, let $r_i = \sum_{j=1}^{a} m_{ij} = \sum_{k=1}^{b} n_{ij}$ be the number of replications of the i^{th} treatment, $\mathbf{r} = (r_0, r_1, ..., r_v)'$ and

$$\mu_{ii'} = \sum_{j=1}^{a} m_{ij} m_{i'j} \quad \text{and} \quad v_{ii'} = \sum_{k=1}^{b} n_{ik} n_{i'k}$$

Define

$$\lambda_{ii'} = \frac{1}{ab}\left[a\mu_{ii'} + bv_{ii'} - r_i r_{i'}\right]$$

Definition 4.1: The necessary and sufficient conditions for a row column design to be BTCRC is that $\lambda_{01} = \lambda_{02} = ,…, = \lambda_{0v} = \lambda_0$ (say) , $\lambda_{12} = \lambda_{13} = ,…, = \lambda_{v-1,v} = \lambda_1$ (say)

4.1 Construction of BTCRC Designs

Method 4.1.1: [Notz (1985)]. Start with a Latin Square of order $w \geq v$ and replace symbols v+1,…,w by the symbol 0 (control). The resulting design is a BTCRC with parameters v, a = b =w, r = w, $r_0 = (w-v)w$, $\mu_{ii'} = v_{ii} = w$, $\mu_{0i} = v_{0i} = (w-v)w$, $\lambda_0 = w-v$, $\lambda_1 = 1$.

Example 4.1.1: Consider the 5 x 5 Latin Square Design. Replacing symbols 4 and 5 to 0, the following design with three test treatments is obtained:

$$\begin{Bmatrix} 1 & 2 & 3 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 \\ 3 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 & 0 \end{Bmatrix}$$

Here $\mu_{ii'} = v_{ii'} = 5$. This design is row as well as column balanced with $\lambda_0 = 2$ and $\lambda_1 = 1$.

Method 4.1.2: Construct a Pseudo-Youden design (PYD) introduced by Cheng (1981). Thus a BTCRC design can be constructed from a PYD in w symbols by changing symbols v + 1, …,w to 0.

Example 4.1.2: Consider a 6 x 6 PYD for 9 treatments. Replacing symbols 7, 8 and 9 by 0's, the following BTCRC design for six test treatments with $\mu_{i0} = 7$ and $v_{i0} = 8$ and $\mu_{ii'} = 3$, $v_{ii'} = 2$ for $1 \leq i \neq i' \leq 6$ is obtained:

$$\begin{Bmatrix} 4 & 0 & 0 & 6 & 0 & 5 \\ 3 & 1 & 2 & 0 & 0 & 0 \\ 2 & 5 & 1 & 3 & 6 & 4 \\ 0 & 3 & 6 & 2 & 5 & 0 \\ 0 & 6 & 0 & 4 & 1 & 3 \\ 5 & 0 & 4 & 0 & 2 & 1 \end{Bmatrix}$$

The design has $\lambda_0 = 7/6$, $\lambda_1 = 7/18$.

Method 4.1.3: The transversal of a Latin Square of order v is a set of v cells such that each row, column and symbol is represented exactly once in this set [Hedayat and Seiden (1974)]. Changing all symbols in a transversal to 0, a BTCRC design with a = b = v, $r_1 = \ldots = r_v = v-1$ and $r_0 = v$ is obtained.

Example 4.1.3: Consider the following Latin Square of order 4 with a transversal parenthesized:

$$\begin{Bmatrix} 1 & 2 & (3) & 4 \\ 3 & 4 & 1 & (2) \\ (4) & 3 & 2 & 1 \\ 2 & (1) & 4 & 3 \end{Bmatrix}$$

Replacing the parenthesized treatments by 0 gives the following BTCRC design with $\mu_{ii'} = v_{ii'} = 2$, $\mu_{i0} = v_{i0} = 3$:

$$\begin{Bmatrix} 1 & 2 & 0 & 4 \\ 3 & 4 & 1 & 0 \\ 0 & 3 & 2 & 1 \\ 2 & 0 & 4 & 3 \end{Bmatrix}$$

This design has $\lambda_0 = 3/4$, $\lambda_1 = 7/16$.

Method 4.1.4: Two transversals in a Latin Square of order v are called parallel if they have no cell in common. Suppose such parallel transversals are identified. First apply Method 4.1.3 to obtain a v x v BTCRC design using the first transversal. Then take horizontal and vertical projections (Hedayat and Seiden 1974) of the second transversal, and add a 0 to complete the design.

Example 4.1.4: Consider the following 4 x 4 Latin Square with two parallel transversals, one parenthesized and other square-bracketed:

$$\begin{Bmatrix} [1] & 2 & (3) & 4 \\ 3 & [4] & 1 & (2) \\ (4) & 3 & [2] & 1 \\ 2 & (1) & 4 & [3] \end{Bmatrix}$$

Replace the parenthesized transversals by 0's to obtain the following BTCRC design using Method 4.1.3:

$$\begin{Bmatrix} [1] & 2 & 0 & 4 \\ 3 & [4] & 1 & 0 \\ 0 & 3 & [2] & 1 \\ 2 & 0 & 4 & [3] \end{Bmatrix}$$

Next project the square-bracketed transversals horizontally and vertically and use the sum composition methods to complete the following square BTCRC design:

$$\begin{Bmatrix} 0 & 2 & 0 & 4 & 1 \\ 3 & 0 & 1 & 0 & 4 \\ 0 & 3 & 0 & 1 & 2 \\ 2 & 0 & 4 & 0 & 3 \\ 1 & 4 & 2 & 3 & 0 \end{Bmatrix}$$

This design has $\lambda_0 = 34/25$ and $\lambda_1 = 14/25$.

Method 4.1.5: This method of Kiefer (1975) can be used to construct large BTCRC designs from smaller ones as shown with the following example:

Example 4.1.5: BTCRC design with 4 test treatments with 9 x 9 array can be constructed as follows:

$$\begin{Bmatrix} d_1 & d_2' \\ d_2 & d_3 \end{Bmatrix}$$

where d1 is 6 x 6 BTCRC design with v = 4 obtained from a Latin square of order 6 by changing symbols 5 and 6 to 0.

$$d_1 = \begin{Bmatrix} 0 & 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 1 & 2 & 3 & 4 \\ 4 & 0 & 0 & 1 & 2 & 3 \\ 3 & 4 & 0 & 0 & 1 & 2 \\ 2 & 3 & 4 & 0 & 0 & 1 \\ 1 & 2 & 3 & 4 & 0 & 0 \end{Bmatrix}$$

d_2 is BTCRC design belonging to the "Euclidean family" of Hedayat and Majumdar (1988).

$$d_2 = \begin{Bmatrix} 1 & 0 & 3 & 4 & 2 & 0 \\ 0 & 3 & 4 & 2 & 0 & 1 \\ 4 & 2 & 0 & 0 & 1 & 3 \end{Bmatrix}$$

And d3 is a 3 x 3 matrix of all 0's. Then design is,

$$\begin{Bmatrix} d_1 & d_2' \\ d_2 & d_3 \end{Bmatrix} = \begin{Bmatrix} 0 & 1 & 2 & 3 & 4 & 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 & 3 & 4 & 0 & 3 & 2 \\ 4 & 0 & 0 & 1 & 2 & 3 & 3 & 4 & 0 \\ 3 & 4 & 0 & 0 & 1 & 2 & 4 & 2 & 0 \\ 2 & 3 & 4 & 0 & 0 & 1 & 2 & 0 & 1 \\ 1 & 2 & 3 & 4 & 0 & 0 & 0 & 1 & 3 \\ 1 & 0 & 3 & 4 & 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 4 & 2 & 0 & 1 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 1 & 3 & 0 & 0 & 0 \end{Bmatrix}$$

This design has $\lambda_0 = 40/9$, $\lambda_1 = 14/9$.

CONCLUSIONS

A general class of incomplete block designs that are appropriate for use in the comparison of test treatment-control problem have been described. These designs are referred as BTIB designs. BTIB designs are balanced with respect to test treatments-control treatment comparisons. The concept of BTIB designs is extended to compare a set of test treatment to a set of control treatments. The designs for comparing two disjoint sets of treatment are called as Balanced Two Disjoint Sets of Treatments (BTDT). A class of row-column designs, which are balanced for test treatments vs. control comparisons referred to as the Balanced Treatment vs. Control Row-Column (BTCRC) can be used for comparing several treatments with a control when heterogeneity is to be eliminated in two directions.

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