Review

A Review of Chaotic Systems Based on Memristive Hopfield Neural Networks

Hairong Lin [1], **Chunhua Wang** [1,*], **Fei Yu** [2], **Jingru Sun** [1], **Sichun Du** [1], **Zekun Deng** [1] and **Quanli Deng** [1]

1 College of Computer Science and Electronic Engineering, Hunan University, Changsha 410082, China
2 School of Computer and Communication Engineering, Changsha University of Science and Technology, Changsha 410114, China
* Correspondence: wch1227164@hnu.edu.cn

Abstract: Since the Lorenz chaotic system was discovered in 1963, the construction of chaotic systems with complex dynamics has been a research hotspot in the field of chaos. Recently, memristive Hopfield neural networks (MHNNs) offer great potential in the design of complex, chaotic systems because of their special network structures, hyperbolic tangent activation function, and memory property. Many chaotic systems based on MHNNs have been proposed and exhibit various complex dynamical behaviors, including hyperchaos, coexisting attractors, multistability, extreme multistability, multi-scroll attractors, multi-structure attractors, and initial-offset coexisting behaviors. A comprehensive review of the MHNN-based chaotic systems has become an urgent requirement. In this review, we first briefly introduce the basic knowledge of the Hopfiled neural network, memristor, and chaotic dynamics. Then, different modeling methods of the MHNN-based chaotic systems are analyzed and discussed. Concurrently, the pioneering works and some recent important papers related to MHNN-based chaotic systems are reviewed in detail. Finally, we survey the progress of MHNN-based chaotic systems for application in various scenarios. Some open problems and visions for the future in this field are presented. We attempt to provide a reference and a resource for both chaos researchers and those outside the field who hope to apply chaotic systems in a particular application.

Keywords: chaotic systems; memristor; Hopfield neural network; dynamical behavior; memristor synapse; electromagnetic induction; image encryption

MSC: 34H10; 65P20; 34C23; 68T07

1. Introduction

Chaos theory is an important discovery of human natural science in the 20th century, which is considered the third revolution of basic science after relativity and quantum theory. Chaotic behavior, which is a type of dynamical behavior, was first observed in meteorology to describe the unpredictability of weather [1]. After that, chaos phenomena are found to be widely existent in many natural and non-natural behaviors, such as biotic population [2], road traffic [3], and the stock market [4]. After half a century of in-depth study, chaos has been found to be very useful and has great potential in many disciplines such as mathematics [5], physics [6], chemistry [7], economics [8], information and computer sciences [9], and so on [10,11]. Over the past decades, scholars have devoted great enthusiasm to chaos generation, and a large number of different types of chaotic systems have been constructed.

In the early days, chaos researchers focused mainly on the design of double-scroll/wing chaotic systems. Since Lorenz presented the first chaotic system with double-wing attractors in 1963 [1], many double-scroll/wing chaotic systems have been developed, such as Chua's system [12], Sprott system [13], Jerk system [14], Chen system [15], Lü system [16], and their modified systems [17–21]. These chaotic systems have greatly promoted the development of chaos theory. With further study on the double-scroll/wing chaotic systems, Suykens and Vandewalle [22], in 1993, constructed the first multi-scroll chaotic

systems by introducing a multi-piecewise-linear function into a Chua's circuit. From then on, the design of multi-scroll/wing chaotic systems (MS/WCSs) has greatly stimulated the researchers' interest. During this period, Yu et al. [23–26] proposed a series of nonlinear polynomial function control methods to realize different MS/WCSs based on double-scroll/wing chaotic systems. Afterward, the theory of MS/WCSs has been developed rapidly, and various MS/WCSs such as Lorenz-system-based MS/WCSs [27–29], Chua's-system -based MS/WCSs [30–32], Sprott-system-based MS/WCSs [33–35], Jerk-system-based MS/WCSs [36–38], Chen-system-based MS/WCSs [39,40], and so on [41–47], have been designed using function control methods. However, recent explorations seem to indicate that it is difficult to make new progress in the design of the MS/WCSs.

Renewed research interest in the design of chaotic systems was generated when a physical nonlinear memristor device was first manufactured in 2008 by Hewlett-Packard Lab [48]. Memristor is a nonlinear element that has many special properties, including programmability, nonlinearity, and memory function [49,50]. Due to the special nonlinearity and memory effect, memristors are often used to construct memristive chaotic systems [51,52]. It is found that memristive chaotic systems have the ability to generate complex dynamical behaviors, especially coexisting behaviors [53] and multistability [54]. Furthermore, the memristive chaotic systems have some advantages in solving dynamic equations [55,56]. Therefore, many scholars have developed a great interest in designing memristive chaotic systems in the past decade. In this endeavor, there are three major efforts: (1) constructing memristive chaotic circuits by using the memristors to replace the resistors of the existing chaotic circuits, such as memristor-based Chua's circuit [57,58], memristor-based Jerk circuit [59–62], and so on [63–65]; (2) constructing memristive chaotic systems by introducing memristors into the existing chaotic systems, such as memristor-based double-scroll/wing chaotic systems [66–69], memristor-based multi-scroll/wing chaotic systems [70–78], and so on [79,80]; (3) constructing memristive chaotic systems based on different memristor models, such memristive chaotic systems [81–87], memristive hyperchaotic systems [88–95], and so on [96–98]. Consequently, the memristive chaotic systems have made extraordinary development, which greatly enriches chaos theory.

In recent years, memristive Hopfield neural networks (MHNNs) with complex, chaotic dynamics have attracted much attention from scholars in the chaos field. The Hopfield neural network presented in 1984 is a brain-like neural network [99], which can exhibit abundant dynamical behaviors, especially chaos [100–106]. Thanks to the inherent memory effect and charge flux relationship, the memristor can be applied in Hopfield neural networks to emulate biological neural synapses or to describe electromagnetic induction effects [107–110]. As a result, a large number of MHNNs have been proposed based on these strategies. It is found that the MHNNs can exhibit complex, chaotic dynamics due to the introduction of the memristors. For example, in 2014, Li et al. [111] constructed the first hyperchaotic MHNN. In 2016, Pham et al. [112] presented the first MHNN with hidden attractors. In 2017, Bao et al. [113] proposed the first MHNN with coexisting attractors. In 2018, Hu et al. [114] presented the first MHNN with hidden coexisting attractors. In 2020, Lin et al. [115] constructed the first MHNN with coexisting infinite attractors. In the same year, Zhang et al. [116] designed the first MHNN with initial-offset behaviors and multi-scroll attractors. In 2022, Lin et al. [117] presented the first MHNN with multi-structure attractors. At the same time, due to their complex dynamics, the MHNNs have wide applications in information encryption and communication security [118–120], especially in the medical image encryption field [121]. Numerous works show that MHNN-based medical encryption schemes have much higher security [122,123]. Therefore, the design of chaotic systems based on MHNNs is becoming a new research hotspot. So far, there have been many important achievements in MHNN-based chaotic systems. However, from the whole research process, the study of the MHNN has just started, and more MHNN-based chaotic systems are still to be explored and discovered. Therefore, a detailed review is needed for the existing chaotic MHNNs. This review aims to address this shortcoming. Compared with the review [108], this review has three advantages: (1) MHNNs are divided into more

types in terms of different functions and positions of memristors in neural networks than just functions; (2) Some important, relevant research results in the last two years have been added; (3) Some of the open questions raised earlier are partially answered in this article.

The rest of this article is organized as follows. In Section 2, some basic knowledge, including HNN, memristor, and chaotic dynamics, is introduced. In Section 3, different modeling methods of the MHNNs are analyzed, and related works are reviewed in detail. MHNN-based application and future work are presented in Section 4. Finally, in Section 5, conclusions are drawn.

2. Introduction of Basic Knowledge

This section first briefly introduces the original model of the Hopfield neural network, then describes the basic concepts and properties of the memristor, and finally gives the basic definitions and classification of the chaotic dynamics.

2.1. Hopfield Neural Networks

Neuroscience shows that the human brain nervous system contains tens of billions of neurons [124]. Neurons are connected to each other by synapses, including chemical and electrical synapses. Each synapse has an adjustable synaptic weight that characterizes the coupling strength between the two neurons. According to these biological principles, the original model of the Hopfield neural network was developed by Hopfield in 1984 [99]. Due to its special network structure and hyperbolic tangent function, the HNN can exhibit abundant dynamical behaviors. The HNN with n neurons can be described by a set of dimensionless nonlinear ordinary differential equations as follows [99]:

$$\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}\tanh(\mathbf{x}) + \mathbf{I} \tag{1}$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{bmatrix}, \mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix}, \mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1j} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2j} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ w_{i1} & w_{i2} & \cdots & w_{ij} & \cdots & w_{in} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & w_{nj} & \cdots & w_{nn} \end{bmatrix} \tag{2}$$

where x_i denotes the *i*-neuron membrane voltage, $\tanh(\mathbf{x})$ is the neuron activation function, and I_i represents *i*-neuron external stimulate current. Furthermore, the **W** represents the synaptic weight matrix, where w_{ij} represents the synaptic weight between the *j*-neuron and to *i*-neuron. Commonly, the synaptic weight is a resistive synaptic weight that can be achieved by a resistor. Over the past decades, many improved models have been constructed based on the original HNN model, such as HNNs with different active functions [125], HNNs with time delay [126], fractional HNNs [127,128], discrete HNNs [129], and so on [130,131].

2.2. Memristor

In the year 1971, Chua proposed the concept of memristor based on the symmetry theory of circuit variables [132]. The memristor is a memory resistor, which describes the relationship between charge and flux. Later on, the memristor concept is extended to include any two-terminal device with a pinched hysteresis loop that always passes through the origin in the voltage-current plane when triggered by a periodic voltage or current signal [50]. In 2008, Hewlett-Packard Lab manufactured the first physical memristor device [48], which opened up a new research field related to memristors. With the development of memristors, many key memristor theories have been built. According to the memristor theories [49,50], a common ideal flux-controlled memristor model can be written by

$$\begin{cases} i = W(\varphi)v \\ \dot{\varphi} = v \end{cases} \tag{3}$$

where v, i are the input voltage and output current, respectively. $W(\varphi)$ is a continuous function of φ, called the memductance, and φ is the flux. For example, in [53], $W(\varphi) = m + n\varphi^2$, where m and n are constant parameters. Moreover, a generic memristor is defined by

$$\begin{cases} i = W(\varphi)v \\ \dot{\varphi} = f(\varphi, v) \end{cases} \tag{4}$$

where $W(\varphi)$ is the memductance, φ is the memristor state variable, and $f(\varphi, v)$ is called the state equation, which is a Lipschitz function. For instance, in [115], $W(\varphi) = \varphi$ and $f(\varphi, v) = \sin(\varphi) + v$.

In recent years, memristors have been widely investigated and applied in various fields [133–135]. Among them, the memristor is often used to construct memristive neural networks due to its bionic characteristics [136–138]. On the one hand, the memristor can be used to describe electromagnetic induction effects in biological nervous systems because of its characteristic of magnetic flux [139,140]. On the other hand, the memristor can be used to emulate neural synapses in nervous systems [141,142].

2.3. Chaotic Dynamics

Chaotic behavior is a type of special dynamical behavior that has many unique properties, such as initial state sensitivity, unpredictability, ergodicity, and topological mixing [1–4]. Over the past few decades, many methods have been proposed to study chaotic dynamics, such as equilibrium point stability, bifurcation diagrams, Lyapunov exponents, phase portraits, Poincare maps, basins of attraction, and so on. In order to better reveal dynamical behavior in chaotic systems, chaotic dynamics with different characteristics are classified and studied. From the perspective of Lyapunov exponents, chaotic dynamics can be divided into chaos, transient chaos, and hyperchaos. Usually, a dynamical behavior with at least one positive Lyapunov exponent on infinite time is considered chaos [12–15]. Transient chaos is a dynamical behavior that the existence of chaos is on finite time [143]. Hyperchaos [88–90], which is more complicated than chaos and transient chaos, is defined as chaos with two or more positive Lyapunov exponents. From the perspective of equilibrium point stability, the chaotic dynamics include self-excited attractors and hidden attractors [144]. If an attractor's basin of attraction does not intersect with any open neighborhood of the system equilibria, it is referred to as a hidden attractor. Otherwise, it is referred to as a self-excited attractor. From the perspective of attractor structure, chaotic dynamics contain single-scroll/wing attractors and multi-scroll/wing attractors [23–26]. The multi-scroll/wing attractors have multiple single-scroll/wing chaos trajectories. Generally, multi-scroll/wing attractors are more complex compared to single-scroll/wing attractors. From the perspective of stability, chaotic dynamics contain coexisting attractors [53], multistability [62], and extreme multistability [58]. The complex dynamical phenomenon of coexisting attractors consists of two distinct types of chaotic behaviors in two different initial conditions. Multistability refers to the simultaneous existence of three or more dynamical behaviors in distinct initial conditions. Multistability implies that a rich variety of stable states exists in chaotic systems, which mirrors the qualities of complex systems. There are two special types of systems with multistability: extreme multistability systems and megastable systems [145]. Extreme multistability is the term for the phenomenon in which infinitely many coexisting attractors. Additionally, the phenomenon of the coexistence of infinite attractors with the same topology structures and different positions is called initial-offset coexisting behaviors [146].

3. Memristive Hopfield Neural Networks

In this section, the MHNNs are divided into four categories according to the different functions and positions of memristors in neural networks. The modeling mechanism of each category is analyzed, and existing MHNN-based chaotic systems are reviewed and introduced.

3.1. Using Memristors to Emulate Neural Synapses

In the traditional HNN model (1), the synaptic weight w_{ij} is a resistive synaptic weight that is realized by using resistors to emulate neural synapses. Compared with

resistors, memristors have many synapse-like properties, including nanoscale, nonlinearity, adjustability, and nonvolatility, which make them more suitable for emulating neural synapses. Therefore, when using the memristors to simulate the neural synapses in the traditional HNN model, the memristive HNN model can be constructed [108]. That is to say that when the resistive synaptic weight w_{ij} is replaced with the memristive synaptic weight $W(\varphi)$, the MHNN is modeled. Due to the introduction of the memristive synaptic weight, the MHNN model is closer to the biological nervous system. As a result, the MHNN can generate more complex dynamical behaviors. Generally, the biological nervous system has two types of neural synapses, namely self-connection autapses and coupling synapses. For the HNN model, there are two types of synaptic weights, namely self-connection synaptic weights w_{ij} ($i = j$) and coupling synaptic weights w_{ij} ($i \neq j$). Therefore, according to different types of memristor synapses, the MHNN can be divided into two categories: memristor-autapse-based MHNN and memristor-synapse-based MHNN.

Category 1: Memristor-autapse-based MHNNs

The models of the memristor-autapse-based MHNNs can be constructed by replacing resistive self-connection synaptic weights with memristive self-connection synaptic weights. Taking $n = 3$ as an example, the connection topology for the memristor-autapse-based MHNN with a memristor autapse is shown in Figure 1. As shown in Figure 1, we replace resistive self-connection synaptic weight w_{22} with the memristive self-connection synaptic weight $W(\varphi)$, and then the original HNN model has an additional differential equation about φ. Thus, its mathematical model can be described in a dimensionless form as

$$\begin{cases} \dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}\tanh(\mathbf{x}) + \mathbf{I} \\ \dot{\varphi} = f(\varphi, v_i) \end{cases} \tag{5}$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}, \mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & W(\varphi) & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \tag{6}$$

where x_i is the membrane potential of the i-neuron, and $\tanh(x_i)$ represents the neuron activity function.
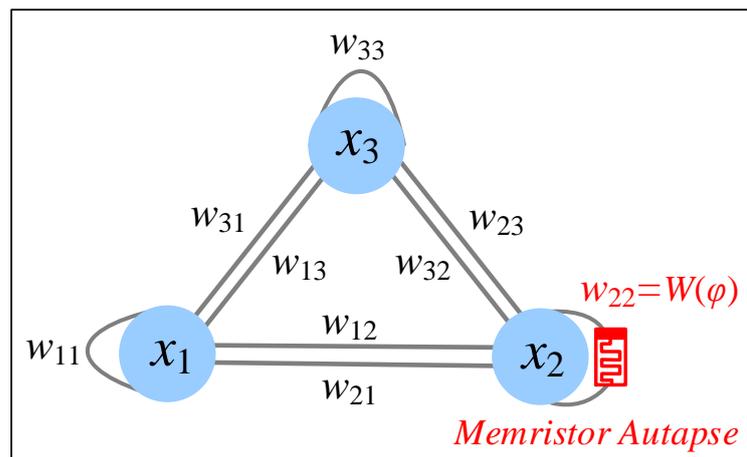

Figure 1. Connection topology of a memristor-autapse-based MHNN.

Over the past years, many different MHNNs with memristor autapses have been constructed, and various chaotic dynamical behaviors have been revealed. For example, Ref. [147] proposed an MHNN model with three neurons by using hyperbolic-type memristor autapses to replace resistor autapses. The authors found that the MHNN can generate abundant dynamical behaviors, including chaos, coexisting attractors, and the Feigenbaum tree. Ref. [148] constructed an MHNN with two neurons by introducing a linear memristor autapse, and bursting firing and chaos were observed. Initial offset coexisting behaviors

and multi-double-scroll attractors have been reported in an MHNN with a multi-piecewise quadratic nonlinearity memristor autapse [116]. Concurrently, some similar MHNNs with multi-scroll chaotic attractors have been reported in [149,150]. By considering memristive self-connection synaptic weight, an MHNN with one neuron has been proposed [151]. Numerical analysis and experimental results show that the MHNN with one neuron can generate multiple firing behaviors like coexisting periodic and chaotic spiking, chaotic bursting, and periodic bursting. Moreover, hidden extreme multistability has been found in a one-neuron-based MHNN with cosine memristor autapse [152]. Infinitely many coexisting hidden attractors have been reported in a two-neuron-based MHNN with a modified hyperbolic-type memristor autapse [153]. In particular, recently, Ref. [154] proposed an MHNN with two memristor autapses. The authors found that the MHNN can exhibit complex initial-offset plane coexisting behaviors. Additionally, the MHNN with multiple memristor autapses has also been investigated [155]. The research results show that with the increase of memristor autapses, the MHNN can generate different dynamical behaviors like exciting neurodynamics or inhibiting neurodynamics.

Category 2: Memristor-synapse-based MHNNs

The model of the memristor-synapse-based MHNNs can be constructed by replacing resistive coupling synaptic weights with memristive coupling synaptic weights. Taking $n = 3$ as an example, the connection topology for the memristor-synapse-based MHNN with a memristor synapse is shown in Figure 2. As shown in Figure 2, we replace a resistive coupling synaptic weight w_{12} with a memristive coupling synaptic weight $W(\varphi)$, and then the original HNN model has an additional differential equation about φ. Thus, its mathematical model can be described in a dimensionless form as

$$\begin{cases} \dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}\tanh(\mathbf{x}) + \mathbf{I} \\ \dot{\varphi} = f(\varphi, v_j) \end{cases} \tag{7}$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}, \mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ W(\varphi) & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \tag{8}$$
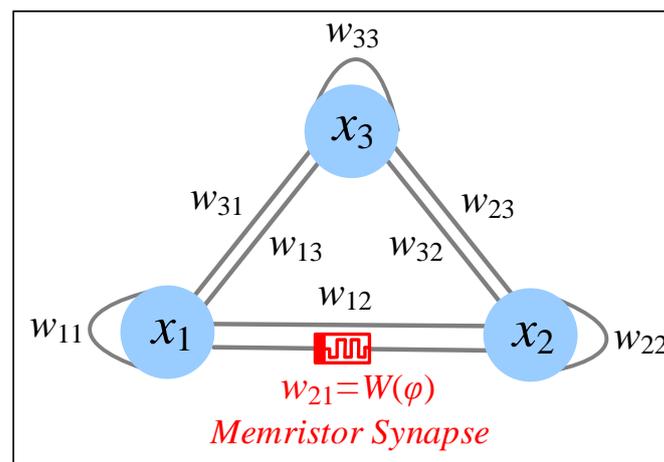

Figure 2. Connection topology of a memristor-synapse-based MHNN.

In recent years, various MHNNs with memristor synapses have been reported. Dynamical analysis and numerical simulation showed that MHNNs could generate complex, chaotic dynamical behaviors. For instance, Ref. [111] proposed an MHNN with three neurons by using a memristive coupling synaptic weight to substitute a resistive coupling synaptic weight. The proposed MHNN can generate hyperchaotic behavior. Hidden attractors have been revealed in a three-neuron-based MHNN with a quadratic memristor synapse [112]. Based on a hyperbolic-type memristor synapse, Ref. [113] found that the

MHNN with three neurons can generate coexisting asymmetric attractors. Particularly, Ref. [156] has shown that the MHNN with an improved hyperbolic-type memristor synapse can exhibit chimera state, synchronization, and oscillation death. Infinitely many coexisting attractors have been observed in a four-neuron-based MHNN with a multi-stable memristor synapse [115]. In [157], a fraction-order MHNN with a hyperbolic-type memristor synapse is proposed. Research results show that the fraction-order MHNN can generate complex dynamical transition, evolving from periodic to chaotic and finally to coexisting attractors. Complete synchronization and anti-phase synchronization have been investigated in two coupled MHNNs with a hyperbolic-type memristor synapse [158]. Considering the influence of synaptic cross-talk in the MHNN with three neurons, multi-stability, asymmetry attractors, and anti-monotonicity have been observed in an MHNN with a novel hyperbolic-type memristor synapse [159]. The complex phenomenon of multi-scroll attractors has been found in two different MHNNs [122,160]. The multi-structure attractors have been reported in a four-neuron-based MHNN with different memristor synapses [117]. Initial-offset coexisting behaviors have been revealed in some MHNNs with memristor synapses [161,162]. Furthermore, Ref. [163] designed an MHNN with two generalized multi-stable memristor synapses. The authors found that the MHNN with two memristor synapses can generate chaos and coexisting asymmetric attractors. Such complex dynamical behaviors have been demonstrated on a DSP platform. Similarly, Ref. [164] designed an MHNN with a hyperbolic-type memristor autapse and a hyperbolic-type memristor synapse, which can generate grid multi-scroll attractors. The MHNN with three hyperbolic-type memristor synapses has been investigated in Ref. [165]. Recently, locally active memristors have attracted much attention in the construction of MHNNs due to their synapse-like local activity. Ref. [166] presented an MHNN with three neurons by replacing a coupling synapse with a tristable locally active memristor. It is found that the MHNN can generate complex bursting oscillation and multistability. Ref. [167] proposed a fractional-order MHNN with a locally active memristor synapse. Research results show that the fractional-order MHNN can exhibit the dynamical behavior of the coexistence of multiple attractors. In addition, multi-scroll chaotic attractors have also been observed in an MHNN with a local active memristor synapse [168].

3.2. Using Memristor to Describe Electromagnetic Induction

The traditional HNN model does not consider the influence of electromagnetic induction. In fact, numerous physical and biological experiments show that the biological nervous systems are often affected by the electromagnetic field generated by internal membrane voltages difference and external electromagnetic radiation [107,108]. According to the physical law of electromagnetic induction, the distribution and density of magnetic flux across the membrane can be changed when a neuron is exposed to an electromagnetic field. Consequently, the electrical activities of the biological nervous system can be changed due to the electromagnetic field. To consider the effects of the electromagnetic field on the dynamics of the nervous system, the flux-controlled memristor is introduced into the traditional HNN model to describe the electromagnetic induction current. When the effect of an electromagnetic field on a neuron is considered as magnetic flux across the membrane of the neuron, the coupling between magnetic flux and membrane potential can be described by using a voltage-controlled memristor [109,110]. Consequently, the nervous systems under an electromagnetic field can be modeled by adding a magnetic induction current in the traditional HNN model. That is to say that an MHNN model can be constructed by considering the effect of an electromagnetic field. Usually, according to different types of electromagnetic fields, the MHNNs can be divided into another two categories: MHNNs under external electromagnetic radiation and MHNNs under an internal electromagnetic field.

Category 3: MHNNs under external electromagnetic radiation

When considering neurons are exposed to external electromagnetic radiation, an electromagnetic induction current can be described by a flux-controlled memristor [107,108]. Thus,

the model of the MHNNs under external electromagnetic radiation can be constructed. Taking $n = 3$ as an example, the connection topology of an MHNN under external electromagnetic radiation is given in Figure 3. The mathematical model of the MHNN under electromagnetic radiation can be written as

$$\begin{cases} \dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}\tanh(\mathbf{x}) + I_M + \mathbf{I} \\ \dot{\varphi} = f(\varphi, x_i) \end{cases} \tag{9}$$
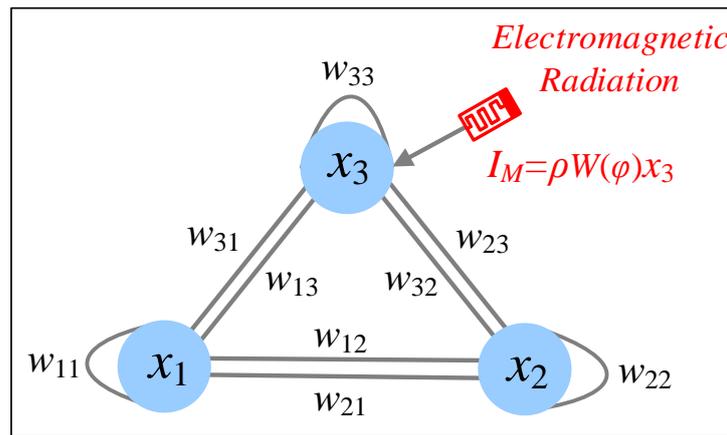
where

$$I_M = \rho W(\varphi) x_i \tag{10}$$

Figure 3. Connection topology of an MHNN under external electromagnetic radiation.

Here the **W** represents the synaptic weight matrix, and I_M represents the electromagnetic induction current caused by external electromagnetic radiation. The parameter ρ represents the coupling strength between membrane potential and magnetic flux. $W(\varphi)$ is the memconductance of the flux-controlled memristor, and x_i is the membrane potential of the neuron under electromagnetic radiation. In this case, x_i is the membrane potential x_3 of the 3-neuron.

Based on this model, the influence of external electromagnetic radiation on chaotic dynamical behaviors in neural networks can be analyzed. For example, complex dynamical behaviors, including coexisting chaos and transient chaos, have been revealed in a three-neuron-based MHNN under external electromagnetic radiation [114]. The effects of external electromagnetic radiation distribution on the chaotic dynamical behaviors of a neural network with n neurons are investigated in an MHNN [169]. The authors found that with the increasing number of neurons under external electromagnetic radiation, the dynamical behavior of the MHNN gradually changes from period to chaos, then to transient chaos, and finally to hyperchaos. Hidden extreme multistability with hyperchaos and transient chaos is discussed in a three-neuron-based HNN under external electromagnetic radiation [170]. Multi-scroll attractors [171] and multi-style attractors [172] have been reported in MHNNs under external electromagnetic radiation. Furthermore, Ref. [123] proposed a ring MHNN under external electromagnetic radiation. It is found that the ring MHNN can generate complex hyperchaotic behavior. Additionally, Ref. [173] found that an MHNN under external electromagnetic radiation can exhibit multistable dynamics, including periodic attractors, quasi-periodic attractors, transient chaotic attractors, and hidden chaotic attractors. Additionally, an MHNN with different external stimuli, including electromagnetic radiation and multi-level logic pulse, has been studied in Ref. [174]. The research results demonstrated that the MHNN with multiple external stimuli could generate complex coexisting attractors and multi-scroll attractors.

Category 4: MHNNs under internal electromagnetic field

When possessing a potential difference between two neurons, an internal electromagnetic induction current appears in the neural network, which can be described by a

flux-controlled memristor synapse [107,108]. Thus, the model of the MHNN under an internal electromagnetic field can be constructed. Taking $n = 3$ as an example, a structure diagram of a three-neuron-based MHNN under an internal electromagnetic field is given in Figure 4. As shown in Figure 4, an induced current is sensed by the internal electromagnetic field caused by potential difference $(x_2 - x_3)$ between two neurons in the HNN, which can be characterized by an electromagnetic induction current IM following through a flux-controlled memristor synapse. Therefore, an MHNN under an internal electromagnetic field is established, which is mathematically described as

$$\begin{cases} \dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}\tanh(\mathbf{x}) + I_M + \mathbf{I} \\ \dot{\varphi} = f(\varphi, (x_i - x_j)) \end{cases} \quad (11)$$

where

$$I_M = \rho W(\varphi)(x_i - x_j) \quad (12)$$

where ρ represents coupling strength between memristor and neuron, $W(\varphi)$ is a memductance function, $(x_i - x_j)$ is the potential difference between two neurons.
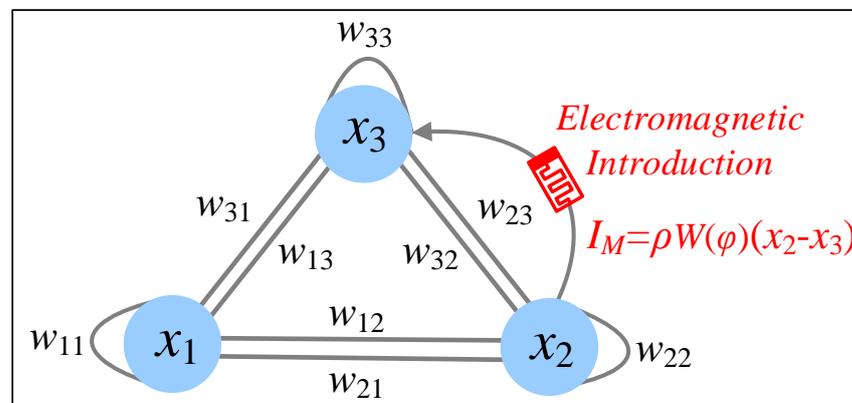

Figure 4. Connection topology of an MHNN under an internal electromagnetic field.

MHNNs under internal electromagnetic fields have been reported in recent years. For example, Ref. [175] proposed a two-neuron-based MHNN by considering an internal electromagnetic induction current. It is found that the MHNN can produce coexisting multi-stable patterns such as spiral chaotic patterns with different dynamic amplitudes, periodic patterns with different periodicities, and stable resting patterns with different positions. Similarly, considering an internal induced current described by a non-ideal memristor synapse, Ref. [176] designed a bi-neuron MHNN with coexisting attractors. The MHNN with different numbers of internal electromagnetic induction currents has been investigated in Ref. [177]. The authors found that when using hyperbolic-type memristors to link different neurons, the MHNN can generate different dynamical behaviors, including periodic and chaotic bubbles, initial-related multistable patterns, and riddled basins of attraction. Utilizing a hyperbolic-type memristor and a quadratic nonlinear memristor to simulate the effects of internal electromagnetic induction and external electromagnetic radiation, a three-neuron-based MHNN with coexisting behaviors has been reported in Ref. [178]. The generation mechanism of chaos has been researched in a ring fraction-order MHNN with an internal electromagnetic induction current [179]. Moreover, the fractional-order MHNN with three internal electromagnetic induction currents has been reported in Ref. [180]. It is found that the fractional-order MHNN can exhibit complex coexisting behaviors. Initial-offset coexisting hyperchaotic attractors have been observed in a coupled MHNN with an induced current [121]. Ref. [181] proposed an MHNN with time delay by using a memristor synapse to emulate the electromagnetically induced current. The complex dynamical behaviors, including coexisting chaos, periodic limit cycles, and stable point attractors, have been revealed in a two-neuron-based MHNN with an internal electromagnetic induction current [182]. Additionally, hyperchaotic multi-

structure attractors have been reported in a memristor-couple asymmetric MHNN with an internal electromagnetic induction current caused by membrane potential difference [118].

To facilitate readers' reading, we summarize different dynamical behaviors in the MHNNs, as shown in Table 1. As can be seen, many complex dynamical behaviors have been revealed from the MHNNs, especially coexisting attractors, hidden attractors, multi-scroll attractors, multistability, extreme multistability, and initial-offset coexisting behaviors. Furthermore, some unfrequent dynamical behaviors, such as the Feigenbaum tree, chimera state, and multi-structure attractors, have also been reported in the MHNNs.

Table 1. Various dynamical behaviors in the MHNNs.

Dynamical Behaviors	References
Transient chaos	[114,169,173]
Hyperchaos	[111,118,123,169]
Feigenbaum tree	[147]
Coexisting attractors	[113,147,157,163,174,176,178,180,182]
Bursting firing	[148,151,166]
Chimera state	[156]
Hidden attractors	[112,152,153,170,173]
Synchronization	[156,158]
Multi-scroll attractors	[116,120,122,149,150,160,164,168,171,172,174,183]
Multi-structure attractors	[117,118]
Multistability	[124,151,155,159,166,167,173,175,177]
Extreme multistability	[115,116,121,152–154,161,162,170]
Hidden extreme multistability	[152,153,170]
Initial-offset coexisting behaviors	[116,121,154,161,162]

4. Application and Future Works

Due to flexible network structure and abundant dynamical behaviors, traditional HNNs have been widely applied in various fields, such as associative memory [101], information protection [102], process optimization [103], and so on [129,130]. Compared with traditional HNN models, the MHNNs have some advantages in terms of chip area, computing speed, and complex dynamical behaviors, which makes them have wider application ranges. For example, Ref. [184] designed a reconfigurable MHNN circuit that can realize associative memory quickly. An efficient combinatorial optimization method has been presented by weight annealing in MHNN [185]. Due to their complex, chaotic dynamical behaviors, the MHNNs can be used to generate random numbers with high randomness. Thus, it is very suitable for information protection and image encryption. For example, Ref. [116] proposed an image encryption scheme based on an MHNN with initial-offset coexisting behaviors. The encryption results show that the proposed encryption scheme has excellent security due to the high randomness of the MHNN. Ref. [186] designed a pseudo-random number generator by using an MHNN with complex, chaotic attractors. At the same time, some image encryption schemes have been presented and verified based on various MHNNs, such as the MHNNs with multi-scroll attractors [150,168,184], the MHNNs with multi-style attractors [172], and the fractional-order MHNN with coexisting attractors [180]. Furthermore, Refs. [118,167] designed color image cryptosystems based on an MHNN with hyperchaotic multi-structure attractors and an MHNN with coexisting multiple attractors, respectively. Additionally, Ref. [183] proposed an audio encryption scheme based on a fractional-order multi-scroll MHNN. Recently, due to their characteristics of artificial intelligence and complex dynamical behaviors, MHNNs have attracted much attention in the field of medical image encryption. For instance, Ref. [121] designed a

biomedical image encryption scheme based on an MHNN with brain-like initial-boosted hyperchaos. Experimental evaluations showed that the designed medical image cryptosystem has some advantages in the keyspace, information entropy, and key sensitivity. Similarly, some other medical image encryption schemes have also been reported based on different MHNNs, such as ring MHNN with hyperchaos [123], MHNN with multi-scroll attractors [122], and MHNN with multi-structure attractors [117]. Moreover, to ensure the information security of the medical data transmitted through the Internet of Things, Ref. [120] proposed a medical data encryption method based on a multi-scroll MHNN.

As reviewed above, the MHNNs with complex, chaotic behaviors have greatly stimulated researchers' interest, and many valuable research results have been reported to date. In particular, some work has answered the questions raised earlier in the review [108]. For example, different from the previous special structure, the MHNNs with ring structure have been proposed in [123,165]. To construct a reliable neural network model, internal electromagnetic induction has been considered in some recent research [175–182]. However, several important questions still remain to be answered. There are at least four aspects that can be further explored. First, it is well known that the nervous system is composed of a large number of neurons. However, the existing MHNN models only consider several neurons, such as two-neuron-based MHNNs [148], three-neuron-based MHNNs [156], and four-neuron-based MHNNs [150]. So, the MHNN with more neurons needs to be further developed and investigated. Second, the biological nervous system has many neural autapses and synapses. So far, the current MHNNs mainly consider a few memristor autapses and memristor synapses, like one-memristor-based MHNNs [162] and two-memristor-based MHNNs [154]. Undoubtedly, the MHNN with multiple memristor autapses and memristor synapses needs to be further designed and researched. Third, the biological nervous system is very sensitive to external stimuli such as electromagnetic radiation, electric field, light, temperature, noise, and so on. At present, other factors are rarely considered in the existing MHNNs besides electromagnetic radiation. Therefore, to model a reliable and realistic neural network model, different external stimuli and internal factors should be added to MHNN models. Fourth, from the viewpoint of chaos-based application, the MHNNs have the features of high dimensional equations, artificial intelligence [187–189], and complex dynamical behaviors [190], which makes them more suitable for application in medical image encryption, video encryption, and pseudo-random number generators. Therefore, MHNN-based applications in information protection can be further discussed in the future. Additionally, most of the current MHNNs are based on the memristor mathematical model. In our opinion, the real nano-memristor is essential if the memristor-based chaotic systems and neural networks are to be applied in practical engineering [191,192]. Undoubtedly, it is a new research direction to construct MHNNs with complex, chaotic dynamics using nano-memristor.

5. Conclusions

In this review, several significant results on MHNN-based chaotic systems are introduced for readers in the field of chaos. First, the basic knowledge of Hopfield neural networks, memristors, and chaotic dynamics is illustrated. Then according to the different functions of the memristors in MHNNs, we divide MHNNs into four different models, namely, the MHNNs with memristor autapses, the MHNNs with memristor synapses, the MHNNs under electromagnetic radiation, and the MHNNs with electromagnetic induction. The modeling mechanism, modeling method, and pioneering works of each type are introduced. Concurrently, we reviewed some recent important papers related to those types. Finally, potential applications of the MHNNs within different areas, especially information encryption, are also introduced, and future work has been discussed. This discussion could be helpful for further investigation of MHNN-based chaotic systems. Although some MHNN-based chaotic systems and their chaotic dynamics have been reported, it is still in the infant stage and needs to be further researched. We hope that this review can provide a good reference for researchers who want to investigate such chaotic systems deeply.

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