A review of *E* infinity theory and the mass spectrum of high energy particle physics

M.S. El Naschie [*]

Solvay Institute, University of Brussels, Brussels, Belgium
Department of Astrophysics, Faculty of Science, University of Cairo, Cairo, Egypt
Department of Mathematics, School of Electronics and Computer Science, University of Surrey, Guildford, UK

Abstract

The essay outlines the basic conceptual framework of a new space–time theory with application to high energy particle physics. Both achievements and limitations are discussed with direct reference to the mass spectrum problem. © 2003 Elsevier Ltd. All rights reserved.

1. The main purpose of the present work

In what follows we would like to give a short account of the so-called *E* infinity ($\varepsilon^{(\infty)}$) theory, the main application of which has been so far in determining coupling constants and the mass spectrum of the standard model of elementary particles. I am afraid I will have to make a long story (which took many years of work) quite short with all of what this entails in reading it. The results of *E* infinity are at present contained in dozens of published papers too numerous to refer to them all, but for the purpose of filling the gaps in the present summary, half a dozen papers which are mentioned at the end may offer good help in overcoming the inevitable shortcomings in a condensed presentation (see Ref. [1–7]).

2. An outline of the conceptual framework of the theory

2.1. General remarks

The main conceptual idea of my work (which is encoded in Figs. 1 and 5) is in fact a sweeping generalisation of what Einstein did in his general theory of relativity, namely introducing a new geometry for space–time which differs considerably from the space–time of our sensual experience. This space–time is taken for granted to be Euclidean. By contrast, general relativity persuaded us that the Euclidean 3 + 1 dimensional space–time is only an approximation and that the true geometry of the universe in the large, is in reality a four dimensional curved manifold. In *E* infinity we take a similar step and allege that space–time at quantum scales is far from being the smooth, flat and passive space which we use in classical physics [1–3]. On extremely small scales, at very high observational resolution equivalent to a very high energy, space–time resembles a stormy ocean [1]. The picture of a stormy ocean is very suggestive and may come truly close to what we think the high energy regime of the quantum world probably looks like (see Figs. 1–4). However such a picture is not accessible to mathematical formulation, let alone an exacting solution. The crucial step in *E* infinity formulation was to identify the stormy ocean with vacuum fluctuation and in turn to model this fluctuation using the mathematical tools of non-linear dynamics, complexity theory and chaos [1,8,9]. In particular the geometry of chaotic

---

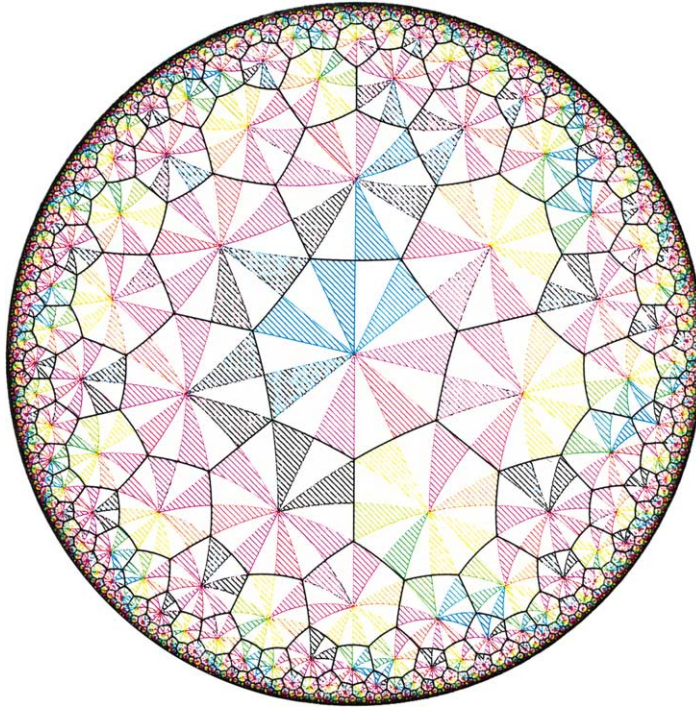[*] Address: P.O. Box 272, Cobham, Surrey, UK.

Fig. 1. Tiling the plane using Klein's modular curve in the Beltrami–Poincare representation. *E* infinity theory alleges that the quantum gravity of space–time is a hyperbolic fractal on a Klein modular group akin to what is shown in the figure. The relevance to high energy physics is more direct than one may suspect. For instance, the Weinberg mixing parameter $\sin^2 \theta_w$ at the electroweak scale is given by the cosine of the $3\pi/7$ angle of Klein's modular curve as $\cos(3\pi/7) \cong 0.2225$ in excellent agreement with the experimental measurement. Similarly the number of triangles gauged in our mass norm gives the approximate value for the constituent mass of the fundamental light quarks. Since we have 168 automorphisms, then the number is (2)(168) and the masses are given by $m_u^* = m_d^* = 336$ MeV for the up and down quarks. In addition the 42 point unique orbit of the original Klein curve corresponds to the inverse of the quantum gravity couplings constant in the non-supersymmetric case.

dynamics, namely fractal geometry is reduced to its quintessence, i.e. Cantor sets (see Fig. 5) and employed directly in the geometrical description of the fluctuation of the vacuum. How this is done and how to proceed from there to calculating for instance the mass spectrum of high energy elementary particles is what I will try to explain and summarise in the following essay.

As is well known, special relativity fused time and space together, then came general relativity and introduced a curvature to space–time. Subsequently Kaluza and later on Klein added one more dimension to the classical four in order to unify general relativity and electromagnetism. From this time on, the dimensionality of space–time played a paramount role in the theoretical physics of unification leading to the introduction of the 26 dimensions of string theory, the 10 dimensions of super string theory and finally the Heterotic string theory [10] dimensional hierarchy 4, 6, 10, 16 and 26. This is all apart from the so-called abstract or internal dimensions of various symmetry groups used, for instance, the 8 dimensions of the SU(3) of the strong interaction.

By contrast, in *E* infinity theory we admit formally infinite dimensional "real" space–time [1,2]. However this infinity is hierarchical in a strict mathematical way and we were able to show that although *E* infinity has formally infinitely many dimensions, seen from a distance, i.e. at low resolution or equivalently at low energy, it mimics the appearance of a four dimensional space–time manifold which has only four dimensions. Thus the four dimensionality is a probabilistic statement, a so-called expectation value. It is remarkable that the Hausdorff dimension of this topologically four dimensional-like "pre" manifold is also a finite value equal to $4 + \phi^3$, where $\phi = (\sqrt{5} - 1)/2$ with the remarkable self-similar continued fracture representation (which is in a sense self-similar as Figs. 1, 2, and 5):

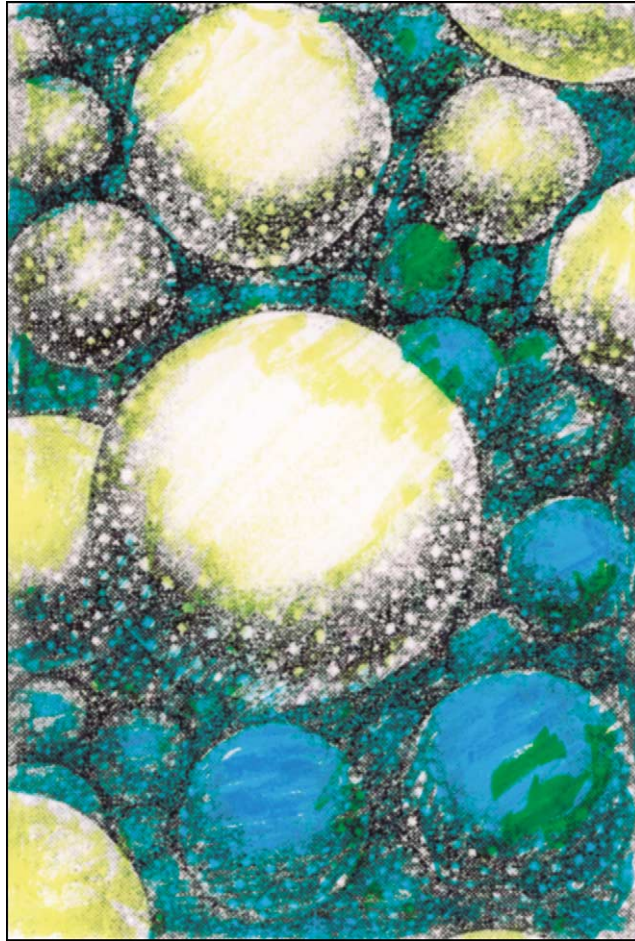$$4 + \phi^3 = 4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \cdots}}}$$

Fig. 2. A depiction of T. Right's cosmos as a form of sphere packing on all scales. This is very similar to the author's $S^{(\infty)}$. This space is related, but by no means identical to the Cantorian E infinity space. One notes the similarity with Fig. 4. However Fig. 4 was published for the first time in Chaos, Solitons & Fractals 11 (2000) 453–464. By contrast, T. Right's cosmos was conceived by him in the middle of the 18th century.

There are various ways for deriving this result which was given in detail in numerous previous publications. However maybe the simplest and most direct way is to proceed from the mathematical definition of E infinity.

2.2. Definition of the E infinity space

Definition

E infinity refers to the limit set of a pre-geometry model of the transfinite extension of a projective Borel hierarchy [11].

From the definition of the above and in particular the definition of Borel sets and projective hierarchy [11], it follows that if the sets involved in the Borel set are taken themselves to be transfinite Cantor sets (see Fig. 5), then the Hausdorff dimension of E infinity could be written as [1,2,8,9]

$$\langle \mathrm{Dim}\, E - \infty \rangle_H = \sum_0^\infty n (d_c^{(0)})^n$$

where $d_c^{(0)}$ is the Hausdorff dimension of the involved transfinite sets where the superscript refers to the Menger–Urysohn dimension of the one dimensional Cantor set, namely $1 - 1 = 0$. Now there is a well known theorem due to

Fig. 3. A fractal-like universe, with clusters of clusters ad infinitum as envisaged by the Swedish astronomer C. Charlier who lived between 1862 and 1934. This work was clearly influenced by the work of the Swedish astrophysicist A. Swedenborg (1688–1772).

Fig. 4. An artistic impression of *E* infinity space–time published by the author almost two and a half centuries after the work of Swedenborg and without any conscious knowledge of this or similar work of the same period. The figure represents a form of space made up of turbulent disorderly packed 3D spheres. *E* infinity space is similar only it has infinitely more dimensions.

Mauldin and Williams which states that with a probability equal to one, a one dimensional randomly constructed Cantor set will have the Hausdorff dimension [1,2] equal to $(\sqrt{5} - 1)/2 = 0.618033$, i.e. the golden mean ϕ. Setting $d_c^{(0)} = \phi$ one finds
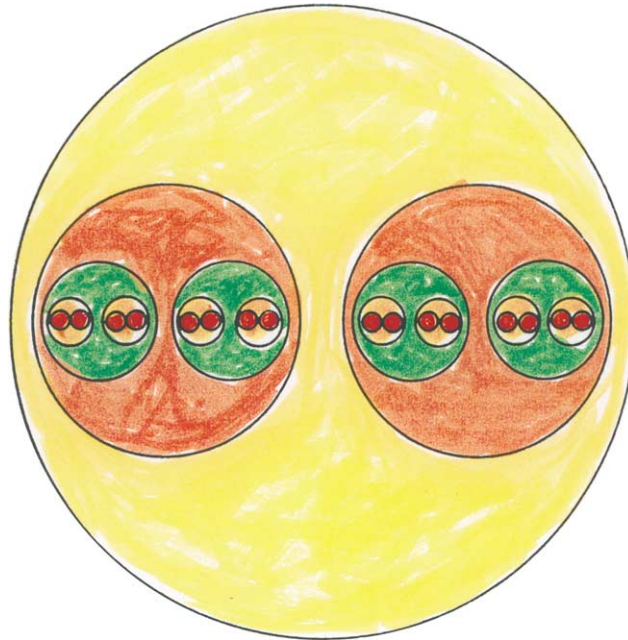
Fig. 5. An alternative two dimensional construction of a topological equivalent to the Cantor set using pairs of circles. The E infinity limit set is very similar but has infinitely many dimensions and not only two like here.

$$\langle \mathrm{Dim}\, E - \infty \rangle_{\mathrm{H}} = (0)(\phi)^0 + (1)(\phi)^1 + (2)(\phi)^2 + (3)(\phi)^3 + \cdots$$
$$= 4 + \phi^3$$
$$= (1/\phi)^3$$
$$= 4.236067977\cdots$$

as anticipated. It is now instructive to contemplate the following. The intersection rule of sets shows that [1,2] we can lift $d_{\mathrm{c}}^{(0)}$ to any dimension n as follows

$$d_{\mathrm{c}}^{(n)} = (1/d_{\mathrm{c}}^{(0)})^{n-1}$$

Thus taking $d_{\mathrm{c}}^{(0)} = \phi$ and $n = 4$, one finds

$$d_{\mathrm{c}}^{(4)} = (1/d_{\mathrm{c}}^{(0)})^3 = 4 + \phi^3 = (1/\phi)^3 = 4.236067977\cdots$$

In other words, we have

$$\langle \mathrm{Dim}\, E - \infty \rangle_{\mathrm{H}} = d_{\mathrm{c}}^{(4)} = 4 + \phi^3$$

which shows that the expectation value of the Hausdorff dimension of E infinity is $4 + \phi^3$ but its intrinsic embedding "expectation" dimension is exactly 4 and that although the formal dimension is infinity. In fact the expression $\sum_0^\infty n(d_{\mathrm{c}}^{(0)})^n$ may be regarded as the sum of the weighed $n = 1$, $n = 2$, $n = 3$, ... dimensions where the weights are the golden mean and its power. That is why E infinity is hierarchical. Note that intrinsic embedding is just another name for the Menger–Urysohn dimension and that our intuitive embedding dimension for $d_{\mathrm{c}}^{(0)}$ is not zero, but one. Similarly for $d_{\mathrm{c}}^{(4)}$ it is 5 and not 4.

2.3. The limit set, Kleinian groups and Penrose tiling

It then turns out that the limit set of any Kleinian-like group is a set which is best described in terms of chaotic Cantor sets (see Figs. 1 and 5) and E infinity [9]. This fact is clear from the work of Mumford et al. [12]. Another surprise was the realisation that

$$\langle \mathrm{Dim} - \infty \rangle_{\mathrm{H}} = 4 + \phi^3 =\sim \langle n \rangle$$

is just twice the isomorphic length of the so-called Penrose-hyperbolic fractal tiling [1,2]

$$l \leqslant \frac{1}{2}(4 + \phi^3)(\varrho)$$

where ϱ is the radius of the circular region considered.

In other words if one projects the space–time of vacuum fluctuation on a Poincare circle we will see a hyperbolic tesselation of this circle with predominantly Klein curve-like geometry [13] which ramifies at the circular boundary exactly as in many of the famous pictures of the Dutch artist M. Echer. It is an important part of our thesis that actual quantum space–time strongly resembles the hyperbolic geometry of the ramified $\chi(3\pi/7)$ Klein curve (see Fig. 1).

We started with the picture of a turbulent sea which we took to model vacuum fluctuation then moved to model the space–time of the vacuum using infinite numbers of unions and intersections of an elementary random Cantor set only to find at the end that this is the limit set of the well known Möbius–Klein transformation of space [9,12,13] which may be represented using the Beltreram–Poincare methods of hyperbolic geometry [1]. In other words, quantum space–time is conceived here as a hyperbolic fractal in which the low resolution major part is the original Klein curve while the high resolution part at the circular boundary may be considered a transfinite correction which may be superimposed to it following certain rules, just like in a perturbation analysis of a weakly non-linear problem. For instance, and as will be shown later on, the dimension SU(5) = 24 will be replaced by $24 - \phi^9 = 24 - 0.013155617 = 23.98684438$ and similarly Dim $E_8 \otimes E_8 = 496$ will become $496 - k^2 = 45.96747752$ where $k = \phi^3(1 - \phi^3)$ and ϕ^9 are examples of such transfinite "corrections".

We will see later on that the Heterotic string hierarchy is also imbedded in E infinity and that the theory is clearly related to A. Conne's non-commutative geometry [14] as well as the four dimensional fusion algebra and M. Friedman's theory of four dimensional topological manifolds [14]. It is worthwhile to note that in all the preceding theories, our Hausdorff dimension $d_c^{(0)} = (\sqrt{5} - 1)/2 = \phi$ plays a pivotal role [14].

2.4. The string connection and KAM theorem

Having mentioned string theory, we should mention that in string theory particles are perceived as highly localised vibration of Planck length strings, so that strictly speaking, within string theory there is no essential difference between a resonance particle and say a meson or an electron. The nice thing about the geometrical–topological picture which E infinity theory offers is that the string picture may be retained yet in another form, namely as a sizzling Cantor set [2], simulating string vibration and that such strings are embedded in E infinity as will be shown. Thus the two theories remain reasonably compatible. However we are running ahead of the logical sequence and we should return to the Cantorian hyperbolic geometry of quantum space–time. It is well known that hyperbolic geometry is highly distortive. Taking the original Klein curve as an example [13], all triangles are essentially the same, yet they are distorted and the further away from the center of the Beltrami–Poincare projection we are, the larger the distortion is. It is here that our basic conjecture regarding the mass spectrum of high energy particle physics come to the fore. We will show that all particles are just different scaling of all other particles as long as we disregard all other aspects and concentrate on energy. E infinity has a set of scaling exponents which distorts the "figure" of any particle so that it all depends on the region which the particle inhabits, or said in another way, it all depends on the way we probe space–time (see Fig. 1).

Exactly as in Einstein's general relativity and even far more so, space–time topology decides on the mass spectrum which we observe. Seen that way one should really expect infinite numbers of elementary particles but this is clearly not the case. It is one of the main pillars of the E infinity theory to hold that the whole issue is that of stability. Only stable particles could be observed. Again it is one of the most important results of E infinity theory to reason that the question of stability of elementary particles is closely related to KAM theory, Arnold diffusion and the vague attractor of Kolomogorove [2] and that the most irrational number that exists, namely $\phi = (\sqrt{5} - 1)/2$ is the secret of the stability of certain elementary particles [9,13]. Vibration simulating particles which do not have a sufficiently irrational winding number dissipate as fast as they are produced.

3. Dimensions and coupling constants

3.1. The fine structure constant and the special orthogonal group SO(n)

Now we need to show some quantitative results. We start by deriving an important dimensionless quantity namely the low energy fine structure constant α_0. Within E infinity theory this α_0 is no different from the expectation value of the Hausdorff dimension of E infinity, namely

$$\langle \mathrm{Dim}\, E - \infty \rangle_{\mathrm{H}} = \sim \langle n \rangle = 4 + \phi^3 = 4.236067977 \cdots$$

and its inverse $\bar{\alpha}_0 = 1/\alpha_0$ may therefore be regarded as a dimension while α_0 itself may be thought of as a probability in the fashion of the interactive theory dealing with massinger particles rather than in the theory of electromagnetic field. In fact in E infinity theory all coupling constants α_0, α_s and so on will be regarded as probabilities to absorb or emit the corresponding massinger particle. Let us give first a formal derivation of α_0. We know that the special orthogonal group SO(32) is similar to the $E_8 \otimes E_8$ group of strings which is the only theory which has a graviton arising naturally from its basic formulation. Now the dimension of SO(32) is equal to that of $E_8 \otimes E_8$ and is given simply by [10,13]

$$\mathrm{Dim}\, \mathrm{SO}(n = 32) = (n)(n-1)/2 = (32)(31)/2 = (16)(31) = 496$$

Next we take n to be the non-integer

$$n = \sim \langle n \rangle = \langle \mathrm{Dim}\, E - \infty \rangle_{\mathrm{H}} = 4 + \phi^3$$

Consequently one finds the expectation value

$$\text{``}\mathrm{Dim}\, \mathrm{SO}(n = 4 + \phi^3)\text{''} = (4 + \phi^3)(4 + \phi^3 - 1)/2$$

Taking 20 copies of that one finds

$$\text{``}20\ \mathrm{Dim}\, \mathrm{SO}(4 + \phi^3)\text{''} = 137 + k_0 = 137.082039325 \cdots$$

where $k_0 = \phi^5(1 - \phi^5) = 0.082039325 \cdots$

This is the value of the global $\bar{\alpha}_0$ which may be written more conveniently as

$$\bar{\alpha}_0 = (20)(1/\phi)^4 = 137 + k_0$$

where $\phi = (\sqrt{5} - 1)/2 = 0.618033989 \cdots$

Clearly we need not justify setting $20 \otimes \mathrm{SO}(4 + \phi^3)$ equal to $\bar{\alpha}_0$ as long as we can show that it gives the right physics and is consistent. Nevertheless, we will return later on to give a more intuitive justification beyond the fair numerical agreement with our "physical" reality. This is done by linking $\bar{\alpha}_0$ to the so-called expectation π meson [2,13].

3.2. Embedding of strings dimensions in E infinity

Next let us show how the dimensional hierarchy of the Heterotic string is imbedded in our E infinity and scale $(\bar{\alpha}_0/2)$ using the golden mean. That way we find [14]

$$(\bar{\alpha}_0/2)(\phi) = 42 + 2k = \bar{\alpha}_{\mathrm{g}}, \qquad (\bar{\alpha}_0/2)(\phi)^2 = 26 + k = \bar{\alpha}_{\mathrm{gs}}$$
$$(\bar{\alpha}_0/2)(\phi)^3 = 16 + k = D^{(16)}, \qquad (\bar{\alpha}_0/2)(\phi)^4 = 10 = D^{(10)}$$
$$(\bar{\alpha}_0/2)(\phi)^5 = 6 + k = D^{(6)}, \qquad (\bar{\alpha}_0/2)(\phi)^6 = 4 - k = D^{(4)}$$

Setting $k = \phi^3(1 - \phi^3) = 0.18033989$ equals zero one finds

$$\bar{\alpha}_{\mathrm{g}} \simeq 42 \simeq (1 + \phi)^3(10)$$
$$\bar{\alpha}_{\mathrm{gs}} = D^{(26)} \simeq 26 \simeq (1 + \phi)^2(10)$$
$$D^{(16)} = \bar{\alpha}_{\mathrm{yu}} \simeq 16 \simeq (1 + \phi)(10)$$
$$D^{(10)} = 10 \simeq (1 + \phi)^0(10)$$
$$D^{(6)} \simeq 6 \simeq 16(\phi)(10)$$
$$D^{(4)} \simeq 4 \simeq (\phi)^2(10)$$

Note that $\bar{\alpha}_{\mathrm{g}}$ can be shown to be the non-super symmetric coupling constant of the unification of all fundamental forces. In the super symmetric case we have $\bar{\alpha}_{\mathrm{gs}} = 26.18033$.

3.3. Isospin and symmetry groups

Since the introduction of isospin theory by W. Heisenberg, groups theoretical considerations play a prominent role in high energy particle physics. However, strictly speaking we just became used to group theoretical arguments although

they remain as abstract as ever, for instance to say we have 12 massless gauge bosons because [2,14] the total dimensions of all the symmetry groups involved in the standard model (SM)

$$\mathrm{Dim}\,(\mathrm{SM}) = \mathrm{Dim}(SU(3) \otimes SU(2) \otimes U(1)) = 12 \simeq \sqrt{\bar{\alpha}_0} = \langle \mathrm{Dim}\,\mathrm{SM} \rangle$$

is not justified by any intuitive physics, only pure mathematics and experimental verification. The situation in string theory is even more abstract. In string theory we accept that we have 496 massless gauge bosons without experimental evidence because

$$\mathrm{Dim}\,SO(32) = \mathrm{Dim}\,E_8 \otimes E_8 = 496 \simeq 496 - k^2 = \langle \mathrm{Dim}\,SO(32) \rangle$$

In E infinity theory we compliment groups by sets in a manner of speaking. Seen at a low resolution, our set is a Kleinian modular group but it is the set character which is important in E infinity and as we will see, it leads to exactly verifiable results regarding the mass spectrum [2,14]. Note that the ratio between in number of mass less gauge bosons in the standard model (SM) and quantum gravity defines the non-super symmetric coupling constant $\bar{\alpha}_g \cong (496)/12 \simeq 41.333 \simeq 42$.

Thus in E infinity theory the dimension corresponding to 496 is in fact $496 - k^2$ where $k = \phi^3(1 - \phi^3) = 0.18033989$ which we can neglect for all practical purposes, but not in principle. In fact, ignoring small numbers can lead as we learnt from chaotic non-linear dynamics to disastrous inaccuracies in certain cases. Nevertheless, $496 - k^2 \simeq 496$ is not only a dimension but is much more than that as will be shown shortly. It is related to the expectation of the mass of the K meson.

4. The expectation mesons and the mass of the nucleons

4.1. The π mesons

The dimension $496 - k^2$ may be obtained from the dimension $137 + k_0$ by scaling as follows

$$\langle \mathrm{Dim}\,SO(32) \rangle = 496 - k^2 = (137 + k_0)(3 + \phi) = (\bar{\alpha}_0)(1/\phi)^2(1 + \phi^2)$$

The deep meaning of the above is the following. It was Sidharth [32] who showed using classical mechanics and non-classical Cantorian space–time that the mass of a π meson is given by [2,14]

$$m_\pi = \bar{\alpha}_0 \; \mathrm{MeV} = 137 \; \mathrm{MeV}$$

Sidharth's calculation is only approximately true because we do have two π mesons

$$m_{\pi^\pm} = 139.57 \; \mathrm{MeV}$$

and

$$m_{\pi^0} = 134.98 \; \mathrm{MeV}$$

To obtain the accurate result we postulate the existence of an expectation π meson given by $\bar{\alpha}_0$ as follows

$$\langle m_\pi \rangle = \bar{\alpha}_0 \; \mathrm{MeV} = (20)(1/\phi)^4 = 137 + k_0 \; \mathrm{MeV}$$

Subsequently we can show that $m_{\pi^\pm}$ and m_{π^0} are just different scaling of another fundamental non-composite particle, say the electron. However before showing that we show that the neutron mass may be obtained as a scaled i.e. in fractal hyperbolic space–time of E infinitely distorted π meson [2]

$$m_N = \langle m_\pi \rangle \frac{\bar{\alpha}_0}{D^{(26)} - D^{(6)}} = \langle m_\pi \rangle \frac{\bar{\alpha}_0}{(26 + k) - (6 + k)}$$

That means the scaling exponent in this particular case is

$$\lambda = \frac{\bar{\alpha}_0}{20} = (1/\phi)^4$$

and

$$m_N = [(137 + k_0) \; \mathrm{MeV}]\left(\frac{137 + k_0}{20} \right) = (20)(1/\phi)^8 \; \mathrm{MeV} = (\bar{\alpha}_0)^2/20 = 939.5742749 \; \mathrm{MeV}$$

which is almost exactly equal to the experimental value. As we define an expectation meson

$$\langle m_\pi \rangle = \bar{\alpha}_0 = [m_N \text{ MeV}] - (8)(10)^2 \text{ MeV} = 137.082039325 \text{ MeV}$$

which came very close to the expected value found from taking the average mass of $m_{\pi^\pm}$ and m_{π^0}

$$\frac{1}{2}(m_{\pi^\pm} + m_{\pi^0}) = \frac{139.57 + 134.98}{2} = 137.27 \text{ MeV}$$

one could define an expectation K meson by scaling $\langle m_\pi \rangle$ using $\lambda = 3 + \phi$

$$\langle m_K \rangle = \langle m_\pi \rangle (\lambda) = \langle m_\pi \rangle (3 + \phi) = (496 - k^2) \text{ MeV} = (\text{Dim} E_8 \otimes E_8 - k^2) \text{ MeV} = (\text{Dim} SO(32) - k^2) \text{ MeV}$$

4.2. The mass of the neutron and the mass of the proton

Now by scaling $\langle m_K \rangle$ we can find again m_N as follows [2]

$$m_N = \langle m_K \rangle \lambda = \langle m_K \rangle \frac{b_2^-}{D^{(10)}} = \langle m_K \rangle \frac{(19 - \phi^6)}{10}$$

That means scaling this time is

$$\lambda = \frac{b_2^-}{D^{(10)}} = \frac{19 - \phi^6}{10} = \frac{(4 + \phi^3)^2 + 1}{10} = \frac{(1/\phi)^4}{3 + \phi} = 1.89442714$$

where $10 = D^{(10)}$ is the dimension of the core of super string embedded in E infinity and $19 - \phi^6 = b_2^-$ which is a Betti number. Consequently one finds

$$m_N = (496 - k^2)(19 - \phi^6)/10 = 939.57 \text{ MeV}$$

To obtain the mass of the proton all what we need is to change $k^2 = (0.18033989)^2$ to $4k$ and find

$$m_p^+ = (496 - 4k)(b_2^-/10) = (496 - 4k)(19 - \phi^6)/10 = 938.269 \text{ MeV}$$

which is almost exactly the best know experimental result. Now we calculate the mass of the electron again as a scaling, this time of the proton by writing

$$m_e = (m_p^+)(\lambda_e) = (m_p^+)\left(\sqrt{10}/(\overline{\alpha_0}\overline{\alpha_g})\right) \cong 0.511 \text{ MeV}$$

where $\overline{\alpha_0} = 137 + k_0$ and $\overline{\alpha_g} = 42 + 2k$ and the scaling is $\lambda_e = (\sqrt{10})/(\overline{\alpha_0}\,\overline{\alpha_g}) = \frac{1}{1836.299330}$.

Now we can determine the $m_{\pi^\pm}$ and m_{π^0} as scaling of the electron

$$m_{\pi^\pm} = (\lambda_{\pi^\pm})(m_e) = (2\overline{\alpha_0} - 1)(m_e) = 139.58 \text{ MeV}$$

and

$$m_{\pi^0} = (\lambda_{\pi^0})(m_e) = (2\overline{\alpha_0} - 10)(m_e) = 134.987 \text{ MeV}$$

in excellent agreement with the experimental evidence.

5. Linking some scaling exponents to string theory and non-commutative geometry

5.1. Hyper tensor

It may be of deep theoretical interest into the structure of the super string theory [10] to note that

$$\lambda_{\pi^\pm} = (2\overline{\alpha_0} - 1) \simeq 273$$

plays a profound role in a necessary condition for anomaly cancellation, namely that the number of hyper tensor and vector multiples satisfies the following condition [10]

$$\eta_H + 29n_T - n_V = 273 \simeq (2\overline{\alpha_0} - 1)$$

This anomaly cancellation is what made Schwarz and Green develop super string by adding super symmetry and gravity to the bosonic string. Seen that way, E infinity theory can give string theory practical predictivity power with considerable accuracy at least as far as the vital mass spectrum is concerned.

Now one could, with considerable justification, ask why did we take these particular scaling exponents and whether we could take any arbitrary factors as a scaling. The answer to this and similar questions, is the following.

5.2. Non-commutative geometry and the golden mean

We could take any exponent and there are very many of them, but there are restrictions. We have a large set of admissible factors but they must be looked at carefully to be taken from the topology of E infinity space–time. First the golden mean and all its powers multiplications and additions may be taken as valid scaling as long as they come out from Connes dimensional function and its extension to higher dimensions. The two dimensional function for instance is given in non-commutative geometry by [2,8,9,13]

$$D = a + b(1/\phi), \quad a, b \in Z$$

Second, all the transfinite version of the Heterotic super string dimensions and their combinations are valid scaling provided the corresponding vibration can be shown to be stable. Thus we may use $10, 6 + k, 6, 16, 16 + k, 26 + k, 26$ and so on. In addition the following dimensions are extremely important and are drawn upon continuously in E infinity theory:

$$\bar{\alpha}_0 = 137.082039325, \qquad \text{Dim}\, E_{8,8} = 128 \cong \bar{\alpha}_{ew}$$

$$\text{Dim}\, E_7 + D^{(4)} = 133 + 4 = 137 \simeq \bar{\alpha}_0$$

$$\text{Dim}\, SU(3) = 8 \simeq \bar{\alpha}_s, \qquad \text{Dim}\, SU(3) = \text{Dim}\, SL(2, R) = 3$$

$$\text{Dim}\, SL(2, c) = 6 \cong D^{(6)}, \qquad \text{Dim}\, U(1) = 1$$

$$\text{Dim}\, E_8 \otimes E_8 = \text{Dim}\, SO(32) = 496 - k^2 \cong 496$$

$$\text{Dim}\, G_2 = 14, \qquad \text{Dim}\, E_6 \otimes E_6 = (2)(78) = 156$$

$$\text{Dim}\, SU(5) = 24 - \phi^9 = (5)^2 - 1 - \phi^9 \simeq 24$$

The last expression gives the dimension of the SU(5) symmetry group needed for GUT i.e. grand unification of all non-gravitational forces which is due to Glashow and Georgi [10].

6. Stability considerations, scaling and the quantum

6.1. The Planck length and the Bohr radius

Besides the preceding conditions the stability condition must be established [2]. Clearly if we know a particle with a certain mass which we have just calculated using a certain scaling, then this particle really exists, otherwise we will not be sure. The problem is that KAM stability and Arnold diffusion in higher dimensions [2] (more than 4) are almost impossible to solve at present. Thus the more direct and obvious the scaling is, the more confidence we will have that such particles will be sufficiently stable to be observed. An example of a direct scaling is for instance the isomorphic length. As we mentioned this length is directly proportional to $4 + \phi^3$

$$l \leqslant \varrho\left(\sim \frac{\langle n \rangle}{2}\right) = \left(\frac{4 + \phi^3}{2}\right)\varrho$$

where ϱ is a radius which can be any number. Consequently $4 + \phi^3$ and 4 are obvious fundamental scaling exponents in E infinity. To show that this is true we give a simple but striking example of how $4 + \phi^3$ and 4 connects the Plank length (which is related to the quantum h by $h = (l_p)^2$ cm^2 in natural units) with the semi-classical scale par excellence, namely the Bohr radius [2,10].

$$(l_p)^{1/4} = \sqrt[4]{(10)^{-33}} \text{ cm} \simeq (0.5)(10^{-8}) \text{ cm} = R_{Bohr}$$

While for the related stony length one finds

$$(l_s)^{1/(4+\phi^3)} = \sqrt[4+\phi^3]{(10)^{-35}} \text{ cm} \simeq (0.5)(10^{-8}) \text{ cm} = R_{\text{Bohr}}$$

Thus 4 and $4 + \phi^3$ are the scaling of the classical ($h = 0$) to the quantum ($h \neq 0$) and visa versa. Other scaling transformation have a direct and obviously appealing physical interpretation and inspire a direct confidence even without experimental verification. An example of this kind is the following coupling equation

$$(\bar{\alpha}_{\text{gs}}/2) = [(\text{Dim}\,E_8 \otimes E_8 - k^2)/\bar{\alpha}_0]^2$$

Thus

$$\bar{\alpha}_{\text{gs}} = 2\left(\frac{496 - k^2}{137 + k_0}\right)^2 = 26 + k = 26.18033989$$

where $\bar{\alpha}_{\text{gs}}$ is the super symmetric coupling constant of quantum gravity, $k = \phi^3(1 - \phi^3)$, $k_0 = \phi^5(1 - \phi^5)$ and $\phi = (\sqrt{5} - 1)/2$. Clearly $\bar{\alpha}_{\text{gs}}$ is the coupling between the graviton field represented by the string group, namely the Lie expectational group $E_8 \otimes E_8$ and the electromagnetic field as represented by the quasi-dimension $\bar{\alpha}_0$. The factor 2 is analogus to $\bar{\alpha}_0/2$ of the ϕ scaling of $\bar{\alpha}_0/2$ to give the Heterotic string dimensional hierarchy discussed earlier and may be interpreted indirectly as a kind of Bose condensation of a Cooper particle at the extremely high energy of some $(10)^{19}$ GeV [10].

6.2. Geometry and topology of the vacuum and quantum gravity

Maybe we still need to explain the deeper origin of the preceding relation. At least historically the relation goes back to the sigma model. In this model and as is explained for instance by 't Hooft, it is the squares of the masses which must be compared [15], a situation which is similar to the Regge trajectories. For the π meson and the K meson, the correct comparison has to use the expectation π and K meson which are defined in E infinity as theoretical intermediate and probably totally unstable particles which need not really exist and the ratio comes indeed near to 14. In fact it is exactly 13.09016995. This value happens to be exactly half of the value of the theoretical super symmetry quantum gravity coupling constant. Thus

$$\langle m_{\text{K}} \rangle = \left(\sqrt{\bar{\alpha}_{\text{gs}}/2}\right)(\langle m_\pi \rangle)$$

Now at the beginning of any new theory, the most difficult things is the new concept. Once this is established then mathematics takes over. Let us clear the concept a little more because it is not immediately obvious how we move from a Hausdorff dimension to mass. The chain is not long. We know that entropy is a measure for complexity. Likewise the Hausdorff dimension is a measure for complexity. This is how the work of Schlögel and Beck should be understood because the Hausdorff dimension is related to thermodynamics. Consequently the Hausdorff dimension is related to energy via thermodynamics and since energy is related through special relativity to mass, the connection of Hausdorff dimension to mass becomes clearer. Now the Hausdorff dimension is predominantly a geometrical–topological devise and the afore mentioned connections mean that geometry is indirectly connect to temperature and mass. In essence there is nothing new in all of that, it is general relativity seen from another maybe deeper view point. On a deep level geometry is paramount and decides on everything including energy and thus mass. The preceding simple thoughts were the basis for a relatively recent work in which the author derived the temperature for a drawing by Pablo Picasso [16,17,36].

6.3. Complexity theory

Having said all that, one should not confuse disorder with complexity (see Ref. [37]). In fact hyper-disorder may be regarded as a form or ergodicity and ergodicity is a completely uniform disorder which has a complexity zero, exactly as complete disorder. Innovation in nature takes place somewhere between the two extremes and I was not astonished to find out that the VAK state has a maximum complexity. Consequently the vacuum has a maximum complexity which is the reason why it is so rich giving rise to quantum physics. I was later informed that maximum complexity is connexted approximately to the number 0.273. Prof. Alan MacKay, a leading British crystalographer was particularly intrigued by this because this number appears continuously in the E infinity theory, (i.e. $(2\bar{\alpha}_0 - 1)/(10)^3 \simeq 0.273$).

So far we have discussed the inter-scaling relationship between π mesons, K mesons, electrons and nucleon, but what about the quarks model for hadrons. Could this model by of any use in E infinity theory? The answer is yes it is and treating the same particles using a combination of the quarks model and scaling gives a deeper understanding of the theory which we do next. In fact some scientists regard the electron as a kind of quark and that was used in our earlier analysis. [33,39].

7. Constructing the neutron and the proton from quarks

7.1. The mass of the quarks

First we give here without derivation, the current and constituent masses of the light and heavy quarks which are consistent with E infinity theory. Needless to say, that these masses are in excellent agreement with the majority of the scarce and difficult to obtain data about the mass of the quarks. It takes only one look at these values for anyone to realise that they form a harmonic musical ladder. In fact, particle physics seen through the eyes of E infinity must resemble a cosmic symphony, even for the most hard nosed so-called realist. Here are the values [18]:

(a) Current mass [14,18]

$$m_u = (1/\phi)^3 + 1 = 5 + \phi^3 \text{ MeV}$$

$$m_d = (2)(1/\phi)^3 = 8.472135954 \text{ MeV}$$

$$m_s = (1/\phi)^6(10) = 179.442719 = \langle m_\pi \rangle (1/\phi)^2 \text{ MeV}$$

$$m_c = (3)(1/\phi)^3(10) = 127.0820393 = (\langle m_\pi \rangle - 10) \text{ MeV}$$

$$m_b = (1/\phi)^3(10)^3 = (4.236067977)(10)^3 \text{ MeV} = 4.236067977 \text{ GeV}$$

$$m_t = (1/\phi)^3(10)^3 = 42.36067977(10)^3 \text{ MeV} = 42.36067977 \text{ GeV}$$

(b) Constituent mass [14,18]

$$m_u^* \simeq (8)(1/\phi)^3(10) = 338.8854384 \text{ MeV}$$

$$m_d^* \simeq (8)(1/\phi)^3(10) = 338.8854384 \text{ MeV}$$

$$m_s^* \simeq (1/\phi)^8(10) = 469.7871382 \text{ MeV} = 0.4697871382 \text{ GeV}$$

$$m_c^* = 2m_s^*(1/\phi) = (1/\phi)^9(10) = (m_N)(1/\phi) = 1520.263114 \text{ MeV} = 1.520263114 \text{ GeV}$$

$$m_b^* = (m_s^*)(10) = 4697.871382 \text{ MeV} = 4.697871382 \text{ GeV}$$

$$m_t^* = (m_s)(10)^3 = (10)^4(1/\phi)^6 = 179.4427193(10)^3 \text{ MeV} = 179.4427193 \text{ GeV}$$

7.2. Constructing the nucleon from quarks

The point is that we know from the classical quark theory that a nucleon is supposed to be made up from three confined light quarks. For the neutron these are two down quarks and one up and for the proton, two up quarks and one down. That way one finds that [16,18]

$$m_N = (m_u + 2m_d)\lambda_N$$

where λ_N is a scaling given by

$$\lambda_N = \frac{(\text{Dim} E_8 \otimes E_8)}{\text{Dim}(\text{SU}(3) \otimes \text{SU}(2) \otimes U(1))}\bigg|_{\varepsilon^{(\infty)}} \simeq \frac{\text{Dim} \, \text{SO}(32)}{\sqrt{\alpha_0}}$$

Thus

$$m_N = [(22 + k) \text{ MeV}]\lambda_N = (22 + k)\left(\frac{496 - k^2}{\sqrt{137 + k_0}}\right) \text{ MeV}$$

where $n_1 \simeq 496$ is the expectation value for the number of massless gauge bosons in the quantum gravity field while $\sqrt{137} \simeq 12$ is the expectation value of the number of massless bosons in the standard model.

That means

$$m_{\mathrm{N}} = (22 + k)(42.36067977) = (22 + k)\overline{\alpha_{\mathrm{g}}} \text{ MeV} = 939.5742755 \text{ MeV}$$

which is exactly the value we obtained previously and which agrees completely with the experimental results. Now we look at the proton which is electrically charged and must therefore be made up of two up quarks and one down quark [16,17]

$$m_{\mathrm{p}}^{+} = (2m_{\mathrm{u}} + m_{\mathrm{d}})\lambda_{\mathrm{p}}$$

where λ_{p} is the scaling

$$\lambda_{\mathrm{p}} = \left[\frac{\mathrm{Dim}\, E_8 \otimes E_8}{(D^{(10)} = 10)}\right] \cos\left(\frac{\pi}{60}\right)$$

Thus we have

$$m_{\mathrm{p}}^{+} = (18.94427)\left[\frac{495.9674775}{10} \cos\left(\frac{\pi}{60}\right)\right] = (b_2^{-}(\mathrm{MeV}))\left(\frac{\mathrm{Dim}\, SO(32)}{D^{(10)}} \cos\frac{\pi}{(D^{(6)} - k)D^{(10)}}\right)$$

$$= (939.5742753)\cos\left(\frac{\pi}{60}\right) \text{ MeV} = 938.286621 \text{ MeV}$$

in full agreement with the experimental evidence namely $m_{\mathrm{p}} = 938.279$ MeV. However we gain from the previous equation a great deal of insight into the relation between m_{N} and m_{p} within E infinity. We notice in the last equation that m_{p} is a projection of $m_{\mathrm{N}} = 939.57427$ MeV. The projection is due to a rotation of an angle equal to π divided by $(D^{(10)})(D^{(6)} - k) = 60$ where $D^{(6)} = 6 + k$ and $D^{(10)} = 10$ as we have known from the ϕ scaling of $\overline{\alpha_0}/2$ corresponding to the dimension of our transfinite version of the Heterotic string theory. One could ask why this rotation? Formally one could answer it is exactly equivalent to the internal rotation of the isospin theory of Heisenberg only more tangible and it gives the right result. However using our hyperbolic distortion picture (see Fig. 1) of the Cantorian E infinity space, we can give the deeper answer that this is the angle at which we look at a neutron and conceive it as a proton as far as the mass is concerned. It is the geometry and topology of space–time all over again. In string theory we know that the mass equations of the "particles" lives in the 6 dimensional part of the 10 dimensional space of the string core embedded in E infinity. This is one of the important results of the theory of super strings [10].

8. Deriving the mass of the meson from the "vibration" of the light quarks

8.1. The expectation π meson

Now we would like to derive the expectation π meson mass (which was never observed experimentally until this point of time, but may be found in the future) using the quarks model. We know that the meson consists of two quarks. For that purpose we take one up and one down quark and find

$$\langle m_{\pi} \rangle = (m_{\mathrm{u}} + m_{\mathrm{d}})\lambda$$

The reassuring thing here is that we find the scaling to be exactly $\lambda_{\pi} = 10$. Thus the $\langle m_{\pi} \rangle$ is ten copies, (to use the terminology of the 10 dimensional super string) of the sum of the two light quarks

$$\langle m_{\pi} \rangle = (10)[(5 + \phi^3) + (8.47213)] = (10)(13.7082039325) = (10)(\overline{\alpha_0}/10) = \overline{\alpha_0} \text{ MeV}$$

In such cases it is instructive to see the calculation as going forward and backward from higher to lower dimensionality and visa versa. That means the masses of the quarks which we perceive are the ones measured here in our 3 + 1 dimensional projection of E infinity. However the combination we talk about takes place in this case in the 10 dimensional super string core of E infinity so that the value we measure in our 3 + 1 projection is the 10-fold of the simple sum of the mass of m_{u} and m_{d}.

8.2. Nested and fractal vibration

So far we have made no direct quantitative reference to the vibrational interpretation of E infinity and that is what we will touch upon now.

Consider a simple two degrees of freedom linear vibration consisting of two masses connected by linear elastic springs and hanging on the ceiling. Setting the masses and the spring constant equal to unity, we obtain a quadratic secular equation with two frequencies as the solution, namely [14,18]

$$\omega_1 = \phi = \frac{\sqrt{5}-1}{2}, \qquad \omega_2 = \frac{\sqrt{5}+1}{2} = \frac{1}{\phi} = 1 + \phi$$

These are indeed the golden mean again. If we now imagine an infinite collection of such two degrees of freedom unit cells connected sequentially and in parallel at random, then we need only to introduce a so-called wired hierarchy in the architecture of our neural network like structure and we would have some reasonable mechanical realisation of E infinity space. In fact, such an infinite collection of possibly nested oscillators has already been considered by L. Crnjac [5] and also by S. Wolfram in his recent book "A new kind of science". I have used in this context the well known Eigen value theorems of Southwell and Dunkerly to show that the expected hierarchy of frequencies of vibration are simple or complex function of the golden mean and may add that many of the results obtained within the theory of N. Wiener and its modern recasting in the theory of spontaneous self-organisation (see Ref. [37]) are of great relevance to E infinity and reproduce partly some of our arguments. This could however take us too far from our present limited objective of an introduction to E infinity and will not be discussed in detail. The important point which we gain from the preceding "N. Wiener" picture is that when we add say two Hausdorff dimensions, for instance

$$\omega_t = \omega_1 + \omega_2 = \phi + \phi = 2\phi$$

we can regard this also as adding two frequencies to find a joint frequency. Similarly we have in the sequential net the second variant of adding two frequencies and that would be

$$\frac{1}{\omega_t} = \frac{1}{\omega_1} + \frac{1}{\omega_2}$$

Thus

$$\frac{1}{\omega_t} = \frac{1}{\phi} + \frac{1}{\phi} = 2\left(\frac{1}{\phi}\right) = 2 + 2\phi$$

This is obviously trivial but things can get quite sophisticated and our approach requires knowledge of advanced modern geometry of the Kähler manifold [10] in particular the so-called K_3. To explain this point let us take a concrete example. Very frequently when writing a scaling exponent using the main dimensions of the Heterotic string we would write something like $-26 + 10 = -16$ and we justify this by saying that the 10 dimensions of $D^{(10)} = 10$ are moving to the right while the 26 dimensions of $D^{(26)} = 26$ are moving to the left. This situation is not as mad as it initially sounds. The point is somewhat similar to what we encounter frequently in the general theory of diffusion where we have a process defined at least mathematically via two diffusion equations, one running forward and the second backward in time. This is a special case of what I have introduced as the complex conjugate time of the quantum world [19]

$$0 \pm it$$

Something similar is used in the theory of Heterotic super strings where we introduce a so-called Minkowski analytical continuation and end with a holomorphic field and anti-holomorphic field. We use then the synonyms for left moving for holomorphic and right moving for anti-holomorphic [10].

9. Quantization and transfinite discretization

9.1. The work of G. Ord

This brings us now to what we should have explained at the beginning but deliberately postponed until this stage. The theory of E infinity would have remained without a strong theoretical foundation if it had not been for the work of the English–Canadian physicist, Garnet Ord [3]. Ord set out to take the mystery from analytical continuation. We should recall that analytical continuation is what converts an ordinary diffusion equation into a Schrödinger equation and a telegraph equation into a Dirac equation. Analytical continuation is thus the short cut quantization. However what really happened is totally inexplicable. It was Ord who showed, using his own (invented) quantum calculus, that analytical continuation is not needed if we work in a fractal-like setting, a fractal space–time if you want. In fact it was Ord who introduced the expression fractal space–time in a formal paper in the eighties. Only recently Ord's work has

gained acceptance in Physic Review Letters and so one is hopeful that his message will be widely understood; it is the transfinite geometry and not quantization which produces the equations of quantum mechanics. Quantization is just a very convenient way to reach the same result fast, but understanding suffers in the process of analytical continuation [3].

9.2. Complex time and transfiniteness

Ord has accepted the limited validity of $0 \pm it$ as dual equations and that quantum mechanics [19] for instance is governed not only by one Schrödinger equation but by a second conjugate complex Schrödinger equation as pointed out by the author [35]. However he has written that this is not going as far as one should in demystifying analytical continuation and replacing it with a deep geometrical understanding. The further development of E infinity take Ord's point completely which he acknowledged in several of his recent papers. So, our slogan for E infinity could be

'Do not quantize and do not merely discretize. You should discretize transfinitely'. This is the right way from M. Kac to P. Dirac.

Once this is done, we are in the middle of hyperbolic Cantor sets and E infinity. Now we come naturally to a totally justified question, namely what happens to h. Ord never needed to look into this question thoroughly because he regards his equation as being totally dimensionless and setting $C = h = 1$ are his natural units system. Later on once he arrived at his Schrödinger and Dirac equations, he restores the situation and h appears again. In my E infinity, I do not need to dwell on h directly, but it is built in there for sure. This is because the dimension $D^{(26)} = 26 + k \simeq 26$ is at the same time the value at which all differences between all fundamental forces completely disappear and we have then one force, the super force so to speak. This situation takes place at an energy of around 10^{19} GeV. This energy is in turn related to the Planck length and to the length at which complete unification takes place. The Planck length and the total unification length are connected via this coupling constant, namely

$$\bar{\alpha}_{gs} = 26 + k = 26.18033989 = (10)(1/\phi)^2$$

On the other hand, h is nothing but the square of the Planck length when measured in centimeters

$$h = (l_p)^2 = (10^{-66}) \text{ cm}^2$$

That is where h is hiding in E infinity. In other words, once we have found h experimentally and once we have accepted it as fundamental and final, we should have at once given up the smooth Euclidean space in favour of something more in harmony with

$$h = (10^{-66}) \text{ cm}^2$$

such as E infinity space time. Now we may return again back to our main concern, the mass spectrum. We have so far converted some particles into others by means of scaling as far as mass is concerned, but we never really explained where mass came from in the first place. In the standard model for instance which E infinity accepts as a valid approximation, mass is explained using the Higgs mechanism. However no one has ever seen a Higgs experimentally and could not be sure that this Higgs really exists. None the less, this is not an argument against the Higgs because no one has ever seen a quark either, I mean a single quark moving freely in space and none the less, we accept the existence of quarks. If the Higgs particle exists, then one could ask again where did the Higgs particle get its mass from? In addition how could the Higgs field hide away its gravitational attraction which should in principle be detectable even with today's technology as emphasized continuously by Veltman.

10. Unification and the mass of the electron

10.1. The unification π meson

Now all these questions are answered within E infinity theory in a fundamentally different way. In E infinity the particles acquire their mass at the unification of all fundamental forces. To explain this point we would like to calculate here the mass of the electron from the condition of unification.

All fundamental forces are unified when all the three fundamental coupling constants intersect with that of gravity [20] at one point in the $\bar{\alpha}_i - E$ space where α_i. stands for the coupling constants of the weak force, the strong force and the electromagnetic force as well as the dimensionless coupling of gravity while E stands for the corresponding energy.

Steven Weinberg gave a highly simplified and lucid account of this subject in the Millennium Edition of Scientific American [21] and one could see from his clearly presented coloured figures that assuming super symmetry the unification coupling constant lies indeed near $\bar{\alpha}_{gs} = 26$ which is very close to our theoretical result $\bar{\alpha}_{gs} = 26.18033989$. Now we remember that we calculated a theoretical intermediate particle which we called an expectation π meson and found it equal to $\langle m_\pi \rangle = \bar{\alpha}_0$ MeV. Remember we also obtained $D^{(26)} = 26.18033$ as a scaling of $(\bar{\alpha}_0)$ namely $26 + k = (\bar{\alpha_0})(\phi^2/2)$.

10.2. The unification electron and the experimental mass of the electron

Thus in analogy with that we would like to introduce formally a hypothetical particle with a mass equal to 26.18 MeV which we will call the unification π meson

$$\langle m_{u\pi} \rangle = 26.18033989 = (10)\left(\frac{1}{\phi}\right)^2 \text{ MeV}$$

However we should keep in mind that this $\bar{\alpha}_{gs} = 26.18$ point is a point at which there is no difference what so ever between gravity and consequently mass and energy and electromagnetic charge nor nuclear forces. Now we know that the dimensionless electric charge is given by $e = 1/\sqrt{\bar{\alpha}_0}$. Consequently we may deduce analogically a unification charge equal to $e_u^* = 1/\sqrt{26.18033}$. However this has to be lifted to 10 dimensions as we have seen before so that the correct expression would seem to be

$$e_u^* = 10/\sqrt{26.18033989}$$

The above relation as it stands is unfortunately not right. It would have been right if it would not have been for a remarkable duality known in string theory as the Olive–Montonen duality, where we have to take the reciprocal value when moving from large scale to the ultra small scale and the correct expression is the reciprocal value

$$e_u = \left(\sqrt{26.18033989}\right)\big/10$$

This value measured as $\langle m_\pi \rangle$ in MeV and is what we call the unification electron mass.

$$m_{eu} = \frac{\sqrt{(10)(1/\phi)^2}}{10} = \frac{1}{(\sqrt{10})(\phi)} = 0.511667273 \text{ MeV}$$

To obtain our $3 + 1$ electron mass we have to project onto $3 + 1$ dimensions and find using $D^{(10)}$ and $D^{(6)}$:

$$m_e = (m_{eu})\left(\cos\frac{\pi}{(10)^2(\phi)}\right) \simeq 0.511 \text{ MeV} \equiv \left[(\langle m_{u\pi}\rangle)^{1/2}/10\right]\cos\frac{\pi}{61.8033989} \simeq 0.511 \text{ MeV}$$

The experimentally found value for m_e is, as is well known, $m_e = 0.511$ MeV.

11. The experimental fine structure constant and the electroweak particles

11.1. The Sommerfeld α_0

Now some may feel uneasy about the introduction of the string dualities [10] as well as the projection but both manoeuvres are routinely used in string theory and we are basing our self on it. One may find more than adequate and detailed coverage in the concerned monumental literature on the subject of strings, which we basically, globally accept as excellent approximation of what is the case in high energy physics. With E infinity theory however, we need not think of projection as more than special scaling to account for the distortion caused by infinite dimensional hyperbolic and fractal topology of quantum space–time. There is also a vital meaning for the procedure of projection connected to the low energy inverse fine structure constant $\bar{\alpha}_0$. We have found $\bar{\alpha}_0$ to be

$$\bar{\alpha}_0 = 2(1/\phi)^4(10) = (20)(1/\phi)^4 = 137 + k_0 = 137 + \phi^5(1 - \phi^5) = 137.082039325$$

but the very accurately measured $\bar{\alpha}_0$ is really

$$\bar{\alpha}_0(\text{exper}) = 137.03598$$

so what is the meaning of this slight but important difference. The explanation is as follows. The $\bar{\alpha}_0 = 137.082$ is a global $\bar{\alpha}_0$ and is a true constant. By contrast, $\bar{\alpha}_0 = 137.03598$ is a projection in $3 + 1$ and may therefore vary slightly with space and time. To obtain the experimental $\bar{\alpha}_0$ we project it using the "quantized" projection angles (see Fig. 1) in this case $\theta = \pi/\alpha_0$ and one finds [2,6]

$$\bar{\alpha}_0(\text{exper}) = (\bar{\alpha}_0) - k_0)/\cos(\pi/\bar{\alpha}_0) = \frac{137}{\cos(\pi/137 + k_0)} = 137.03598$$

in complete agreement with the experimental value.

11.2. The electroweak theory

In fact the cosine of, "quantized" angles plays a very important role in E infinity and may be thought of as a diffraction-like effect such as that found in crystallography. For instance, the Weinberg mixing parameter

$$\sin^2 \theta_w(\text{exper}) \simeq 0.2225$$

is identified in E infinity theory with the cosine of the angles of the triangles which make up the original Klein curve $\chi(3\pi/7)$ which forms the major part of E infinity as seen in the hyperbolic Poincare–Beltrami disc (see Fig. 1)

$$\cos(3\pi/7) = 0.2225 \simeq \sin^2 \theta_w(\text{exper})$$

With this value at our disposal, we can determine the masses of the $W^\pm$ and Z^0 of the electroweak. For this purpose we look at the $m_{W^\pm}$ as a scaled m_t^*, that is to say the constituent mass of the top quarks where

$$m_{W^\pm} = m_t^* \lambda_t, \qquad m_t^* = (1/\phi)^6 (10)^4 \text{ MeV}$$

and

$$\lambda_t = \frac{1}{(\sqrt{5}) \cos\left(\frac{\pi}{(2)(26.18033)}\right)}$$

That way one finds

$$m_{W^\pm} = m_t^* \lambda_t = 80.39388 \text{ GeV}$$

The best experimental value is $m_W = 80.4$ GeV. To obtain the M_z we use the same formalism of the Glashow–Salam–Weinberg theory but use $\cos(3\pi/7)$ instead of $\sin^2 \theta_w$ and one finds

$$m_{Z^0} = \frac{m_{W^\pm}}{\sqrt{1 - \cos(3\pi/7)}} = \frac{80.39388}{\sqrt{1 - 0.2225}} = 91.1778 \text{ GeV}$$

The best experimental value is 91.18 GeV. Incidentally the coupling constant of the electroweak is also easily found from

$$\bar{\alpha}_{ew} = \bar{\alpha}_0 - (D^{(10)} - 1) = \bar{\alpha} - (10 - 1) = \bar{\alpha}_0 - 9 = 128 + k_0 = 128.082039325$$

which agrees with the P-adic expansion of $\bar{\alpha}_0$, namely

$$\|137\|_{p=2} = (2)^8 + (2)^3 + (2)^0 = 128 + 8 + 1 = 128 + 9$$

Thus 128 may be interpreted as being $\bar{\alpha}_0$ at the electroweak scale

$$128 = 137 - 9 \simeq \bar{\alpha}_0 - 9 = \bar{\alpha}_{ew}$$

while 8 is the inverse of the strong coupling $\bar{\alpha}_s = 8$. The one left may be related to quantum gravity in the P-adic theory.

The relation between P-adic numbers, fractal and E infinity was discussed by many authors. We should also note that 137 is the 33 prime number while 127 is the 31 prime number. We may also note that since the mass of our theoretical π meson is $\langle m_\pi \rangle = \bar{\alpha}_0$ MeV we could interpret 128, 8 and 1 as masses measured in MeV.

12. Continued fraction and stability

There is an important point which we did not discuss so far and which is important for our vital quarks model interpretation of E infinity. We have reason to think that in our E infinity theory we must have

$$(m_u/m_d) = \phi = \frac{\sqrt{5}-1}{2} = 0.618033989\cdots$$

This is indeed the case as can be verified from

$$(m_u/m_d) = (5+\phi^3)/(8.4721359) = \phi$$

This relation is extremely important because all permanent matter is made of m_p^+ and m_N^0 i.e. protons and neutrons. Therefore in any realistic model protons must be the most stable particle. In string terms as well as in E infinity terms, this must be the most stable "string" vibration. Since according to KAM this will entail the most irrational frequency ratio possible, the ratio of m_u and m_d must be the most irrational number possible which is the golden mean, as is well known from number theory [12].

Now we should contemplate the following. The proton is the most stable composite particles we know of and this particle is made of two m_u and one m_d, so we have

$$\frac{m_d}{2m_u} = \frac{8.4721359}{(2)(5+\phi^3)} = 0.809850375 = \cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cdots}}}}$$

By contrast, for the unstable neutron we have

$$\frac{m_u}{2m_d} = 0.309016994 = \cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cdots}}}}$$

The question we hope someone can answer precisely one day is the following. Is it possible simply from looking at the continued expansion of the ratio of two elementary or sub-elementary particles or the logarithmic scaling of the ratio to judge the relative stability of the concerned particle from a criterion connected to the continued fractional expansion of these quantities.

We have a strong feeling that such criteria may be possible and this would simplify KAM theory and Arnold diffusion calculation beyond our present hopes, something which is, as far as we are aware, completely lacking at present.

13. Present mathematical limitations imposed on a general theory

Maybe it is now the place where we should discuss the limitations of our present theory. The customary thing to do in classical physics is either to establish the differential equation using Newtonian physics or what is completely equivalent to find a variational principle for which a Lagrangian is needed. This standard procedure is kept, as far as possible also in the standard model. However, as we can see, inconsistencies force us to give up smooth space–time and it was the French astrophysicist, Nottale [7] who investigated the consequence of giving up differentiability and came to his by now reasonably well known theory of scale relativity and conclusions similar to ours. Nottale of course did not give up continuity but only differentiability and this was difficult enough [7]. However in our case we have to find a way to integrate infinitely many times something which is classically non-integrable in order to find the stationary points corresponding to our Eigen values, i.e. masses and coupling constants, then we have to find the first variation (i.e. we differentiate) of the non-existing Lagrangian and set it equal to zero

$$\delta(L(VAK)) = 0$$

However if this could be done, we still do not know anything about the stability of this solution unless we require that we take the second variation (if it exists) and insist that

$$\delta^2(L(VAK)) > 0$$

for stability and only then can we find the stable particles which could be observed.

It is clear that we would need for this program a mathematics which does not exist yet and the only hope for an exact solution would be a super, super computer, i.e. a quantum computer.

E infinity theory may be regarded thus as an attempt to go around all these difficulties as far as possible and extract as much exact information as we can using a flexible strategy of applying almost everything we have in mathematics simultaneously. In particular we do not use only group theoretical consideration but also set theory and number theory as well physical considerations such as the nested vibration model [5], which was probably inspired by the pictures of self-similar universe due to Swedenborg, Charlier and Right (see Figs. 2–5).

14. The meuon and the mass spectrum

Now we may turn our attention once again to the calculation of the mass spectrum and it is time to consider the meuon. Being an electron in every respect except for being 206 times more heavy, the meuon should be regarded within the classical quarks theory as non-composite. Such notions, I mean "non-composite", are only relative within *E* infinity theory and depend on the resolution which is used but for all practical purposes, we may regard the meuon as non-composite. The best is then to regard it as scaling (distortion) of a quark and we take it to be the up quark. That way we may write

$$m_\mu = (m_u)\lambda_\mu$$

where

$$\lambda_\mu = (26 + k - 6) = (D^{(26)} - D^{(2)}D^{(3)}) = 20 + k$$

Consequently

$$m_\mu = (5 + \phi^3)(20 + k) \text{ MeV} = 105.6656315 \text{ MeV}$$

On the other hand the meuon is clearly a scaled electron

$$m_\mu = \lambda_\mu(m_e) \simeq \left((3/2)\bar{\alpha}_0 + \frac{\sqrt{\bar{\alpha}_0}}{10} \right)(m_e) = \bar{\lambda}_\mu(m_e)$$

where

$$\lambda_\mu = \left[\frac{\text{Dim SO}(32)}{\text{Dim SU}(5)} D^{(10)} \right]_{\varepsilon^{(\infty)}} = \left(\frac{496 - k^2}{24 - \phi^9} \right)(10) \simeq \bar{\lambda}_\mu = \frac{3\bar{\alpha}_0}{2} + \frac{\sqrt{\bar{\alpha}_0}}{10}$$

Consequently

$$m_\mu = (206.766458)(m_e) = 105.65766 \text{ MeV}$$

Setting for m_μ the value found earlier namely $m_\mu = 105.6656315$ MeV, the electron mass is readily found to be

$$m_e = (m_\mu)/\lambda_\mu = \frac{105.6656315}{206.766} \simeq 0.511 \text{ MeV}$$

exactly as expected. Now we may need to discuss the scalings λ_μ. Most of the time these scaling involve the ratio of the largest symmetry group we have, namely that which contains super strings and consequently gravitation, Dim $E_8 \otimes E_8 = 496$. The second value $24 - \phi^9 \simeq 24$ could be interpreted in different ways. First it is the dimension of SU(5) of the GUT unification.

The ϕ^9 is the transfinite so-called "correction" which reminds us that we are dealing with Cantor limit sets. On the other hand, if we take the 496 to be the number of massless gauge bosons of string theory, then the 24 should be taken to be the number of instantions which is equal to the second Chern class for $K_3 \otimes T_2$ as well as $E_8 \otimes E_8$. That means [10]

$$C_2(E_8 \otimes E_8) = 24$$

Multiplication with 10 is taking it to the 10 dimensions of the super string core. I do of course appreciate that the preceding explanation itself needs explanation but this would take us deep into the topology of super strings and string field theory which is definitely not the purpose of the present introduction.

15. Possible experiments

A question of great interest for any true physicist is obviously the following: could we have direct experimental verification for E infinity theory? The answer is probably yes but probably also not so direct. In my opinion, if it can be done at all, calculating the Hausdorff dimension of a quantum path may be our best bet [33]. Such an experiment should at the end say that the Hausdorff dimension of a quantum path is larger than the classical topological dimensional one. To find that the average Hausdorff dimension is exactly two would be a definite confirmation for all the postulates of E infinity theory. Although such sophisticated experiments are completely outside my range of expertise, I have given this question some though and think it will involve reconstruction of quasi-phase space using Ruell Takens method as well as deep laser cooling but this is still too vague to write about it here [33]. It seems that H. Kröger [34] in Canada attempted to find the Hausdorff dimension of a quantum path experimentally but so far, no real experiments were ever made.

Another possibility is to find a deviation in Newton's gravity law which could not be explained except with the existence of five and more dimensions for space–time. Such a possibility is being pursued by a team in CERN [31]. Two more predictions of $E - \infty$ could be tested experimentally. First the existence of the expectation π meson $\langle m_\pi \rangle = 137.08$ MeV as well as the expectation K meson $\langle m_K \rangle = 495.9674$ MeV.

16. Additional points of interest—the mass of the neutrino

We hope the preceding discussion helps to clarify the basic idea behind our approach although we have ignored some important aspects related to sphere packing in higher dimensionality, Leech lattices, quantum calculus and scale relativity as well as loop quantum mechanics and knot theory. Some of these subjects were discussed by the writer and other authors in many previous publications [22–29], for instance Saniga [26], consider the relation between E infinity and projective geometry whereas Agop et al. [29] considers super conductivity and E infinity. Discussing all these aspects would take a considerable space and we reserve them for coming occasions but one more remark regarding the neutrino may be essential. Any new theory for particle physics is tacitly expected to say something about the mass of the neutrino. E infinity can do that and predict the mass of the neutrino on the basis of the energy of the microwave background radiation energy [30] to be of the order of 10^{-4} eV which agrees well with the scarce experimental evidence [30]. We also should draw attention to a recent interesting paper by Koschmieder [22]. A work which is similar in it's philosophy is that of Sternglass [39].

17. Intermediate summary of the results

Let us summarise the most important formulas found so far for the different masses to reassure ourselves of their simplicity and elegance which excludes any possibility of interpreting these as any thing but true.

$$\bar{\alpha}_0 = (20)(1/\phi)^4 = (10)(4 + \phi^3)(3 + \phi^3) = 137 + \phi^5(1 - \phi^3) = 137.082039325$$

$$m_N = (\bar{\alpha}_0)^2/20 = \langle m_\pi \rangle (1/\phi)^4 = 939.57 \text{ MeV}$$

$$m_p^+ = (m_N)\cos\left(\frac{\pi}{60}\right) = 938.28 \text{ MeV}$$

$$\langle m_\pi \rangle = \bar{\alpha}_0 \text{ MeV} = (20)(1/\phi)^4 \text{ MeV} = 137 + k_0 = 137.082039325$$

$$\langle m_K \rangle = \langle m_\pi \rangle (3 + \phi) = \text{Dim SO}(32) - k^2 = \text{Dim } E_8 \otimes E_8 - k^2 = 496 - k^2 \cong 496$$

$$m_{\pi^\pm} = (2\bar{\alpha}_0 - 1)(m_e) = (2\bar{\alpha}_0 - 1)(0.511) = 139.58 \text{ MeV}$$

$$m_{\pi^0} = (2\bar{\alpha}_0 - 10)(m_e) = (2\bar{\alpha}_0 - 10)(0.511) = 134.98 \text{ MeV}$$

$$m_e = (m_p^+)\frac{\sqrt{10}}{\bar{\alpha}_0 \bar{\alpha}_g} = \sqrt{\frac{1}{(10)(\phi)^2}}\cos\left(\frac{\pi}{61.8033989}\right) = 0.511 \text{ MeV}$$

$$\bar{\alpha}_g = (10)(1/\phi)^3 = \sim \langle n \rangle (10) = 42.36067977 = 42 + 2k$$

$$\sin^2\theta_w(\text{exper}) \cong \cos(3\pi/7) \simeq 0.2225$$

$$m_\mu = \frac{\langle m_K \rangle}{\text{Dim SU}(5)} (D^{(10)}) = \left(\frac{496 - k^2}{24 - \phi^9} \right)(10) = 105.6656 \text{ MeV}$$

$$m_{W^\pm} = (m_t^*) \Big/ \left[\sqrt{5} \cos \left(\frac{\pi}{2\alpha_{gs}} \right) \right] = 80.39 \text{ GeV}, \quad \bar{\alpha}_{gs} = (10)(1/\phi)^2 = 26.18$$

$$m_{Z^0} = m_{W^\pm} \Big/ \left[\sqrt{1 - \cos(3\pi/7)} \right] = 91.177 \text{ GeV}$$

$$m_u/m_d = \phi, \qquad m_u + m_d = \frac{\langle m_\pi \rangle}{10} = \frac{\bar{\alpha}_0}{10} \text{ MeV}.$$

18. Symmetry breaking of $E_8 \otimes E_8$, the fundamental mass norm and $\bar{\alpha}_0$

We could arrive at $\bar{\alpha}_0$ via unification argument. Such an argument relies heavily upon quantum field theory and strings formulation and readers not familiar with both subjects may just disregard the reasoning of this section and note only the final conclusion.

One of the accepted scenarios for moving from $E_8 \otimes E_8$ with its 496 massless bosons to the standard model SU(3) SU(2) U(l) with its initially 12 massless gauge boson is to assume that $E_8 \otimes E8$ brakes into the smaller exceptional Lee group $E_6 \otimes E_6$

$$E_8 \otimes E_8 \Rightarrow E_6 \otimes E_6$$

where $\text{Dim} E_6 = (\frac{1}{2})(156) = 79$.

Let us recall first the approximate integer value of the fine structure constant, namely $[\bar{\alpha}_0] = 137$. Thus we may write that

$$\text{Dim} E_6 = (\tfrac{1}{2})(156) = \tfrac{1}{2}(137 + 19)$$

where 19 may be interpreted as the Bitti number of K_3, namely $\bar{b}_2 = 19$. Our symmetry breaking may thus be written symbolically as

$$
\begin{array}{ccc}
E_8 \otimes E_8 & \Rightarrow & E_6 \\
\Downarrow & & \Downarrow \\
\text{Dim} E_8 \otimes E_8 = 496 & \Rightarrow & \tfrac{1}{2}(137 + 19) \simeq \tfrac{1}{2}(\bar{\alpha}_0 + b_2^-)
\end{array}
$$

Now recall that the mass of the two intermediate "theoretical" particles, namely the expectation meson $\langle m_\pi \rangle$ and the expectation Kaon were given by

$$\langle m_\pi \rangle = \bar{\alpha}_0 \text{ MeV}$$

and

$$\langle m_K \rangle = (\text{Dim} E_8 \otimes E_8 - k^2) \text{ MeV}$$

so that the following theoretical "decay" is suggested by the preceding symmetry breaking

$$
\begin{array}{ccccc}
\langle m_K \rangle & \Rightarrow & \tfrac{1}{2}\langle m_\pi \rangle & + & \tfrac{1}{2}\langle 2m_u + m_d \rangle \\
\updownarrow & & \updownarrow & & \updownarrow \\
(469 - k^2) & & (\bar{\alpha}_0/2) & & (b_2^-/2)
\end{array}
$$

We see that $\bar{\alpha}_0/2$ the electromagnetic fine structure constant for a Cooper pair arises naturally from the preceding symmetry breaking and in addition we have

$$\frac{b_2^-}{2} = \frac{2(5 + \phi^3)}{2} + \frac{8.46135954}{2} = 9.47235955$$

Consequently this may be interpreted as

$$\frac{b_2^-}{2} = (2)(\sim \langle n \rangle) + (\dim \varepsilon^{(1)} = 1) = (2)(4 + \phi^3) + (1) = (8.47235955) + (1) = 9.47235955$$

but we also know that

$$\frac{b_2^-}{2} \Rightarrow \frac{1}{2}(2m_u + m_d) = 9.47235955 \text{ MeV} = (m_d + 1) \text{ MeV}$$

Consequently the dimension-like value dim $\varepsilon^{(1)} = 1$ corresponding to 1 MeV. This may be a cumbersome way to state a trivial but deeply surprising and immensely useful fact. In E infinity space every dimension corresponds to 1 MeV in the mass space.

19. The Higgs and E infinity

We have already mentioned that our approach to the mass problem is quite different from that of the standard model and the Higgs mechanism. However the Higgs picture could be in general interpreted in a way useful for E infinity. The mere fact that if we do not involve self-interaction of the Higgs field in order to give the Higgs particle mass, then we must assume that there is a second Higgs field which gives the particles of the first their mass and so on indefinitely is a statement about fractalness. In this picture and as mentioned by Veltmann, the Higgs would be just a new level of finer description of particle physics. (see Figs. 1 and 5)

19.1. The fine structure constant revisited

General remarks and alternative rationalisation

The reader may have long noticed the central role played by the fine structure constant α_0 in $\varepsilon^{(\infty)}$. One could say that the value $\bar{\alpha}_0 = 137.082039325$ range second in the line of importance just after the Hausdorff dimension

$$\langle \text{Dim}\, \varepsilon^{(\infty)} \rangle_H = 4 + \phi^3 = 4.236067977$$

A well meaning critic which I take very seriously for more than many very good reasons besides being one of the best theoretical physicists of the past century, remarked that he expected α_0 to come at the end of a general theory as a final conclusion and not at the beginning. This remark hits the nail on the head. Indeed, this is the point. In order to be able to achieve what we set out to do, I had to turn the classical way of attacking the problem on its head. The rationale behind this reversed strategy is found in the topological–geometrical conception of E infinity theory. Once we take the topologicalization program seriously, then α_0 follows from its interpretation as a probability. In our case as a geometrical–topological probability. It is this deceptively simple move which made everything fall into place and laid bare the deep harmony underlying the golden mean mass spectrum of high energy particle physics. To explain this let us start ab initio.

We have already established that E infinity is a kind of probability space. However E infinity is strictly speaking a "prespace" and therefore we should be very careful in using words like space and probability. All the same, we need to define what we mean with probability. In our particular case we have a formaly infinite dimensional Cantor set with unaccountably infinitely many Cantor points in a "prespace" without a metric because the Lebesgue measure of E infinity is zero. As a consequence of this situation, combinatorial probability can be ruled out because the probability in all events will be

$$P_{\text{com}} = \frac{n_1}{n_t} = \frac{n_1}{\infty} = 0$$

In such a case one would usually attempt to define probability geometrically but also in this case we find

$$P_{\text{geo}} = \frac{V_1}{V_t} = \frac{0}{0}$$

The only way left is to attempt to define a topological probability using the Hausdorff dimension and the embedding dimension

$$P_{\text{Top}} = \frac{\text{Dim(set)}}{\text{Dim(embeding of set)}}$$

This means

$$P_{\text{Top}} = \frac{\text{Dim Hausdorff for a Cantor Set}}{\text{Dim Topological for a Line}}$$

In other words, we have,

$$P_{\text{Top}} = d_c^{(0)}/d_T = \frac{d_c^{(0)}}{1} = d_c^{(0)}$$

For a Mauldin–William random Cantor set one finds

$$P_{\text{Top}} = \frac{(\sqrt{5}-1)/2}{1} = \phi$$

That means that the multiplication and addition theorems may be applied to $P_{\text{Top}} = d_c^{(0)} = \phi$. For instance

$$P = \phi^3 = \phi \otimes \phi \otimes \phi$$

is the probability that event with a probability ϕ takes place three times simultaneously. On the other hand the probability that only one event of three events takes place is given by

$$P = 3\phi = \phi \otimes \phi \otimes \phi$$

Applying these elementary ideas to reality we Consider once more the fine structure constant $\bar{\alpha}_0 = 1/\alpha_0$. We interpret $\bar{\alpha}_0$ as in atomic physics in a quite elementary fashion as a cross-section for the interaction of two electrons. A cross-section is a nuclear engineering conception but is actually a marvellous geometrical concept ideally suited to the entire philosophy and structural concepts of E infinity. Thus α_0 would be thought of primarily and in contrast to the classical definition of α_0 as a probability. It is the probability for two electrons to interact for instance. It is also a probability for an electron to emit or absorb a photon. Thus it is a measure of the strength of the electromagnetic field interactivity. Now in the strain of positivistic philosophy we could define $\bar{\alpha}_0$ at will to be

$$1/\alpha_0 = \bar{\alpha}_0 = (20)(1/\phi)^4 = 137 + k_0 = 137 + \phi^5(1 - \phi^5) = 137.082039325$$

We disregard for the moment the slight difference from the experimental value

$$\bar{\alpha}_0 = 137.03598$$

The only thing we need to show is that defining $\bar{\alpha}_0$ in this way leads to rational and particularly economic way of describing physical phenomena without contradicting well established other theories nor of course contradicting well established experimental facts. However at least as far as the present author is concerned, this positivistic operational philosophy is not entirely satisfactory and we would like to give a deeper explanation as to why we fine tune $\bar{\alpha}_0$ to be $\bar{\alpha}_0 = (1/\phi)^4(20)$. Now $(1/\phi)^4$ could be interpreted as

$$(1/\phi)^4 = (1/\phi)(1/\phi)(1/\phi)(1/\phi)$$

Since ϕ is a probability of finding a Cantorion point in a one dimensional Cantor set, then ϕ^4 is also a probability. It is the probability of finding a Cantor point, a so-called Cantorion in all four topological dimensions simultaneously. That means $\bar{\alpha}_0$ or a part of $\bar{\alpha}_0$ is totally an inextricably connected to four dimensional space. However E infinity goes further than that. We have the 10 dimensional core of the super string space $D^{(10)} = 10$ which we have shown to be part of E infinity and embedded in it. Now the probability $(1/\phi)^4$ penitrates into the "string space" through the non-massive section, namely the $(26 + k) - (6 + k) = 20$ dimensions and that on the basis of the addition theorem, so that the total fine structure constant becomes

$$\bar{\alpha}_0 = (D^{(26)} - D^{(6)})(1/\phi)^4 = (20)(1/\phi)^4 = 137 + k_0 = 137.082039325$$

There are numerous ways to convince oneself with the inevitability of setting $\bar{\alpha}_0 = (20)(1/\phi)^4$. However they all have a feel of ad hocness to them. For instance one could see $\bar{\alpha}_0$ as the intersection of $3 + \phi^3$ with $4 + \phi^3 = 3 + \phi^3 + 1$ living in the union of the $D^{(10)} = 10$. That means

$$\bar{\alpha}_0 = (3 + \phi^3)(4 + \phi^3)(10) = (10)(12 + 3\phi^3 + 4\phi^3 + \phi^6) = (10)(12 + 7\phi^3 + \phi^6) = (10)(13.7082039325)$$
$$= (10)(\bar{\alpha}_0/10) = 137.082039325$$

19.2. The E infinity interpretation of the 26 dimensions of super strings

One must have noticed by now that the philosophy of D. Finkelstein and his school regarding that a process is more fundamental than space–time and that a particle creates its own space time has at least some indirect application in *E* infinity theory. In a sense that is the reason why particle masses and dimensions are so much interrelated within this theory. It is here where *E* infinity theory can give an intuitive rationalisation for the need for some 26 dimensions for space–time. To explain this I may use a lucid and clear presentation of the number of the free parameters. Such a presentation was given to the author on request by A. Goldfain and is a follows:

First we have three coupling constant of SU(3), SU(2), U(1). Second, we have the two parameters of the Higgs mass and its vacuum expectation value. Then we have the mixing angle of the instanton contributions. That brings us to six parameters. Next we have $(N_q^2 + 1)$ quark parameters made up of $2N_q$ quark masses for N_q generations and $(N_q - 1)^2$ mixing angles (Cabbibo) and phases. For $N_q = 3$ we have the 10 parameters in addition to the previous 6 making the 16 free parameters. Finally we add another $(N_l^2 + 1)$ lepton parameters giving for generation number $N_l = 3$ another 10 parameters and consequently we end with 26 free parameters. This may be found directly from the formula $2(N_f^2 + 1) + 6$ when setting $N_f = 3$.

The situation is just like in elementary linear algebra where 26 equations are needed to find 26 unknowns. Consequently we need 26 degrees of freedom in our Lagrangian and these 26 degrees of freedom are our quasi-dimension.

The attentive reader may have noticed that we made use of $b_2^- = 19 - \phi^6 \simeq 19$ in our mass formula.

Now this is a geometrical quantities related to the Betti number of the Kahler manifold K_3. However it could also be interpreted as the number of quasi-dimensions when we set in our standard model massless neutrinos and dispose of the leptonic mixing angle. This is again a valid approximation depending on the resolution in keeping with the basic philosophy and concepts of *E* infinity. Finally in the so-called minimal model we could reach the minimum number of 18 free parameters which is what is commonly quoted in text books. By contrast in string theory one normally reads the sentence that the standard model possesses about 20 free parameters.

Thus from our *E* infinity view point we think that we should think of the 26 dimensions of string theory as being the expectation value of the number of needed free parameters for a consistant theory. This alone should give us yet another strong argument to believe that the neutrinos must have a non-vanishing mass. Finally we should link the hierarchical structure of the mass spectrum with the number of dimensions and the fact that the volume of *n* dimensional sphere vanished as *n* goes to infinity.

Now, in *E* infinity theory the exact value is an expectation value namely 26.18033989 rather than just 26. In addition the number of fundamental forces is not just 5 for electric force, magnetic force, weak force, strong force and gravity, but an expectation value $5 + \phi^3 = 5.236067977$. Consequently the total number of free parameters is

$$n = (5 + \phi^3)(26.18033989) = 137.082039325$$

In other words we have gained yet another derivation and interpretation of $\bar{\alpha}_0$ as dimension and number of free parameter at a higher resolution namely

$$\bar{\alpha}_0 = n = 137.082039325$$

20. Symmetry breaking and the Higgs field

There seems to be some misunderstanding about the role of symmetry breaking in connection with the Higgs field which we would like to explain briefly.

If we restrict ourselves to the very elementary case of a skeleronomic and holonomic conservative system described by a potential energy then there are only three types of symmetry breaking bifurcation points. The stable symmetric, the unstable symmetric and the asymmetric or Poincare exchange of stability. Even here we do not have the case of indifferent equilibrium which must therefore be classified as unstable. Thus the massless particles are indifferent to any "potential" and thus unstable. Once the particle "absorbs" a Higgs from the surrounding Higgs field, then it puts on a weight, i.e. it acquires a mass. This mass will in the ball analogy lead to either a stable or an unstable position depending on the shape of the potential. Thus we are not really dealing with a true symmetry breaking bifurcation, neither in the sense of Poincare nor Hopf nor in fact that of Pexito structural stability. We may note on passing that the field associated with *E* infinity is a fractal field. Thus it is not a classical field like the scalar field of the Higgs nor the Vector field of Electromagnetism. It is also not a tensor field like in general relativity. It is far more complex.

21. Cantor space and Newtons non-dimensional gravity constant

For a projective Borel hierarchy, one comes to the notion of a Cantor space as follows:

Definition

Let the space A^N be viewed as the product of infinitely many copies of A with discrete topology, be completely metrizable and countable. In the case of $A = 2 = \{0, 1\}$ and $A = N$, we call the space

$$\varepsilon = 2^N$$

a Cantor space. The amazing and amusing fact which follows from the above is the following. Taking heuristically N to be

$$N = 137 - 10 = 127 \simeq \bar{\alpha}_0 - D^{(10)} \simeq \bar{\alpha}_{\mathrm{ew}}$$

which is the 31 prime number, then one finds

$$\varepsilon = (2)^{127} \cong (1.7)(10)^{38}$$

The dimensionless Newton gravity constant is given by

$$\alpha_G = (\hbar c)/(G m_{\mathrm{p}}^2) = (1.7)(10)^{38}$$

The agreement between ε and α_G is remarkable and although suggesting and indicative of the deep relation between physics and E infinity theory, we do not want to over estimate its importance nor are we at present in a position to give a rational consistent physical explanation for it. We have of course some intuition for the problem derived for instance from comparing the square of the Planck mass and the proton mass which leads to the same pure number when squared

$$(m_{\mathrm{pl}}/m_{\mathrm{p}})^2 \simeq (2)^{127} = (1.7)(10)^{38}$$

While $m_{\mathrm{pl}}/m_{\mathrm{p}} \simeq (1.3)(10)^{19}$ when measured in GeV gives us the unification scale of quantum gravity. Note also that $[(\alpha_G)(\mathrm{GeV})^2]^{-1}$ gives us the right gravitational constant order of magnitude, namely

$$G_{\mathrm{N}} = [(2)^{127}(\mathrm{GeV})^2]^{-1} \simeq (5.8)(10)^{-39} \ \mathrm{GeV}^{-2}$$

The experimental value is $G_{\mathrm{N}} = (6.70784)(10)^{-39} \ \mathrm{GeV}^{-2}$. Similar conclusions were reach in a couragious work by the prominent Stanford experimental physicist Noyes [38].

22. The mass spectrum revisited

Having established the mass norm relating quasi-dimensions to MeV units in $\varepsilon^{(\infty)}$ it is an amazingly simple task to estimate the masses of the some 200 or so known elementary particles. Here we will restrict our attention to only some of the more well known particles and resonances.

The following particles are simply multiples of $\bar{\alpha}_0 = \langle m_\pi \rangle = (137 + k_0)$ MeV and the results are in excellent agreement with experiments

$$m_\eta = 4\bar{\alpha}_0 = 548.32815 \ \mathrm{MeV}$$
$$m_{\eta'} = 7\bar{\alpha}_0 = 959.5742751 \ \mathrm{MeV}$$
$$m_{\gamma_1} = (69)\bar{\alpha}_0 = 9458.66 \ \mathrm{MeV}$$

The experimental values are 548.8, 957.5, 9460.3 MeV. Particularly interesting is the mass for the expectation Σ particle. This is given by

$$\langle m_\Sigma \rangle = m_{\mathrm{N}}(3\bar{\alpha}_{\mathrm{g}} - k_0/D^{(10)} = (939.57)(127)/10 = 11932.53 \ \mathrm{MeV}$$

The experimental value is 11932.8 MeV. The mass of the well known J/ψ can also be found easily as

$$m(J/\psi) = (m_{\pi^\pm})(\bar{\alpha}_{\mathrm{gs}} - 4) = (m_{\pi^\pm})(22 + k) = 3096.08 \ \mathrm{MeV}$$

The experimental value is 3096.9 MeV. For m_Ω we also have a simple formula

$$m_\Omega = (m_{\pi^\pm})(8) = 1116.6 \ \mathrm{MeV}$$

The experimental value is 1115.63 MeV. Similarly for Δ (1232), m_s (770) and m_ω (783) one finds

$$m_\Delta(1232) = \langle m_\pi \rangle (b_2^- - 1)/2 = (137 + k_0)(19 - \phi^6 - 1)/2 = 1229.918 \text{ MeV}$$

$$m_\varrho(770) = (m_n)(5 + \phi^3) = 770.131 \text{ MeV}$$

and

$$m_w(783) = (m_{\pi^\pm})(5 + \phi) = 784.13 \text{ MeV}$$

The experimental values are 1230, 770 and 782 MeV respectively. A particularly neat expression is found for the tau particle by scaling the proton using

$$\lambda = \frac{b_2^-}{D^{(10)}} = \frac{19 - \phi^6}{10}$$

Proceeding that way one finds

$$m_\tau = (\lambda)(m_p^+) = \frac{b_2^-}{D^{(10)}}(938.27 \text{ MeV})\left(\frac{19 - \phi^6}{10}\right)(938.27) = 1777.4842 \text{ MeV}$$

The experimental value is 1777 MeV according to D. Perkins. We could come to a similarly accurate estimation by scaling the expectation π meson, K meson or the constituent mass of the t quark m_t^*:

$$m_\tau = \langle m_\pi \rangle (3 + \phi)^2 \cos\left(\frac{\pi}{26 + k - 3}\right) = \langle m_K \rangle (3 + \phi)^2 \cos\left(\frac{\pi}{23 + k}\right) = (\bar{\alpha}_g)^2 \cos\left(\frac{\pi}{23 + k}\right)$$

$$= \left(m_t^* \cos\frac{\pi}{23 + k}\right)\Big/(10)^2 = 1777.972449 \text{ MeV}$$

In conclusion we may give the mass of the Exi minus and Exi plus particles

$$m_{\text{Exi}^-} = \langle m_\pi \rangle (10) \cos\frac{\pi}{\sqrt{\bar{\alpha}_0}} = 1321.7 \text{ MeV}$$

and

$$m_{\text{Exi}^+} = \langle m_\pi \rangle (10) \cos\frac{\pi}{10 + 1} = 1315.25 \text{ MeV}$$

The experimental values are 1321.32 and 1314.9 MeV.

23. A brief history of ideas leading to the *E* infinity concept

If we focus our attention on hierarchy and self-similarity (see Figs. 2–5) rather than on mathematical transfiniteness, then one may be surprised to see an unsuspected long history of ideas which bear a striking resemblance to the geometrical concept of *E* infinity.

The idea of hierarchy and self-similarity in science first started in cosmology before moving to the realm of quantum and particle physics. It is quite possible that a clergyman, T. Right was the first to entertain such ideas (see Fig. 2). Later on the idea reappeared in the work of the Swedish scientist Emanular Swedenborg and then much later and in a more mathematical fashion, in the work of another Swedish astrophysicist, Carl Charlier (see Fig. 3). This Swedish school may have inspired the work of the eminent British scientist Lord Kelvin on the space–time foam and then in turn together with the work of the Swedish school may have reached Zyldovich in the former Soviet Union.

My own work was done independently and until very recently without any knowledge of the above starting from non-linear dynamics as applied to turbulence (see Fig. 4). Subsequently I became acquainted with Wheeler space–time foam as well as the work of G. Ord and then L. Nottale, K. Svozil, B. Sidharth and finaly the Swedish School.

24. Conclusions

Seen through the eyes of transfinite sets and the golden mean renormalization groups the mass spectrum of high energy particles resembles a non-linear dynamical symphony where everything fits with everything else. We could start

virtually any where and drive everything form everything else. Once we manage to familiarise ourselves with the mass norm, everything falls into place. In fact it takes very little effort to be able to memorise the masses of the most important particles and derive the corresponding formulas with remarkable ease. If we take the words of *E.* Mach seriously, that understanding may be equated with "denk" economy, then one could say that *E* infinity theory furnishes us with such economy of thoughts and thus understanding of the mass spectrum.

Acknowledgements

References

[1] El Naschie MS. Superstrings, knots and non-commutative geometry in *E*-infinity space. Int J Theoret Phys 1998;37(12).
[2] El Naschie MS. The VAK of vacuum fluctuation, spontaneous self-organisation and complexity theory interpretation of high energy particle physics and the mass spectrum. Chaos, Solitons & Fractals 2003.
[3] Ord GN, Mann RB. Entwined paths, difference equations and the Dirac equation. Phys Rev A 2003;67:0121XX3.
[4] Datta DP. The golden mean, scale free extension of real number systems, fuzzy sets and l/f spectrum in physics and biology. Chaos, Solitons & Fractals 2003;17:781–8.
[5] Marek Crnjac L. On mass spectrum of elementary particles of the standard model using El Naschie's golden field theory. Chaos, Solitons & Fractals 2003;15:611–8.
[6] Goldfain E. Derivation of the fine structure constant using fractional dynamics. Chaos, Solitons & Fractals 2003;17:811–8.
[7] Nottale L. Scale relativity and non-differentiable fractal space–time. In: Sidharth B, Altaisky M, editors. Frontiers of Fundamental Physics. New York: Kluwer; 2001. p. 65–79.
[8] El Naschie MS. Nonlinear dynamics and infinite dimensional topology in high energy physics. Chaos, Solitons & Fractals 2003;17:591–9.
[9] El Naschie MS. Complex vacuum fluctuation as a chaotic limit set of any Kleinian group transformation and the mass spectrum of high energy particle physics via spontaneous self-organisation. Chaos, Solitons & Fractals 2003;17:631–8.
[10] Polchinski J. String theory, vol. I and II. Cambridge, MA: Cambridge University Press; 1998.
[11] Kechris AS. Classical descriptive set theory. New York: Springer; 1995.
[12] Mumford D, Sevies C, Wright D. Indra's Pearls. New York: Cambridge; 2002.
[13] El Naschie MS. VAK, vacuum fluctuation and the mass spectrum of high energy particle physics. Chaos, Solitons & Fractals 2003;17:797–807.
[14] El Naschie MS. On a class of general theories for high energy particle physics. Chaos, Solitons & Fractals 2002;14:649–68.
[15] 't Hooft G. In search of the ultimate building blocks. New York: Cambridge; 1997.
[16] El Naschie MS. Modular groups in Cantorian *E* infinity high energy physics. Chaos, Solitons & Fractals 2003;16:353–66.
[17] El Naschie MS. Kleinian groups in *E* infinity and their connection to particle physics and cosmology. Chaos, Solitons & Fractals 2003;16:637–49.
[18] El Naschie MS. On the exact mass spectrum of quarks. Chaos, Solitons & Fractals 2002;14:369–76.
[19] El Naschie MS. On conjugate complex time and information in relativistic quantum theory. Chaos, Solitons & Fractals 1995;5(8):1551–5.
[20] El Naschie MS. Theoretical derivation and experimental confirmation of the topology of transfinite heterotic strings. Chaos, Solitons & Fractals 2001;12:1167–74.
[21] Weinberg S. A unified Physics by 2050. Sci Am 1999;(December):36–43.
[22] Koschmieder EL. The standing wave model of the mesons and baryons. Chaos, Solitons & Fractals, in press.
[23] Sidharth BG. The nature of quantum space–time and the Cantorian proposal. Chaos, Solitons & Fractals 2002;(December):1325.
[24] Castro C. Fractal strings as an alternative justification for El Naschie's Cantorian space–time and the fine structure constant. Chaos, Solitons & Fractals 2002;(December):1341.
[25] Castro C. On p-adic stochastic dynamics, super symmetry and the Riemann conjecture. Chaos, Solitons & Fractals 2003;(January):15.
[26] Saniga M. A further note on a formal relationship between the arithmetic of homoloidal nets and the dimensions of transfinite space–time. Chaos, Solitons & Fractals 2002:1571–3.
[27] Gottlieb I, Ciobann G, Buzea CCh. El Naschie's Cantorian space–time, Toda lattices and Cooper–Agop pairs. Chaos, Solitons & Fractals 2003;(August):789.

[28] Zmeskal O, Nezadal M, Buchnicek M. Fractal Cantorian geometry Hausdorff dimension and the fundamental laws of physics. Chaos, Solitons & Fractals 2003;(July):113.

[29] Agop M, Strat M, Strat G, Nica TP. Cantorian $\varepsilon^{(\infty)}$ structures in discharge plasma double layers. Theoretical and experimental aspects of basic processes. Chaos, Solitons & Fractals 2002;(June):1541.

[30] El Naschie MS. The mass of the neutrinos via the energy of the cosmic background radiation of the VAK. Chaos, Solitons & Fractals, in press.

[31] Abel S, March-Russell J. The search for extra dimensions. Physics World 2000;(November):39.

[32] Sidharth BG. Chaotic Universe. New York: Nova; 2001.

[33] El Naschie MS. The fractal dimension of space–time—remarks on theoretical derivation and experimental verification. Chaos, Solitons & Fractals 1988;9(7):1211–7.

[34] Kröger H. Generalized Aharonor–Bohm effect, homotopy classes and Hausdorff dimension. Phys Lett A 1997;226:127–34.

[35] El Naschie MS. A note on quantum mechanics, diffusional interference and informions. Chaos, Solitons & Fractals 1995;5(5):881–4.

[36] El Naschie MS. Determining the temperature of the microwave background radiation from the topology and geometry of space–time. Chaos, Solitons & Fractals 2002;14:1121–6.

[37] Strogatz S. Sync. Penguin Books, London, 2003.

[38] Noyes HP, Bastin T, Amson J, Kilmister C. On the physical interpretation and the mathematical structure of combinatorial Hierarchy. Int J Theoret Phys 1979;13(7):445–88.

[39] Sternglass EJ. Before the big bang. New York: Four Walls Eight Windows; 1997.