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A review of experimental investigations and assessment methods for masonry arch bridges

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Abstract

Masonry arch bridges constitute a significant proportion of European road and rail infrastructures. Most of them are well over 100 years old and are supporting traffic loads many times above those originally envisaged. The inherent variation in their constituent materials, the traditional design criteria and methods used for their construction, their deterioration over time caused by weathering processes and the development of other defects, significantly influence the mechanical response of these historic structures. A deep understanding on the numerous factors that affect the structural behaviour of masonry arch bridges and on the analysis methods to assess the life expectancy of such bridges and inform maintenance and strengthening strategies is essential. This paper provides a critical review of the experimental studies that have been carried out and of the assessment approaches that have been developed in the last three decades to these aims. The current knowledge is established and areas of possible future research work are identified, with the aim of providing students and researchers, asset managers and bridge owners, and practitioners with a guidance for research activities and maintenace strategies.

Keywords: Masonry, Arch bridges, Structural assessment, Limit state analysis, Experimental investigation, Finite Elements, Distinct Elements.

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1. INTRODUCTION

The masonry arch dates several thousand years back. In 3,500 BC, the Sumerians knew how to assemble stones in the form of an arch in order to construct roofs for their buildings (Favre and de Castro san Roman, 2001). Although true arches were already known at their time, the Romans were the first to realise the potential of arches for bridge construction. The development of transport infrastructure for the movement of armies, trade and communication, as well as for the water-supply to built-up areas, was vital to the spread and successful administration of the empire. Bridge building was a key part of the underlying Roman infrastructure. Since then, and up to the XIX Century, many masonry arch bridges, tunnel linings and viaducts have been built to aid the development of transport infrastructure in Europe.

During the early 1900s, the introduction of new construction materials such as iron, steel and later that of the reinforced and prestressed concrete has reduced the development of masonry arch bridge construction. Today, however, there are still many thousands remaining stone and brick masonry arch bridges around Europe, most of which were built between the second half of the XIX Century and the first decades of the XX Century. These bridges form a vital part of the road, rail and waterway infrastructure. Restrictions to the operation of bridges or their closure can result in local disruption as well as economic and political consequenses. For example, only in the UK, there are about 40,000 masonry arch bridges in daily use on highways, railways and canals, representing an estimated 40-50% of the total bridge stock (Page, 1993). In Italy, there are nearly 10,000 masonry arch bridges only along the railway network, 20% of which having span between 2 m and 5 m, 11.5% between 5 m and 10 m and 8.5% over 10 m, most of which dating back to the period 1860-1920 (De Santis and de Felice, 2014b). In Spain, along the railway network, there are more than 3,000 masonry arch bridges and viaducts, corresponding to 45% of the total, which have been built between 1850 and 1920 (Martin-Caro, 2013).

Most of these bridges are still in service (Melbourne et al., 2007), despite the current traffic loads are much higher than those assumed in the original design, which was carried out on the base of empirical criteria or simple design rules (Brencich and Moribducci, 2007; Oliveira et al., 2010; De Santis and de Felice, 2014b). However, masonry arch bridges are deteriorating over time after being subjected to a prolonged exposure to traffic loads, large vibrations, foundation settlements, environmental conditions (wind, rain, frost attack, high/low temperature cycles, moisture) and extreme natural events (earthquakes, river overflows, floods) (Olofsson et al., 2005; Modena, 2015). The combined effect of these factors progressively induces material deterioration (decrease of mechanical properties), damage development (opening of joints and ring separation in arch barrels, cracks in piers, wing walls and parapets, loss of bricks) and deformations (distortion of the arch profile, out-of-plane rotation of spandrel walls). Inspection and long-term monitoring have also revealed the possibility of occurrence of multiple damage and failure modes in the same bridge (Page et al. 1991; Helmerich et al., 2012; Pellegrino et al., 2014; Modena et al., 2015; Rota et al. 2005; Stablon 2011; Kaminski and Bien, 2013; Harvey et al., 2013; Zampieri et al. 2015). Therefore, on the one hand, there is still today the need for a deeper understanding of the structural behaviour of masonry arch bridges and for a more aware choice of the analysis method to assess their load-carrying capacity, safety level and life expectancy, in order to inform maintenance, repair and strengthening strategies. On the other hand, the wide existing knowledge needs to be reviewed in order to provide researchers with the fundamental references for the study of masonry arch bridge and to orient future research activities.

This paper provides a critical review of the experimental investigations and of the assessment methods developed in the last three decades. First, the basic principles of the structural behaviour of masonry arch bridges are recalled, starting from the historical treatises dating back to the XVIII Century, up to the well-estabished theories of the recent past. Then, an overview of the experimental studies carried out in the last 20-30 years is provided. These investigations were mainly devoted to the mechanical characterization of the materials and to the structural behaviour of masonry arch bridge models (in the laboratory) and real structures (in the field). Some of these studies have already been incorporated in the ordinary activites of maintenance and appraisal, but many important issues (such as the fatigue behaviour and the effect of material deterioration or the contribution of fill and spandrel walls in the structural assessment) are still today under investigation and a deeper knowledge needs to be gained at the research level and then transferred to the engineering practice. Finally, the methodologies for the assessment of the load-carrying capacity of masonry arch bridges, ranging from semi-empirical and equilibrium based methods to finite element and distinct element approaches, are described and their advantages and disadvantages are discussed with reference to their perspective applications in the engineering practice.

2. STRUCTURAL ELEMENTS AND MATERIAL PROPERTIES OF A MASONRY ARCH BRIDGE

2.1. Structural elements of a masonry arch bridge

The main structural elements of a masonry arch bridge are shown in Figure 1. Clearly, the primary element is the arch barrel. Arch barrels built between the second half of the XVIII Century and the beginning of the XX Century generally had a segmental profile, while semi-ellipse and parabolic arches were quite rare. The arches were generally built up in one ring of large cut stones or in several concentric rings or layers of bricks, crossed by headers to promote interlocking. In few cases, multi-ring arches, consisting of several concentric rings of bricks without headers, were built especially in the UK. Compacted fill soil was placed on top of the arch barrel to provide a level formation. The fill distributes the load from the road or rail surface over a larger area of the arch extrados and contributes to the load-carrying capacity and stiffness of the whole structure. In order to retain the fill over the arch barrel, two external spandrel walls were built at the edges of the arch barrel and extended into the wing walls beyond the abutments. Examples of large bridges with inner spandrel walls that sustain the roadway allowing for a reduction of the fill weight have been also documented (Harvey, 2012).

The arch barrels, the piers and the walls of most early bridges were built with either stones or bricks assembled with lime mortars. Brickwork was used particularly where a supply of stone was not available locally. The materials had often relatively poor mechanical properties and were susceptible to deterioration over time (McKibbins et al., 2006). In the last decades of XIX Century, as the technology of brick production improved further and started to become mechanised, stronger and more durable clay bricks and cement based mortars were used for the construction of bridges.

Usually masonry arch bridges have been built perpendicular to a crossing. However, there were cases where masonry arch bridges had to span obstacles at an angle or otherwise at a skew. The construction of skew masonry arch bridges requires construction difficulties and precise stone cutting (Hodgson, 1996). The three most common methods of construction of a segmental arch spanning a 45° skew are shown in Figure 2 (Page, 1993).

2.2. Strength of historic masonry under compression and bending

Since the stress state is usually relatively low with respect to the compressive strength of masonry, the collapse of the barrel vaults and the piers is generally induced by loss of equilibrium. In some cases, local crushing failure may however occur due to the stress concentrations induced by the high eccentricity of the axial load, especially in structures with weak or deteriorated materials. For this reason, the behaviour of masonry subjected to centred and eccentric axial load has been widely investigated over a long period of time. The first works date back to 1970s and 1980s and focused on the parameters influencing the compressive strength and the stiffness of brickwork, such as the strength of the units and the deformation mismatch between units and mortar (Francis et al., 1971; Shrive, 1985; Page, 1981; 1983). The results of the main experimental investigations of these years were summarised by Hendry (1998):

(i) the strength of brickwork in compression is much smaller than the nominal compressive strength of the bricks;
(ii) the strength of brickwork may greatly exceed the crushing strength of the mortar;
(iii) brickwork loaded in compression usually fails by the development of tensile cracks parallel to the axis of loading, as a result of the radial tensile stresses that arise at the interface between brick and mortar. This is due to the mismatch of stiffness, which restrains the lateral deformation of the mortar in the bed joints.

Based on these experimental observations, formulations for the compressive strength of brickwork have been developed. These were determined directly from the results of compressive tests on separate material samples (units and mortar), small masonry prisms (Rots, 1997; Cavalieri et al 2005) or on small cores extracted for existing structures (Pech and Zach, 2009). Over the last 15 years, and with the spread use of numerical methods and computational tools, experimental research focused on the development of constitutive models for masonry. Olivito and Stumpo (2001) carried out extensive experimental tests to investigate the mechanical response of brickwork under compression along both material directions for the identification of constitutive models for both unconfined and confined clay brick masonry prisms. Displacement controlled tests on both stone and brick masonry as well as on their components (sandstone and clay bricks) were carried out to investigate strength, stiffness, brittleness, energy dissipation and deterioration (Venu Madhava Rao et al., 1997; Oliveira et al., 2006). In order to gain an improved knowledge on the stress state experienced by the material under traffic conditions and earthquakes, the cyclic behaviour of masonry under axial load was also investigated (AlShebany and Sinha, 1999; Roberts et al., 2006).

Referring to masonry bridges, some data on the mechanical properties of bricks, natural stones and mortar may be found in historic treatises (Rondelet, 1802; Curioni, 1874; Donghi, 1905; Séjourné, 1913; Gay, 1924) and in a few more recent experimental studies (Barbi et al. 2002). From these works, it was found that the compressive strength of historic bricks may vary between 10 N/mm^2 and 35 N/mm^2, while that of the mortar is between 3 N/mm^2 and 15 N/mm^2. As a rough approximation, the corresponding strength of brickwork ranges from 5 N/mm^2 to 20 N/mm^2.

The condition of combined axial load and bending moment, which is the typical stress state of the cross section of a masonry arch, has been investigated by several authors since the 1980s (Hamid and Drysdale, 1982). A wide experimental campaign is described in Brencich and Gambarotta (2005), in which the tests were carried out on clay brick masonry specimens, as well as on their components; crack pattern evolution, acoustic emissions and cross-section deformation were monitored. Most contributions deal with modern, rather than historic, masonry, which may behave differently to the modern one, due to both brick and mortar properties, as well as to the material deterioration. Few experimental investigations have been carried out to date on historic brickwork under compression (Aprile et al., 2001) as well as under compression and bending (Brencich and de Felice, 2009; de Felice and De Santis, 2010), showing that historic masonry may display lower compressive strength and Young's modulus and slower post-peak deterioration than modern one.

From the above studies, it was shown that the experimental response of brick masonry under both centric and eccentric compression displays an initial elastic phase, followed by a reduction of stiffness before the peak stress is reached. The post-peak behaviour is characterized by a softening phase; the residual strength can be often neglected. Unloading-reloading cycles generally show nearly the same stiffness of the initial one, and some studies even revealed an increase due to the compaction of mortar in the bed joints. Masonry is capable to sustain cyclic loading even if they are performed in the softening phase; the monotonic response curve well represents the envelope of unloading-reloading cycles. Finally, the hysteretic dissipation is generally very low.

Recently, high-cycle loading tests showed that brick masonry may display fatigue failure for relatively large stress range (in the order of 50-60% of monotonic strength) both under compression and under shear (Tomor et al., 2013). Such deterioration is induced by the development and accumulation of micro-cracks starting from mortar joints and progressively extending to brick units, well before they become visible to the naked eye (De Santis and Tomor, 2013). Further research is needed on this topic, in order to develop a deeper understanding of fatigue deterioration under more complex stress states (e.g. combined compression and shear and eccentric compression) and to incorporate fatigue failure in assessment procedures (Casas, 2011). The available experimental outcomes indicate that accurate and reliable structural health monitoring techniques are expected to become a precious tool to identify critical damage development during condition assessment and long-term monitoring of masonry arch bridges.

2.3. Tensile and shear strength of historic masonry

The strength of brickwork in tension is mainly influenced by the unit/mortar bond strength, which depends on the consistency and water retentivity of the mortar, the brick absorption, the brick texture and the workmanship (Page, 1983). Due to the poor mechanical properties of the materials (especially of the mortar) and their deterioration over time, the tensile strength of historic brickwork is generally very low, insomuch that it is often neglected in structural calculations.

Brick masonry structures are frequently subjected to racking shear in addition to compressive loads. Such stress state mainly involves masonry walls, thus being relevant for the structural analysis of spandrel walls, wing walls and parapets. The shear strength of brickwork is essentially due to friction in the bed joints of mortar, thus strongly depending on the load normal to the joints (the vertical load in walls, the component of the force orthogonal to the joints in the cross section of a masonry arch). Mohr-Coulomb is a common criterion that has been used extensively by many researchers to describe the response of masonry under shear and normal stresses. According to such criterion, the shear strength (τ) is provided by the expression: $\tau=\sigma_0\cdot\tan(\varphi)+c_0$, σ_0 being the normal stress on the failure plane, φ the friction angle and c_0 the cohesion. Typical values for φ in masonry are comprised between 25° and 35°, while the c_0 is often considered null, as said before. In the case of the co-existence of shear to compressive stresses (orthogonal to the bed joints), four failure modes may occur (Mann and Muller, 1982), such as (under increasing normal stress): bond tensile failure (a), bond shear failure in the bed joint (b), tensile failure of the bricks/blocks (c), and compression failure of masonry (d) (Figure 3).

3. EXPERIMENTAL STUDIES

3.1. Failure mechanisms of arch barrels

3.1.1. Earlier tests on small-scale and medium-scale bridge models

Many model tests were carried out in the past without records being kept. The first to record the results of a series of model tests was Gautier in 1717 (Hendry, 1998). On the attempt to determine the magnitude of the abutment thrust, half-arches were built made of wooden blocks and piled up other blocks at the springing in order to maintain equilibrium. Backing blocks were incrementally removed and their weight was recorded until failure of the arch. Later, in 1846, Barlow carried out a series of tests on model voussoir arches with the intention of determining the exact mode of collapse (Barlow, 1846). From the experimental testing it was found that, if the thickness of an arch contains a line of thrust that does not touch its edges in four sections (i.e., does not correspond to a failure condition), then more than one such curve could be drawn, each of which is as possible as any other. This means that the problem of the stability of a masonry arch is statically indeterminate as was proved by testing an arch model in which voussoirs were separated by joints from several wood blocks (Figure 4), showing that many different combinations of wood blocks could be removed from the joints whilst preserving equilibrium.

In 1930s, Pippard carried out a series of 23 tests on concrete voussoir arches with either lime or cement mortar . The dead load of the fill was represented by hanging equivalent weights at the centre of each voussoir and all arches were supported en-caste. It was found that the voussoir arch behaved as an elastic arch-rib and that the arch failed when four hinges developed (alternating at the extrados and at the intrados), turning the structure into a mechanism. Also, it was observed that after the first crack occurred, there was a significant amount of reserve strength before collapse. Slip between voussoirs occurred only when crushing and spalling happened. Finally, it was revealed that the line of thrust was often well outside the middle third before tensile cracking was observed. On the base of this work, Pippard developed an elastic method of analysis, according to which tensile stresses can arise provided that the line of thrust does not leave the middle half of the section. In addition, a permissible compressive stress was prescribed (Pippard, 1948). Pippard’s work was later incorporated into the MEXE method.

Additional experimental work was carried out by Pippard and Chitty (1951) on small scale voussoir arches in order to clarify the failure mechanisms of masonry arches. In the test models, the formation of successive hinge points along the arch was demonstrated and it was observed that the critical loading position for a free standing masonry arch (built on pinned abutments, and with no fill on top) was in the region close to the quarter span. In addition, in the tests on model arches built of concrete voussoirs it was found that the limited tensile strength of the mortar between the units could delay the appearance of a crack and raised the ultimate load beyond that calculated when assuming zero tensile strength. This indicated that the assumption of no tensile strength may provide underestimated resistance values. On the other hand, some evidence of crushing failure was observed, showing that assuming an infinite compressive strength of the material may lead to an overestimate of the actual load-carrying capacity.

3.1.2. Field tests on masonry arch bridges

Most of the early experimental work carried out in the laboratory was mainly devoted to testing models made out of arch barrels and abutments only. The other components of an arch bridge, such as the spandrel walls and the wing walls, were not considered and fill was assumed to act as a vertical load on the barrel. The first experimental results on the actual behaviour of real masonry arch bridges were achieved through field testing. Between 1984 and 1994 the TRRL (Transport and Road Research Laboratory), now called TRL, in the UK, carried out eight tests on masonry and broken stone arches, to identify the failure modes of masonry arch bridges and their load-carrying capacity (Page, 1993; Page, 1995). A 750 mm wide load was used in most cases to avoid localised failure of the soil, with the hydraulic jacks reacting against ground anchors embedded into the ground below the bridge. Table 1 show the basic results of the tests and indicates the failure modes observed in each case.

From the above experimental work, Page (1995a) made the following notations regarding the behaviour of masonry arch bridges subjected to vertical loading.

(i) Four-hinge mechanism. When a load is applied at or near the quarter span of an arch, four cracks or hinge points gradually form with the increase of load. These cracks normally occur one at either abutment, one under the point load and one approximately half way between the point load and the far abutment (Figure 5). This failure mode becomes more complicated with the introduction of the spandrel walls and the fill material, and becomes less clear when the arch ring is constructed with weaker and less homogeneous materials.

(ii) Crushing of masonry. The failure of the material of the arch ring under the loading point can be caused by the compressive stresses over a relatively small portion of a cross section experiencing high bending. Such kind of failure may occur in arches built in masonry with poor mechanical strength, in slender arches (thin with respect to the span), or shallow arches (with small rise with respect to the span). Furthermore, if crushing of masonry occurs, this happens under concentrated loads (which may be experienced by the arch barrel if the fill depth is small) and just below the point of application of the load.

(iii) Falling out of bricks. Punching shear due to high loads may cause sections of the arch ring to fall out. This failure mode may activate under concentrated forces parallel to the mortar joints in the cross section of the arch experiencing relatively low compression (e.g., at about quarter span, with small fill).

3.2. Arch-fill interaction and contribution of the fill to the load-carrying capacity

Many researchers tried to understand the incidence of the soil-structure interaction in a masonry arch bridge and the factors affecting it. A number of experimental tests were carried out to this purpose. Melbourne (1991) undertook several experimental tests to identify the contribution of the fill to the load-carrying capacity. From the tests, it was found that when a simple arch barrel backfilled with soil is loaded from the fill surface at a certain position (e.g., at quarter span), the following happens (Figure 6):

(i) The load applied on the fill surface disperses through the fill and onto the arch barrel.

(ii) The barrel section at the load side moves away from the backfill whilst the barrel section on the other side moves into the fill.

(iii) The pressure distribution increases on the opposite side of the arch to the load as the load increases.

Therefore, the earth pressure tends to be fully active beneath the applied load, with a destabilising effect, while it tends to be fully passive at the opposite part of the arch barrel, stabilising the arch, as demonstrated in the experiment at Prestwood Bridge (Figure 7) (Page, 1993).

Darvey (1953) carried out a series of load tests on masonry arch bridges up to failure and found that the interaction between arch barrel and fill soil significantly increased the capacity of the bridge when compared to the case in which the soil strength was ignored. Other experimental investigations devoted to the study of the interaction between the fill and the arch barrel were carried out by Harvey et al. (1989), Melbourne and Walker (1990), Melbourne (1991), Fairfield and Ponniah (1994), Harvey et al. (1994), Melbourne and Gilbert (1997), Hughes et al. (1998), and Gilbert et al. (2006).

In Harvey's 1989 test, relatively small soil pressure was found. Gilbert (1993) suggested that this might be due to the interface of the retaining walls built close behind the springing. Fairfield and Ponniah (1994) carried out 88 tests on 0.7 m span, semi-circular and segmental model arches made of timber voussoirs. The fill consisted of uniform graded dry silica sand, restrained by two standing glass walls. The tests aimed at investigating the dispersal of surface load and the mobilisation and redistribution of earth pressure acting on the arch. The parameters investigated included the end wall position, the density and the depth of the fill, and the load position. The movements of the arch and fill which could mobilise active and passive earth pressures were observed by using still photography and a video camera, while no instrumentation was used to record pressure. It was found that the collapse load increases with increased fill depth.

Harvey et al. (1994) carried out a series of tests on model arches to investigate soil-structure interaction effects. Instrumentation to measure interface stresses between the backfill and the arch barrel was not feasible since installation of the stress cells would have caused significant disturbance to the fill. Thus, soil-structure interaction was derived from pressure changes recorded by pressure cells mounted on the arch extrados. The cells showed high pressures directly beneath the line load at 1/3 span (up to 300 kPa at failure), but much smaller pressures elsewhere (less than 50 kPa). The interaction between arch barrel and soil produced the movement into the fill material on the side of the arch away from the load. However, significant changes were also seen across the full width of the structure and it was found that a large proportion of the stabilising forces required by the arch were provided by the spandrel walls, rather than by the fill soil. These tests highlighted the importance of taking into account the transversal redistribution of the loads to achieve an accurate estimate of the actual load-carrying capacity of a masonry arch bridge, especially on wide arch barrels with small and/or very deformable fill on top. This issue is still today open and more research is needed, from both an experimental and numerical point of view, to develop reliable assessment methods to be incorporated in standard codes and guidelines.

Hughes (1998) carried out a series of centrifuge tests on 1/6 scale models of masonry arch bridges with fill. Centrifuge tests allowed the self-weight of the model to be varied to produce stresses as in a full-scale test. Models were built in order to replicate at each stage all the scale effects, such as fill, brick and mortar sizes. Instruments measured stresses, strains and deflections of both the fill and arch barrel. Hughes' small scale experiments replicated large scale tests carried out at Bolton, UK (Gilbert 1993). The failure load and formation of hinges of the two experimental investigations showed very good agreement. The effects of changing the brick, mortar and fill properties were investigated. Test results indicated that reducing the strength of brick and mortar produces a reduction in the failure load while changing the fill type also has a significant effect on the load-carrying of the arch bridge. More specifically, the experimental results indicated that the limestone backfill which was denser and had a substantially higher friction angle significantly strengthened the arch bridge when compared to a less dense, lower friction angle, sand backfill.

Gilbert et al. (2006) carried out two experimental tests on brickwork arch bridges with two different fill materials, namely limestone and clay, to investigate the arch-fill interaction. Very stiff, transparent, low friction tanks were used to support the fill material at both sides. Measurements were taken with the use of displacement traducers, soil pressure cells and acoustic gauges. Also, the movement of the soil was recorded with photographic digital images (Figure 8) processed through the Particle Image Velocimetry (PIV) technique. Both bridges failed in hinged mechanisms, although some movement at the unconstrained supporting skewbacks was recorded (especially in the clay filled bridge, probably because it is less stiff than limestone). Test results showed the significant effect of the fill material on the behaviour of the bridge, as the ultimate load was nearly double when limestone fill material was used as opposed to clay.

The research on the effect of the presence of fill soil on the load-carrying capacity of masonry arch bridges is still nowadays ongoing. Laboratory testing on full-scale models of masonry arches with fill on top were recently carried out within a research project on the ultimate and permissible limit state behaviour of soil-filled masonry arch bridges, led by the University of Sheffield. Specimens under testing had 3 m span, 0.75 m rise and 21.5 cm thickness (one brick), and were built on concrete abutments, which were allowed to move apart, and filled with either granular or clay soil, laterally constrained by tank walls. The friction between lateral walls and soil was minimized in order to eliminate three-dimensional components and study the composite behaviour of the arch barrel and fill soil in the longitudinal plane only. Specimens were subjected to a set of vertical loads, which were cyclically varied in time to simulate rail traffic. After a certain number of cycles (varying from test to test, but always in the order of one million), loads were increased up to failure. Test results confirmed the important role played by the fill in the behaviour of the bridge and also showed that cyclic loads of relatively low intensity do not strongly affect the load-carrying capacity (Swift et al., 2013; Gilbert et al., 2013). Other issues are currently under investigation within this research project, such as the effect on the behaviour under cyclic loading regimes and on the ultimate load-carrying capacity of fill properties, of the number and range of cyclic loading and on the presence of reinforcement devices (such as ballast injection with resins and horizontal slabs under the road surface to distribute loads).

3.3. Contribution of the spandrel walls and the backfill to the load-carrying capacity

Numerous experimental studies are available in the scientific literature on the contribution of the spandrel walls and backfill to the structural behaviour of masonry arch bridges. Darvey (1953) reported results of load tests on 22 existing masonry arch bridges tested to failure. The experimental work aimed at determining the amount of load dispersion through the backfill, investigating the transverse distribution of load within the arch barrel, assessing the contribution of the backfill and spandrel walls, and examining the effect of spreading abutments. Darvey found a significant contribution of the backfield on the load-carrying capacity showing that: *"when the load is above the abutment, then they move inwards which may be accompanied by an upward movement of the crown"*, and *"when the load is above the span they move outwards"*. From the analysis it was also found that the non-uniform movement of the abutments is a function of the quality of the backfill and of the foundations. In addition, transverse cracking between voussoirs occurred under a relatively low load. These cracks closed when the load was removed. Therefore, it was concluded that the presence of cracks did not result in collapse and that the ultimate load was far more than that required to cause the first crack.

Melbourne and Walker (1990) carried out a full scale model test on 6m span brickwork arch bridge to identify the effect of the spandrel walls and the backfill material on load-carrying capacity and failure mode. A diffused four-hinge mechanism took place, which was facilitated by ring separation. From the outcome of the test, it was concluded that the backfill provided a significant restraint to the deformation of the arch ring thus increasing the load-carrying capacity with respect to a free standing arch.

Royles and Hendry (1991) tested 24 model arches, consisting of (i) arch barrel only, (ii) arch barrel and fill material (no spandrel walls), (iii) arch barrel and fill material and unrestrained spandrel walls, and, finally, (iv) arch barrel and fill material and restrained spandrel walls and wing walls. A substantial increase in the load-carrying capacity of the bridge was observed when spandrel and wing walls interacted with the arch barrel. The maximum load was 100 kN on the arch with fill and no spandrel walls, 150 kN with unrestrained spandrel walls, and, finally, 320 kN with restrained spandrel walls. Furthermore, the lower was the span-to-rise ratio, the greater were the strengthening effects produced by additions to the simple arch, such as spandrel walls and wing walls (Figure 9).

Two field tests were carried out up to failure by Hendry and coworkers on Bargower and Bridgemill bridges, to investigate the contribution of the fill soil on the load-carrying capacity. It was also found that soil structure interaction is more important in deep than in shallow arches. The

Bridgemill arch at Girvan, Scotland is a shallow single arch bridge made of 62 sandstone blocks with a significant span of 18.29m and an 8.30m width; the fill in crown is only 20cm thick. The Bargower masonry arch ring had a semi-circular profile, built up of regular, cut to shape, sandstone voussoirs. The Bargower masonry arch had a span of 10m, rise at mid span equal to 5.18m and arch thickness 0.58 m. The shallow Bridgemill arch (Hendry et al., 1985) derived 50% of its strength from the arch barrel alone while the deeper Bargower masonry arch (Hendry, 1986) derived only 8% of its strength from the barrel and was considerably strengthened by the fill.

Fairfield and Sibbald (1997) carried out destructive tests on a brickwork model arch with spandrel walls, loaded at 1/4 span. Failure was initiated by separation of the spandrel walls and the arch ring before the formation of any visible hinges within the arch ring. The final failure occurred when the spandrel walls rotated outwards and overturned.

Laboratory tests on three large-scale models of 3-span brickwork arch bridges are described in (Melbourne et al., 1997). The study aimed at investigating the effect of the presence of fill and spandrel walls on the load-carrying capacity and collapse mechanism in multi-span bridges. It was found that both the fill and the spandrel walls contributed largely to the strength of the bridge. In particular, the presence of the spandrel walls on top of the arch vault led to an increase of 70% of ultimate load with respect to an identical bridge in which the spandrel walls were not connected to the arch barrel but just built next to it. The collapse mechanism activated with the development of hinges and involved not only the loaded span but also the adjacent ones. The effect of horizontal backfill pressures, although contributing to the load-carrying capacity of the bridge, was less important than in single span bridges, due to the activation of more complex failure mechanisms causing the relative movements of the tops of the piers. Due to the presence of fill and spandrel walls, the critical loading position was not at quarter span (as it is expected on arches without fill) but close to the middle span, as also observed by Gilbert et al (2006) after load tests on large-scale bridge models.

3.4. Effects of ring separation

Failure by ring separation may occur when the arch barrel is built by superimposition of thinner rings. Many researchers have carried out experimental tests to identify the factors that influence the occurrence of this type of failure. Melbourne et al. (1989) carried out an experimental study on 1m span bridge models, whose barrels were built with half scale bricks. Two sets of models were tested, one with two rings of brickwork in the barrel bonded normally with mortar (but without brick bond) and a second with the two rings of brickwork separated by a layer of damp sand. In both cases, the bed faces of the bricks were oiled prior to laying, to minimise the effects of bond on the arch behaviour. The models with artificial ring separation showed a 50% reduction in strength compared to the normal models.

In 1991, the British Rail Research undertook a study at Bolton Institute to assess the incidence of cracks between brick rings on the failure of a masonry arch bridge by ring separation (Melbourne and Gilbert, 1992). Two 3m span arch barrels with two brick rings were built with spandrel walls detached. One of the barrels was built normally, while the other barrel was built with ring separation by replacing the mortar material between the two rings by a layer of sand. The results from the two tests showed that the introduction of ring separation reduced the arch strength by 33%. Two larger models with 5m span and four brick rings with detached spandrel walls were also tested to collapse. The effect of the ring separation was much more significant than for those of smaller models as, in this case, the strength of the arch reduced by 71%.

Melbourne and Gilbert (1995) carried out six large scale model tests on brickwork masonry arch bridges to investigate the effect of ring separation. More specifically, they tested (i) two bridges with 2 rings and 3m span (with detached spandrel and wing walls), (ii) two bridges with 2 rings and 3m span (with attached spandrel and wing walls), and (iii) two bridges of 4 rings and 5m span (with detached spandrel and wing walls). Ring separation was simulated in three of the models using damp sand between the rings, while in the other three models lime mortar was used. The failure load of the bridges with the built-in defect of ring separation were 1.5-3 times lower than those of the bridges without the defect. Also, the strength of the bridge models with separated spandrel walls was about 25% lower with respect to those having the spandrel walls connected to the arch barrel.

Melbourne and Tomor (2005) carried out a series of tests at the University of Salford on multi-ring brickwork free standing arches, to investigate the effect of weak/deteriorated masonry on the behaviour and load capacity of arch barrels (Figure 10). Two 5 m span arches were built, one with weak and one with strong bricks, and tested under static loading at 1/4 span. The tests indicated that the weak bricks lowered the capacity of the arches by 20%, compared to arches built by strong bricks. A slight difference in the failure mode was also observed as the ‘weak’ arch failed by ring separation in the middle section (Figure 10), while in the ‘strong’ one ring separation occurred between the 1/4 span and the nearest abutment. Test results confirmed that ring separation was reliant on the shear capacity of the brick-mortar interface. Since no crushing occurred and the same mortar was used in both arches, the lower shear capacity was caused by the poorer quality of the brick surface.

4. ASSESSMENT METHODS

The need to predict the in-service behaviour and load-carrying capacity of masonry arch bridges has led researchers to develop several methods with different levels of complexity, ranging from expeditious procedures based on empirical rules (such as MEXE), to limit state analysis based approaches (Heyman, 1997), to the most advanced non-linear computational formulations (e.g. finite element and discrete element methods). The selection of the most appropriate method to use depends on, among other factors: (i) the structure under analysis; (ii) the level of accuracy desired; (iii) the knowledge of the material properties and the experimental data available; (iv) the financial resources; (v) the time requirements and the experience of the modeller (Lourenço, 1996). Furthermore, for a numerical model to adequately represent the behaviour of a real structure, both the constitutive model and the input material properties must be selected carefully to represent the non-linear response of masonry. It should also be expected that different methods should lead to different results depending on the adequacy of the approach and the information available, and not always an increase of the level of complexity leads to a more refined estimate of the actual load-carrying capacity (Gibbons and Fanning, 2012). Preferably, the approach selected to model masonry arch bridges should provide an acceptable degree of accuracy and within sustainable time and cost efforts.

A review of the existing strategies for the structural assessment of masonry arch bridges is presented in the next sections, starting from the historic development of the structural analysis, showing the assessment methods that are currently used in the engineering practice, up to the more refined strategies and modelling tools developed for research purposes. The main approaches developed for the seismic assessment of masonry bridges are also presented. Finally, the incidence of material deterioration and damage condition is discussed.

4.1. Historic development

The explicit arch theory originated from Hooke in 1675, who realised that the statics of an arch can be represented by that of a flexible cable carrying suspended weights. Hook stated: *"as hangs the flexible line, so but inverted well stand the rigid arch"*. About two decades later, Gregory in 1697 suggested that the theoretical correct shape for the centreline was obtained by turning upside down Hooke's catenary (Figure 11). Gregory stated: *"an arch would be stable provided that the cable representing the structure could be contained within its thickness"*. Both the concepts of Hooke and Gregory were adopted by Poleni, who, whilst working on St Peter's dome in 1748, stated that the stability of the structure would be assured if "*our chain can be found to lie entirely within the thickness of the arched dome"*. Poleni proved such theory by loading a flexible chain with weights proportional to the self-weight of each segment of the vault.

The failure mode of masonry arches on buttresses was first studied by La Hire in 1712, who proposed the subdivision of the structure at collapse into portions separated by failure planes, whose stability could be investigated by recurring to the principles of equilibrium. By doing so, a relationship was derived to check the overturning stability of the buttresses subjected to the moment produced by the self-weight and by the thrust of the middle portion of the arch sliding downwards. In La Hire's solution, however, friction was neglected, so sliding was allowed in the joints, leading to an inexact not understanding of the problem. La Hire's approach was recalled by Couplet in 1729, who proposed a failure mechanism with five hinges, and finalized by Coulomb in 1775, who introduced friction and sketched out the problem of determining the horizontal thrust at the crown in a rigorous physical way, even if it was not solved correctly.

Gauthey in 1771, Mascheroni in 1785, and Lamé and Clapeuron in 1823 carried out other studies devoted to the identification of all the theoretically possible failure mechanisms of a symmetric masonry arch and the corresponding shape of the line of thrust. Navier in 1826 showed that for an arch made of a linear elastic material and whose plane sections remain plane, tensile stress could be avoided by ensuring that the thrust line lays within the middle third of the section. In 1840 Mery provides an approach for the construction of the line of thrust by means of a graphical procedure (Figure 12), and a method for the design consisting in the well-known middle third rule. The method requires to check that the line of thrust is comprised within the middle third of the arch, to eliminate tensile stresses and avoid cracking. Barlow in 1846 demonstrated that there was no unique thrust line associated with a stable arch but there were many possibilities.

In 1875, Castigliano introduced the principle of the minimum elastic work in the analysis of masonry arches and solved the problem of analysing indeterminate structures using the strain energy method (Castigliano, 1875). His proposal to determine the position of the thrust line was based on a sequence of elastic solutions, in which the tensile zone was removed and the calculation iterated until no tensile stress was present at any point in the arch. Pippard finalized the concepts of the middle third rule and developed a theory to assess the service limit as the load that induces the first crack (Pippard et al., 1951). This approach has proven to be extremely conservative for Ultimate Limit State analysis, as the load that produces the first crack is much lower than that causing the failure of the structure.

Only in the second half of the XX Century, thanks to the concepts of plastic analysis, some fundamental principles of the mechanics of masonry arches were established for assessing their load-carrying capacity. The first contribution was provided by Kooharian (1952), but a comprehensive and general formulation was reached by Jacques Heyman (1982; 1998) who proposed the application of limit state analysis utilising the same concepts of plastic hinges, already developed for steel structures (Heyman, 1982). Heyman's approach assumes infinite compressive strength and no tensile resistance of the masonry and neglects the possibility of sliding between voussoirs (infinite friction). Heyman was able to demonstrate, within the framework of plastic theory, that the arch is able to sustain the given load provided that a line of trust exists which lies entirely within the arch thickness. According to this approach, a plastic hinge develops at the section where the line of thrust touches either the intrados or the extrados. A collapse mechanism occurs when at least four hinges are formed, making the safety of a masonry arch a purely geometrical matter. The theory developed by Heyman is still today the reference one for the numerous assessment approaches based on the so-called mechanism method.

Nowadays, there are mainly three methodologies for the structural analysis of masonry arch bridges and the assessment of their load-carrying capacity. These are:

(i) semi-empirical models (the prime one is MEXE method),

(ii) equilibrium based models (i.e., mechanism method),

(iii) numerical models (i.e., finite elements and distinct elements methods).

A comprehensive comparison between the available methods for the structural analysis of masonry arch bridges is provided by Annex A of UIC Code 778-3R, while a comparison between them for applications to stone arch bridges is presented in (Gibbons and Fanning, 2012).

4.2. Semi-empirical methods

The prime empirical method, which is still used today, is the MEXE method. It was derived by the Military Engineering Experimental Establishment based on the work done by Pippard (1948; 1951) and is classified as empirical because it is based on the classic elastic theory and a series of experimental studies. The assumptions made in the MEXE method are:

(i) The arch is parabolic;
(ii) It has a span-to-rise ratio of four;
(iii) Both abutments are pinned;
(iv) The masonry has a unit weight equal to 21.97kN/m^3;
(v) The arch is loaded at the crown with a transverse line load;
(vi) The permitted maximum arch compressive stress is 1.4N/mm^2 and the maximum tensile stresses is 0.7N/mm^2.

The MEXE method starts from the evaluation of a provisional axle load (PAL), which is calculated from the depth of the arch ring, the depth of the fill material at the crown and the span of the arch, according to the equations provided by BA 16/97 (Highway Agency, 2001). The PAL is then adjusted by a series of modification factors taking into account the geometry, the material and the condition of the arch bridge. Finally, it is multiplied by the axle factors to convert it to single and multiple axle loads, which are then translated into maximum vehicle weights.

Due to the fact that Pippard's equations neglected the effects of axial thrust in evaluating the strain energy, the current version of MEXE overestimates the load-carrying capacity of thick and short span bridges, especially those with large span-to-rise ratios (shallow arches). A modified version of the method has been recently proposed by Wang and Melbourne (2010). In this work, the effects of axial strain energy are incorporated to assess the load-carrying capacity of small span bridges. Also, the work shows that the limitation of the compressive stress at the crown can be exceeded under the dead load only and for larger spans. Comparisons of the advantages against the disadvantages of the MEXE method are collected in Table 2.

4.3. Limit state analysis based methods

Limit state analysis methods assume the arch is on the verge of collapse and there are four or five hinges in the arch barrel. The development of these hinges turns the arch into a mechanism, which is a statically determinate structure (Liversley, 1987). The classical assumptions made by limit state analysis based methods are those proposed by Heyman:

(i) The arch has no tensile strength,

(ii) The arch has infinite compressive strength,

(iii) Sliding cannot occur.

According to the mechanism method, the arch is divided into small segments which are acted upon by an assumed configuration of live and dead loads and the lateral forces of the backfill (Figure 13). Static equilibrium equations are then derived to determine the collapse load and the reactions of the abutments. The backfill pressure coefficient is generally taken as constant, i.e., independent from the arch deflection, although some recent researchers included deflection into consideration (Ng, 1999). In order to estimate the failure load, an optimization problem has to be solved to determine the position of the hinges that leads to the lowest strength. Due to its simplicity and reduced number of constitutive parameters, the mechanism method is gaining in popularity as more and more computer programs are available, including graphical tools for the understanding of the arch behaviour and determining possible equilibrium states (Block et al., 2006). The main advantages and disadvantages of limit state analysis based methods are recalled in Table 3, while an extensive review of this approach applied to masonry arch bridges is presented in (Gilbert, 2007).

Several authors developed assessment procedures based on the principles of limit state analysis. In most of them, the calculations were performed with the principle of virtual works rather than the static equilibrium equations (i.e. a kinematic approach was used instead of the static one). Doing so could lead to an overestimate of the ultimate collapse load if not all the failure modes are taken into account, especially for arches where the soil resistance is important, since, in this case, it is more difficult to identify the weakest failure mode (the one associated to the lowest load multiplier).

Chrisfield (1987) developed a program to perform the structural analysis of masonry arch bridges in which the plastic hinges were represented by yield blocks and the lateral backfill forces were also included. Later, a code known as MARCH was developed by Davies (1998), based on Heyman's method with the addition of an iterative procedure to obtain a thrust line occupying the whole arch ring. Four different distribution patterns of lateral soil forces are incorporated in the program, and is up to the user to select one of them. Load dispersion angles can also be defined and a fixed backfill pressure configuration is to be assumed in a speculative manner. Also, this program is not suitable for solving steep haunched arches due to its inherent difficulty in allowing the thrust line to reach the springers without heading outwards into the backfill.

A program known as CTAP was developed at the University of Cardiff starting from the elastic approach initially proposed by Castigliano (1996). In order to determine the thrust line and the load-carrying capacity, tensile zones in the arch ring are eliminated, which results in progressive development of hinges, and loads are applied until the ultimate limit state is reached, and therefore the actual collapse load cannot be determined exactly as it lies between the last two load increments. Thus, a small load increment needs to be allowed in order to overcome such problem. Unlike the MEXE and the other mechanism methods, which are based on the infinite stiffness assumptions, this method considers the deformability of the material, and thus predicts not only the collapse load, but also provides some prediction of arch deflections. It is also able to model snap-through buckling failure. However, an accurate description of the material properties to be assigned as input may be very difficult and a low elastic modulus generally needs to be assigned for arch with weak mortar and to simulate large deflections due to rotation of arch segments (Ng, 1999). Similarly to other limit-analysis based approaches, this method assumes the material to have unlimited ductility, which may lead to an overestimate of the actual load-carrying capacity.

Harvey et al. (1994) developed a computer program based on the principles of limit state analysis called ARCHIE, in which the thrust line is calculated for a given applied load acting on the barrel vaults of a masonry arch bridge, with the purpose of estimating its load-carrying capacity. This program was used by the DoT (Department of Transport) in the UK to analyse ten full-scale bridges (Highway agency, 2001a), including multi-span viaducts. The program includes the effect of the interaction between structure and fill soil, through a load dispersal angle, taking into account the passive pressure distribution. However, a fixed soil pressure has to be defined before the analysis is run, which means that the load capacity is only pertinent to that pressure.

In order to provide a better representation of the arch-fill interaction and of its incidence on the load-carrying capacity of a masonry arch bridge, more refined limit-analysis based strategies have been developed in the last decade, in which the fill soil is modelled explicitly. Gilbert et al. (2006) developed a rigid block model for the analysis of masonry structures that applies the upper-bound theorem of the theory of plasticity to determine the collapse load. The solution procedure is based on linear programming techniques and is implemented in RING software (Figure 14). In this model the requirement of no overlap between blocks is enforced by the constraint equations and a no tension criterion is adopted. The "no sliding" restriction is removed, which increases the generality of the method, assuming frictional interfaces. However, there are limitations associated with the assumption of normality rule and from the small deflection theory. In order to include the contribution of the fill soil, as well as to solve other geotechnical problems, a technique called Discontinuity Layout Optimization (DLO) was proposed and its reliability was proved by comparison with experimental tests on large-scale models (Callaway et al., 2012).

Cavicchi and Gambarotta (2005) proposed a two-dimensional modelling approach to describe arch-fill interaction effects (Figure 15). The constitutive properties of the materials and the solution algorithm are based on the principles of limit state analysis, and the model is implemented in the framework of finite elements. The arch is represented by monodimensional elements, with no tensile resistance and elastic-plastic compressive behaviour. The fill is modelled with triangular plane elements connected by interfaces, characterized as a Mohr-Coulomb material modified by a tension cut-off under plane strain conditions. The model was then refined by closing the strength domain of the soil in compression to provide a lower bound of the load-carrying capacity (Cavicchi and Gambarotta, 2007).

4.4. Continuum modelling computational approaches

Semi-empirical methods and limit state analysis based methods are useful for initial assessments and are quick to use, but give no information on expected displacements under traffic loads nor at failure (Table 3). A more comprehensive analysis of the structural behaviour of masonry arch bridges requires more complex computational tools with non-linear models and the use of incremental resolution algorithms. The Finite Element Method (FEM) has become more and more popular over the last few decades for modelling masonry arch structures. In FEM, a macro-modelling approach is followed, which means that masonry is described as an equivalent continuum and its mechanical properties are either determined by experimental investigations or derived through homogenization techniques. The user-friendly mesh generation tools developed by a number of practice oriented FE software programs made FEM widely used not only for research but also for assessment purposes. Most applications, however, have a number of limitations, such as the assumptions of isotropic and continuum material, which do not reflect the heterogeneous anisotropic composite nature of masonry, the strong dependency of the results on the accuracy of the constitutive models, on the material parameters and boundary conditions, and the numerical problems related to mesh dependency and ill conditioning (Table 4).

4.4.1. Modelling masonry arch bridges with 1D elements

The simplest way to model a masonry arch structure with the FEM consists in representing the barrel as a segmental beam, made up of monodimensional elements. Towler and Sawko (1982) used frame elements to estimate the load deflection curve and the collapse load of a single-span masonry arch. Choo et al. (1990) and Gong (1992) worked in conjunction with the British Rail Research at Nottingham University to develop a finite element program for arch bridge assessment, known as the MAFEA suite (which includes not only 1D but also 2D and 3D finite elements). In MAFEA, tapered beam elements were used, in which the effective depth of a section was defined by eliminating yielded zones in the arch ring, and the progressive development of hinges was allowed by eliminating tensile zones in the arch, as originally theorised by Castigliano. More recently, Brencich and de Francesco (2004) modelled multi-span masonry arch bridges with 1D finite elements by recurring to a step-wise iterative procedure, based on the assumptions that masonry is elastic-perfectly plastic in compression and no tensile resistant. The method proved to take into account the interaction between the spans of a multi-span masonry arch bridge.

Fibre beam elements have been successfully used to model masonry arch bridges and estimate the load-carrying capability (de Felice, 2009), assess their seismic capacity (De Santis and de Felice, 2014a) and applied to a large set of existing multi-span arch bridges (De Santis and de Felice, 2014b). Following this approach, the constitutive characterization is easily made by assigning to the fibres of the cross-section the stress-strain relationship of masonry derived from compression tests (de Felice and De Santis, 2010). Thanks to the fact that the effective material properties are accounted for (including pre-peak non-linearity, post-peak deterioration, cyclic behaviour), 1D finite elements allow to overcome the main drawback of limit state analysis, related to the assumption of elasto-plastic material behaviour, resulting in a significant reduction in the effective load carrying capacity for large-span bridges.

4.4.2. Simplified representations of arch-fill interaction with 1D elements

The main drawback of 1D models is that the contribution of the spandrel walls and of the fill material is either not taken into account or considered in a very simplified way. Crisfield (1985) used non-linear spring elements (Figure 16), which were initially pre-compressed to the equivalent at rest pressure, and whose maximum horizontal pressure was limited by the active or passive pressures depending upon the type of the movement of the arch in order to produce realistic collapse loads.

Similarly, non-linear truss elements are used to represent the backfill, the abutments and the spandrel walls (Figure 17), in order to represent the interaction between adjacent spans and predict multi-span failure modes under both traffic loads and seismic action (De Santis and de Felice, 2014a,b).

In the MAFEA suite, the effects of soil-structure interaction were taken into account with a Mohr-Coulomb failure criterion and the live load was distributed over the fill under a fixed dispersal angle. Extensive numerical analyses were carried out on different arch bridge geometries and a good agreement was obtained with experimental results.

4.4.3. Modelling masonry arch bridges with 2D and 3D elements

In order to improve the description of the bridge geometry, including fill soil and spandrel walls, 2D and 3D models have been developed, which are mainly used for research purposes at present time. With respect to 1D approaches, these methods require a higher effort for the construction of the model and longer runtime. Furthermore, they still present intrinsic difficulties related to the sensitivity to boundary conditions and input parameters, whose calibration may be affected by strong uncertainties.

Two-dimensional elasto-plastic FE models were developed to explicitly simulate the presence of the fill soil and of the spandrel walls and evaluate quantitatively their effects on the structural response and load-carrying capacity (Audenaert et al., 2008; Gago et al., 2011; Kishen et al., 2013).

Fanning and Boothby, in 2001 proposed one of the first modelling studies with three-dimensional finite elements. Non-linear constitutive relationships were assigned to both the masonry of the bridge and the fill soil. The possibility of cracking occurrence in the masonry and of sliding between fill soil and barrel vault were also included. Aiming at reproducing the damage pattern induced by transversal effects, a three-dimensional FE model was developed by Fanning et al (2001) to predict the longitudinal cracks in the barrel vault under truck loading. Garity and Toropova (2001) modelled a typical brick arch highway bridge (Figure 18a), taking into account the effects of the fill over the barrel, spandrel walls, patch loading and the presence of cracks. The authors highlighted some possible limitations of FE analysis for design or assessment of masonry arch bridges related to the fact that: (i) the constitutive models for masonry in standard FEA packages are not representative for the material behaviour, (ii) it is difficult to obtain reliable/representative material parameters, and (iii) considerable uncertainties with fill-structure interaction properties.

The transverse behaviour of a masonry arch subjected to concentrated loads was investigated in (Fanning et al., 2005), showing that neglecting three-dimensional effects may lead to an overestimate of the actual load-carrying capacity of the bridge. More recently, Milani and Lourenço (2012) proposed a three-dimensional modelling approach, developed with a non-commercial software, with eight-noded parallelepiped finite elements separated by non-linear interfaces accounting for crack opening and deterioration (Figure 18b). The comparison with experimental tests on full scale bridges and with simulations with 2D finite elements confirmed the importance of describing transversal effects to gain an estimate of the load-carrying capacity and the deformed configurations of masonry arch bridges.

4.5. Discrete modelling computational approaches

In discrete element methods, a micro-modelling approach is followed, in which masonry is represented as an assembly of distinct units. The mortar joints are modelled as zero thickness interfaces between the blocks, which represent the preferential crack location where tensile and shear cracking occur. Non-linear relationships between contact force and relative displacement are defined for joints, while blocks are usually considered simply as rigid or elastic bodies. An explicit integration procedure is followed in the time domain allowing for the non linear kinematics to be considered. On the one hand, discrete element modelling allows for a detailed representation of the geometrical and mechanical characteristics of the masonry. On the other hand, it requires a high computational cost, such that it may be hardly applicable to large 3D structures and for engineering practice purposes (Table 4).

Starting from the 1970s, different strategies have been developed to model masonry arch bridges with discrete elements, including particle models with circular and spherical elements, non-smooth contact dynamics, distinct element method (also combined with finite elements), and discontinuous deformation analysis. These formulations, which are described in the following paragraphs, differ from each other because they derive from different fields, ranging from rock mechanics to structural analysis or engineering mechanics.

4.5.1. Particle models with circular and spherical elements

Particle models are assemblies of discrete circular or spherical elements. They were initially proposed to analyse the micro-mechanical behaviour of soils and other granular materials (Cundall and Hart, 1989). The solution follows the standard explicit time-stepping algorithm, allowing for the computationally efficiency and for a straightforward detection of contact between particles. Beyond applications to geotechnical engineering, this method appears particularly suitable to analyse the backfill of a masonry arch bridge and its interaction with the arch barrel, this latter being modelled by clusters of fully bonded particles representing the stone units. Thavalingam et al. (2001) used particle models to simulate an experimental test carried out on a backfilled masonry arch bridge (Figure 19). The comparison revealed a good capability of numerical simulations to predict not only the load-carrying capacity, but also the entire load-displacement response and the post peak structural behaviour of the bridge.

4.5.2. Non-smooth contact dynamics

The non-smooth contact dynamic (NSCD) approach, implemented in the LMGC90® code, is particularly suitable to investigate the dynamic behaviour of arched structures built in stone masonry (mainly made out of large blocks), when the relative displacements between the units or their detachment is expected. Differently from other discrete element approaches, in this method the velocities of the mechanical system are allowed to undergo jumps at certain time instants due to possible impacts (Jean, 1999) and there is no need to resort to artificial damping in order to secure the numerical stability. Rafiee (2008) used the NSCD method to analyse the seismic behaviour of the Arles aqueduct, which was partially destroyed in 150 AD (Figure 20a). The numerical simulation indicated that a large number of block detachments occurred (Figure 20b), which was in good agreement with the damage pattern surveyed in the field.

4.5.3. Distinct element method

Among the different discrete element approaches, the distinct element method is currently the most used one for the structural analysis of masonry arch structures. It was developed from rock mechanics and is characterized by the following features:

(i) blocks may be rigid or deformable;

(ii) a soft contact approach is adopted (i.e., contact forces are obtained from the relative displacements between blocks, given the joint normal and shear stiffness properties);

(iii) an explicit integration procedure in the time domain is followed which ensures the numerical stability provided that the time step is adequately small.

The most widely used code for this analysis is UDEC (Universal Distinct Element Code, ITASCA, 2004), which evolved from Cundall’s original work (Cundall, 1971) for 2D modelling of rock mechanics and later improved to model 3D structures by the program 3DEC (Hart et al., 1998). Both in UDEC and 3DEC, various types of block assemblies can be generated and the detection and update of contacts is automatic. Many applications to masonry structures have been reported so far (see, amongst others: Rots, 1997; Lemos, 2007; Dimitri et al., 2011; Sarhosis et al., 2014a; 2015; Giamundo et al., 2014). Structural reinforcement elements can also be simulated, with wide potential applications for rehabilitation design purposes.

Tóth et al. (2009) analysed a stone masonry arch with the discrete element software UDEC. The aim of the study was to investigate the effect of the backfill on the mechanical behaviour of single and multi-span masonry arch bridges. The UDEC software proved capable of predicting the response of the bridges under vertical static loading and from the sensitivity analysis it was concluded that the stiffness, the friction angle and the cohesion of the backfill largely influenced the load bearing capacity of the structure.A 3D rigid block model was employed by Lemos (1998) to study the out-of plane failure modes of circular and pointed arches, as well as intersecting arches, with different cross-sections, under seismic loading (Figure 21). Later, this model was used to evaluate the load-carrying capacity of a stone masonry arch subjected to multiple cycles of quasi-static horizontal loading (Lemos, 2001) and to assess the distribution of the live load across the width of a masonry arch and the contribution of the spandrel walls to the collapse load (Figure 22).

Schlegel and Rautenstrauch (2004) carried out a 3D analysis of an old masonry arch bridge with spandrel walls and no fill, using both continuum and discontinuous models to show the influence of constitutive modelling on the limit load and on the collapse mechanism. It was concluded that one of the most important effects in the non-linear stress-strain behaviour is the interaction between spandrel walls and masonry arch. Both the continuum and discontinuous models showed good agreement, but since the latter includes the blocking effect of the masonry assembly, the ultimate load increased by about 5%.

More recently, Sarhosis et al. (2014b) investigated the effect of the angle of skew on the load-carrying capacity of twenty-eight stone masonry arches with different geometry. The variables investigated were the arch span, the span-to-rise ratio and the skew angle. A full width vertical line load was applied incrementally to the extrados at quarter span until collapse. At each load increment, the crack development and vertical deflection profile were recorded. The results were compared with similar “square” (or regular) arches, showing that an increase in the angle of skew increases the twisting behaviour of the arch and causes failure to occur at a lower load than in a straight arch. It was also shown that the effect of the angle of skew on the ultimate load that the masonry arch can carry is more significant for segmental arches than for circular arches.

4.5.4. Combined discrete/finite element methods

In order to simulate fracturing occurrence in the units of the masonry, modelling approaches that combine Distinct Element and Finite Element Methods have been developed, named finite-discrete element method (Munjiza, 2004), that include deformable blocks represented by a mesh of triangular elements, which may split and separate in the course of the analysis, based on fracture mechanics criteria. An application of this approach to masonry arch bridges was proposed by Mullett et al. (2006) to assess the load-carrying capacity and identify the suitability of a strengthening system consisting in inserting and grouting stainless steel reinforcing bars into the masonry (Figure 23).

4.5.5. Discontinuous deformation analysis (DDA)

The DDA method was developed for rock engineering problems and applied to model 2D stone arch bridges by Ma et al. (1996) and Bićanić et al. (2003). According to the method, all blocks are taken as deformable but each block is assumed to be in a state of uniform strain and stress. Contacts are continuously updated and enforced as the solution progresses, according to either a hard or a soft contact approach. Similarly to the distinct element method, DDA can simulate the behaviour of interacting discrete bodies, and recognize new contacts between bodies during calculations. The system of equation in DDA is derived from minimizing the total potential energy of the system to guarantee that equilibrium is enforced, allowing for larger time steps than DEM. Bićanić et al. (2003) presented a DDA model of an arch bridge in which the backfill was represented by a system of deformable blocks created by a random pattern of joints (Figure 24).

4.6 Comparative studies

Only a few studies have been carried out to compare the previously described computational approaches in order to assess their capabilities and limitations, by comparison with experiments. A significant contribution was provided by Thavalingam et al. (2001) in the analysis of a backfilled masonry arch bridge. Both a non-linear finite element technique with joint/interface elements (FEM based DIANA) and a discrete element approach using either discontinuous deformation analysis (DDA) or particle flow code (PFC) were used to simulate an experimental test. The load displacement curves obtained from the numerical simulations are shown in Figure 25. Of the three models, PFC seems to give better predictions of the collapse load obtained from the experimental study. The curves also show the ability of the discrete models to simulate the post peak structural response of the masonry arch bridge.

A further comparison between continuous and discrete approaches was carried out in (Schlegel and Rautenstrauch, 2004). A continuum model (ANSYS) was compared with a discontinuum model (3DEC) to simulate the behaviour of a masonry arch bridge with spandrel walls under vertical in-plane load. It was found that the ultimate load capacity of the bridge predicted using ANSYS was 5% less than that predicted using 3DEC. According to the authors, the difference in the results arises from the interlocking effect among the block units which also depends on the stone size, since when this latter is increased the ultimate load increases as well. The results of the study are presented in Figure 26.
Based on the literature review, the main differences between continuous and discrete modelling approaches, in terms of basic assumptions, input parameters and field of application, are collected in Table 4.

4.7 Seismic assessment approaches

Despite to date most research works have been devoted to the assessment of masonry arch bridges under vertical loads, a number of studies have recently started to explore the dynamic response of masonry arches and assessing the seismic behaviour of bridges. Due to the large number of existing masonry arch bridges, their seismic assessment has become a priority for ensuring the safe operating conditions of road and rail infrastructures in earthquake-prone areas.

The dynamic behaviour of masonry arches has been primarily investigated by making use of the mechanism method, taking advantage of the simplifying assumptions of no tensile resistance and infinite compressive strength of the material (Heyman, 1982). If the seismic action is simply described by horizontal static body forces, the arch fails as soon as four hinges develop, turning the structure into a mechanism. A limit analysis approach (De Luca et al., 2004; Zampieri et al., 2015) cah be used in this case, providing a lower bound estimate of the actual capacity under earthquake ground motion (Da Porto et al., 2015).

A properly dynamic approach was proposed by Oppenheim (1992), in which the equation of motions of the mechanism made out of four bars are derived and solved for impulse base motion. The failure domain of the arch was obtained under pulses (DeJong et al., 2008; Mauro et al., 2015), free vibrations and sine base accelerations (Clemente, 1998) also taking into account damping during cycles at impacts (De Lorenzis et al., 2007).

As an alternative to analytical models, the distinct element method (DEM) has been used to study the dynamic response of masonry arches (De Lorenzis et al., 2007), also including the presence of buttresses (Dimitri et al., 2011). However, in addition to the difficulty of describing large structures with a discrete element modelling approach (already discussed in Section 4.5.), the explicit integration scheme may be very sensitive to contact stiffness and damping values in time history analyses (de Felice and Mauro, 2010). For these reasons, finite elements (FE) and macro-elements are often preferred for the seismic assessment of real case studies in the current practice. 2D and 3D FE models (Pelà et al., 2013) allow for representing the effect of fill soil, spandrel walls and backings on the dynamic behaviour of the bridge, as revealed by field testing and dynamic monitoring (Brencich and Sabia, 2007; Mautner and Reiterer, 2007). Nevertheless, the high computational effort required by 3D FE models for time-step simulations makes them unfeasible for applications to large multi-span bridges, especially in current design practice. Nonlinear time-step analyses under accelerograms can rather be carried out with macro-elements (Resemini and Lagomarsino, 2007) or fibre beam elements (De Santis and de Felice, 2014a), which are computational efficient and allow for the assessment of the seismic performance with limited runtime.

On the one hand, the studies on the seismic assessment have confirmed the empirical observation that masonry arch bridges are generally sufficiently safe against earthquakes. Very few examples have been reported of collapses involving a large portion of a masonry arch bridge after a severe seismic event. Speaking about global collapse mechanisms, the most vulnerable bridges are those with very high and slender piers subjected to transversal seismic action. Differently, local collapse mechanisms have been observed, consisting in the movement/loss of voussoirs, the development of vertical cracks in the piers and the activation of out-of-plane overturning of spandrel walls (Rota et al., 2005). On the other hand, recent works (De Santis, 2015) have raised the question about the possibility of applying to masonry arch bridges the seismic assessment methods that have been conceived for reinforced concrete and steel frame structures and are recommended by current standard codes. A code procedure for the evaluation of the safety level against earthquakes specific for masonry arch bridges does not exist yet, due to the difficulty of defining an equivalent single degree of freedom system for a masonry arch bridge and to the uncertainties related to hysteretic dissipative behaviour.

4.8. Incidence of material deterioration and damage condition

As a general trend, experimental tests, numerical simulations and inspection of existing structures indicate that almost no masonry bridge fails because of traffic loads (see, amongst others, Page, 1993; Oliveira et al., 2010; Harvey, 2012; De Santis and de Felice, 2014b). However, prolonged exposure to traffic loads and vibrations, environmental conditions (wind, rain, frost attack, high/low temperature cycles, moisture), extreme natural events (earthquakes, river overflows, floods) and impacts progressively induces deterioration and damage development, which, in turn, may significantly affect the actual safety level (Melbourne and Tomor, 2005; Hulet et al., 2006; Modena et al., 2015). The main deficiencies can be classified as follows (Modena et al., 2015):

(i) material deterioration with decrease of mechanical properties (compressive strength and stiffness). This is due to both chemical aggression, which causes corrosion (visible by the development of dips and holes), or cyclic loading, which produces micro-cracks.
(ii) damage development: opening of joints and ring separation in arch barrels, cracks in piers, wing walls and parapets, displacement or loss of bricks or lack of pointing. These may be due to penetrating vegetation, infiltration of water and freeze-thaw cycles.
(iii) permanent deformations: distortion of the arch profile from its original shape (due to compaction of fill soil, differential settlements of piers or abutments), out-of-plane rotation of spandrel walls (under horizontal pressure of fill soil and seismic loads). Due to the stiffness and extremely tensile strength of masonry, permanent deformations are generally associated to cracking

Prior to evaluating the safety level of an existing masonry arch bridge, its structural health condition should be assessed (Pellegrino et al., 2014). To this purpose, the following activities can be undertaken:

(i) visual inspection, to detect deficiencies on superstructure elements (piers, arch barrels, spandrel, parapet and wing walls) and confirm the dimensions and the presence of any strengthening device (e.g., ties, plates, etc.) indicated by drawings and other available documents;
(ii) surveys, to detect information on inner structural elements that cannot be directly measured from outside (thickness of arch barrel, spandrel wall, and external leaf of the pier by endoscopy, inhomogeneities, vois, internal cracking, moisture content, hidden structural elements by sonic tests and georadar, stone arrangement and cracks, wet areas, indentification of materials by infrared thermography) (Orban and Gutermann, 2009; Bergamo et al., 2015).
(iii) field testing, to determine quantitative information on material properties (stress state, compressive stiffness and strength by single and double flat-jack test), and extraction of small samples (e.g., cores) to be tested in the laboratory. From cores, small samples of mortar are also extracted for micro-chemical analyses.
(iv) monitoring, to record modification occurring in the long-term (e.g., measurement of crack opening with displacement transducers or extensometers), derive additional information on the structural health condition (by acoustic emission monitoring technique, Invernizzi et al., 2010; De Santis and Tomor, 2013) or on the dynamic behaviour (by use of accelerometers and dynamic identification techniques, Brencich and Sabia, 2007).

On the one hand, proper and careful maintenance (waterproofing, repointing, removal of vegetation) and periodical inspection and monitoring are fundamental for ensuring the continuous safe service conditions of the infrastructures. On the other hand, more research is needed to improve the existing knowledge on the effect of ageing on the structural reliability. Despite some recommendations exist that are specifically addressed to this topics (e.g., UIC Code 778-3R), the assessment of the actual incidence of ageing and damage state on the structural behaviour of a bridge still mainly relies on engineering judgement. Ultimate Limit State and life expectancy assessment procedures are nevertheless affected by strong uncertainties due to:

(i) scarce representativeness of the quantitative data derived from a small number of tests (necessarily limited by constrains related to time, cost, accessibility and need for service interruption), such that, in the professional practice, the mechanical properties listed in historical treatises or recommended by standard codes are often considered.

(ii) difficulty of determining the geometry and the mechanical properties of inner elements (e.g., fill soil, foundations), which are often assumed on the base of available documents and on the rules of the building state of the art.

(iii) complexity of achieving a suitable representation of deficiencies in the numerical model used for structural assessment. Depending on the modelling strategy (micro or macro, continuum or discrete modelling scheme) different strategies could be chosen, for instance modifying the parameters of constitutive relationships in the portions of the structure where a deterioration has been observed, or neglecting the connection between adjacent elements to represent a crack (Garity and Toporova; Kamiński & Bień, 2013), or allowing for the update of contacts during analyses to account for progressive damage development (Munjiza, 2004).

5. CONCLUSIONS

Masonry arch bridges have proved to be reliable, enduring structures and remain a vital part of the European road, rail and waterway infrastructure. Today, most of these bridges are well over 100 years old and are facing a number of challenges related to their extended period in service and the changing requirements of modern transport systems. The load-carrying capacity and the failure mode of a masonry arch bridge depends on a number of factors, including the mechanical properties of its materials, the complex interaction among the structural elements than constitute the whole bridge structure, and that between the structure and the foundation soil, the loading regime, the deterioration of materials and damage development over time, the maintenance conditions. In order to ensure the continued efficient use of these assets in the future it is necessary to manage and keep them carefully, establish and improve the existing knowledge (whose development has started more than two Centuries ago and still presents important issues needing a deeper understanding), and identify shared and reliable methods for the structural assessment. Three main areas of research were reviewed in this paper related to the characterization of the materials, the experimental testing on model bridges and real structures, and the structural assessment methods.

Numerous experimental studies have been devoted in the last 40 years to the investigation of the mechanical properties of historic masonry, which still remains one of the critical elements in the assessment of bridge structures. The difficulty of achieving a reliable constitutive characterization for structural assessment purposes lies, on the one hand, on the heterogeneity and anisotropy of the material, on its fatigue response (which has not been fully investigated yet) and its deterioration over time (induced by traffic loads and environmental aggression) and, on the other hand, on the problems related to the extraction of representative samples and to the identification of appropriate test methods, both in the laboratory and in the field. Innovative non-destructive methods have lately been proposed to provide information on structural health condition (e.g., making use of the Acoustic Emission Technique) and dynamic identification parameters, but their straightforward application for Ultimate Limit State assessment appears unfeasible.

The first experimental works on small/medium scale bridge models were mainly devoted to the identification of the failure modes, the critical load position and the ultimate strength of free-standing arches, either with pinned abutments or built on pillars. More recently, medium/large scale tests have clearly shown the importance of improving the understanding of the contribution provided by the fill (not only as a load-spreading mean, but also as a constrain to the arch deflection), and by the spandrel walls (including the inner ones), of the possible activation of multi-span mechanisms, of the incidence of ring separation or foundation settlements, of cyclic/fatigue behaviour (with the aim of identifying a Permissible Limit State), of the response of skew arches, and of the effectiveness of reinforcement measures. Further investigations, especially in the field, are still needed to gain a better and more reliable knowledge on the incidence of all these factors on the in-service response and load-carrying capacity of masonry arch bridges.

Many masonry arch bridges that are today in service along the infrastructures were designed with empirical rules or simplified methods based on graphical statics. Their safety level needs now to be assessed under current traffic loads and according to in force standards. Empirical methods based on inspection and engineering judgment (such as MEXE) and limit state analysis approaches implemented in practice oriented software codes (such as RING, ARCHIE and others) are currently mainly used by practitioners, at least for most standard assessment and maintenance purposes. Refined computational tools have recently been developed for the non-linear analysis of arch bridge structures, which can be used for much more complex situations, but whose application is limited by the sensitivity to the material parameters and to the boundary conditions (which may be difficult to establish), and to the high computational cost. Continuum approaches are suitable for the analysis of large structures offering at the same time a reasonable compromise between accuracy and efficiency. 1D and 2D finite elements can be used to accurately account for the (macroscopic) mechanical properties of the materials, with some simplifications on the bridge geometry. Conversely, the use of 3D elements allows for a faithful description of all the elements of the bridge, but not for a robust and detailed constitutive characterization. On the other hand, the individual units of the masonry and the mechanical properties of its constituents can be represented in detail with discrete modelling approaches, which lead to a reliable description of the crack development process that characterizes the collapse behaviour of masonry arch bridges. These approaches, however, require particularly high computational efforts, especially for large structures or if 3D elements are used. Scientific research trends point to the integration of the continuum and discrete modelling approaches, in order to appropriately represent the complex behaviour of masonry arch bridges under both in-service and ultimate limit state conditions, accounting for the interaction between its elements, for the occurrence of deformations and cracks both in the joints and in the units, and for the presence of pre-existing damage.

Despite the number of studies carried out for the experimental investigation and the structural analysis of masonry arch bridges, shared assessment approaches have not been identified yet that provide reliable information on the safety level of a bridge and on its residual service life under traffic loads also considering the effects of ageing, deterioration and fatigue. Recent research outcomes have not been fully incorporated in standard codes and guidelines addressed to practitioners and bridge owners. This need will be one of the main challenges to be faced by the academic and the professional communities in the next future.

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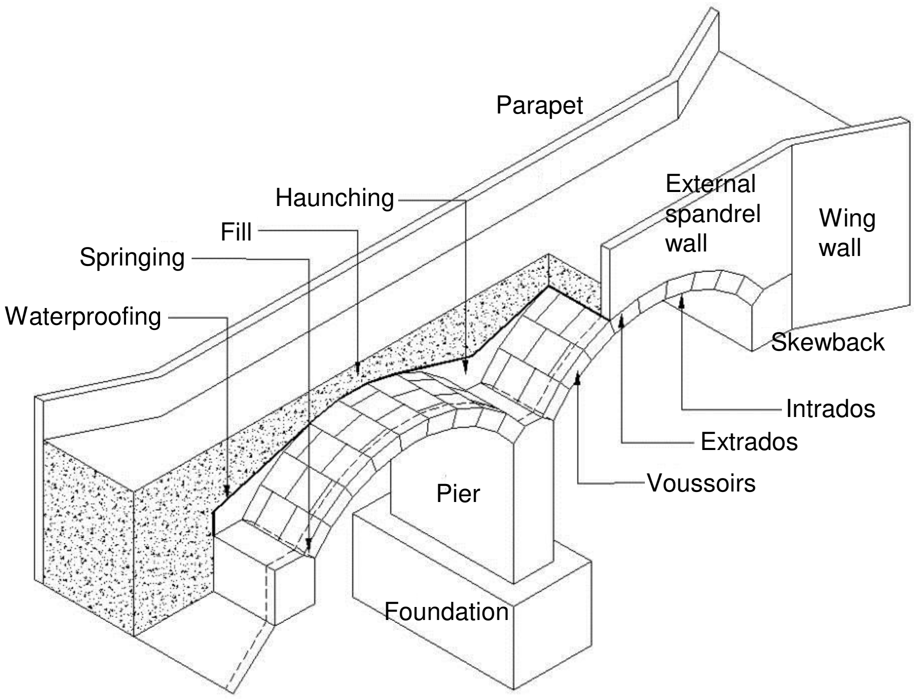

Figure 1. Main elements of a masonry arch bridge (UIC Code 778-3R). Note: *The reader is addressed to the glossary of UIC Code 778-3R for a complete list and more detailed descriptions.*

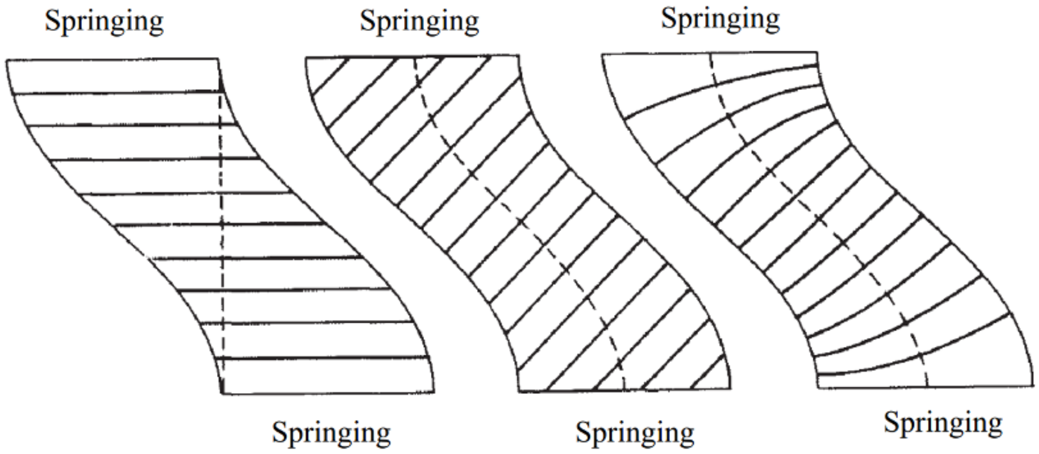

Figure 2. Intrados of an arch spanning at 45° skew (Page, 1993).

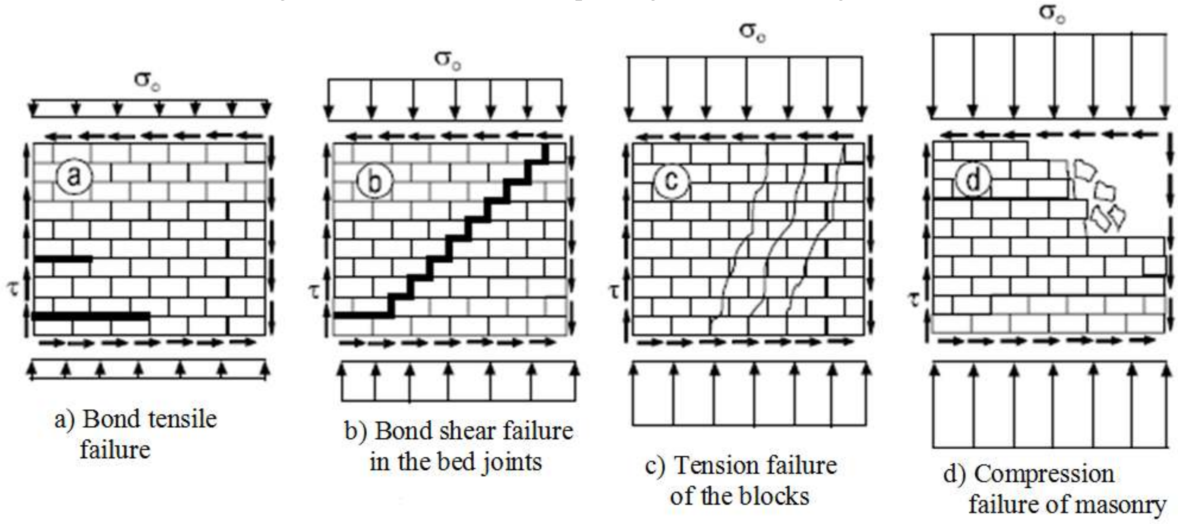

Figure 3. Classification of failure mechanisms in masonry under compression and shear (Mann and Muller, 1982).

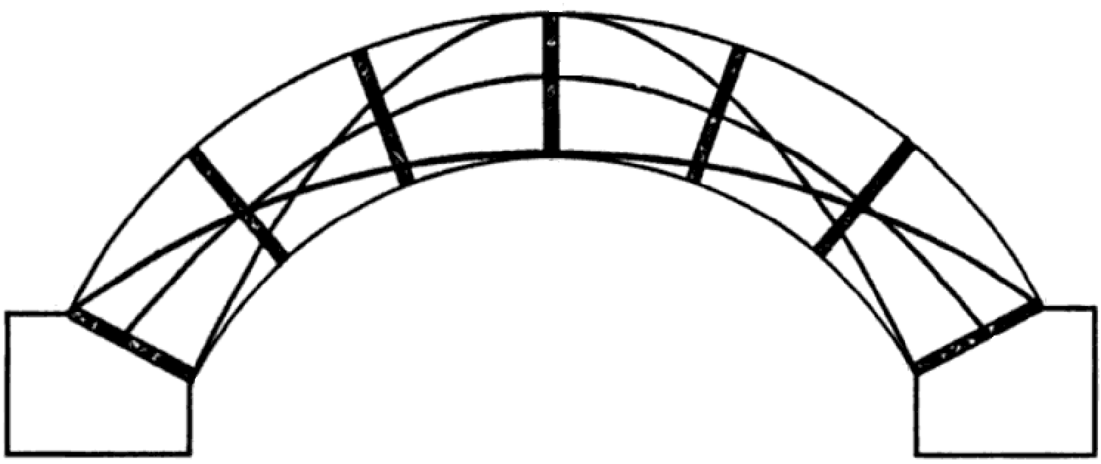

Figure 4. Voussoir arch model tested by Barlow in 1846 showing alternative positions of the thrust line

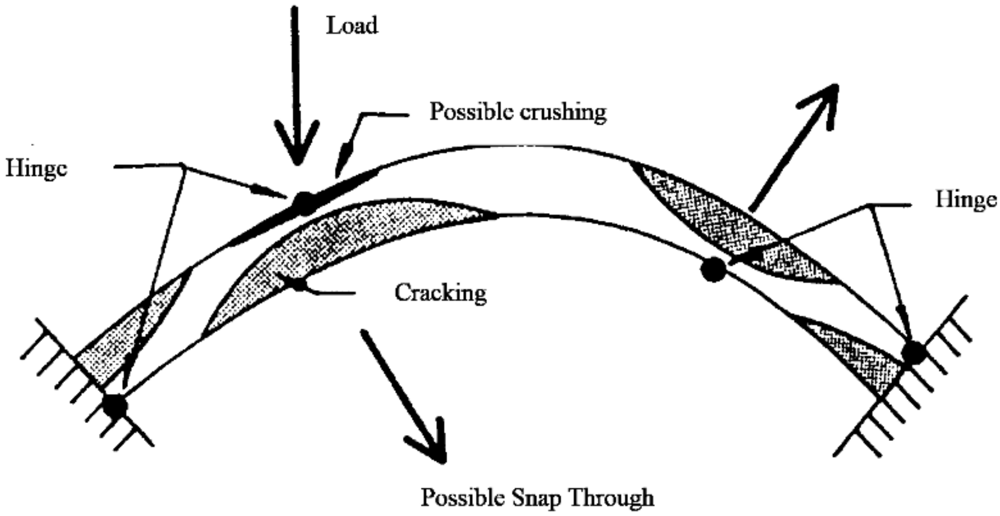

Figure 5. Failure modes in masonry arch bridges.

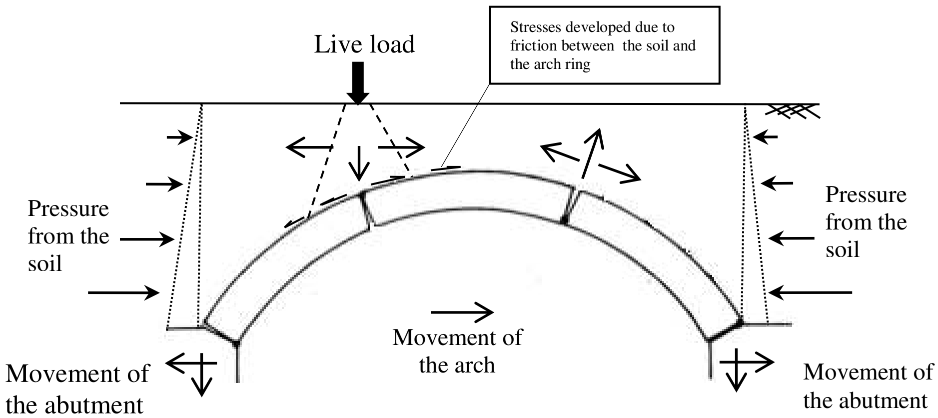

Figure 6. Sketch of the arch-fill interaction mechanism.

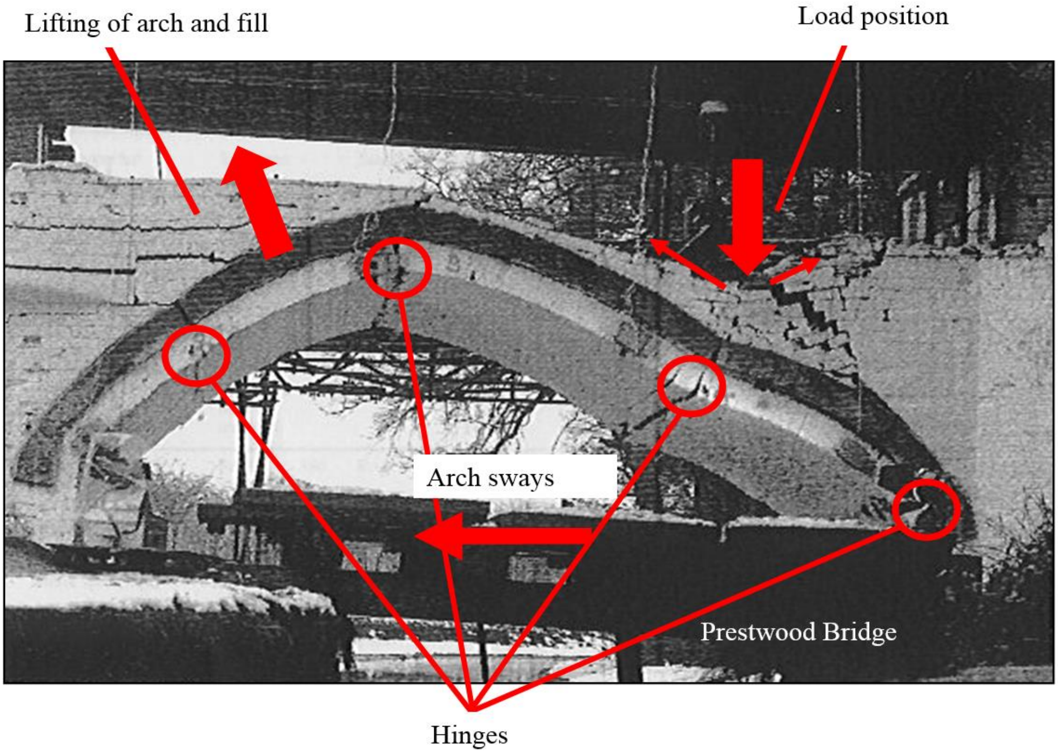

Figure 7. Field testing on Prestwood Bridge (Page, 1993).

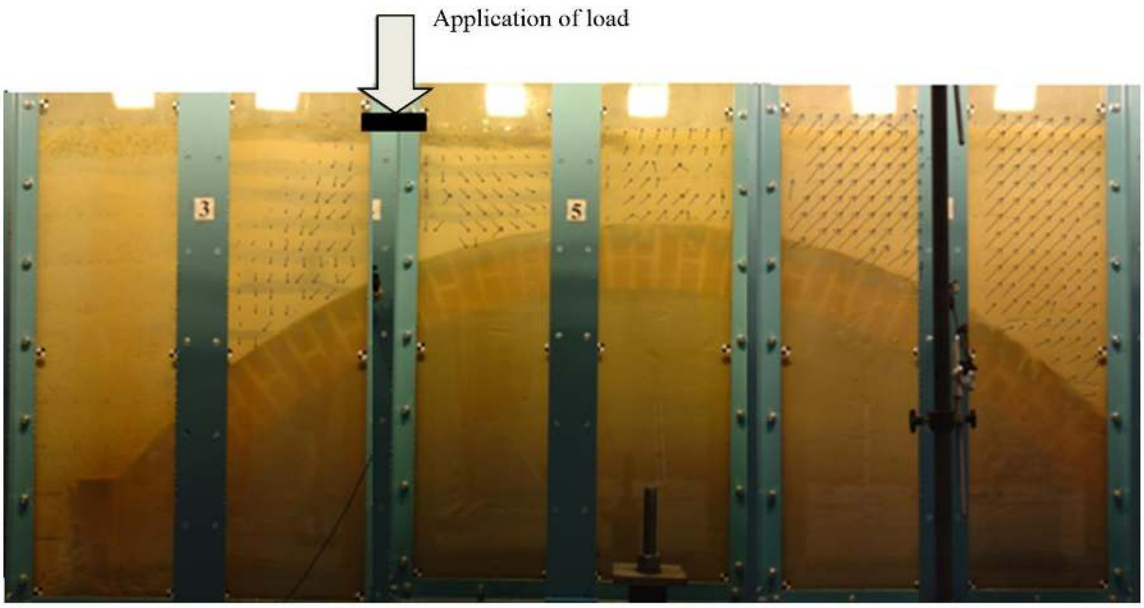

Figure 8. Large-scale laboratory testing on filled masonry arch bridges (Gilbert et al., 2006).

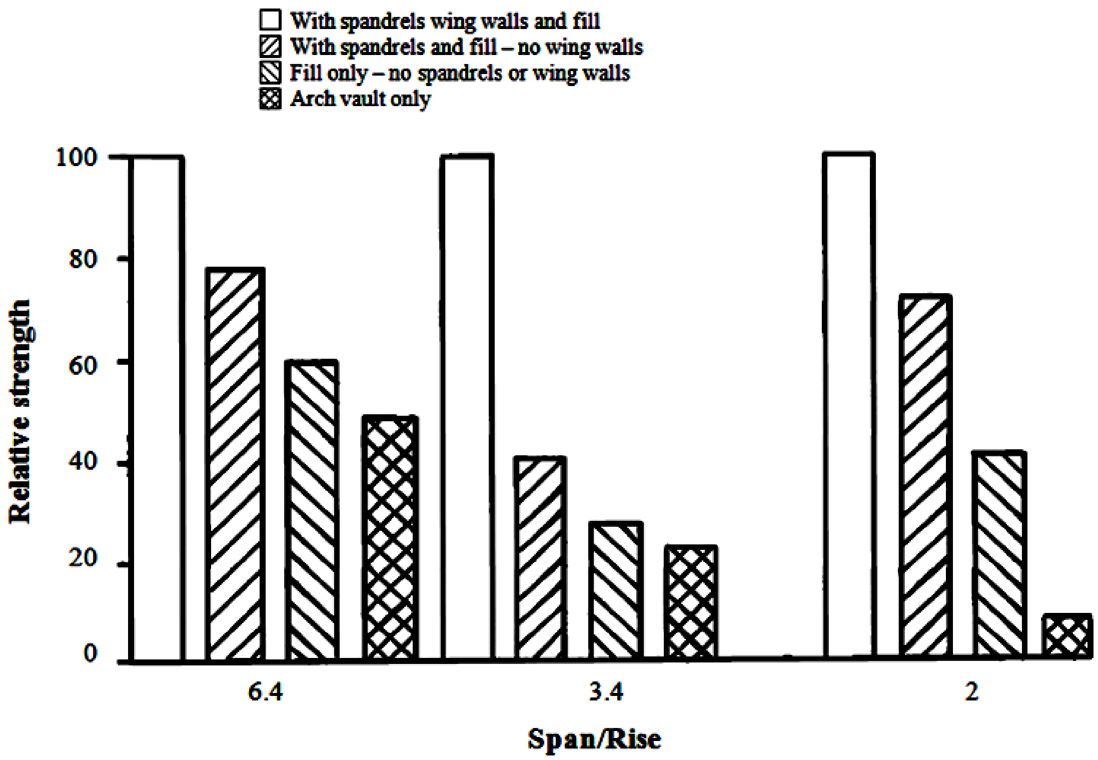

Figure 9. Effect of wing walls, spandrel walls and fill on the load-carrying capacity (Royles and Hendry, 1991).

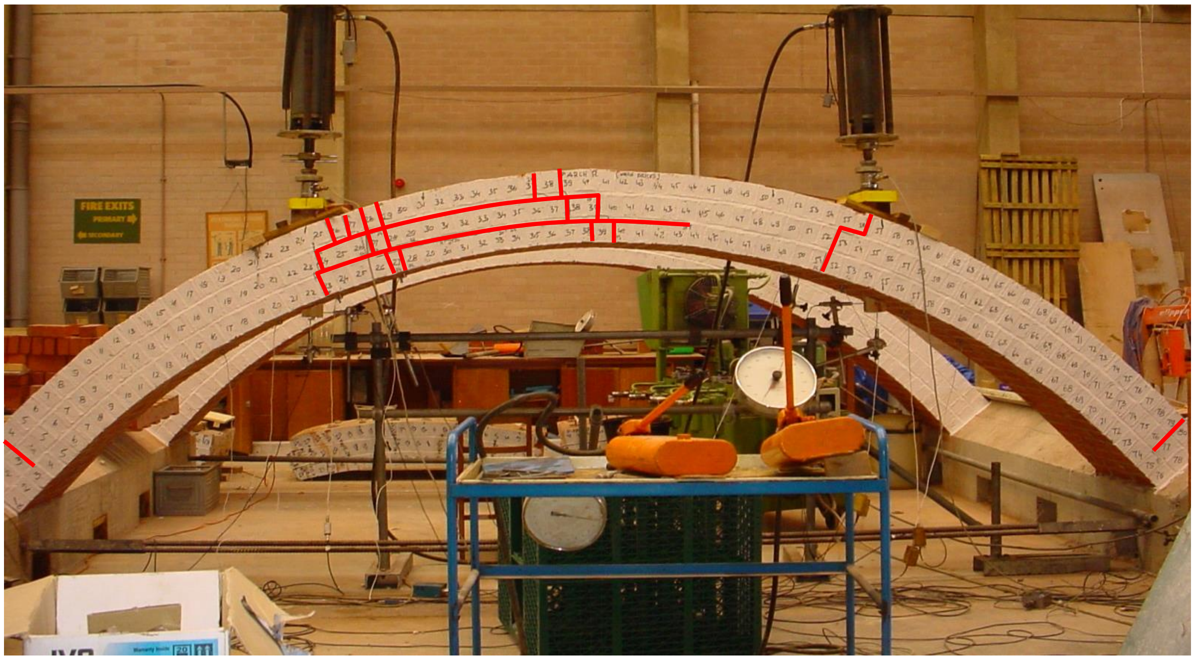

Figure 10. Failure mode of weal arch ring under static loading (Melbourne and Tomor, 2005).

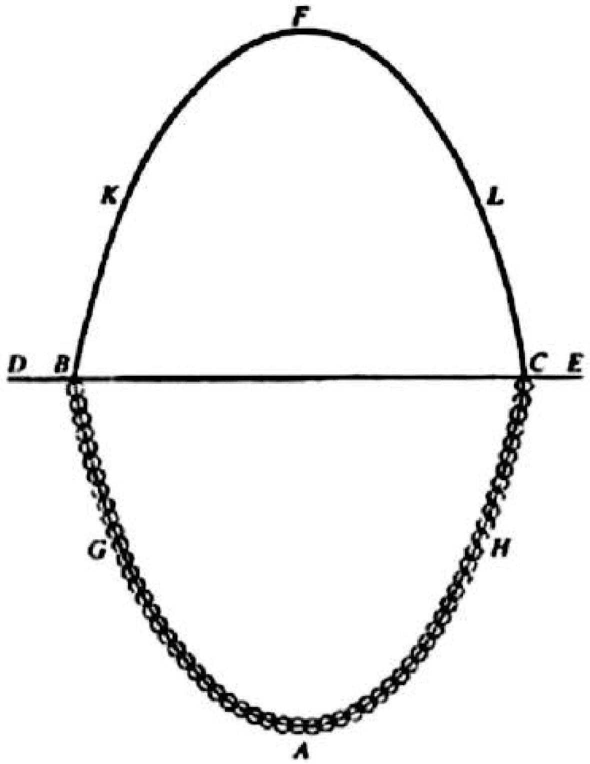

Figure 11. Hanging chain analogy: Poleni's drawing of Hooke's analogy between an arch and a hanging chain

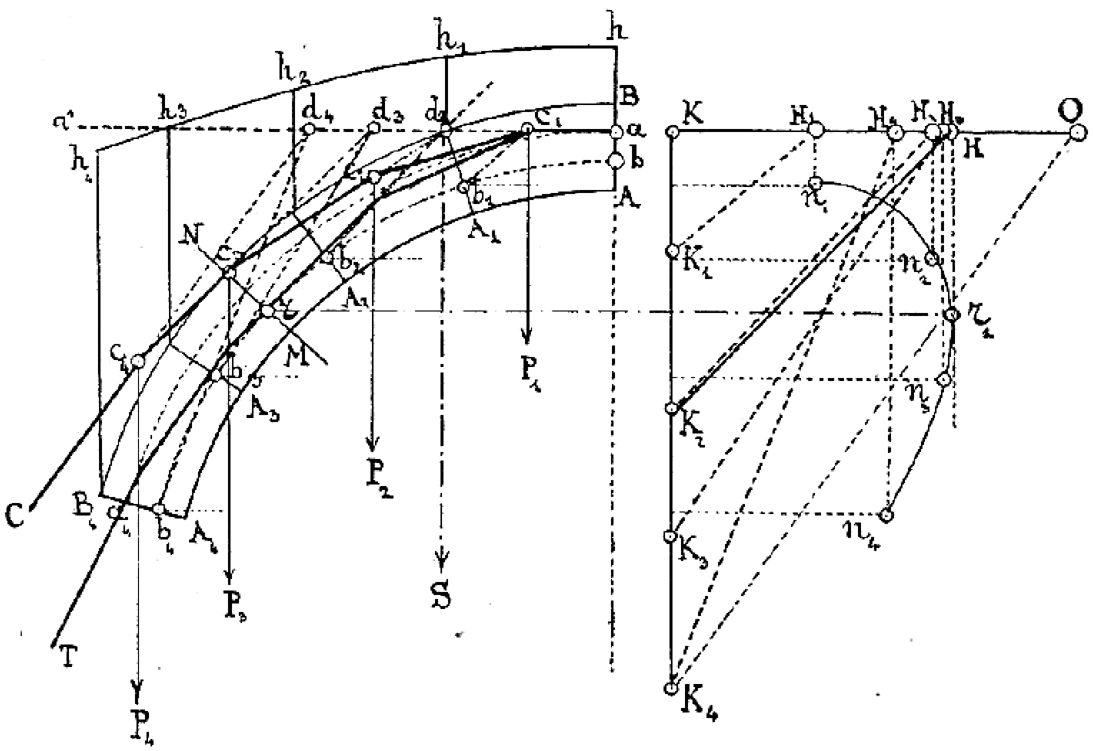

Fig. 230.

Figure 12. Mery's Method for the design of an arch (Jorini, 1918).

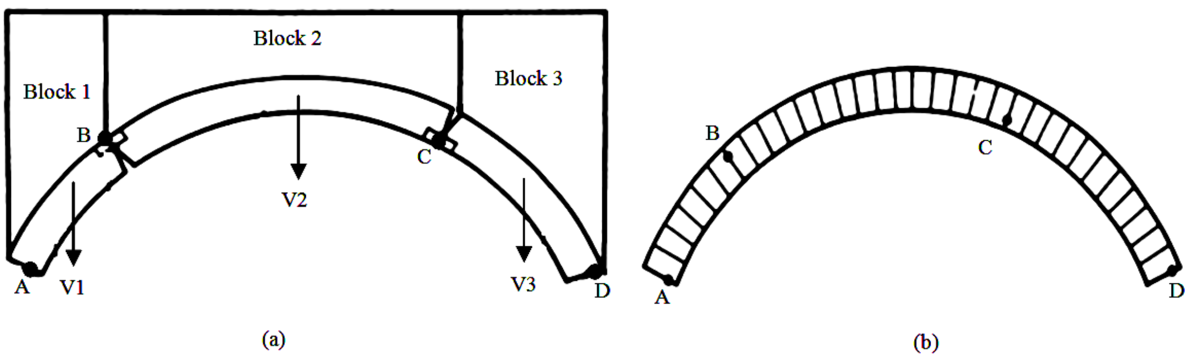

Figure 13. Principles of mechanism method: (a) mechanism with equilibrium forces; and (b) arch divided into elements (Crisfield and Packham, 1987).

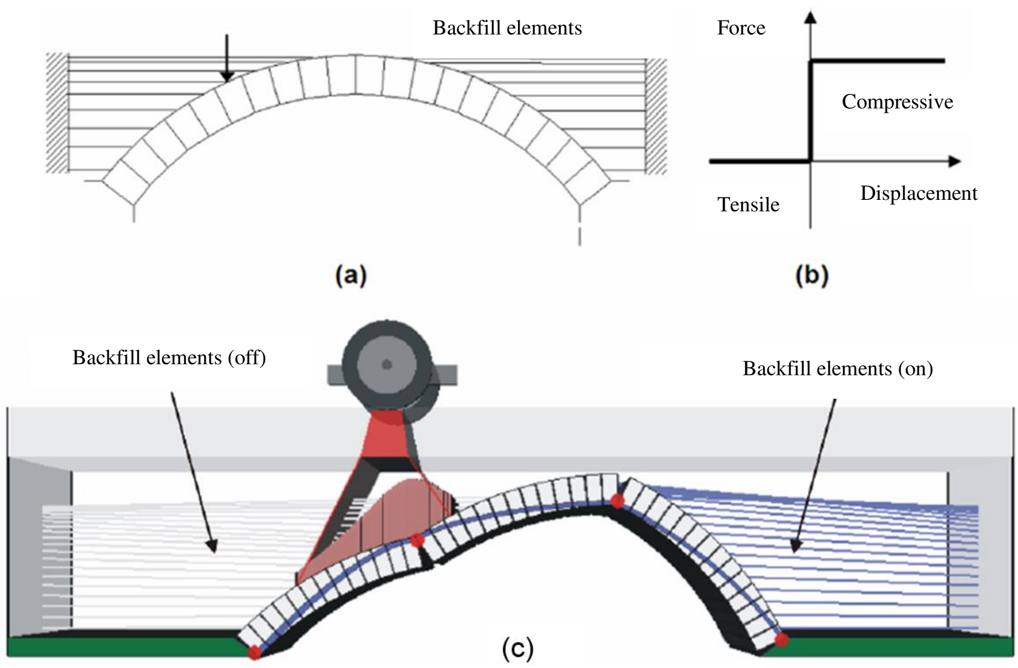

Figure 14. RING software with soil pressures included :(a) Arch restrained with uniaxial backfill elements; (b) backfill element response; (c) LimitState: RING representation of backfill elements (LimitState:RING Manual 2014).

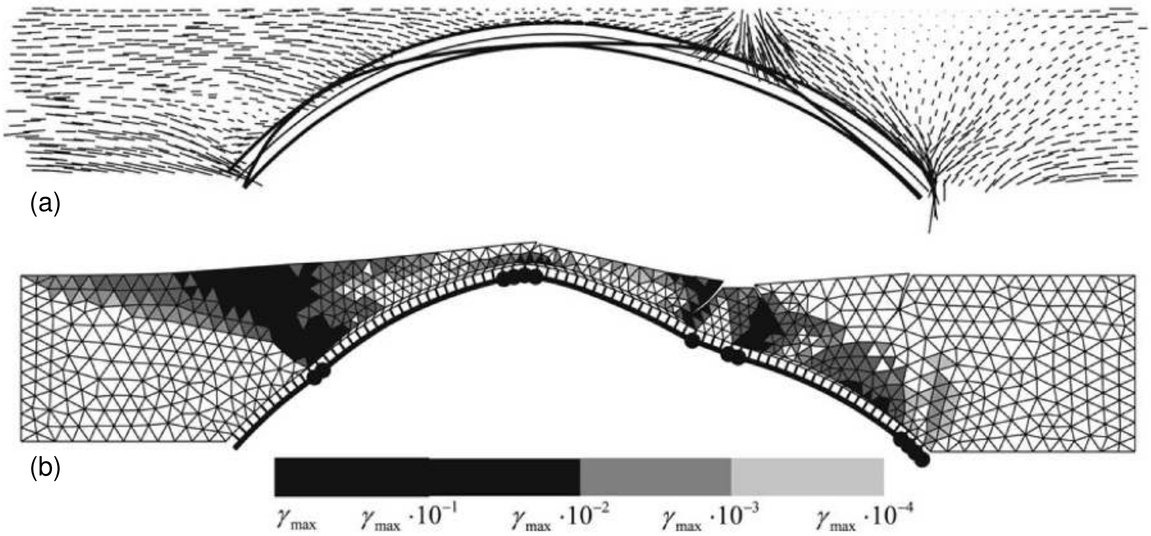

Figure 15. Modelling of masonry arch bridges with 2D finite elements proposed by Cavicchi and Gambarotta (2007) for the investigation of arch-fill interaction: principal stress field (a) and collapse mechanism and contour plot of the maximum shear strain rate (b).

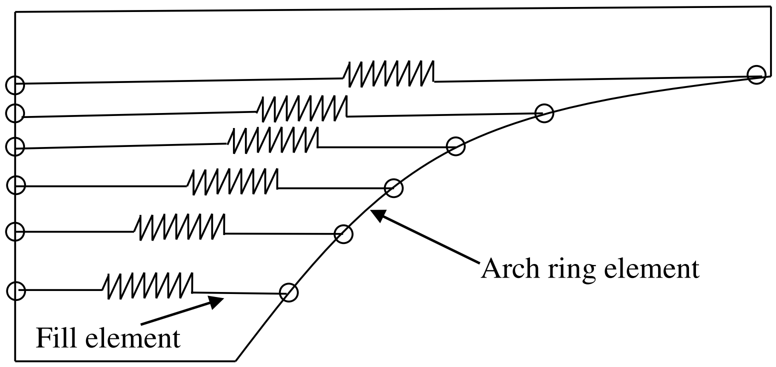

Figure 16. Modelling of masonry arch bridges with 1D finite elements proposed by Crisfield (1985).

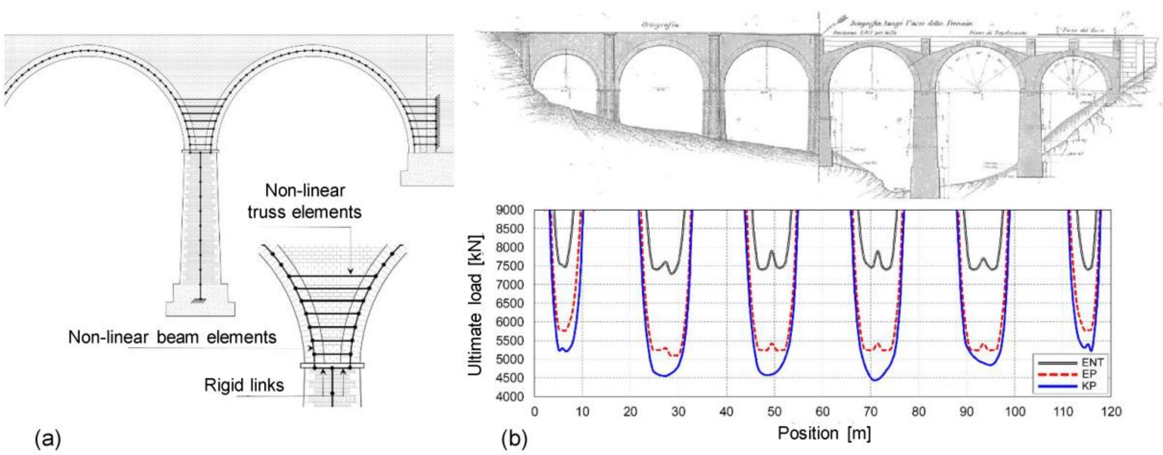

Figure 17. Modelling of masonry arch bridges with fibre beams (De Santis and de Felice, 2014a) (a) and evaluation of the load-carrying capacity of an historic railway viaduct (De Santis and de Felice, 2014b) (b).

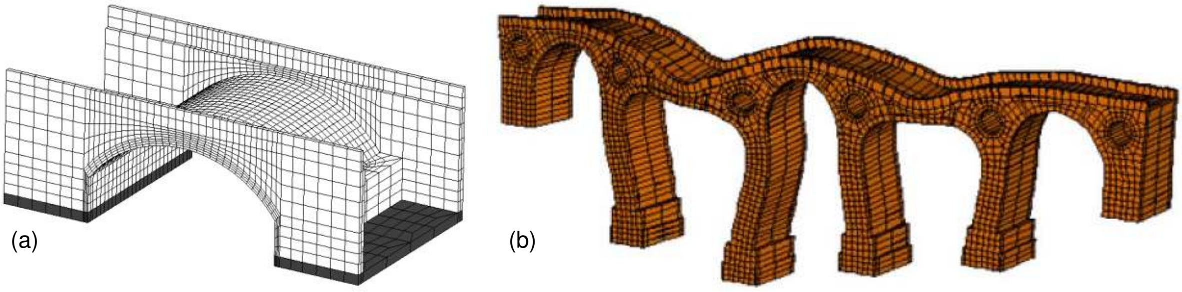

Figure 18. Modelling of masonry arch bridges with 3D finite elements proposed by Garrity and Toropova in 2001 (a) and by Milani and Lourenço in 2012 (b)

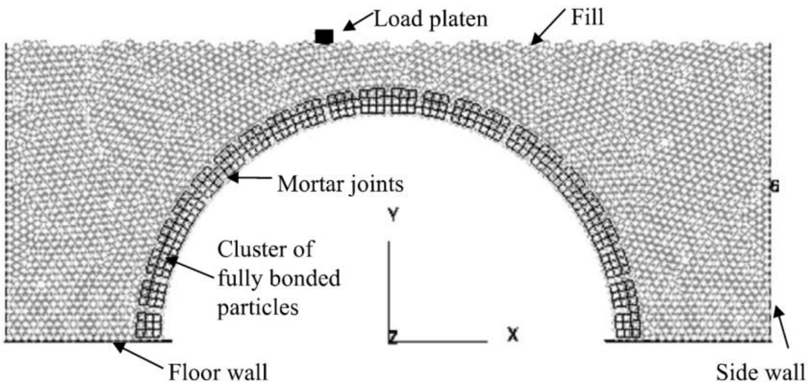

Figure 19. Model of a backfilled semi-circular masonry arch with PFC3D (Thavalingam et al., 2001).

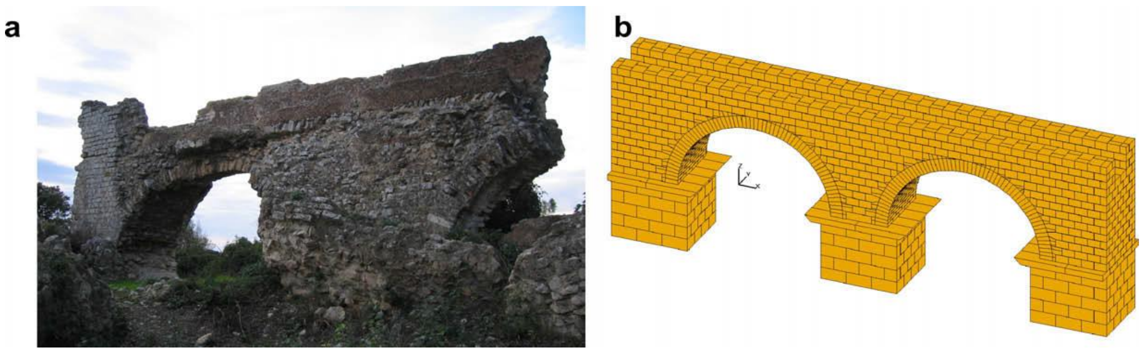

Figure 20. Application of the NSCD method to analyse the dynamic behaviour of Arles aqueduct (Rafiee, 2008).

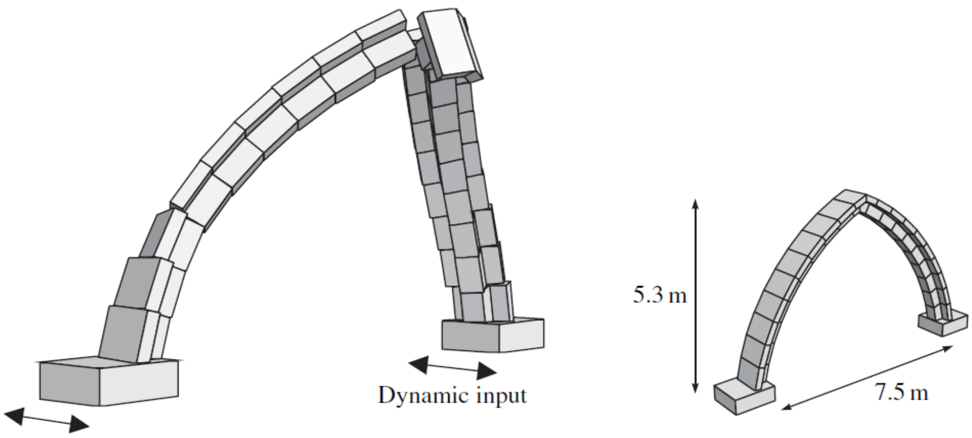

Figure 21. Collapse of pointed arch, modelled with a 3D rigid block discrete element model, under out-of-plane seismic action (Lemos, 1998).

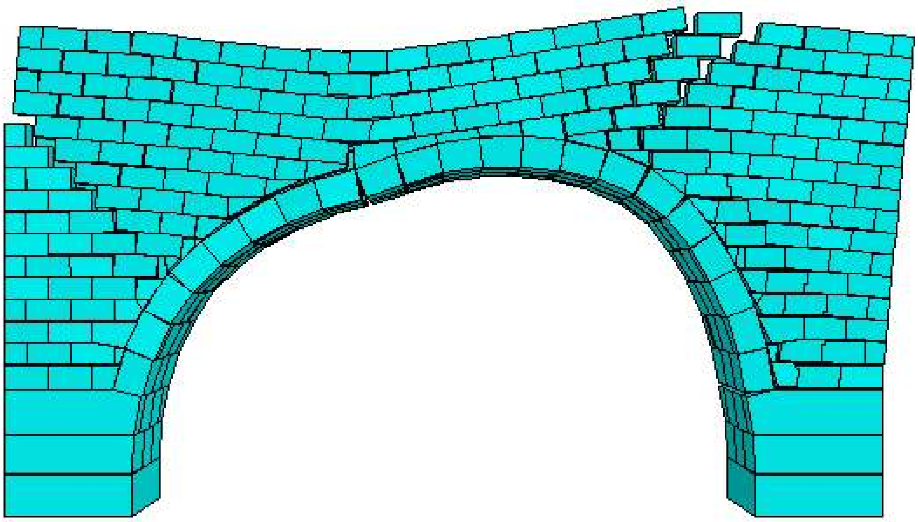

Figure 22. The deformed shape of a masonry arch bridge modelled using 3DEC (Courtency of J. V. Lemos).

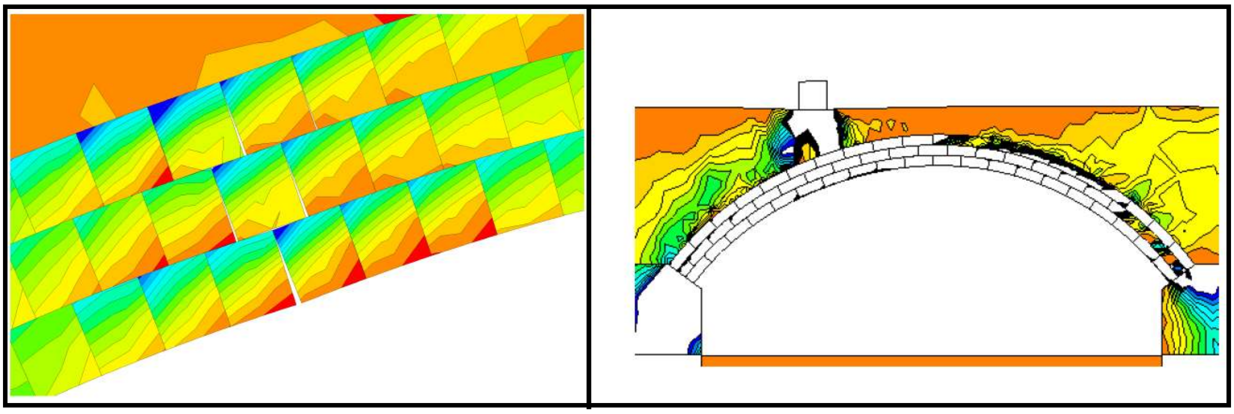

Figure 23. Finite/discrete element method applied to a masonry arch bridge (Mullett et al., 2006).

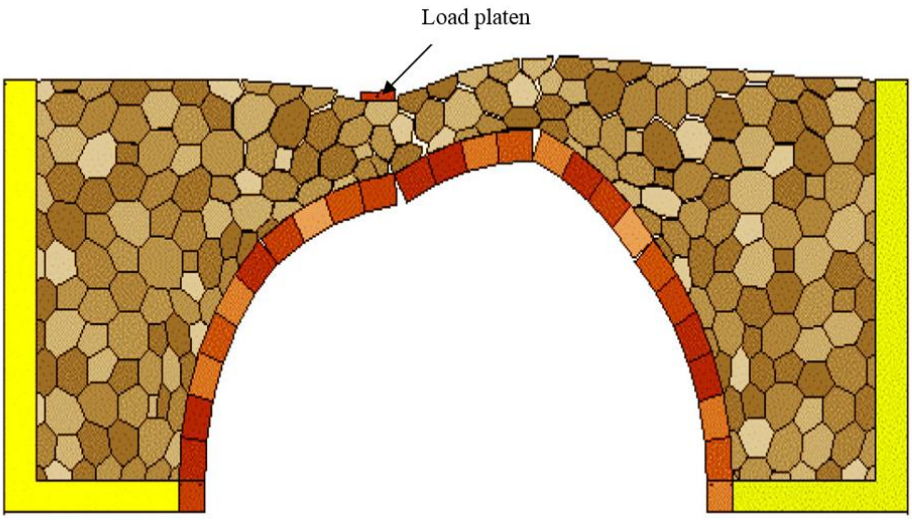

Figure 24. Deformed shape of an arch bridge using discontinuous deformation analysis with simplified deformable blocks (Bićanić et al., 2003).

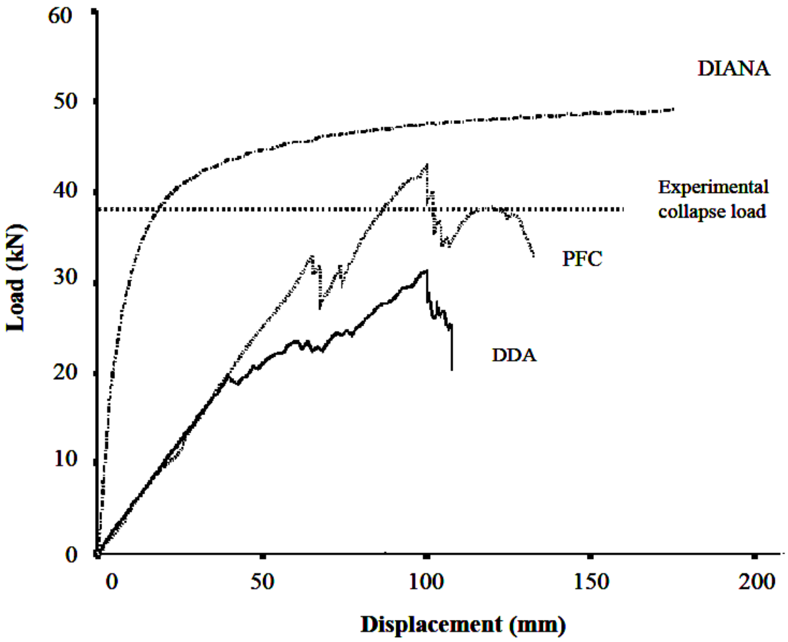

Figure 25. Comparison of experimental against numerical results (Thavalingam et al., 2001)

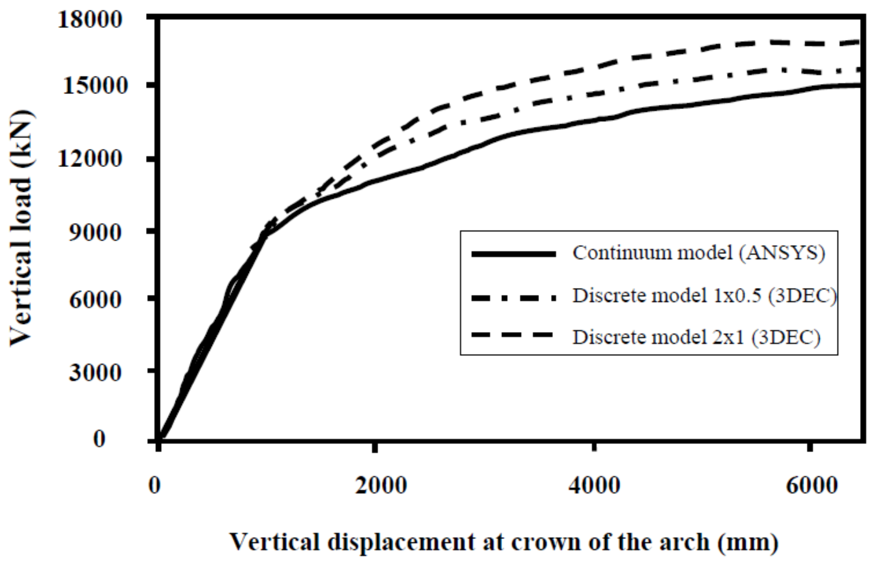

Figure 26 Load against displacement relationship (Schlegel and Rautenstrauch, 2004)

Table 1. Failure modes of test bridges (after Page 1993 & 1995).

Name	**Square span [m]**	**Depth of fil at crown [m]**	**Thickness of the cross section [m]**	**Rise at mid span [m]**	**Shape**	**Failure load [kN]**	**Failure mode**
Prestwood	6.55	0.16	0.22	1.43	Segmental	228	Formation of four hinges
Bolton model	6.00	0.30	0.22	1.00	Segmental	1170	Formation of four hinges
Shinafoot	6.16	0.21	0.39-0.77	1.18	Segmental	2500	Formation of four hinges, complicated by random brick
Torksey	4.90	0.27	0.34	1.15	Segmental	1080	Three pinned snap through
Bargower	10.36	1.20	0.56	5.18	Segmental	5600	Crushing failure below load
Preston	5.18	0.38	0.36	1.64	Elliptical	5600	Crushing failure below load
Strathmashie	9.42	0.41	0.60	2.99	Segmental	1325	Not well defined, material falling out of existing longitudinal crack
Barlae	9.87	0.30	0.45	1.69	Segmental	2900	Heavily skewed bridge. Three pinned snap through followed by shear failure in the spandrel

Table 2. Advantages and disadvantages of the MEXE method.

Advantages	Disadvantages
• Simple, quick and computationally inexpensive • Based on visual inspection • There is no other widely used approximate method available	• Significantly underestimates the load-carrying capacity (but not always) • Limited to certain structures and conditions (e.g., not available for skew arches; increasingly conservative when assessing the load-carrying capacity on bridges with a longer span than 12 m) • Cannot be used for serviceability check

Table 3. Advantages and disadvantages of limit state analysis based methods.

Advantages	Disadvantages
• Useful for initial assessment • Easy to use • Small number of parameters needed	• The actual load-carrying capacity may be overestimated due to the assumption of unlimited ductility • No information on stresses, strains and displacements are provided • The positions of the plastic hinges needs to be known a priori or derived by solving an optimization problem

Table 4. Comparison between continuous and discrete modelling for masonry arch bridges.

	Continuous modelling	Discrete modelling
Basic assumptions	• Masonry assumed as a homogenous isotropic or anisotropic material • Unit, mortar and unit-mortar interface are smeared out in the continuum over the entire masonry structure • User friendly mesh generation (depending of the specific software used)	• Masonry assumed as a composite of its individual components, i.e., brick and mortar • Units and mortar in the joints are represented by continuum elements whereas the unit mortar interface is represented by discontinuous elements • Approach suits for small size models. Because of the complexity of modelling the current computers cannot perform the analysis in the economical time ranges
Input parameters and requirements	• A relationship between average masonry strains and average masonry stresses is required • Reduced time and memory requirements. Used when compromise between accuracy and efficiency is needed • Number of needed parameters to characterize masonry is high. It needs comprehensive testing results of large masonry part which contains adequate unit and mortar combinations to determine the assembling property of masonry units and mortars under different loading conditions (i.e., compression/compression and	• The geometry of the model needs to be represented in detail (i.e., brick by brick) • A large number of parameters is required in order to characterize the materials. Individual properties of the brick, mortar and brick-mortar interface are required

	bending/shear, monotonic/cyclic)	• Large computational effort required
Field of application and limits	• Can be applied for the large scale models so that the stresses across or along a macro-length will be essentially uniform • Provide an understanding about the global behaviour of the structure • Useful for large multi-span bridges and viaducts for a preliminary assessment of the load-carrying capacity, the detection of multi-span failure modes (i.e., configurations at collapse that involve more than one span due to the interaction between adjacent spans), the assessment of the structural response to earthquakes and the estimate of the seismic capacity • Localized conditions such as cracks along the interface cannot be represented sufficiently nor realistically enough through a homogenization of entire structure • Some failure modes (such as de-bonding of bricks, shear sliding, ring separation) cannot be captured, due to simplicity of the modelling • Used for both research and design practice purposes.	• Used when there is need to localize the initiation of cracks and investigate crack propagation up to failure • Provides a deep understanding about the local behaviour of masonry structures • Used for exact localization of maximum tension zones in the materials, cracks along the joints or through the cross-section of the units • At present, mainly used for research work on masonry structures