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# A Review of Operations Research Applications in Workforce Planning and Potential Modelling of Military Training

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## **ABSTRACT**

This report presents the review of workforce planning applications of operations research and explores potential modelling of military training. We classify the operations research techniques applied in workforce planning into four major branches: Markov chain models, computer simulation models, optimisation models and supply chain management through System Dynamics. For each of these, we outline the underlying mathematical formalism and concepts, overview models published and discuss potential limitations. The prospect of modelling training forces via System Dynamics is demonstrated by a causal-loop analysis of the military officer system and a simulation model based on a stock-flow diagram for a one-rank officer training system.

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## Executive Summary

“To provide the *right* (required) number of the *right* (qualified) personnel at the *right* (specified) time at the minimum cost” is the goal for any workforce planning, or military training more specifically, system. Various operations research (OR) techniques have been applied in modelling of workforce or training systems. As the first step in the construction of models for military (army in particular) training, the report surveys the OR techniques applied in the field.

Decomposing the OR models in workforce planning captured in this report into four major categories – Markov chain models, computer simulation models, optimisation models and supply chain management through System Dynamics – we review, for each category, the underlying mathematical formalism and concepts, and highlight advantages and potential limitations. We summarise published models, especially those in military workforce planning such as training models for the US Army, to demonstrate what can be expected from OR models.

The potential value of modelling of training forces via System Dynamics is demonstrated by a causal-loop analysis of military training systems and a simulation model based on a stock-flow diagram for a one-rank officer training system. The ‘chain-training-demand’ and ‘worse-before-improvement’ effects due to the ‘hierarchy’ and ‘closedness’ features of military training systems are exposed qualitatively and illustrated, in part, numerically. In doing so, it highlights the further development of OR models in military training to help understanding, to facilitate analysis and to assist policy design, for a robust and adaptive training system.

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# Glossary

ADF	Australian Defence Force
ASCAR	Accession Supply Costing and Requirements
CAPT	Captain
CFSM	Combat Force Sustainment Model
CPL	Corporal
DP	dynamic programming
DSTO	Defence Science and Technology Organisation
FLTLT	Flight Lieutenant
GP	goal programming
IP	integer programming
LP	linear programming
LTCOL	Lieutenant Colonel
MANSIM	Workforce Simulation by Individual Movements
NO	normal unit
OR	operations research
SD	System Dynamics
SLAM	Simulation Language for Alternative Modelling
TC-A	Training Command - Army
TOPLINE	Total Officer Personnel Objective Structure for the Line Officer Force
TFSM	Training Force sustainment Model
UK	United Kingdom
US	United States (of America)
WO	Warrant Officer
WP	workforce planning

# 1. Introduction

This report reviews the applications of operations research (OR) in workforce planning (WP) to search for possible methodologies applicable in building a Training Force Sustainment Model (TFSM). TFSM is envisaged as a tool to help Training Command-Army (TC-A) in identifying the critical resource and planning issues to meet the training demand effectively and efficiently.

While the ultimate aim is to construct models for military training systems, the scope of this literature survey is extended to the field of modelling of WP. We believe that it is possible to transplant OR techniques applied in WP to serve our needs because the purpose of WP [1] is the same as that of any training system [2, 3], i.e., “to ensure that the right people are available at the right places and at the right times to execute corporate plans with the highest levels of quality[1]”, and a training system is, after all, a workforce system.

Effective WP is essential for all organisations because it ensures that an adequate supply of trained employees exist to meet both existing and new demands. For the Australian Defence Force (ADF), and the Army in particular, effective WP means “there will continue to be sufficient people with the required competencies to deliver the capability output required by the Government at affordable cost” [4]. The military workforces are characterised by their closed nature and strict hierarchy, i.e., only recruiting from the lowest rank and filling all vacancies at higher ranks internally by promotion. These characteristics make military WP distinct from WP in civilian organisations such as universities or research institutes where vacancies, even at senior ranks, are open for outside applicants by advertisements. Moreover, two kinds of dynamics in promotion of military WP practice should be recognised: a push flow and a pull flow. A pull flow refers to the WP policy in which recruitment and promotion occur only when vacancies are available, while in a push flow the constraint of vacancy availability does not apply. For example, officers in the Air Force are promoted on the basis of *Time Promotion*, which works as a push flow, for ranks up to and including Flight Lieutenant (FLT), and then they are promoted on the basis of *Selective Promotion*, which works as a pull system to fill vacancies in the constrained establishment [5]. Promotion for an Army officer “will only be approved when a suitable establishment vacancy exists at the next rank [6]” and therefore works only as a pull flow. Due to these unique features of military WP, the use of OR models to gain insight into the system and to facilitate the planning process is essential.

There is a rich variety of publications on OR applications in WP using various models, collected in books and published proceedings [7-13]. We group the captured models into four classes, i.e., models based on Markov chain, computer simulation, optimisation and System Dynamics (SD). In the sections below, when we review each class of models, we first introduce the mathematical framework underlying the model construction, then overview the published applications especially in military WP, and conclude each section



with discussion of potential limitations of the technique involved. Moreover, we have applied the causal-loop diagramming technique of SD to analyse typical features of military training systems and constructed a template of an SD simulation model for a one-rank officer training system, which we believe can be used as the starting point for SD modelling of military training.

## 2. Markov Chain Models of Workforce Systems

### 2.1 Background of Markov chain theory

Markov chain theory is one of the mathematical tools used to investigate dynamic behaviours of a system (e.g. workforce system, financial system, health service system) in a special type of discrete-time stochastic process in which the time evolution of the system is described by a set of random variables [14, 15]. It is worth mentioning that variables are called random if their values cannot be predicted with certainty and discrete-time means that the state of the system can be viewed only at discrete instant rather than at any time[14].

The type of discrete-time stochastic process applicable by Markov chain theory is called Markov process [15] which is defined as

$$P(X_{t+1} = x_{t+1} | X_0 = x, X_1 = x_1, \dots, X_t = x_t) = P(X_{t+1} = x_{t+1} | X_t = x_t), \quad (1)$$

where  $\{X_i = x_i, i = 0, 1, 2 \dots\}$  means that the random variables  $X_i$  (uppercase) have the value  $x_i$  (lowercase) at the time  $i$ , and  $P$  is the conditional probability distribution of the system. Equation (1) simply states that the conditional probability of the system in the state  $x_{t+1}$  at the moment, given by the left hand side of Equation (1), is independent of the states occupied before  $t$  [15]. In other words, the probability of which state a Markov process will be in at the next moment is determined by the current state only and is independent of evolution history of the system.

The right hand side of Equation (1),  $P(X_{t+1} = x_{t+1} | X_t = x_t)$ , is called a one-step transition probability which describes the chance a system can transit to state  $x_{t+1}$  at the next moment  $t + 1$  given  $x_t$ , the status of the system now at time  $t$ .

Denoting  $P_{ij}(t)$  as the transition probability of a system from state  $i$  to state  $j$ , the transition matrix is defined as

$$\mathbf{P}(t) = \begin{bmatrix} P_{11}(t) & P_{12}(t) & \dots & P_{1k}(t) \\ P_{21}(t) & P_{22}(t) & \dots & P_{2k}(t) \\ \dots & \dots & \dots & \dots \\ P_{k1}(t) & P_{k2}(t) & \dots & P_{kk}(t) \end{bmatrix}, \quad (2)$$

where  $k$  is the number of exclusive and exhaustive states of the system [15]. The matrix  $\mathbf{P}(t)$  is called stochastic for

$$0 \leq P_{ij}(t) \leq 1; \text{ and}$$

$$\sum_{j=1}^k P_{ij}(t) = 1, \quad i, j = 1, 2, \dots, k.$$

We note that some authors use the name “Markov chain” for those with time independent (or stationary) transition probability [14-16]. The chains with transition probability varying with time are named non-homogeneous Markov chains [17-19].

Define a row vector of state probability  $\mathbf{s}(t)$  [20]

$$\mathbf{s}(t) \equiv [s_1(t), s_2(t), \dots, s_k(t)],$$

where  $s_i(t)$  ( $i = 1, 2, \dots, k$ ) is the probability that the system will be in state  $i$  at time  $t$  (or after  $t$ -step transitions from the beginning  $t=0$ ) with  $\sum_{i=1}^k s_i(t) = 1$  ( $t=0, 1, 2, \dots$ ), then

$$\mathbf{s}(t+1) = \mathbf{s}(t) \mathbf{P}(t), \quad t=0, 1, 2, \dots \quad (3)$$

Note that we have used bold letters for vectors or matrices.

For a process with stationary  $P$ , the relation between the initial state probability vector  $\mathbf{s}(0)$  and  $\mathbf{s}(t)$ , the state probability vector  $t$ -step away, could be obtained recursively [20] from Equation (3) as

$$\mathbf{s}(t) = \mathbf{s}(0) \mathbf{P}^t, \quad t=0, 1, 2, \dots \quad (4)$$

Equation (4) simply states that the state distribution at arbitrary time  $t$  is the product of the initial state distribution and the  $t$ -th power of the *stationary* transition matrix. A Markov chain with stationary transition matrix is called homogeneous [21].

While it is straightforward to calculate the  $t$ -th power of a matrix with modern computing facilities, there is an elegant algorithm to calculate Equation (4) by the so-called  $z$ -transformation [20] which we present in Appendix A.

After collecting the essential ingredients of the Markov chain theory, we outline its application in workforce modelling in Section 2.2.

## 2.2 Workforce modelling by Markov chain theory

The population in a workforce system is divided into classes according to staff’s common characteristics and attributes (such as rank, trade, age, or experience). The classes should be exhaustive and mutually exclusive so that an individual must belong to one but only one class at any time [22]. It is assumed that there is no difference between members in the

same class as far as transitions between classes are concerned [22], i.e., staffs in the same class have the same transition probability.

Workforce systems could be described by the terminology: stocks and flows [16]. The stock  $n_i(t)$  is the expected number of people in class  $i$  at time  $t$ . The flow  $n_{ij}(t) = n_i(t)p_{ij}$  denotes the expected number of members moving from class  $i$  to class  $j$  in an interval of unit length of time from  $t$  to  $t + 1$  with  $p_{ij}$  being the transition probability that an individual in class  $i$  at the start of the time interval sitting in class  $j$  at the end [16].

The basic equation<sup>1</sup> for a  $k$ -class workforce system using Markov chain theory is [16]

$$\mathbf{n}(t) = \mathbf{n}(t-1) \{ \mathbf{P} + \mathbf{w}'\mathbf{r} \} + \Delta N(t) \mathbf{r}, \quad (5)$$

where  $\mathbf{n}(t) = [n_1(t), n_2(t), \dots, n_k(t)]$  is the row stock vector. The number of new positions created due to expansion of the organization is expressed as  $\Delta N(t) = N(t) - N(t-1)$ , with

$N(t) = \sum_{i=1}^k n_i(t)$  representing the total number of staff in the system. The row wastage vector  $\mathbf{w} = [w_1, w_2, \dots, w_k]$  and the row recruitment vector  $\mathbf{r} = [r_1, r_2, \dots, r_k]$  are composed of the probabilities of staff losses or gains, constrained by

$$\sum_{j=1}^k p_{ij} + w_i = 1, \quad (6)$$

and  $\sum_{j=1}^k r_j = 1$ , respectively. Denoting the transpose of a vector or a matrix by prime,  $\mathbf{w}'$  is a column vector and  $\mathbf{w}'\mathbf{r}$  is a matrix with the element  $(\mathbf{w}'\mathbf{r})_{ij} = w_i r_j$ . Equation (5) could be written in a more aggregated form as [16, 23-26]

$$\mathbf{n}(t) = \mathbf{n}(t-1) \mathbf{P} + R(t) \mathbf{r}, \quad (7)$$

where  $R(t) = \Delta N(t) + \mathbf{n}(t-1) \mathbf{w}'$  is total number of recruitment and  $R(t) \mathbf{r}$  is the vector of new-entrant distribution. The transition probabilities  $P_{ij}$  could be estimated from the historical data of stocks and flows using the method of maximum likelihood [16]:

$$\hat{P}_{ij} = \sum_t n_{ij}(t) / \sum_t n_i(t),$$

where  $n_{ij}(t)$  is the flow, i.e., the observed number of staff moving from class  $i$  to class  $j$  during the time interval of  $(t, t + 1)$ ,  $n_i(t)$  is the stock, i.e., the observed number of staff in class  $i$  at the beginning of the time period  $(t, t + 1)$ , and the summation is taken over the time period of available historical data [16]. The wastage and recruitment probabilities could be estimated in the same way as the estimation for transition probabilities [16].

---

<sup>1</sup> Following the reference [16], we do not display time-dependence in  $\mathbf{P}$ ,  $\mathbf{w}$ , and  $\mathbf{r}$  in Equation (5).

The Markov chain model expressed by Equation (5), or its special case  $\Delta N(t) = 0$  for a fixed size system, could be used to forecast workforce profiles of organisations, e.g., staff distributions in classes  $\mathbf{n}(t)$  for given inputs in initial conditions  $\mathbf{n}(0)$ , policies in internal transitions  $\mathbf{P}$ , recruitments  $\mathbf{r}$  and  $\Delta N(t)$ . On the other hand, the model can also be used to help design of policies in promotion and recruitment to maintain a given class proportion structure  $\{n_i(t)/N(t), i = 1, \dots, k\}$  for an organisation in expansion ( $\Delta N(t) > 0$ ) or contraction ( $\Delta N(t) < 0$ )[27].

As described briefly in Section 1, flows in workforce modelling are classified into pull flows and push flows. Push flows are those described in Markov models where there is no vacancy-availability constraint on promotion and recruitment, e.g., staff are automatically pushed (promoted) to higher ranks after a specified length of service or completing training [16, 23-26]. Markov models are suitable for forecasting the future behaviour of workforce systems in which personnel transitions are not specifically controlled [24]. Pull flows, which are described by Renewal models [16, 24], mean that promotion and hiring are directed at filling vacancies, i.e.,  $\{n_i(t), i = 1, \dots, k\}$  are fixed. While two kinds of dynamics coexist in the WP of the Air Force [5], the flow in the Army WP is of pull type according to management principles and guidelines[6]. The KENT model, initially built at the University of Kent and further developed by the UK Civil Service, can model mixtures of push and pull flows[28].

Markov chain theory is the basis for the construction of the CFSM (Combat Force Sustainment Model) model [29] of the Australian Army, TOPLINE (Total Officer Personnel Objective Structure for the Line Officer Force) models of US Air Force [30] and has been employed in the development of some educational models [31].

### 2.3 Potential limitations

We list some potential limitations in Markov chain modelling of workforce systems.

First, Markov models are classified as “descriptive” [24] or “exploratory” [25] or simply non-optimisation [30] models because of the lack of internal mathematical-programming techniques to optimise outcomes such as minimising operational costs or maximising productivity. The workforce models with optimisation algorithms implemented are called normative [24, 25] or optimisation models [30] which are discussed later.

Secondly, Markov models are linear in the sense that they do not incorporate the feedback mechanism (such as the influence of promotion possibility on wastage) [16] which might be important in practice. There are reported studies which demonstrate that non-linear models perform better than simple linear models [32, 33]. An alternative way to model workforce systems with feedback mechanisms is supply chain modelling via SD (see Section 5).

Finally, there is a requirement on sample size for statistical reasons [34]. The number of people in classes must be large otherwise the transition probability estimates will tend to be unstable [35]. Typically, classes with staff number fewer than 100 are not suitable for aggregate-based techniques [26] such as Markov chain. It is pointed out that the total size of 500 is about the lower limit for a KENT model application [34]. Qualitatively speaking, workforce systems sized in tens are not appropriate for Markov chain modelling [16]. Small workforce systems are better studied by simulation techniques, such as Monte Carlo, where individual transitions of each staff are simulated explicitly instead of working on aggregates. The MANSIM (Workforce Simulation by Individual Movements) model, also developed by the UK Civil Service, is a Monte Carlo simulation model for workforce systems of a maximum of 1000 members [36].

We discuss simulation models of workforce systems in Section 3.

## 3. Simulation Models of Workforce Systems

### 3.1 Background of simulation

Simulation is a technique based on mimicking the operations of real-world systems in a computer. Simulation is based on models that are composed of mathematical or logical relationships among constituent parts of the system at hand [14]. In contrast to so-called analytic models where answers to the questions of interest could be found by mathematical techniques such as algebra, calculus or probability theory, simulation models are those which generally cannot be solved analytically [37]. Simulation processes, resembling doing experiments on computers, answer “what would be the system behaviour if ...” types of questions by displaying likely system performance under different input parameters.

Simulation models are classified along three different dimensions [37]: simulations are called dynamic (or static) if they imitate system evolutions over time (or at a particular time point) [14, 37]; simulations are called stochastic or deterministic depending on whether the model variables and parameters are probabilistic or known with certainty [15]; simulations are called discrete or continuous subject to whether model variables can change only at discrete time points or continuously over time [14, 37]. Unlike analytical models, such as Markov chain models introduced in Section 2 and optimisation models outlined in Section 4, which can be represented by some fundamental equations, there are no *universal* mathematical or logical relationships to express simulation models since simulation by nature is system specific.

Having introduced simulation technique qualitatively, we now review the simulation models constructed in military WP.

## 3.2 Simulation models in military workforce planning

Designed as a long-range planning tool for a time context up to 20 years, a series of simulation models, consisting of models for pre-commissioning training, undergraduate pilot training, advanced pilot training and corresponding cost estimations, have been constructed for the US Air Force by the RAND pilot training study [38, 39]. Driven by pilot requirements in terms of aircraft system, year and type of pilot, the models simulate the flow of pilots through various training stages, output the training loads as numbers of trainees and annual graduates required, and calculate physical, personnel and financial resources of training [38, 39]. The simulation is deterministic with the input attrition-rate in training derived from historical or estimated data [38].

A simulation model has been built for the US Army to evaluate the effects of Title IV<sup>2</sup> on officers' career paths and implications of the new policy on the officer supply to meet the capability requirements [40]. The model, constructed by the simulation software SLAM (Simulation Language for Alternative Modelling) II [41], represents the career development of officers, from 2nd Lieutenant toward qualified Colonel<sup>3</sup> via various training and assignments, by a network flow diagram with nodes depicting decision points for promotions, commands and military schools [40]. The simulation identifies potential officer shortfall due to the new policy and examines possible remedial policies [40]. The simulation is stochastic with probabilistic distributions of officer continuation rates, assignment timing and duration, etc., derived from historical data [40].

## 3.3 Potential limitations

While simulations are powerful and flexible in studying systems that are too complex for analytical models, the following limitations are noted:

- Constructions of simulation models, like design of experiments, can be time-consuming and costly [15].
- For stochastic simulations, output data needs statistical analysis due to random variability [14]. Output data from simulation are quite often auto-correlated, which requires special statistical techniques to make inferences [14]. "There are still several output-analysis problems for which there is no completely accepted solution, and the methods available are often complicated to apply" [37].

Simulations are good at answering "what happens if ...?" type of questions but are not based on optimisation algorithms [15]. Simulation models, like Markov models in the last section, are descriptive in the sense that they do not provide advice on the "best" policies. Optimal policies are normally found by various mathematical programming techniques outlined in Section 4.

---

<sup>2</sup> The part of Title IV concerned is the requirement of a minimum three-years service in a "joint" duty assignment for US military officers prior to promotion to the rank of Brigadier General [40].

<sup>3</sup> A qualified Colonel is the one eligible for selection to Brigadier General [40].

## 4. Optimisation Models of Workforce Systems

### 4.1 Background of optimisation techniques

We briefly introduce the mathematical formalisms of optimisation techniques used in workforce models.

The main optimisation techniques used in workforce models are linear programming (LP), integer programming (IP), goal programming (GP) and dynamic programming (DP).

#### 4.1.1 Linear Programming

LP finds solutions to decision problems in which the objective function and the constraints are linear functions of decision variables. Mathematically, the LP problem is to find the set of  $n$  positive decision variables, as components of a vector  $\mathbf{x} = [x_1, \dots, x_n]$ , which optimises (maximise or minimise) the objective function  $Z = \mathbf{c}\mathbf{x}'$ , subject to  $m$  constraints  $\mathbf{A}\mathbf{x}' (=, \geq, \leq) \mathbf{b}'$ , where  $\mathbf{c} = [c_1, \dots, c_n]$  is the cost vector,  $\mathbf{b} = [b_1, \dots, b_m]$  is called the requirements vector and  $\mathbf{A}$  is a  $m \times n$  matrix with each element  $a_{ij}$  ( $i = 1, \dots, m, j = 1, \dots, n$ ) called the technological coefficient [42]. For example, in a study using LP for minimisation of the total cost of an industrial training organisation in recruitment, training, transfers and redundancy [43, 44], the decision variables (unknowns) could be number of staff employed, recruited and left. The components of the requirement vector  $\mathbf{b}$  are required workforce stocks.  $\mathbf{A}$  is the transition matrix and the components of cost vector  $\mathbf{c}$  are known costs per staff in hiring, training, redundancy, or transfer. LP problems could be solved by the *simplex* algorithm or its variants using computer packages [14, 15].

#### 4.1.2 Integer Programming

IP is the technique to optimise an objective function with constraints (whether linear or non-linear) when the decision variables are integers [15]. It is called “pure integer programming” if all decision variables are integers, “mixed integer programming” if some variables are integers, and “zero-one or binary programming” if all variables are binary [15]. Algorithms solving general IP problems exist and computer packages are available [15].

#### 4.1.3 Goal Programming

GP is largely a kind of variation of LP<sup>4</sup> to solve decision problems involving multiple objectives (such as targeted costs or recruitment quotas) which normally cannot be realised exactly at the same time [15]. GP problems are always expressed as minimisation problems. Mathematically, the task of any GP is to find positive decision variables

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<sup>4</sup> We limit our discussion to linear GP only in this report.

$\mathbf{x}=[x_1, \dots, x_n]$  and positive deviational variables  $\mathbf{d}_{+/-}=[d_1^{+/-}, \dots, d_m^{+/-}]$  with  $+$ ( $-$ ) representing overachievement (underachievement) of stated goals, which minimise the objective function  $Z=\mathbf{p}^+ \mathbf{d}'_+ + \mathbf{p}^- \mathbf{d}'_-$ , subject to goal constraints  $\mathbf{A}\mathbf{x}' + \mathbf{d}'_- - \mathbf{d}'_+ = \mathbf{b}$  where  $\mathbf{A}$  and  $\mathbf{b}$  have the same meaning as in the LP case (Section 4.1.1). The  $\mathbf{p}^{+/-}=[p_1^{+/-}, \dots, p_m^{+/-}]$  in the objective function are numbers, named as priority factors, assigned empirically by decision makers according to the priority of stated goals [15]. Note that the goals in a decision problem are stated as constraints and the non-zero solutions of  $\mathbf{d}_{+/-}$  mean that the goals are achieved with deviations. The GP problems could be solved with computer packages using a simplex-based algorithm [15] or with network codes based on network optimisation algorithms [45].

#### 4.1.4 Dynamic Programming

DP is a method to solve multistage decision problems in which a series of decisions need to be made at each stage to reach an optimal solution. More specifically, DP decomposes a large, cumbersome problem into a series of smaller, more tractable problems in single stages, makes decisions at each stage one at a time, according to the stage-specific optimisation objective and then composes the sequence of stage-decisions into an overall answer to the problem [14, 15]. The stage-optimal solutions are chained together by the recursive equations which could be solved by either backward (preferred in the literature) or forward recursions [15]. The backward (forward) recursion starts analysis with the last (first) stage and works backward (forward) to the first (last) stage of problem. The optimal solution at each stage is obtained by optimisation techniques such as LP, IP or classical optimisation by calculus, depending on the characteristic of the problem [15]. There are no general algorithms, like the simplex method for LP, to solve DP problems [15].

## 4.2 Optimisation models in military workforce planning

Based on the optimisation techniques outlined above, the following applications in workforce planning, especially in the military arena, are summarised from the literature.

An LP model built for the UK steel industry could be used to find optimal workforce policies in hiring and redeployment at a minimal cost [43, 44]. LP models are also built for a simplified two-rank workforce system aimed at the Royal Air Force of UK [46], and a four-rank pilot training-employment subsystem intended for the Canadian Forces [47]. The models [46, 47] are built to help workforce planning in the design of optimal policies in promotion, recruitment, redundancy, ..., via optimising objective functions, such as costs of recruiting, training, ..., under various constraints such as establishment requirements, limitations on promotion, .... In these LP models [46, 47], integer requirement on some variables, such as number of promotion, is relaxed since IP calculations are more computationally involved.

In a study to develop a cost-effective training strategy, "The Army Training Mix Model" [48] is constructed to find optimal training modes, i.e., mixing of various training devices,



simulators and simulations and conventional training methods, to minimise training cost under the constraint of proficiency requirement. The underlying mathematical technique is mixed IP since some decision variables (such as the numbers of repetitions of various training exercises per soldier) are strictly treated as integers [48]. A scenario is given for design of a battalion training strategy using the MIP model and it also justified that the LP relaxation, i.e., discarding the integer restrictions on decision variables, produces good solutions [48].

While LP or IP is for a decision problem with only one objective, GP is the tool for decision problems with multiple objectives, which are often the cases in reality. A model based on the combination of the Markov process and GP has been developed for the US Navy [49]. The model provides optimal policies in controlling grade sizes under a given budgetary constraint [49]. In another model for a pilot training-employment subsystem of the Canadian Forces [47], four objectives of LP models were combined into a single GP formalism, which, while no prototype example is given, is intended to find policies in recruitment, promotion, post assignment or mission assignment for goals in minimisation of cost, maximisation of decision-maker quantified value judgments in selective promotion benefits, morale benefits and mission effectiveness [47]. A different GP model was also developed for officers, distributed among eight ranks, within the Canadian Forces to find optimal promotion policies under 622 constraints [50]. The US Office of the Assistant Secretary of Defence has used a GP model, named "The Accession Supply Costing and Requirements (ASCAR) model" [51], extensively. The ASCAR model has about 400 constraints and 500-600 variables, and is developed for the evaluation of the annual supplies of recruits in order to attain personnel levels and strength requirements for the All Volunteer Armed Forces [51]. GP is also employed to formulate the optimisation part of a non-homogeneous Markov chain model of hierarchical workforce system [19].

As a framework of step-by-step optimisation using information from previous steps [15], DP is also used in workforce planning especially for policy design involving a sequence of decisions in time. A DP model for scheduling of personnel training of Canadian Forces was formed to determine, for different time period of planning time-horizon, optimal numbers of starting trainees with a minimum cost of operation under the constraint of required service level [52]. Again based on DP, a model was built for the US Army's basic combat training program [53]. The US Army model is aimed at replacing the manual trial-and-error process in training schedules and resource planning optimisation, such as optimal numbers of trainees assigned and training time load allocated to training companies [53]. The model formulates its objective function using a performance measure of "training quality" based on instructor-to-student ratio [53].

### **4.3 Potential limitations**

An LP or IP model is structured to solve decision problems with single purposes in a sense that decision variables are determined by optimising one objective function while real-world workforce planning has, more often than not, several desired objectives [24, 25].

GP is well suited for decisions with multiple goals but objective functions for GP problems always involve the weights, or empirical ordinal priority factors. These weights, possibly numbering in the thousands [54, 55], have to be subjectively assigned by decision makers, which is non-trivial and possibly not feasible [15]. Moreover, final solutions can be very sensitive to the assigned weights [55].

While DP is powerful in dealing with processes involving a sequence of decisions and is flexible for applications in various optimisation problems, it needs high skills in practice to translate a decision problem into a mathematical DP formalism [15].

All the techniques introduced so far share a common drawback – they “are all essentially linear in character and are not able to capture the dynamic nature of important processes in the real world [56]”. A technique, called Systems Dynamics (or Business Dynamics), capable of addressing dynamics and non-linearity of complex systems, is introduced in Section 5.

## 5. System Dynamics Models of Workforce Systems

### 5.1 Background of System Dynamics modelling of supply chains

A supply chain is simply defined as a “set of structures and processes an organisation uses to deliver an output to a customer” [57]. Products in a supply chain flow through various stages from the beginning as acquired raw materials, to processing stages experiencing transformations, then to the final stage in the form of customer-required products. If trained personnel are considered as the product in a workforce supply chain, it is conceivable that techniques used in modelling and analysing supply-chain management are readily applicable in workforce planning, especially in training.

While various techniques, such as inventory models of classic OR methods [58] and simulation techniques using complex adaptive system theory [59], have been employed for supply-chain modelling and analysis, we are interested here in the application of SD in this area. The intrinsic strength of SD is in its holistic approach in investigating complex dynamic behaviours of systems.

SD, first born with the name “Industrial Dynamics” [60], originated from the theory of non-linear dynamics and feedback control of mathematics, physics and engineering [57]. It allows investigation of the dynamic behaviour of supply-chain systems (actually any managed systems) by analysing structures and interactions of feedback loops<sup>5</sup>, and time delays between actions and effects [62]. The dynamic behaviours of supply chains include: demand amplification [57, 63] of ordering and production; oscillations in production and inventories; and phase lags between the start of intervention and its effect [57]. We note

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<sup>5</sup> A feedback loop is a closed sequence of causes and effects [61].

that demand amplification means the enlarged demand at the product–provider end of a supply chain due to minor increased demand at the customer end.

Two main diagramming tools in SD practice are causal loop diagrams and stock and flow diagrams [57].

Causal loop diagrams are used to qualitatively capture the structures and interactions of feedback loops. A causal-loop diagram consists of cause and effects variables (letters) and causal links (arrows). A causal link connects a cause variable near the tail of the arrow to an effect variable near the head of the arrow. Each causal link is assigned a sign either positive (+) or negative (-), called link polarity. A positive link from one variable X to another variable Y means that either X adds to Y, or a change in X results in a change in Y in the same direction [64]. Similarly, a negative link from X to Y means that either X subtracts from Y or a change in X results in change in Y in the opposite direction [64]. In addition to each link polarity, a complete loop, i.e., a closed path of action and information, can also be given a sign, which is determined by the signs of all links that make up the loop. More specifically, a loop is called positive (or reinforcing) if it contains an even number of negative causal links; a loop is called negative (or balancing) if it contains an odd number of negative causal links [57].

Stock and flow diagrams are the basis for building SD simulation models for quantitative analysis of system dynamic behaviours. Two ingredients in this type of diagram are stock and flow variables [57].

Stock variables (also called state variables or levels) describe the states of the system, such as the number of trainees being trained, while flow variables (also called rate variables) depict the rates of change of stocks, such as the recruitment or graduation rates. Stocks are accumulations of their flows and mathematically are calculated as the integration of net inflows, i.e.,

$$Stock(t) = \int_{t_0}^t [Inflow(s) - Outflow(s)]ds + Stock(t_0)$$

with  $Inflow(s)$  and  $Outflow(s)$  denoting the values of the inflow and outflow at any time  $s$  between the initial time  $t_0$  and the present time  $t$  [57]. Conversely, the net flow determines the rate of change of any stock, i.e., its time derivative, by the differential equation [57]:

$$d(Stock)/dt = Inflow(t) - Outflow(t).$$

It is noted that “the stock and flow notation provides a general way to graphically characterize *any* business process” [61].

We note, however, that there is a third kind of graphical notation, dubbed the influence diagram, which is the combination of the first two with variant symbolic conventions [65, 66].

After reviewing the published SD applications in workforce planning in Section 5.2, we illustrate the possible SD application in modelling of military training (Section 5.3) by the following two exercises: first, we analyse the peculiar features of Army's officer training system by causal loop diagrams; second, we build a template simulation model for a one-rank officer training system using stock-flow diagrams.

## 5.2 System Dynamics models in military workforce planning

While there has been much recent interest in SD modelling of supply chains [62], not many published works on workforce supply chains are captured in this study. There are highly simplified, education-oriented workforce-supply models in SD textbooks [57, 65, 66], SD applications in Naval human resource planning [67], the Army Black Hawk Model [68], and one reported application in US Army's workforce management [69].

As an augmentation of the US Army's decision support models of the Enlisted Personnel Management System, an SD simulation model is constructed to forecast responses of the enlisted-force personnel system to changes in recruiting, training, retention, and promotion policies [69]. The model examines the feasibility of a 26% reduction of combat-ready force personnel in a period of three years for given constraints in recruiting policy, initial training spaces in basic and advanced individual training schools, and loss rates [69]. The simulation shows that [69]:

- the aimed-strength of combat-ready force cannot be reached in the required three year period; and
- there are oscillations around the desired strength, fluctuations in recruiting, accessions and number of trainee soldiers.

Several possible enhancements and refinements of the model are suggested [69].

Aimed at better understanding the Army aviation and employment for Black Hawk pilots [68], SD is used to build the Army Black Hawk Pilot Model (by contractors) "to predict the number of pilots at each stage of the pilot training and employment system, and the training resource requirements for aircraft flying hours and flying instructors" [68]. The model is also used as part of Naval Workforce Planning of Maritime Platform Division of DSTO [67].

As a demonstration of an SD application in workforce management, a textbook model of a consultancy firm is built to help the planning of trainee recruitment [65]. This model is aimed at recruitment policy analysis in order to have the right number of qualified consultants for consultation projects. The model assumes given constraints in training time, the tutor to trainee ratio, and average service time of consultants, etc. The results are: (1) there is an oscillation in supply of qualified consultants in response to oscillatory demand from the business cycles; and (2) there is phase lag between the demand and supply curves. Some new policies, testable using the model, are recommended.

### 5.3 Possible applications of System Dynamics in modelling of training

#### 5.3.1 A causal loop model of the Army's officer training

It is well known that the military personnel system is “a closed system in that a trained, experienced workforce is usually produced within the system rather than recruited from outside, constituting what is essentially a closed labour force. Personnel enter the forces at the most junior grades in the officer and enlisted ranks” [30]. The closed nature of military personnel (training in particular) systems and possible oscillatory response of these systems to an unexpected increase in qualified personnel are also analysed qualitatively [2, 3].

Besides the closedness as mentioned above, it is noted here that the training system of the Army (Navy and Air Force as well) is a strictly hierarchical system, i.e., higher rank trainee soldiers (from Corporal (CPL) up to Warrant Officer (WO)), and higher rank trainee officers (from Captain (CAPT) up to Lieutenant Colonel (LTCOL)) must be from personnel (the next rank below) serving in normal units (NU). Because of this closedness and the hierarchical nature, the training force is highly convoluted with NU. The change in training demand at one rank creates, in theory, chain training-demand in other ranks. This convolution between training force and NU requires that TFSM must be constructed, at least in principle, in a ‘whole-of-army’ approach by including NU as part of the model. We illustrate the above points by a causal loop diagram (Figure 1).

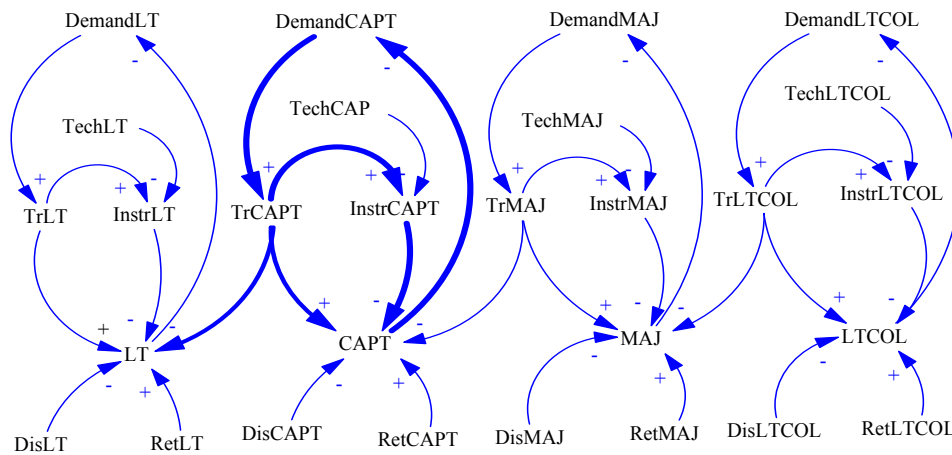


Figure 1 The causal-loop diagram for Army's officer training

Figure 1 represents the relationships among trainee officers, instructors and officers serving in NU at four different ranks. The variables included at the current level of aggregation are: demand for officers (named DemandRank), trainee officers (TrRank), instructors (InstrRank), officers serving in NU (Rank), technology insertion in training (TechRank), discharge rate (DisRank) and retention rate (RetRank). Notice that we have assumed instructors are from one rank above the rank of trainees, e.g., InstrCAPT are Instructors (at the rank of Captain) for trainee Captains (at the Rank of Lieutenant). We

explain the diagram using the CAPT rank as an example, where thicker links are used for guiding the eyes only.

A shortfall in CAPT officers increases the demand for CAPT that results in an increase in the number of trainee CAPT; more trainee CAPT creates, after some time delay in training, more CAPT officers which decrease the demand for CAPT. This is the balancing (or goal-seeking) feedback loop because this loop helps us reach our goal in training, i.e., demand satisfied. But an increase in trainee CAPT asks for more instructors, who we assume must be from CAPT officers serving in NU due to the closedness of military training. The demand for more instructors reduces the number of CAPT serving in NU, which further increases the demand for more CAPT. This loop is reinforcing (reinforcing demand with even more demand) and vicious because it worsens the shortfall in CAPT rank. We note that this reinforcing loop is caused by the closedness of military training system, i.e., instructors are usually from within the system.

Moreover, we see more trainee CAPT will deplete the assets of LT officers serving in NU, which creates training demand for LT. Therefore training demand in one rank causes chain training demands in other lower ranks because of the hierarchical nature of military training system. Of course the chain demands do not necessarily occur in reality as long as there are officer inventories (e.g., officers in desk duties) available for lateral transfer to provide required instructors without creating further shortfalls in NU.

Besides the workforce part of the training system, we also included other possible influences such as discharge rate, and novel training-technology insertions, e.g., employment of simulators or on-line education, which may relieve the demand for instructors.

In summary, the causal loop diagram in Figure 1 exposes two features of the military officer training system and two consequent potential effects:

- Closedness, which has a 'worse-before-improvement' effect on the training system, i.e., be prepared to have fewer service officers first before more qualified officers are available for service (e.g., more Captains? Be ready for fewer service Captains first, since the need for more instructors reduces the number of service Captains).
- Hierarchy, which has a chain effect on the training system, i.e., increased training demand in one rank will increase training demand in other ranks below (e.g., more Captains? Be prepared to train more Lieutenants since the increased number of trainee Captains reduces the number of Lieutenants in service).

We now turn to stock-flow diagrams to build simulation models for quantitative analysis.

### 5.3.2 A template simulation model for a one-rank officer training system

We build a highly simplified one-rank officer training model to see what one can expect from SD simulations. The model is based on the generic structure of stock management of supply chain and labour supply chain [57, 66]. Figure 2 presents the stock-flow diagram constructed by the demonstration version of SD software POWERSIM constructor.

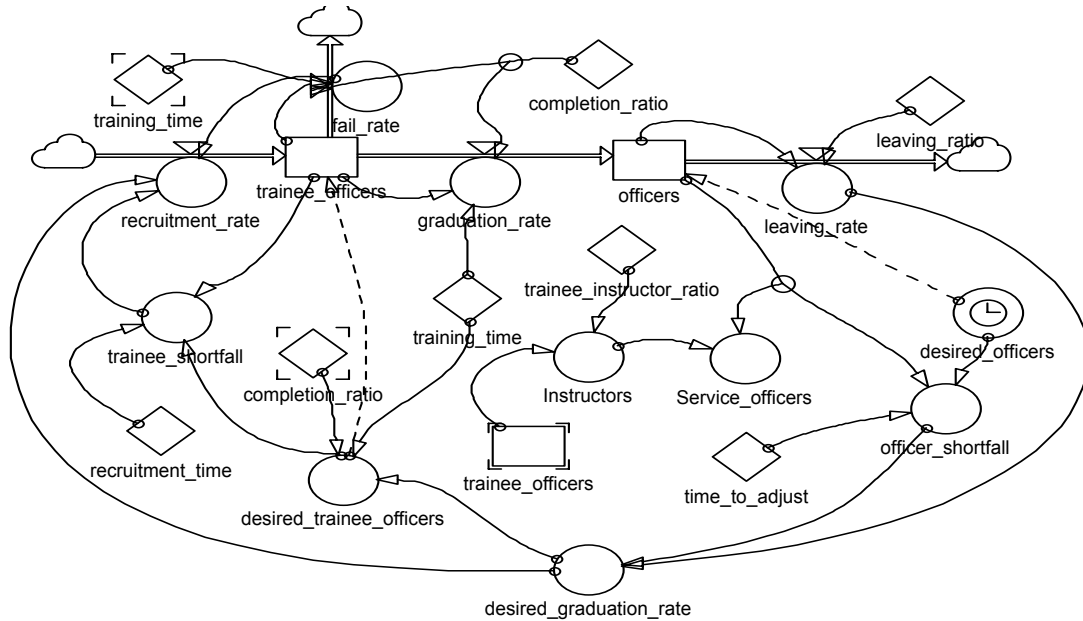


Figure 2 The stock-flow diagram for one-rank officer training. Squares: stocks. Pipe-like arrows : flows. Circles: auxiliary variables. Rhombuses: constants. Links represents influences.

Figure 2 depicts the following *fictitious* training system: Officers are recruited as trainee officers who have to be trained for two years to become qualified officers. There is one instructor for every five trainees and 90% of trainees can complete training successfully. Qualified officers leave the service at a fixed leaving ratio of 2% for various reasons, such as promotion, retirement or natural attrition. The recruitment policy is designed to replace officers who left and maintain the stocks of trainee officers and qualified officers at desired levels. Note that the system works as a pull system since the number of recruits is constrained by the vacancies. It takes one year on average to recruit trainees and five years to adjust the officer shortfall. It is assumed that currently there are a total of 1000 qualified officers either in service or as instructors, which is the desired number. The system is assumed in equilibrium initially, i.e., there is always the right number of trainee officers to replenish the qualified officers to maintain it at the desired level. The question is how the system will respond to an unforeseen 50% step increase in officer demand in year five from now. With the fictitious data assumed, the SD simulation forecasts the system response in the next 30 years as shown in Figures 3 - 5.

Figure 3 shows that the system responds to the step increase in officer demand by first increasing the quota of trainees (desired trainee officers) and then (after a time delay in recruitment) bringing in more trainee officers. The increased training load, as shown by the dashed line, requires more instructors. Figure 4 displays the variation of the number of instructors.

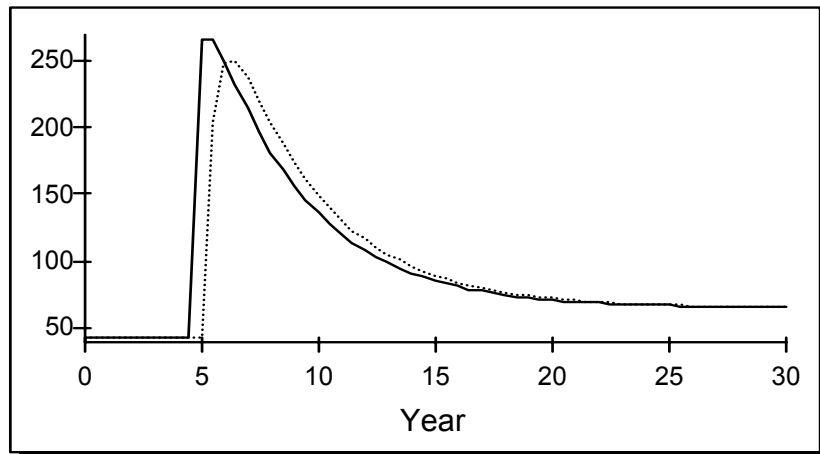


Figure 3 The change in number of trainees in response to a step increase in officer demand. Solid line: desired number of trainees. Dashed line: actual number of trainees.

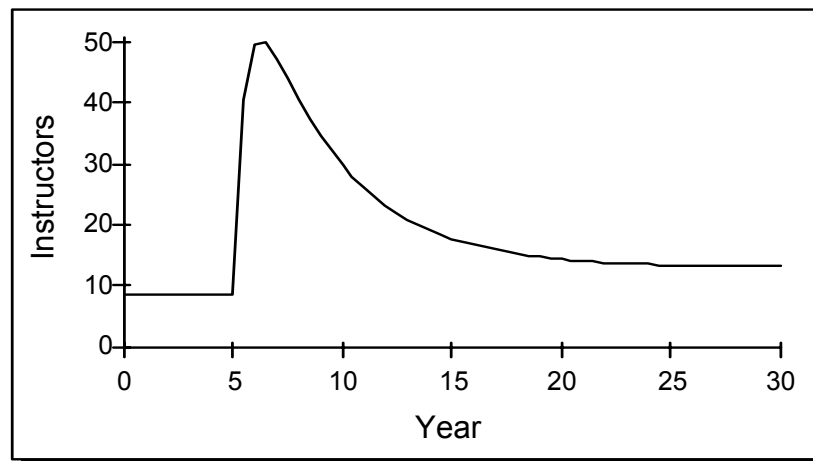


Figure 4 Variation of number of instructors in response to the step increase in officer demand.

The curve in Figure 4 describes the change in the number of instructors required to train the increased number of trainees. There is a sharp increase right after year five and then a gradual decrease after the peak. The system shifts about 40 officers out of service to work as instructors right after year five and then moves back unwanted instructors to service as the training load lessened after the peak of the trainee curve in Figure 3. Note that the added instructors after year five must be from the service officers because of the closed nature of the military training system.

Finally, Figure 5 depicts the behaviour of the training system to deliver the required number of qualified officers. For comparison, the step increase in officer demand and the change in the number of service officers are also presented.



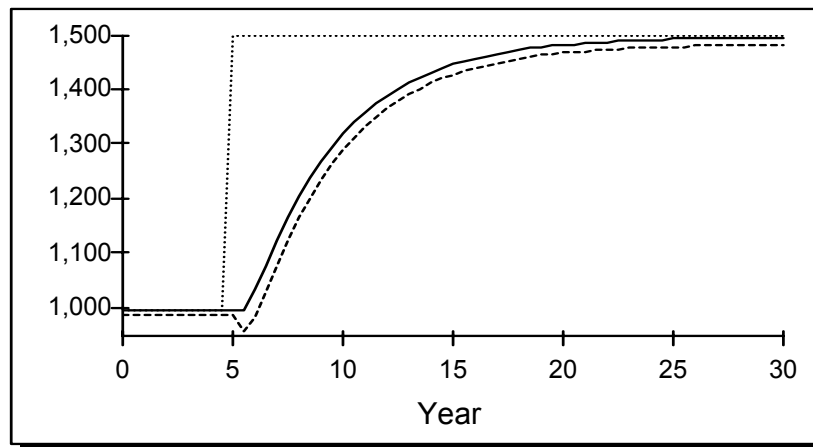


Figure 5. Dashed line: desired officers. Solid line: officers. Long dashed line: service officers.

Figure 5 illustrates how the training system responds to the step increase in officer demand from 1000 up to 1500 in year five. The number of officer grows smoothly and steadily, with a time delay, reaching the targeted level between the year 20 to 25. Note the dip in the number of service officers between year five and six. This dip, corresponding to the peak of instructor curve in Figure 4, displays the so-called “worse-before-improvement” effect due to the closed nature of the military training system. The gap between the officer level and service-officer level is the number of instructors shown in Figure 4.

We note that the template model can only exhibit the ‘worse-before-improvement’ effect of the military training system, which is exposed by the analysis using the causal-loop diagram in the last sub-section, because only one rank is modelled. We leave the multi-rank training model to subsequent work.

We note again that the stock numbers in the model are not constrained to integers, which leads to non-integer trainees and officers. While discrete SD models could be built to deal with exact integers [70], non-integer outputs are generally acceptable where the modelling purpose is for strategic policy design, not for getting precise values. Actually SD is often limited in strategic analysis due to the nature of the systems studied. The potential limitations of SD are discussed next.

## 5.4 Potential limitations

As a powerful tool in understanding and describing the dynamics of systems, SD uses qualitative and quantitative simulation models to reveal feedback loop structures and causes of undesired behaviours.

Both qualitative and quantitative SD modelling starts with diagramming (causal loop or influence diagrams for qualitative analysis; influence or stock-flow diagrams for quantitative simulation) to conceptualise the real situation into models composed of cause and effect or stock and flow variables. This model conceptualisation process is crucial because it lays the basis for qualitative and quantitative analyses. While there is literature trying to formalise the process [57, 65, 66, 71], “the conceptualisation phase in SD has rested heavily on past experience gained from working with ‘canned’ models, apprenticeship in working with experienced modellers, and from trial-and error learning” [56]. Therefore, the lack of formal procedures in conceptualisation makes SD modelling hard to start for newcomers.

The second potential limitation is from the nature of some systems involving ‘soft’ variables. Soft variables are those that are hard to quantify and include in models [72], such as morale, customer satisfaction, environment attractiveness and staff motivation, to name a few. Most of the systems SD designed to simulate involve decision making and human behaviour where soft variables are important and cannot be omitted in modelling [73, 74]. Since modelling soft variables brings uncertainties in simulation outcomes, SD is mainly for strategic analysis in the sense that it concerns itself with overall behaviour of the system under the influence of policies [65] rather than fine details for bookkeeping-type daily management.

## 6. Summary

As shown by this review, the OR applications in workforce planning involves a vast range of OR techniques, which we have classified into four categories: Markov chain, computer simulation, optimisation and SD<sup>6</sup>. These techniques are aimed at different aspects of workforce planning processes. Markov chain, computer simulation and SD are structured to predict what will happen to the system if present policies continue, while optimisation techniques are designed to find what kind of policies should be adopted for given goals. All these techniques face a number of potential limitations as we indicated in the text. Among the four classes of techniques reviewed, SD is the one specifically suitable for the study of system dynamic behaviours where effects of feedback and nonlinearity are vital. With a simple causal-loop model, we expose the peculiarities of military training systems due to their closedness and hierarchical nature. Moreover, we have proposed a template SD model for officer training systems, which could be extended to build the Training Force Sustainment Model if the SD route is to be pursued.

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<sup>6</sup> Strictly speaking, SD is one of the computer simulation techniques.

## 7. Acknowledgements

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## Appendix A: Application of z-transformation in homogeneous Markov chain

The z-transformation for a function  $f(t)$  of discrete, integrally spaced and nonnegative variable  $t$ , is defined as [20]

$$Z[f(t)] \equiv F(z) = \sum_{t=0}^{\infty} z^t f(t), \quad (\text{A.1})$$

where we have introduced the transformation operator  $Z$  and the transformed functions  $F(z)$ .

It can be shown by definition (A.1) that [20]

$$Z[f(t+1)] = z^{-1}[F(z) - f(0)]. \quad (\text{A.2})$$

Substitute (A.2) into the right hand side of z-transformation of Equation (3) and rearrange terms involved[20], we have

$$\mathbf{S}(z) = \mathbf{s}(0)(\mathbf{I} - z\mathbf{P})^{-1}, \quad (\text{A.3})$$

where  $\mathbf{I}$  is identity matrix with the same dimensionality as the transition matrix  $\mathbf{P}$  and the z-transform of vectors or matrices is done by the z-transform of every component of the array [20]. In obtaining Equation (A.3), the time independence of  $\mathbf{P}$  has been assumed.

By Equation (A.3), the z-transform of Equation (4) reads [20]

$$(\mathbf{I} - z\mathbf{P})^{-1} = Z[\mathbf{P}^t].$$

Consequently [20],

$$\mathbf{P}^t = Z^{-1}[(\mathbf{I} - z\mathbf{P})^{-1}] \equiv \mathbf{H}(t), \quad (\text{A.4})$$

where  $\mathbf{H}(t)$ , the inverse z-transform of the matrix  $(\mathbf{I} - z\mathbf{P})^{-1}$ , is named as the response matrix and Equation (4) becomes [20]

$$\mathbf{s}(t) = \mathbf{s}(0)\mathbf{H}(t). \quad (\text{A.5})$$

Therefore, instead of calculating the  $t$ -th power of the transition matrix, the z-transform provides a way to find  $\mathbf{s}(t)$ , the state-probability vector of a Markov system with stationary  $\mathbf{P}$  at time  $t$ , by postmultiplying the initial state-probability vector  $\mathbf{s}(0)$ , with the response matrix  $\mathbf{H}(t)$ .



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Jun Wang

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19. ABSTRACT This report presents the review of workforce planning applications of operations research and explores potential modelling of military training. We classify the operations research techniques applied in workforce planning into four major branches: Markov chain models, computer simulation models, optimisation models and supply chain management through System Dynamics. For each of these, we outline the underlying mathematical formalism and concepts, overview models published and discuss potential limitations. The prospect of modelling training forces via System Dynamics is demonstrated by a causal-loop analysis of the military officer system and a simulation model based on a stock-flow diagram for a one-rank officer training system.					