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A review of subdomain modeling techniques in electrical machines: Performances and applications — Source link

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***Abstract*—This paper reviews the recent developments of semi-analytical subdomains modeling techniques to compute the flux density distribution in electrical machines by the exact solving of Maxwell equations.**

It is shown that with an appropriate development methodology and numerical implementation, these harmonic models break the traditional compromise between accuracy and computation time that must be done using finite element or other analytical methods. Besides that, subdomains model development techniques have been improved to overcome its topological limitations This fact is demonstrated on three different subdomains models in comparison with finite element methods in terms of accuracy and processing time. The first one is a subdomains model of a surface permanent-magnet synchronous machine, the second one is for an inset permanent-magnet synchronous machine, and the third one is for a squirrel-cage induction machine. Thanks to an efficient implementation method, a very low computation time is obtained. The robustness of the subdomains on the geometrical assumptions is also demonstrated.

***Index Terms*—Magnetic field, Electric machines, Analytical model, Harmonic analysis, Performance analysis, Analytical model, Reviews.**

I. Introduction

It is often necessary to estimate the rated power, efficiency, magnetic losses and even magnetic vibrations of electrical machines to optimize their design. This estimation relies on an accurate computation of the machine characteristics, such as electromagnetic torque, magnetic losses and air gap Maxwell forces. All these quantities can be computed if the magnetic field inside the machine is fully determined, meaning the space and time distribution for both radial and tangential components.

For this purpose, different methods have been developed and can be grouped in four main categories: analytical, semi-analytical, numerical, and hybrid methods which result from the combination of the three first. Numerical methods are very flexible to various geometries, include non-linear and non-homogeneous materials, and enable coupling with other physics. Yet, this high level of complexity induces very time-consuming simulations which slows down the design process. Simplified analytical models are consequently used for the first design steps as they are very fast and may give more physical insights, while FEM is more interesting for final validations.

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A semi-analytical method named ”Subdomains Model” (SDM) has been recently developed to compute the magnetic field with great accuracy and much faster than numerical methods such as Finite Elements Method (FEM), representing an interesting alternative between analytical and numerical methods.

The first part of this paper presents the principles of the subdomain modeling technique and reviews the works related to SDM development and applications in electrical machines, showing how these works have enlarged the application range of SDMs. Then, a more detailed comparison with FEM is done to demonstrate the performance of the SDMs despite their modeling assumptions, especially in terms of accuracy, computing speed and robustness to geometry. Finally, some future applications of subdomain models are discussed.

II. State of the art

A. Principles

SDM is a semi-analytical method that consists in dividing the problem into physical regions named subdomains in which Maxwell governing equations can be solved analytically. The main processing steps to obtain the magnetic field in each subdomain are presented in Fig. 1. For this purpose, the subdomains must fulfill specific conditions on geometry physics.

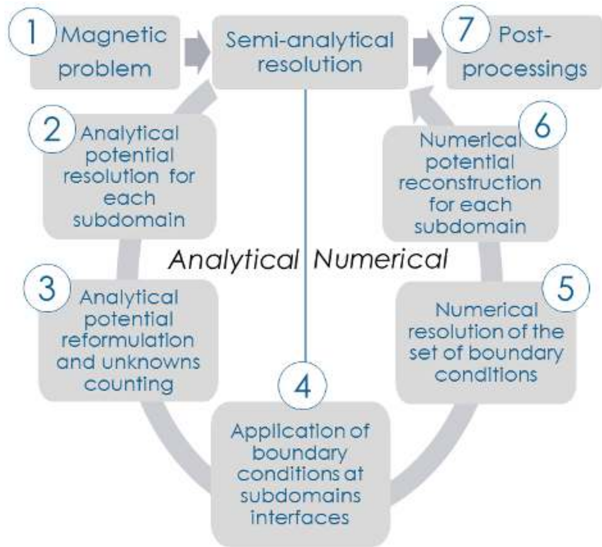

Fig. 1. Methodology graph of Subdomains Models in 7 steps.

In Fig. 2, the problem is composed of one air gap subdomain, stator slot subdomains and rotor slots subdomains, and limited by rotor and stator iron cores.

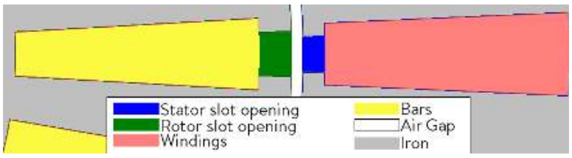

Fig. 2. Division in polar subdomains for an induction machine.

Then, Maxwell equations are written in each subdomain for scalar potential or vector potential, from which the flux density is derived in step **(1)** in Fig. 1. The vector potential formulation can always be used whereas the scalar potential formulation may be used only if the magnetic field is irrotational, meaning there is no current density. Maxwell equations are finally solved using the Fourier method consisting in the separation of variables method **(2)**. This analytical resolution gives in each subdomain the potential in Fourier series in function of several unknown constants. The analytical potential solutions may be then reformulated **(3)** in equivalent expressions for readability and numerical optimization.

By expressing the boundary conditions of potential and magnetic field continuity at each interface between two subdomains, a linear system of analytical independent equations linking all the unknown constants **(4)** can be obtained. If the potential and the magnetic field of both subdomains are expressed in a different Fourier basis, it is necessary to project one Fourier basis on the other.

Then, assuming a finite number of Fourier harmonics in each subdomain, the linear system of equations is put into matrix form and solved numerically **(5)**.

$$\mathbf{M} * \mathbf{X} = \mathbf{S} \tag{1}$$

The matrix **M** is often called "topological matrix", and **S** the "source vector". Solving the linear system gives the numerical value of the unknown constants **X**, which enables to compute the magnetic field's spatial and temporal distribution in each subdomain **(6)**. Because both analytical and numerical resolutions are successively accomplished, SDM may be classified as "semi-analytical" model. A similar methodology is also proposed in [1].

It is important to mention that the formulation of the analytical solution **(3)** may strongly differ from an author to another. In this review, two main formulations are distinguished : the formulation **A** in [2], [3] and the formulation **B** in [4], [5]. Formulation **B** presents the interest of giving dimensionless expressions and a topological matrix **M** with only 1 on the diagonal and more 0 elsewhere. Specific algorithms are dedicated to optimize the inversion of such matrix.

Hence the choice of formulation is a real matter to design a SDM as it may compromise the numerical resolution during **(5)** if the topological matrix obtained after **(4)** is ill-conditioned. Such numerical problems are frequently pointed out in the SDM literacy, though only few articles such as [6] analyze the analytical formulation as regards on the numerical performances.

B. Development history

The main difficulties encountered by any analytical models are how to take into account slotting effect, as air gap length variations strongly influence the magnitude and shape of the magnetic field. The first methods based on the formal resolution of Maxwell equations were developed in the 1980's for both slotless Permanent Magnet Synchronous Machine (PMSM) [7] and Induction Machines (IM) [8], and have been improved by several approaches to better account for slotting effect and radial and tangential air gap flux components .

In 1984, [7] used *Carter's coefficients* to transform a slotted stator into an equivalent slotless one. In 1993, [9] introduced a *relative permeance* which modulates the radial air gap flux density previously computed without slotting effect. Another permeance model was developed in 1997 by [10]. The *relative permeance* method was extended by [11] in 1998 to take into account both radial and tangential components. In 2003, [12] used *conformal transformation* and more specifically Schwarz-Christoffel mapping to model slotting effect. This method was also adopted by [13] in 2006, which applied the conformal transformation to the relative permeance model and deduced a *complex permeance model*, giving better accuracy for both components. One can refer to [6], [14], [15], [16] for their exhaustive history in PMSM analytical modeling, and to [17] for IM modeling.

The first SDM for SPMSM were developed in 2008-2009 by [14], [18] and [19], although the Fourier projection between subdomains was already used by [20] and [21] a few decades ago. Also in 2008, [22] used the same method for a linear actuator. The term "Subdomains model" appeared in 2010 in [15], [23] and was adopted by then in several contemporary major publications. This method is also referred as "exact analytical model" [4], "semi-analytical harmonic model" [24] or "Fourier-based Model" [25]. Compared to the previous analytical models, SDMs provide both components of the magnetic field by exactly taking into account slotting effect and the influence between slots. In 2010, [23] developed an elementary model to give a better understanding of slotting effect in SDMs.

C. Available topologies

1) Introduction: Due to the PMSM popularity in the past decades, most of SDMs deal with them, at the detriment of IM. This can also be explained because of more complex physics. Besides, SDMs for other types of machines have been developed. A complete review on SDMs done until 2014 is presented in [26]. Some examples of existing topologies are illustrated in Fig. 3.

2) Geometry aspects: The geometry is usually in two-dimensional (2-D), but have already been extended to 3-D such as in [27], [28]. In 2-D, axial and radial 3-D end-effects are neglected.

The problem is either expressed in polar or Cartesian coordinates. In case of polar coordinates, every subdomain geometry is approximated by a polar geometry. For example, the rectangular teeth are supposed to have radial edges with orthoradial tooth tips, as it is illustrated in [2]. In case of Cartesian coordinates, the air gap is unrolled by considering an infinite radius of curvature, giving an equivalent rectangular topology such as in [29]. The different topology approximations according to the chosen coordinate systems are gathered in the aforementioned methodology [6].

Besides, SDMs can be applied to internal or external rotor [15], [30]. Moreover, semi-closed slots may be used for a more realistic model [4].

3) Physics aspects: As said previously, the physics is also approximated. The iron is considered to have infinite relative permeability, resulting in homogeneous boundary conditions at the interfaces between subdomains and the iron. The saturation is consequently neglected. For PM machines, magnets have an isotropic and homogeneous relative permeability and a linear $\mathbf{B}(\mathbf{H})$ curve. For induction machines (IM), the rotor bars are assumed to have a homogeneous electrical conductivity.

SDM can model both magnet and current sources. It accounts for any magnetization shapes such as radial, parallel or Hallbach magnetization. Concerning the armature reaction fields, the windings are usually designed by a connection matrix which enables to use (non)-overlapping single/double layer windings. In fact, the magnetic sources are expanded into Fourier series and injected in Maxwell equations at step **(2)**. It results in a linear superposition of stator and rotor fields, hence the possibility to solve everything at once or separately.

4) Synchronous Machines (SM): Several models exist for each topology of PMSM, depending on the chosen modeling level. For SPMSM with armature reaction field and semi-closed slots, one can refer to [2], [4], [30]. Inset PMSM (IPMSM) models with armature reaction field and semi-closed slots can be found in [1], [3], [31].

Besides the above topologies, more singular SM have been modeled by the subdomain technique. For instance, SDMs exist for flux switching SM [32], double excitation SM [33], axial flux SM [29], PMSM with noches [34], pseudo direct drives SM [35] and Switch Reluctance Motor (SRM) [36]. The SRM model illustrates the difficulties to transform any geometry into a polar one.

5) Induction Machines (IM): IM SDMs have both common points and differences with SM ones. Assuming an internal rotor topology, stator slots subdomains and air gap subdomain remain the same as for PMSM. Though, the level complexity is increased because of the the induced current modeling in the rotor bars, and of the existence of two asynchronous frequencies as well as the space harmonics. The former analytical models of IM were designed for laminated solid rotor, such as in [8], [37], and extended by the SDM in [38].

The first model of Squirrel Cage IM (SCIM) in [39] accounts for rotor bars with induction and a current sheet at stator inner bore. A complete SCIM SDM was developed in [17], [24], using Electrical Equivalent Circuit (EEC) to drive the feeding current as a function of the slip value.

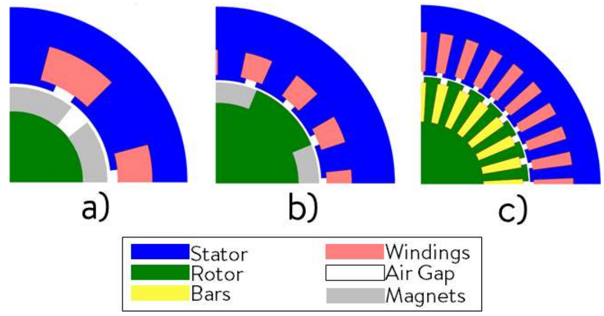

Fig. 3. Three idealized topologies of existing SDMs. a) SPMSM ; b) IPMSM ; c) SCIM. The pictures are provided by MANATEE software.

6) Linear or tubular PM machines: SDMS of linear actuators can be found in [22], [40]. These topologies are well suited to SDMs because the geometry can be directly implemented in Cartesian coordinates without simplifications.

7) Missing machines: To the author's knowledge, no paper specifically deals with SDM of Direct-Current Machines (DCM) but the models used for synchronous machines can be applied with DC current sources. There is either no Buried PMSM (BPMSM) SDM, because of the saturation in the bridges between magnets and the difficulty to approximate interior magnets with polar geometry.

D. Accounting for time parameter

In SDMs, time is differently taken into account according to the machine type. Its influence on the topological matrix $\mathbf{M}$ and the source vector $\mathbf{S}$ must be carefully established. In fact, sources variations impact on $\mathbf{S}$ whereas reluctance variations modify $\mathbf{M}$.

Particularly, for SPMSM with internal rotor and magnet permeability equal to 1, there is no reluctance variation with rotor rotation. Only the magnetization distribution changes with the rotation and this results in a source matrix whose columns are the source vector $\mathbf{S}$ for each time step. Consequently, a single linear system resolution can give the constants $\mathbf{X}$ for every time step.

For IPMSM, both reluctance and magnetization distribution change over time. In this case, it is the same principle as for FEM : $\mathbf{M}$ and $\mathbf{S}$ have to be evaluated at each time step and one linear system resolution may only give one time step.

It is a different method for IM. Every stator quantity, respectively rotor quantity, is first expressed as a phasor of pulsation ω, respectively $slip{*}\omega$. Magnetic potentials are then solved in terms of complex amplitude and finally modulated with their respective pulsation. It implies that one system resolution gives every time step for one chosen slip value.

E. Current applications

Though it may have been restricted to torque and Back-ElectroMotive Force (BEMF) at the beginning, SDMs have been derived for many other applications.

1) Magnetic Forces and Flux linkage computation: Each local and global electromagnetic quantity may be derived from the magnetic potential using well-known physics laws. Electromagnetic forces as well as electromagnetic torque including torque ripple - such as cogging torque - may be directly computed from the magnetic field using the Maxwell stress tensor with a great accuracy. The flux linkage is computed by applying the Stokes theorem to the magnetic potential of each stator slot. Flux linkage knowledge enables to deduce the BEMF generated by the rotating field.

Thanks to the Fourier series formulation, these previous temporal integrations are converted in faster and more accurate summations on the magnetic field's harmonic components. The limit of any 2-D magnetic models is the fact that 3-D axial end-effects have been neglected so the torque and the BEMF may be overestimated.

2) Fault simulation: It is possible to simulate unbalanced magnetic sources distribution by injecting the corresponding Fourier series at the step **(2)**, such as in [41], and deduce the Unbalanced Magnetic Force (UMF) - or Pull (UMP). UMF is also caused by rotor eccentricity. The effect of rotor eccentricity is modeled in [42] by adding a first-order perturbation component to magnetic potentials, whereas [43] introduces it with a superposition method. For SCIM, it is possible to simulate defective bars as in [17] by decreasing their conductivity.

3) Losses: Eddy-current losses may be computed using Helmholtz equation in the PM subdomains [5], [44] and in the windings [45], [46]. In [26], a shielding cylinder is added at the surface of the PM to reduce the eddy-current losses in magnets.

4) EEC parameters: The potential solution is used to compute leakage flux at stator slots [47] and due to end-windings [48]. A method to compute primary and secondary impedances for SCIM is developed in [17], [24]. It is also shown that the estimated EEC parameters enable to compute and check previous quantities such as torque, back-EMF, power losses, etc. with another approach.

III. Advantages and Drawbacks in Comparison with FEM

A. Introduction

In most of SDMs papers, the model is validated by comparing its accuracy with a parallel FEM analysis using the same approximations. For the same modeling level, SDM is naturally as accurate as FEM, since the former is an exact resolution of Maxwell equations. It is actually more exhaustive than FEM because the solution is continuously defined in each subdomain and not only at the mesh points. Tab. I shows the qualitative performances criteria in comparison with FEM.

TABLE I
QUALITATIVE COMPARISON ON PERFORMANCES CRITERIA.

	SDM	FEM
Geometry Complexity	-	+
Non homogeneity, non isotropy	-	+
Saturation	-	+
Mesh sensitivity	+	-
Computation time	+	-

B. Comparison on the model limitations

1) Sensitivity to mesh in the FEM: Significant problems due to the meshing quality exist, as regards on computing derived magnetic quantities. This has been studied a lot for the evaluation of vibrations due to magnetic forces and cogging torque. For example, Fig 2.38 in [49] states a vibration variation up to 4 dB below 10 kHz between different meshing methods in Flux3D [50]. These problems may be solved by refining the mesh in the air gap, but it significantly slows down the computation.

2) Robustness to geometry in SDMs: One drawback of SDM is the geometry simplification. However by defining an equivalent polar geometry as in Fig. 4, the air gap flux computation is still accurate. In Fig. 5, a comparison has been done using MANATEE simulation environment [51] on the SPMSM 6s/4p presented in [4].

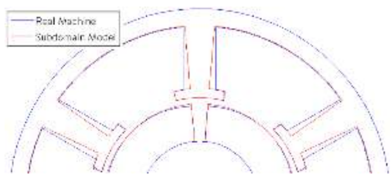

Fig. 4. Polar approximation of constant tooth width and curved magnet.

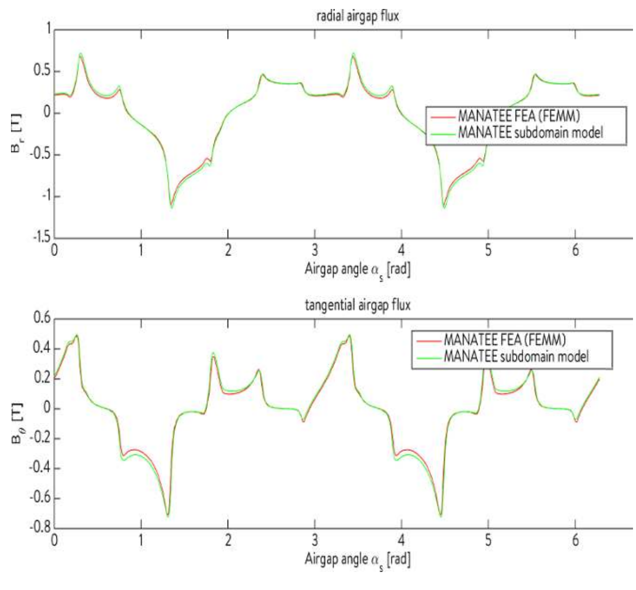

Fig. 5. Magnetic field comparison between idealized and real geometries.

3) Saturation in SDMs: Saturation is still a limitation that prevents from modeling several machine topologies. The first challenge is to compute the magnetic field assuming a finite iron permeability. This is done for example in [38] where the magnetic field is computed in the rotor iron core. In [52], a method is proposed to account for linear soft-magnetic iron with finite permeability, but there exists no SDMs which account for non-linear materials to the authors' knowledge.

However, it is possible to couple SDM with Magnetic Equivalent Circuit (MEC) [53], [54], [55] or FEM [56] in an iterative way to account for saturation.

C. Comparison on computing performance of SDMs

1) SDM numerical optimization: An efficient implementation is necessary to take advantage of SDM performances. To illustrate this, the SDM recently presented in [17], a SCIM 36s/4p at no-load condition has been developed and optimized. However, the analytical resolution at **(3)** is done with reformulation **B** to improve numerical performances, and the numerical implementation uses only vectorized operations.

Besides, the choice of the harmonics number in each subdomain at step **(4)** relies on a compromise between accuracy and computation time [57]. Fig. 6 shows the variation of the Mean Squared Error (MSE) between the magnetic field and the reference obtained with 1000 air gap harmonics, the computation time and the memory used by **M** in function of the air gap harmonics number. Harmonics numbers in the other subdomains are the same as in [17].

Fig. 6 illustrates the fast convergence of the Fourier series, as 150 harmonics gives a MSE of 0.2% compared with 1000 harmonics, within 0.34 seconds. In [17], the SDM computation time is 3.24 minutes and very close to the FEM computation time.

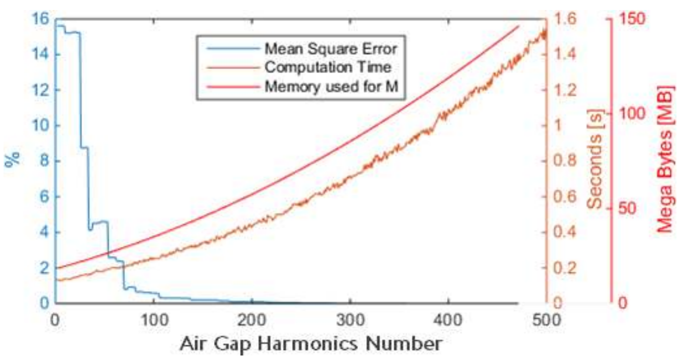

Fig. 6. MSE, computation time and memory used in function of the air gap harmonics number.

Furthermore, a few SDMs for SPMSM accounts for periodicity and one slot per pole approximation [15] [44] to reduce computation time.

The air gap harmonics number can be reduced by a previous analytical study of the magnetic field's harmonic content. For instance, in a SPMSM with Z_s stator teeth and $2p$ poles, the fundamental at no load is at $2p$ and harmonics are linear combinations of $2p$ and Z_s. Hence any other harmonics ranks can be ignored to improve performances.

2) Comparison with FEM: An efficient implementation of SDMs can lead to much lower computation time than FEM, as presented in [58]. Tab. II shows a quantitative comparison on performances between FEM using Femm [59] and three particular SDMs : the SPMSM 6s/4p without armature load presented in [4], the IPMSM 15s/4p without armature load in [31], and the previous SCIM 36s/4p in [17].

For the SDM, the computation time includes building the linear system, solving it, and reconstructing the air gap magnetic field in the air gap subdomain. For the FEM, it includes building the topology, meshing it, solving the problem in the entire topology and extracting the air gap field. Both models are linear and without any symmetry or periodicity simplifications.

TABLE II
COMPARISON ON THE ELAPSED SIMULATION TIME FOR ONE TIME STEP.

	SPMSM	IPMSM	SCIM
SDM			
Computation time [s]	0.109	0.199	0.7337
Air gap harmonics	150	250	300
Number of unknowns	1572	1909	2298
FEM			
Computation time [s]	6.152	10.11	15.27
Number of nodes	45888	64605	73003
Number of elements	91054	128488	145284

When computing temporal quantities such as cogging torque, FEM computation time values in tab. II are multiplied by the number of time steps. As said previously, it is not the case for an SPMSM model whose computation time is very less sensitive to the number of time steps. For instance, considering the same topology of SPMSM 6s/4p, the computation time for the SDM is 0.139 seconds whereas it takes around 347 seconds for the FEM because of the 50 time steps, meaning 50 meshings and resolutions.

SDMs are implemented in the Matlab scientific environment -version R2014b. The computer is equipped with a CPU Intel Xeon E5 1620 v2 @ 3.70GHz and 24GB RAM DDR3 @ 797 MHz. All results in this paper have been repeated 10 times and then averaged for robustness purpose.

IV. CONCLUSION

The paper presents a review of the semi-analytical subdomains models applied to electrical machines. The methodology is first explained step by step, including both analytical and numerical aspects such as model assumptions and the choice of formulation. Then, a state of the art presents the main existing models and their numerous applications. Advantages and drawbacks of the method are finally illustrated in comparison with FEM for three machines topologies. The results validate the model assumptions and show great computation time performances especially when accounting for rotor rotation.

Furthermore, an efficient implementation of subdomains models strongly reduces the computation time and allows to increase the electromagnetic model complexity. This can be done to include:

- the effect of magnet shaping using harmonic superposition
- the effect of winding space harmonics in induction machines using field superposition
- 3-D effects such as fringing flux and skewing
- strong coupling with electrical circuit (calculation of equivalent circuit parameters iteratively)
- integration in complex multiphysics models, especially for vibroacoustic analysis [60].

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