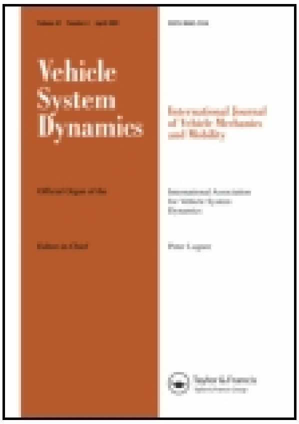

Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/nvsd20

A review on bicycle dynamics and rider control

A. L. Schwab [a] & J. P. Meijaard [b]

[a] Laboratory for Engineering Mechanics , Delft University of Technology , Mekelweg 2, 2628 CD , Delft , The Netherlands

[b] Olton Engineering Consultancy , Enschede , The Netherlands

To cite this article: A. L. Schwab & J. P. Meijaard (2013): A review on bicycle dynamics and rider control, Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, 51:7, 1059-1090

To link to this article: http://dx.doi.org/10.1080/00423114.2013.793365

Vehicle System Dynamics, 2013
Vol. 51, No. 7, 1059–1090, http://dx.doi.org/10.1080/00423114.2013.793365

Taylor & Francis
Taylor & Francis Group

A review on bicycle dynamics and rider control

A.L. Schwab[a]* and J.P. Meijaard[b]

[a]*Laboratory for Engineering Mechanics, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands;* [b]*Olton Engineering Consultancy, Enschede, The Netherlands*

(*Received 9 March 2013; final version received 29 March 2013*)

This paper is a review study in dynamics and rider control of bicycles. The first part gives a brief overview of the modelling of the dynamics of bicycles and the experimental validation. The second part focuses on a review of modelling and measuring human rider control, together with the concepts of handling and manoeuvrability and their experimental validation. The paper concludes with the open ends and promising directions for future work in the field of handling and control of bicycles.

Keywords: bicycle; dynamics; manual control

1. Introduction

The four main parts of which a bicycle consists are the two wheels that are nominally placed behind each other, and the rear and front frame, to which the wheels are connected by revolute joints and that are interconnected by a vertical or inclined hinge. The two wheels contact a road surface, which is mostly assumed to be flat and level. An integral part of the bicycle system is the rider, who has the larger part of the total mass. The rider can be modelled to be rigidly attached to the rear frame or be able to move laterally and longitudinally and exert control forces on the handlebar. Because of the two contact points between the bicycle and the supporting road surface, the bicycle is inherently unstable, like an inverted pendulum. The main way to stabilise the bicycle is by shifting the support point to the side the bicycle is inclined to fall after a perturbation by steering to the side of the impending fall, like an inverted pendulum can be stabilised by horizontally moving the support point. To make this way of balancing possible, the hinged connection between the rear frame and the front fork and a forward speed are necessary. Indeed, it is very difficult to balance a bicycle that is standing still or has a locked steer. The corrective steering can be done in basically two ways:

(i) the rider can supply the required steering input, or
(ii) the steering input is automatically supplied by the dynamics of the bicycle.

For the first case, the rider supplies a steering torque. If the torque is used to obtain a desired steering angle, this can be modelled as a position input, but if the static feedback gain or the bandwidth is small, this simplification cannot be made and the input is modelled as a torque.

---

*Corresponding author. Email: a.l.schwab@tudelft.nl

In the second case, the upright forward motion can be stable without any steering torque input or body motion relative to the frame of the rider; this is called self-stability. Furthermore, the dynamics of the bicycle can reduce the control effort if torque control is used to make the balancing partly automatic.

Several reviews on the dynamics and control of two-wheeled single-track vehicles, that is to say, bicycles and motorcycles, have appeared. Sharp [1–4] has given a series of accounts of the state-of-the-art over time. A clear historical overview is given in [5], whereas Popov *et al.* [6] concentrate on rider control for motorcycles. Reviews specifically directed to the dynamics of bicycles and their control can be found in [7], which concentrates on the linearised equations for idealised bicycles, and [8], which gives a broader summary of bicycle dynamics and control. A historical review of thoughts about self-stability is given in [9].

The history of the bicycle, as related by Herlihy [10], started with the invention of the velocipede by von Drais around 1817. The propulsion of this machine was by stepping with the feet on the ground, like with a scooter, thus limiting the forward speed. von Drais was apparently well aware of the way in which balance is maintained, as well as the need for initial countersteer to initiate a curve, as is shown in the following excerpt from [11] cited in [12].

> Then one makes, by means of lightly putting the feet to the ground, big but initially slow steps in the direction parallel to the wheels, and keeps the heels not inward, in order that one does not come with them under the rear wheel, and when one has later got some speed, and has lost by accident the balance a bit, one can usually help oneself with the feet, or by steering if one steers a little towards the direction in which the centre of gravity of the whole leans, and if one wants to take a turn, one directs the centre of gravity immediately before a little to the inside and steers right after that to that side.

In 1866, Lallement filed a patent for the innovation of adding propulsion by cranks powered by the rider's feet to the front wheel [13]. The same kind of machine was developed by Michaux in France. The bicycle evolved further by increasing the size of the front wheel and finally replacing the front-wheel propulsion by a rear-wheel propulsion with a chain drive and two wheels of equal or nearly equal size in the safety bicycle and the use of pneumatic tyres, as shown in Figure 1.

One of the earliest papers on the balancing of a bicycle was written by Rankine [15]. He discusses the balancing by a steering angle control, which leaves the roll angle as the only dynamic degree of freedom. He noticed the need for countersteering for entering a curve. Furthermore, he is the first to introduce a handling criterion, called the *promptitude*, which measures how fast the support point below the centre of gravity moves laterally for a given steering angle input. Furthermore, the difficulty of riding a bicycle with rear-wheel steering is qualitatively discussed.

Later, simple dynamic models with steering angle control were elaborated by Bourlet [16], who included the influence of the gyroscopic effects of the rotating wheels. A simplified model which only included the mass of the rear frame with the rider rigidly attached to it, and even a model with a single point mass, was further discussed by Boussinesq [17,18]. His discussion about the lateral motion of the rider's upper body is of little practical interest. Essentially the same models were discussed by Bouasse [19] and in the textbook by Timoshenko and Young [20].

Essentially correct linearised equations of motion for a bicycle were derived by Carvallo [21], with some simplifying assumptions on the mass distribution of the front fork, and Whipple [22] at the end of the nineteenth century. Whipple explicitly studied the range of speeds for which the bicycle was self-stable and the possibility to increase this range by applying a steering torque control or a lateral shift of the centre of mass of the rider. Carvallo's model was used by Klein and Sommerfeld [23] (in a part actually written by F. Noether) to discuss the influence of the gyroscopic moments of the wheels; however, they overestimated the contribution of these moments to the self-stability due to some sign errors and erroneously

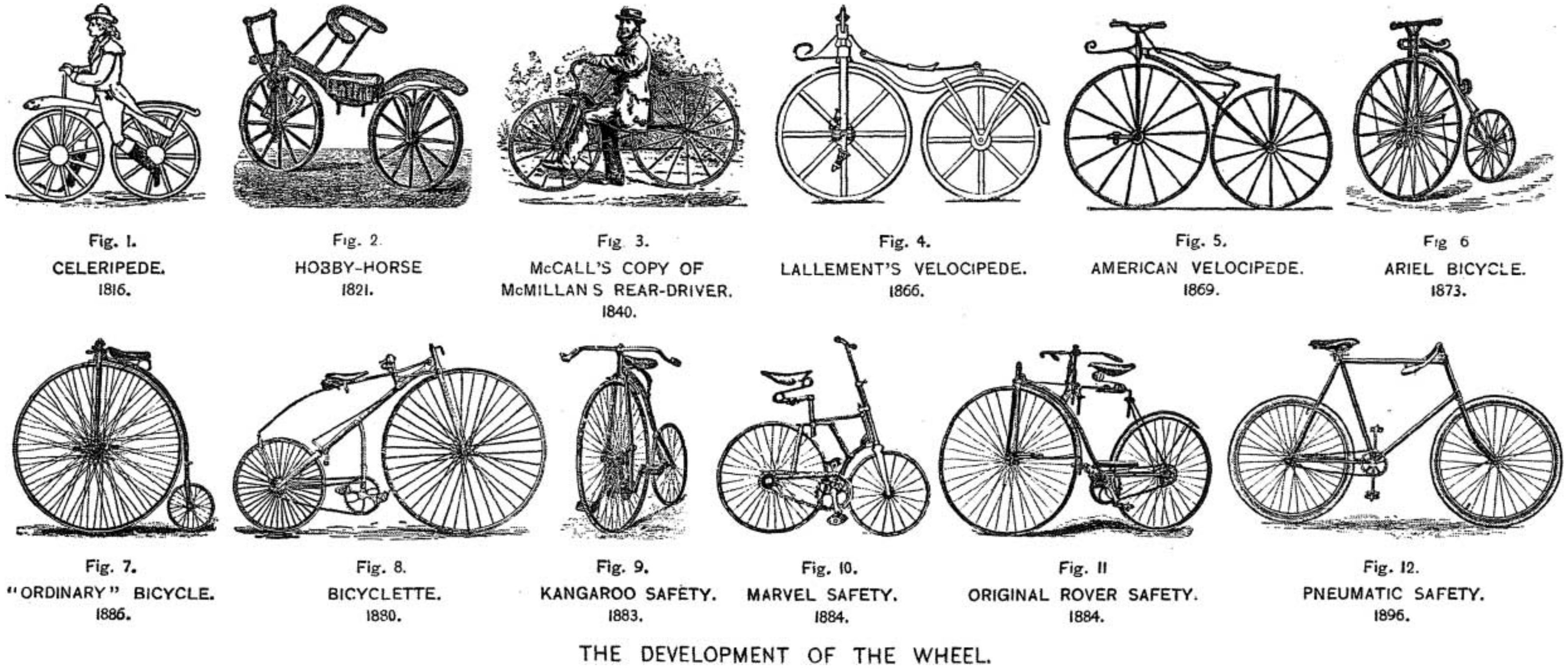

Figure 1. The evolutionary development of the bicycle from the velocipede to the safety bicycle; from *The Aeronautical Annual 1896* [14].

concluded that these were indispensable for typical bicycles, as was pointed out in [9]. Indeed, a self-stable bicycle with the gyroscopic effects of the wheels cancelled was recently demonstrated [24]. Döhring [25,26] extended Klein and Sommerfeld's model to include a fully general mass distribution of the front fork and used it for a motorcycle. Apparently independently, Herfkens derived similar equations for a bicycle [27]. Also, Sharp [28] discusses the model with two degrees of freedom in his seminal article, though its main contribution is showing the influence of realistic tyre force models on the dynamics of motorcycles and identifying the main modes as capsize, weave and wobble, after which the modern era of modelling motorcycle dynamics has begun.

In the next section, the Carvallo–Whipple model and its relevance for the modelling of dynamics of bicycles will be further discussed. Also, refinements that lift some of the limitations of the model will be introduced. The interaction with the rider and ways to model a realistic active and passive rider control is the subject of Section 3. This review ends with some conclusions and an outlook to further developments and needs.

2. Bicycle dynamics

This section starts with a description of the Carvallo–Whipple bicycle model (Figure 2). Next, extensions to this model with toroidal wheels and forward acceleration are discussed. Then, the curving and non-linear models are reviewed. The following parts discuss bicycle tyre force models and wobble motions, and the section ends with an overview of the experimental validations of the various bicycle models.

2.1. *Carvallo–Whipple model*

For small deviations from the nominal upright position, the dynamics of the longitudinal and the lateral motion can be treated sequentially, as the lateral motion has no appreciable influence on the forward motion. The longitudinal motion concerns the propulsion, acceleration and braking of the bicycle with the rolling resistance and the aerodynamic drag forces acting on the bicycle. If ideal contact between rigid wheels with knife-edge rims and the road surface is

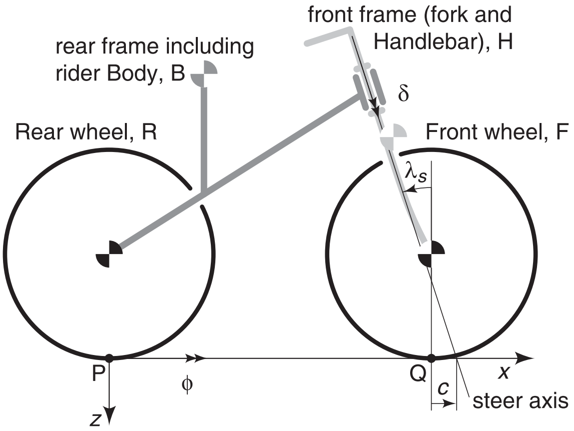

Figure 2. Carvallo–Whipple bicycle model with the four main parts: the rear wheel, the rear frame which can include a rigid rider, the front frame and the front wheel. The lateral degrees of freedom for this model are the rear frame roll angle ϕ and the front frame steering angle δ. Figure adapted from [7].

assumed, with no indentation or longitudinal tyre slip, a single degree of freedom is sufficient to describe the forward motion. If the lateral tyre slip is also assumed to be zero, the lateral motion can be described by two degrees of freedom, the roll angle of the rear frame, denoted by ϕ, and the steering angle, denoted by δ. For a full specification of the configuration of the bicycle, also the rotation angles of the wheels for the longitudinal motion and the lateral displacement and the yaw angle are needed, which depend on the configuration coordinates by first-order differential equations.

The linearised equations for the lateral motion if the bicycle is moving at a constant forward velocity v constitute the Carvallo–Whipple model [21,22], which have the structure

$$\mathbf{M}\ddot{\mathbf{q}} + v\mathbf{C}_1\dot{\mathbf{q}} + (v^2\mathbf{K}_2 + g\mathbf{K}_0)\mathbf{q} = \mathbf{f}, \tag{1}$$

where $\mathbf{q} = [\phi, \delta]^{\mathrm{T}}$ is the vector of degrees of freedom, $\mathbf{f} = [T_\phi, T_\delta]^{\mathrm{T}}$ is the vector of applied generalised forces, $v\mathbf{C}_1$ is the non-symmetric velocity sensitivity matrix that is linear in the velocity and $v^2\mathbf{K}_2 + g\mathbf{K}_0$ is the non-symmetric stiffness matrix that consists of a part that is quadratic in the velocity and a symmetric part that linearly depends on the acceleration of gravity g. Expressions for the entries of the matrices in terms of geometric and mass parameters of the bicycle can be found in [7], and for the benchmark bicycle presented therein the rounded values of the matrices in standard SI units are

$$\begin{aligned} \mathbf{M} &= \begin{bmatrix} 80.817 & 2.319 \\ 2.319 & 0.298 \end{bmatrix}, \quad & \mathbf{C}_1 &= \begin{bmatrix} 0 & 33.866 \\ -0.850 & 1.685 \end{bmatrix}, \\ \mathbf{K}_2 &= \begin{bmatrix} 0 & 76.597 \\ 0 & 2.654 \end{bmatrix}, \quad & \mathbf{K}_0 &= \begin{bmatrix} -80.95 & -2.600 \\ -2.600 & -0.803 \end{bmatrix}. \end{aligned} \tag{2}$$

From this, it is seen that the mass and stiffness associated with the steering angle are much lower than the corresponding terms for the roll angle, but that there is a fairly strong coupling between the two equations. The coupling from the roll angle to the steering equation comes from the trail of the front wheel, the gyroscopic terms from the rotating wheels, the steering head inclination and the mass distribution of the front fork assembly. The coupling from the steering angle to the roll equation depends on the same parameters, though the main contribution comes from the lateral shift of the front-wheel support point due to steering and forward motion if the forward speed is not too small.

The eigenvalues of these linearised equations for $g = 9.81\,\mathrm{N/kg}$ and variable forward speed v are shown in Figure 3. For very small speeds, there are two negative eigenvalues, which represent stable motions, and two positive real eigenvalues, which represent unstable motions. The positive real eigenvalues decrease with increasing speed, coalescing at about 0.7 m/s, after which they represent an oscillating motion with increasing amplitude, which involves roll as well as steer and is called weave. At a speed of about 4.3 m/s, this weave motion becomes stable and the bicycle is self-stable. At about 6.0 m/s, the bicycle becomes unstable as a real eigenvalue becomes positive. The corresponding capsize motion is dominated by the roll angle and consists of a veering-off of the bicycle in a spiral with decreasing radius of curvature until it stabilises in a circular motion due to non-linear terms in the full non-linear equations of motion or it falls down. For a further increase in speed, the bicycle remains mildly unstable with the positive real eigenvalue approaching zero. The large, real and negative eigenvalue corresponds to the very stable castering mode, which is dominated by steer in which the front ground contact follows a tractrix-like pursuit trajectory, like the straightening of a swivel wheel under the front of a grocery cart.

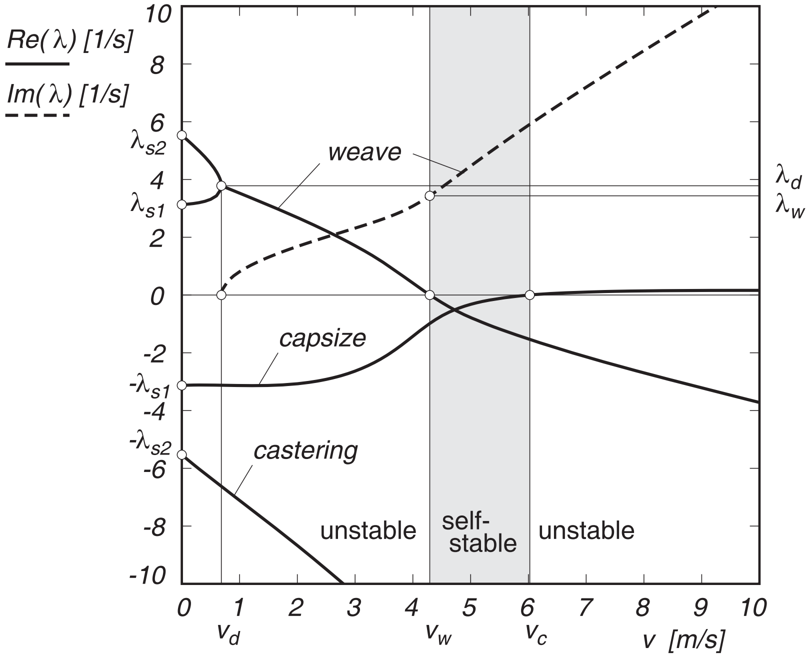

Figure 3. Eigenvalues λ from the linearised stability analysis for the benchmark bicycle [7], where the solid lines correspond to the real parts of the eigenvalues and the dashed line corresponds to the imaginary part of the complex eigenvalues, in the forward speed range of $0 \leq v \leq 10$ m/s. The speed range for the asymptotic stability of the benchmark bicycle is $v_{\mathrm{w}} < v < v_{\mathrm{c}}$. The zero crossings of the real parts of the eigenvalues are for the weave motion at the weave speed $v_{\mathrm{w}} \approx 4.3$ m/s and for the capsize motion at the capsize speed $v_{\mathrm{c}} \approx 6.0$ m/s, and there is a double real root at $v_{\mathrm{d}} \approx 0.7$ m/s.

2.2. *Extension with toroidal wheels and forward acceleration*

Some extensions can be made to the Carvallo–Whipple model without increasing the number of degrees of freedom. A first extension is to assume that the wheels do not have knife-edge rims, but have rims with a toroidal shape, which is a local approximation for a general rim shape valid for a linear analysis. This assumption makes the analysis only a little more complicated. An analysis of this kind was done for the linearised equations in our paper [29]. The effect of the toroidal shape is mainly reducing the arm of the gravity force, thereby slowing down the dynamics and easing the balance task. This was used in adapted bicycles for teaching children to ride a bicycle [30]. Frosali and Ricci [31] analysed the non-linear kinematics for a bicycle with toroidal wheels and compared their results for the pitch angle with the exact quadratic approximation, a new approximation they proposed and an approximation used by Cossalter [32]. For large roll and steering angles, large differences from the quadratic approximation were found.

Toroidal wheel rims were also used by Åström *et al.* [30] and Peterson and Hubbard [33]. More references on wheel rims with toroidal shape can be found for motorcycles, where they are usually combined with tyre force models as discussed below.

The influence of a constant acceleration or deceleration of a motorcycle was first investigated by adding inertia forces to the system [34], as if the motorcycle rode on an incline. Later, more complete models were developed [35,36], which were based on computer algorithms for multi-body system dynamics. A model that can be seen as an extension of the Carvallo–Whipple model was developed by the authors [29], which also included toroidal wheel rims and some tyre contact force effects, which are of no further consequence. The acceleration adds stiffness terms to the linearised equations of motion that are linear in the accelerations. The exact linearised equations depend on the way in which the accelerating forces are applied: driving or braking torque at the wheels and aerodynamic drag have different contributions to the equations of motion. In particular, a front-wheel braking torque with a compensating rear-wheel driving torque, which results in a zero acceleration, gives rise to an additional stiffness term due to the steering torque of the longitudinal tyre forces for non-zero steering angles. Aerodynamic drag forces can influence the damping of the lateral motion. This work on the accelerated bicycle was continued by Sharma [38], Limebeer and Sharma [37,39], who also considered curving behaviour.

As acceleration and deceleration take place over a finite time, the usual concepts of stability cannot be applied directly, as we deal with a linear system with time-varying coefficients. If the magnitude of the acceleration is small, the ‘frozen’ eigenvalues can be used with the coefficients taken as their current values and the usual stability criterion can be used. The influence of a moderate acceleration on the range of speeds with self-stability in this sense can be quite high. For higher values of the acceleration, the concept of practical stability was further developed by Meijaard and Popov [36], which considers expected values of perturbations and allowable motions.

2.3. *Curving and non-linear models*

Steady-state curving for a curve with a large radius of curvature can be described with the linearised equations in Equation (1), with a steering torque in the right-hand side. In the speed range with self-stability, the required steering torque is opposed to the direction of the turn. If the radius of curvature becomes smaller, non-linear equations have to be used, as was shown by Franke *et al.* [40]. A hands-free case was studied by Åström *et al.* [30], Lennartsson [41], Basu-Mandal *et al.* [42] and Basu-Mandal [43]. Around the capsize speed, a steady-state turning without steering torque input can be possible, in which case we have a limit cycle, which might be stable or unstable. Other disconnected families of steady-state turning solutions are present. Peterson and Hubbard [33] studied steady-state turning with an applied steering torque.

An experimental study was performed by Cain and Perkins [44,45], who compared their results with those of the linearised equations and found a good agreement.

2.4. *Tyre force models*

The Carvallo–Whipple model was experimentally validated by Kooijman *et al.* [46] for the autonomous dynamics of a riderless bicycle at low speed up to about 5 m/s. From Sharp’s [28] study, it became clear that tyre slip and tyre force models need to be included to describe the behaviour at high speeds realistically and to be able to describe wobble oscillations. For a general overview of tyre models, we refer to Pacejka [47]. It should be noted that for bicycles, the tyre forces depend not only on the slip values and the tyre properties, but also on the stiffness of the wheel on which the tyre is mounted.

There is a dearth of experimental data on tyre force properties. Still the most important source is the report by Roland and Lynch [48]. They found dimensionless side-slip tyre force coefficients in the range 0.8–1.5 and dimensionless camber force coefficients in the range 0.15–0.60. The coefficients increased with decreasing tyre load and increasing inflation pressure. An ingenious and simple machine to measure lateral bicycle tyre forces was described by Cole and Khoo [49]. Only forces due to side-slip could be measured. The results for one tyre type of 20″ diameter and 2.125″ width and an inflation pressure of 2.4 bar showed that the side-slip force coefficient depended strongly on the load and was in the range 0.9–1.5, in agreement with Roland and Lynch. Another simple measuring apparatus for bicycle tyre forces was recently described by Dressel and Rahman [50]. They found side-slip force coefficients in the same range and camber force coefficients in the range 0.7–1.0. Doria *et al.* [51] used a turn-table type tyre tester to measure tyre force properties, which adds an additional normal spin input motion that was absent from the other studies. For their three ordinary tyres, they found side-slip coefficients in the range 1.0–1.4 and camber force coefficients in the range 1.3–1.5. An increased tyre load and a decreased inflation pressure gave rise to a decrease of the coefficients. They also tested a tyre with markedly lower coefficients.

It appears that all reported measurement data give side slip coefficients in the same range. However, they disagree on the camber force coefficients. Roland and Lynch have remarkably low camber force coefficients, whereas Doria *et al.* report remarkably high values. Dressel and Rahman have values that are between those of the others. A further investigation of the camber force is called for. Only Dressel and Rahman discussed relaxation lengths, which were of the order of 0.1 m.

2.5. *Wobble*

As many other things, wobble in bicycles was first envisaged by H. G. Wells in his bicycling idyll ‘The Wheels of Chance’ in 1896. At present, wobble in single-track vehicles is a fast oscillation of mainly the front wheel around the steer axis, like shimmy in swivel wheels and aircraft landing gears. This mode was identified by Sharp [28] and is influenced by the tyre force properties for non-ideal tyres and frame flexibility [47,52] and can be suppressed by a steering damper. Roe *et al.* [53] elaborated more on the idea of the front wheel assembly as a caster or swivel wheel. Wobble in bicycles can easily be observed in riding with hands off the handlebar at moderate speeds and has usually a small amplitude and is harmless. There is anecdotal evidence that wobble can also occur at high speed. Very few studies in wobble for bicycles are available. Most recently, Plöchl *et al.* [54] performed some theoretical investigations and experimental validations. They found that the frame flexibility, the rider lean and the relaxation length of the tyres play a role.

2.6. *Experimental validation*

Of all the various bicycle models proposed only a few have been experimentally validated. The first was Döhring [25,26,55], who did some eigenvalue stability analysis and experiments on a motor-scooter and two different motorcycles to validate his results. Döhring uses the Carvallo–Whipple model and derives the linearised equations of motion, which are the first fully correct presented in the open literature. He finds good agreement between measured time responses of the lateral motions of the motorcycle after perturbations by the rider and simulated results on the model.

The first real experimental validation of the Carvallo–Whipple model was done by Kooij-man *et al.* [46], who used a riderless bicycle to validate the model. They found very good agreement in the frequency and damping of the weave motion in a forward speed range of 3 to 6 m/s. After that Stevens [56] used a variable geometry riderless bicycle to validate the Carvallo–Whipple model for a wide variety of bicycle geometries. He also found good agreement in the frequency and damping of the weave motion in a forward speed range of 3 to 6 m/s for all the various geometries. Tak *et al.* [57] investigated both theoretically and experimentally the effect of various parameters on the stability of a bicycle, and again found agreement between the Carvallo–Whipple model prediction and the experiments. In order to carry out rider control experiments on a treadmill, Kooijman and Schwab [58] experimentally investigated the dynamics of the bicycle on the same treadmill to confirm that no significant slip takes place at the wheel–treadmill contact. Recently, Kooijman *et al.* [24] built a bicycle without gyro and trail to experimentally validate what the Carvallo–Whipple model already predicted: that bicycles can be self-stable without the gyroscopic effect of the front wheel and without positive trail on the front wheel.

In a large experimental study on bicycle rider control at the University of California at Davis, Moore [59] built an instrumented bicycle and performed a large number of experiments with various riders, riding in a large sports hall and riding on a narrow treadmill. The results of the rider control identification process are discussed in the next section. In a separate set of experiments he tried to identify the Carvallo–Whipple bicycle model. The coefficient in the linearised equation for the roll motion agreed well with the model, but for the coefficients in the steer equation he could not find a good agreement. The author attributes this to poor understanding of the interaction of the tyre and the ground and the rider's complicated coupling to the bicycle frames in which the rider is free to use more than one actuation for control. As mentioned above, Plöchl *et al.* [54] performed some theoretical investigations and experimental validations on the wobble motions. They extended the Carvallo–Whipple model with lateral-slipping tyres and frame compliance, and also a simple pendulum-like passive rider model was added. They found that with the model they were able to identify design parameters and effects that promote an unstable wobble mode.

3. Rider control

Most of the research in rider control for single-track vehicles is devoted to motorcycles, see, for instance, the review by Popov *et al.* [6]. From a safety point of view, this makes sense because of the higher forward speeds and the larger mass of motorcycles compared with bicycles. However, recent studies [60] have shown that, although, in general, the death toll in traffic has decreased over the last decade, among cyclists this remains constant. Moreover, the number of seriously injured cyclists increases. This clearly indicates a need for rider control research in cycling.

Initially, rider control investigations were driven by curiosity: how does a rider stabilise the lateral unstable motions? After that, two major tasks were identified in rider control, namely stabilising and tracking, and researchers started to develop rider control models to understand the manual control tasks at hand. The understanding of this manual control in cycling is still in its infancy and aspects such as handling qualities and manoeuvrability of a bicycle are still under investigation. Recently, several initiatives have been started to develop rider robots for autonomous bicycles. Interesting projects in themselves, they shed little light on how humans control a bicycle.

This section starts with a review on rider control observations. Next the modelling of human rider control is discussed. Then, the experimental validation of these controller concepts are reviewed. The next part discusses the concepts of handling qualities in bicycling, and the section ends with a discussion.

3.1. *Rider control observations*

The inventor of the velocipede, von Drais [11], in 1820 was already well aware that to balance a bicycle (in this case, the Draisine or velocipede) one has to steer into the undesired fall.[1]

Prior to the invention of the safety bicycle around 1890, balance by rider steering control had also been described by many others [9]. In 1869, Rankine [15] already described how a leaned forward-moving bicycle is primarily righted by the lateral acceleration of the support line due to steering. Rankine also compared bicycle balance to that of the motion of an ice skater who, similar to a bicycle, cannot exert a lateral force without rolling over due to the single line of contact. To manoeuvre, riders manipulate this falling: to turn right they first countersteer left, inducing a lean to the right, and then later steer right in the direction of the induced fall.

Quantitative rider control observations started in the early 1970s by van Lunteren *et al.* [61], who were interested in modelling the human control actions, and chose to do this using a bicycle simulator. They had modelled the rider under normal circumstances and validated the model by system identification techniques as discussed in the following section. Even though the correctness of their simulator dynamics was questionable, they used it to determine the effect that four different drugs have on human control behaviour for stabilising the bicycle simulator. Two drugs, secobarbitali natricum (Seconal Sodium) and ethyl alcohol (vodka), showed a marked effect, increasing the time delays between the input and output of riders and strongly acting on the remnants, increasing the upper-body motion and decreasing the handle bar action. The two other drugs tested, chlordiazepoxydi hydrochloridum (Librium) and perphenazinum (Trilafon), did not have a marked effect on the riders' control actions. Interestingly, they noticed that the time delay of the handlebar action was always about one and a half times that of the upper-body action (handlebar control was found to have a time delay of 150 ms and upper-body control 100 ms), suggesting that the upper-body control is governed by hierarchically lower centres of the central nervous system than those which are involved in the control of the handlebar action. Stassen and van Lunteren [62] also performed experiments where they restrained the upper body of the rider and experiments where the upper body was free to move. By comparing the two they concluded that rider body motions are important in normal bicycle riding, however 'perhaps they are not consciously intended as a contribution in the stabilization of the bicycle', but rather are intended to control the rider's head position and orientation in space.

At the same time, a significant multi-year, multi-researcher scientific programme focused on single-track vehicle stability and control was carried out in the early 1970s by the Cornell Aeronautical Laboratory (later renamed Calspan). Much of the work on bicycles was carried out for the Schwinn Bicycle Company and has only just become publicly available. The work consisted of both modelling and experimentally measuring bicycles and motorcycles (with tyre force models) [63,64] and their control [48,65] and the comparison of experimental manoeuvres with time-series computer simulations. This was quite a feat considering the (analogue) computer system technology available at the time. Elaborate (complete system) state measurements on bicycle–rider systems have been carried by Roland and Lynch [48], Roland and Massing [63] and Roland [66], who measured rider lean, steer torque, steer angle, roll angle, yaw rate and lateral acceleration, and compared this with results from their computer simulations. The modelling of the rider will be discussed in the next section.

Almost two decades later, in 1988, Doyle [67] investigated bicycle rider control to understand to what extent motor skills necessarily involve higher functions of the cerebral cortex. Doyle investigated balance control during normal cycling for two situations: first, on a normal bicycle and second, on a bicycle where presumed self-stability factors such as gyro, trail and head-angle had been removed, which was called a destabilised bicycle. The roll angle and steering angle were recorded on both bicycles.

To get such a bicycle with no self-stability factors, he followed Jones's [68] reasoning for an 'unrideable bicycle' and constructed a machine that had the presumed front frame stabilising factors removed – a vertical head angle and no trail, no gyroscopic effect accomplished with a counter-rotating wheel and no mass offset from the steer axis by adding a counterweight. Doyle reasoned that 'without these, all movements of the front wheel come exclusively from the human control system'. More specifically, he reasoned that it eliminates the lean to steer coupling, so on the destabilised bicycle body movements have no effect on the overall motion and the system becomes a single (steer) input system.

By comparing the steer and roll angles, their rates and accelerations for entering and exiting a circular path at about 13 km/h for the normal bicycle, Doyle found that there was a 120 ms lag between the roll and the steer action, indicating that steering follows rolling. Doyle was not sure if the control is achieved through control of the rider's arms or through the bicycle's self-stability and coupled upper-body motion. Therefore, he continued with experiments on the destabilised bicycle.

The experiments with the destabilised bicycle were carried out at about 7 km/h. The riders were told simply to stabilise the bicycle and not to track a path. To assist in this, the riders were blindfolded, yet all riders tended to remove any turns automatically so that the general direction at the start was maintained. The recorded data showed a 0.2 Hz signal and a 1.0 Hz signal in the roll angle. Again, the steering signals followed the roll signals with a mean 120 ms delay. Particularly, the steering acceleration signal follows the roll acceleration signal. Thus, Doyle concluded that the basic rider control mechanism feeds back the roll acceleration, multiplied by some constant, the gain, as an angle-independent force at the handlebar. Interestingly, the recorded data indicated that the rider 'pumps' energy into the system regardless of the control requirements, which Doyle suggests is in order to increase the system output values such that they go above a threshold below which the rider cannot detect the value. He concluded that:

> Because the system delay in the roll rate is so short it is evident that the output from the vestibular system must go almost directly to the controlling muscles making little or no demand on higher cortical processes for this part of the system.

Two decades after the research by Doyle, understanding what rider control actions are performed, in particular for stabilising without a significant tracking task but also during normal cycling, was explored by Kooijman *et al.* [69]. They used an instrumented bicycle and carried out initial experiments on the open road among traffic. Extracting good data from these trials proved rather difficult owing to all the external factors influencing the control such as wind, sleeping policemen, bumps, traffic, chasing dogs, etc. Therefore, they continued their experiments indoors under controlled conditions on a large treadmill ($3 \times 5\,\mathrm{m}^2$). The bicycle was ridden by two riders of average skill at various speeds. Each rider was given enough time to adjust to riding on the treadmill before the measurements started. Three riding cases were considered: normal bicycling, towing and normal bicycling with lateral perturbations. These last-mentioned experiments were carried out to identify the effect of the pedalling motion and the effect of upper-body motion on the control. The bicycle was equipped with a camera system facing the rider and connected to the rear frame, making it possible to make a qualitative investigation of the rider's motion on the bicycle. It was concluded that very little upper-body lean occurs and that stabilisation is done by steering control actions only. However, they also observed a second control action at very low forward speed: knee movement. Moreover, they noted that all control actions, except those at very low forward speeds, are performed at the pedalling frequency, and that the amplitude of the steering motion is inversely proportional to the forward speed. Moore *et al.* [70] then repeated the treadmill experiments with a motion capture system and quantitatively confirmed the qualitative conclusions from [69].

3.2. *Rider control modelling*

In order to understand and predict the stability and handling of a bicycle–rider system, a model for the complete cycling system is required. In other words, a model of both the bicycle and rider is required. The lateral dynamics of a bicycle is well described by the Carvallo–Whipple model as benchmarked and reviewed by Meijaard *et al.* [7] and this section, therefore, focuses on the proposed rider models. The rider's influence on the system can be split into two aspects: the control action and the added system dynamics. The added system dynamics, for example caused by the rider moving relative to the bicycle, could require the vehicle model to be expanded to include these extra dynamics, such as adding a (controlled) pivoted point mass pendulum to the vehicle to simulate upper-body lean.

For the modelling of the human controller, authors have followed three roads for the design and development thereof. First, there is the classical control approach which has been extensively applied to pilot aircraft modelling. This approach is based on observations and the control is determined by system identification techniques and includes rider time delays. At the cross-over frequency (the frequency at which the magnitude of the transfer function is unity), the gain roughly has a 20 dB drop-off per decade. Continuous feedback control systems with human neuromuscular properties (dynamics) are usually included in these models. The second road that authors have travelled down is optimal control, where the rider is assumed to be an optimal controller. The method uses optimal control criteria by weighing control effort against the error in the control task. The third road is a rest class of other control strategies including fuzzy logic, neural network and very simple intuitive controllers. Authors for both the optimal control and the other control strategies have not limited their research to mimicking a human rider, but have also taken advantage of these control strategies to develop autopilots.

All three routes have been reviewed by other authors. These include [71–73], who reviewed the driver models for general road going vehicles (mostly automobiles). Popov *et al.* [6] reviewed the modelling of the control of motorcycles, while Sharp [8] reviewed the work on the control of bicycles. Here, an updated broad overview of all three routes is given for rider control in bicycling.

3.2.1. *Classical control system design*

In classical control, feedback of the outputs is used to create a closed-loop controlled system. The systems are usually multiple input–multiple output (MIMO) and linear or linearised about a given state. Sometimes, time delays are introduced. McRuer was the first to develop the classical control system approach for modelling human control. He applied it successfully to pilot control of aircrafts [74–77].

Such a classical feedback control system is shown in Figure 4. According to McRuer, experimental data for a wide variety of single and multi-loop situations show that the operator (i.e. pilot, driver or rider) adjusts his/her transfer function, Y_e^c, in each feedback loop such that the open-loop transfer function, $Y_e^c Y_c^m$, comprising the effective vehicle dynamics, Y_c^m, has, in the vicinity of the gain cross-over frequency, ω_c, the approximate form

$$Y_e^c Y_c^m = \frac{\omega_c}{i\omega} e^{-i\omega\tau}, \tag{3}$$

where τ is an effective pure time delay that includes rider neuromuscular dynamics as well as any net high-frequency vehicle dynamic lags and i is the imaginary unit. The cross-over

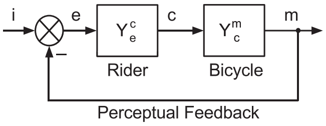

Figure 4. Single-loop control. Here, i is the reference control value; m is the actual control value; e is the control value error; Y_e^c is the rider transfer function; c is the rider output control variable; Y_c^m is the machine transfer function.

frequency ω_c depends on the product of the rider and vehicle gains. The form of Equation (3) emphasises that the rider's characteristics are optimised to the specifics of the control situation and the vehicle. However, McRuer also found that the human controller is limited in its physical control capabilities among others by the muscle dynamics and neural transport resulting in a delay. He found that a human controller can be described by the transfer function

$$Y_e^c = K_\mathrm{p} \frac{(T_\mathrm{L} i\omega + 1)}{(T_\mathrm{l} i\omega + 1)} \, \mathrm{e}^{-i\omega\tau}, \tag{4}$$

where T_L is a lead time constant, T_l is a lag time constant, K_p is a static gain and τ is an effective time delay. This in essence makes the human controller a lag–lead system with time delay and limits the systems that a human operator can control.

Although the classical control method is very promising with respect to determining the performed rider control in individual control loops, it is mathematically less suited for performing or determining multi-loop control. As bicycles and motorcycles require multi-loop control (stabilisation and path following), only a few authors have delved into this method.

Classical control methods where applied by van Lunteren *et al.* [61], Stassen and van Lunteren [62,78] and van Lunteren and Stassen [79–81], in the early 1970s in a study on the differences in rider control under specific circumstances (effect of drugs, alcohol, etc. on rider control). For the rider model, they assumed position control and carried out system identification experiments on a bicycle simulator that had been developed by themselves. The simulator was a device that could roll, steer and pedal, and initially it had no visual feedback. With the simulator, they experimentally determined the rider control parameters that fitted their position (angle) control rider model. Unfortunately, they only simulated and measured at one fixed forward speed, namely 4.2 m/s. They measured the simulator's roll angle, steer angle and the rider's lean angle. Stassen and van Lunteren concluded that the human stabilising control can be described by a proportional and derivative (PD) controller with a time delay for which the input is the frame roll angle and the outputs are the steer angle and upper-body lean angle.

The simulator was later extended with visual feedback with which they showed the rider's deviation from a pre-specified path. Further experiments then led Stassen and van Lunteren to conclude that the tracking task does not significantly alter the controller parameter values found for the stabilising task only.

Also, in the early 1970s, Roland and Lynch [48], Roland and Massing [63] and Roland [66] developed a bicycle (including a tyre force model) and rider model to study the effect of design parameters on bicycle stability and control where the end goal was to be able to perform simulations of bicycle manoeuvres. They developed a rider model that incorporated a steer and lean torque, delayed proportional and integral and derivative (PID) controller. It was implemented as a simplification for a human lead–lag controller model based on the literature [82]. The developed controller was not well documented, but it had both tracking and stabilising control loops [48]. The rider lean torque and steer torque were the outputs for both the stabilising and tracking controller. The stabilising controller inputs were the roll angle, roll rate and roll acceleration. For tracking control, the vehicle path and heading error information were also required. The tracking controller predicted the path based on the current state and comparing this with the desired path and then generated an additional roll angle that was added to the desired roll angle. Roland tuned the coefficients of the stabilising controller by investigating the system's response to driving straight ahead and applying a 20° command roll angle, thus simulating driving straight ahead and going into a curve of constant radius. However, even for the best controller he had an offset between the desired and obtained roll angle.

To our knowledge, Roland never used the rider model that he developed for comparing real manoeuvres with simulations, but Rice [83] later used the controller for the simulation of the same manoeuvre with a motorcycle and riders of different levels of experience. The

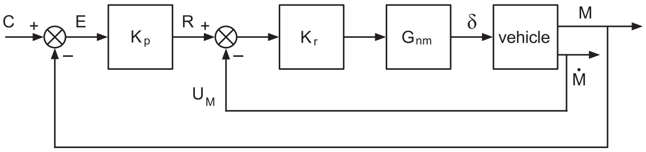

Figure 5. Proposed bicycle rider model by Hess *et al.* [84] for a single-axis tracking task. Here, M and $\dot{M}$ are the bicycle output and output rate response for the variable being controlled, and C is the desired value of M. G_{nm} represents the rider neuromuscular dynamics (highly simplified). The gains K_{p} and K_{r} are chosen such that a specific bandwidth (K_{p}) and a specific level of damping at an oscillatory mode (K_{r}) are achieved.

model, however, did not compare well with experiments, except in the transient stage of the manoeuvre.

Finally, in a recent theoretical study to introduce a task-independent handling-quality metric (HQM) to bicycle control, Hess *et al.* [84] directly applied a HQM from aircraft handling studies using the classical control method to bicycling, see the model in Figure 5. They propose that handling qualities of bicycles can be reflected in the maximum magnitude of the transfer function between the inner-loop rate feedback of a variable (U_M) and the command input (C). To remove the effects of control sensitivity, they normalise the equation with the magnitude of state feedback gain K_{p}.

$$\mathrm{HQM} = \left|\frac{U_M}{C}(i\omega)\right| \frac{1}{|K_{\mathrm{p}}|} \quad \left[\frac{1}{s}\right]. \tag{5}$$

Hess *et al.* directly import highly skilled and trained pilot properties from aircraft handling research into the cycling situation including pilot gains and time delays. In this study, different bicycle models were evaluated on handling for a double-lane change manoeuvre but no significant differences were found among them. It is unclear if such a direct implementation of the pilot is possible in the cycling situation as there are definitely differences between the tasks of a bicycle rider and those of an aircraft pilot. Certainly, turning the handlebar is different from controlling the (joy)stick. However, the methodology is encouraging and hopefully will be validated experimentally in the near future.

3.2.2. *Optimal control*

Optimal control deals with finding a control law for the system such that it optimally fulfils certain criteria. The control law minimises a cost function, which is a function of the state and the control inputs. The optimal control concept is capable of treating multi-variable systems within a single conceptual framework using state-space techniques [85]. A drawback of these optimal control methods is the weighting aspects for the desired input and output signals in the formulation of the cost function, and therefore the objective optimal control method can have a subjective nature if the cost function cannot be defined on the basis of objective criteria.

Kleinman *et al.* [86] and Baron *et al.* [87] were the first to fully develop the idea of optimal control for describing human (manual) control back in 1970. They hypothesise that the human operator works in an optimal manner when carrying out a compensatory control task, but that the actions of the human are bound by human limitations such as time delay and neuromuscular lag. They derived linear feedback for MIMO human operator models based on the gains calculated from minimising the cost function and comparing these with actual measured tasks performed by aircraft pilots.

Interestingly, the skateboard model by Hubbard [88] shows dynamics that is similar to that of the bicycle, with a coupling between the lean and the steer and a dynamically unstable speed range. Hubbard [89] applied full state feedback linear quadratic regulator (LQR) to the stabilising and tracking control of the skateboard. The human control of the skateboard is modelled by body lean relative to the skateboard. No human limitations are set and the dynamics of the skateboard itself are neglected. The roll angle of the skateboard is taken as the control input. The analytically derived results were compared with some experiments, which shows qualitative agreement in the time series. Future plans are to apply system identification techniques in order to determine the feedback gains.

Only a few optimal control investigations for bicycles have been performed. Schwab *et al.* [90] used a similar LQR controller as Hubbard [89] with full state feedback, which was implemented in two different models to investigate the effect of a leaned upper body on the control required to stabilise a bicycle. In the first model, they investigated a rider rigidly attached to the frame of the bicycle and they showed that the system can be easily stabilised through steer torque control, but that at low speeds, the roll feedback gains become unrealistically large. In the second model, the rider was modelled with a leaning upper-body as an inverted pendulum. They found that adding a pivoted upper body does not greatly affect the eigenvalues or eigenmodes for the uncontrolled system. However, at low speeds, the upper-body lean requires large upper-body lean feedback gains and, similar to the rigid rider case, large roll feedback gains are required for the steer torque. Unlike the rigid rider case, they found that at high speeds significant steer and rider lean feedback gains are required for both the upper-body lean control and the steer torque control. Furthermore, they found for the situation where the stabilisation only takes place by the upper body (hands-off situation) that huge unrealistic feedback gains on all states are required at low speeds, suggesting that lean is unlikely to be used when steering is possible (hands-on situation).

Connors and Hubbard [91] investigated the effect of pedalling on the steering control torque for a recumbent bicycle and modelled the rider's control to balance the bicycle as LQR steer torque optimal control. They found that for a recumbent bicycle, the oscillating legs can drastically increase the roll angle sensitivity and the steer torque required to balance the bicycle. Basing on their findings, they devise a gear-shifting strategy to lower the pedal cadence frequency at higher speeds to reduce the control effort at high speeds ($>15\,\mathrm{m/s}$).

Sharp theoretically investigated LQR optimal control tracking, with preview, for bicycles [8,92]. This study was based on his motorcycle research [93,94], but now implemented on the benchmark bicycle [7]. For the path following simulations, he looked at two different tests: a random road and a straight section into a circular path (90°) followed by another straight section. Different weighting factors for tracking errors against control power were investigated. The feedback gains are clearly speed dependent but again become unrealistically high with reduced speed. Sharp concludes that the necessary preview time, as opposed to the motorcycle case, depends very little on speed. Therefore, for bicycles the preview distance is roughly proportional to the speed. Furthermore, he concludes that tight (precise) control requires about 2.5 seconds of preview independent of the forward speed.

Finally, we mention the mixed H_2/H_∞ controller design for the stabilisation of a bicycle robot using gyroscopic precession by Thanh and Parnichkun [95]. They chose to apply the control method to a flywheel to ensure that the bicycle can balance at all speeds including zero speed. The decision to apply an H_2/H_∞ controller was motivated by the good robustness of the optimal controller for systems with uncertainties and not intended to mimic a rider in any way. However, optimisation of such a controller ends up as a complex non-convex problem and for this reason, they apply a particle swarm optimisation algorithm as it enables fast and structured optimisation routes. The control was implemented on a real bicycle and the system was shown to be stable when the bicycle was both stationary and moving slowly in forward and backward directions.

3.2.3. *Other control*

A variety of different control methods have been proposed by authors to model riders. These are discussed below.

Intermittent control: Doyle [67] used bicycle stabilisation observations to develop a stabilising bicycle rider model with intermittent control (as discussed here in Section 3.1). He developed classical controllers for the dynamic model of the bicycle and applied numerical integration to get solutions in time. He found that a stabilisation of the bicycle that worked in a similar manner to that observed with real riders can be achieved by steer torque control with continuous feedback of the roll rate and roll acceleration, in combination with intermittent roll angle feedback. The intermittent feedback takes place in the form of a pulse torque that is triggered when the roll angle exceeds a certain threshold value, which he finds to be 1.6° in his observations.

Intuitive control: An intuitive bicycle controller was developed by Schwab *et al.* [90] for balancing a bicycle using the principle of 'steer into the undesired fall'. They investigated two situations: in the first, the rider is rigidly attached to the rear frame and in the second, the rider has a movable upper body modelled as an inverted pendulum connected to the frame with a passive torsional spring. In both cases, they applied a simple steer torque control law: at low speed, they applied proportional feedback of the roll rate with the gain increasing with decreasing speed and above the stable speed range proportional feedback of the roll angle with the gain increasing with the speed. For both situations, they showed by an eigenvalue analysis that the system can have marginal stability for almost the complete forward velocity range. Furthermore, they found that the controllers in both situations require far more realistic steer torque feedback gains than the values found with an LQR optimal control technique for the same models.

Fuzzy logic: Chen and Dao [96–98] developed a number of steer torque controllers based on fuzzy logic of increasing complexity for a bicycle. First, they developed a PID steer torque controller for stabilising the bicycle where the PID gains are set but the control values are determined via the fuzzy logic controller placed in series. Then, they investigated roll angle tracking by introducing a second fuzzy logic controller placed in parallel to the stabilising fuzzy logic controller. Finally, in [98], they optimised the fuzzy logic controllers using a genetic algorithm. They proposed a strategy to optimise the fuzzy logic controllers by keeping the rule table fixed but tuning their membership functions and by introducing scaling factors and deforming coefficients. In this way, the number of parameters to be trained can be reduced to speed up the learning process. They verified their control schemes with simulations and found good correspondence.

Neural network: Cook [99] devised a neural network controller with only two neurons as an example of a simple human bicycle tracking controller. The first neuron is a proportional controller on the heading with a threshold function. It outputs a desired roll angle, which is an input for the second neuron which in turn outputs a steer torque based on PD control. The desired heading is set using way points enabling the bicycle to perform complex tracking tasks. He finds that the controller is relatively robust as gain values weakly depend on speed and do not have to be perfectly adjusted to the specific bicycle. The controller works at a range of velocities but it fails at low speed.

Inverse dynamics: Controllers with a state observer can be used to predict the future motion of the vehicle based on the current state and inputs. Inverse dynamics is used to determine the forces required to pursue a desired course based on the current state. Getz [100] developed an inverse dynamics method he calls dynamic inversion which he applies to bicycle control [101,102]. The controller determines, on the basis of the desired path or roll angle of the bicycle as a function of time, the forces that have to be applied to the steering system as a function of time. Getz illustrated the potential of his controller on an oversimplified bicycle model in a number of exemplary paths such as a straight path at constant speed, a sinusoidal path, a circular path at constant speed and a figure-eight trajectory. Each example starts with an offset from the track and all show countersteering effects. Such an inverse dynamic method is certainly of interest for determining the performed control by a rider based on a traversed path, but it is probably less suited for determining the control a rider will perform based on a current state and some roughly described desired path.

Forward dynamics: In forward dynamics, the motion of the system is determined from the applied forces and the equations of motion, which then include the controller. Von Wissel and Nikoukhah [103] applied forward dynamics to investigate the control of a bicycle with a multiple-stage obstacle-avoiding optimisation method. They find trajectories for a bicycle in a complex environment consisting of a grid with ordinary cells, forbidden cells and end cells. The bicycle has a constant forward speed and a number of discrete steer torque manoeuvres can be applied. The selection of the manoeuvres uses multiple stages. Therefore, a large tree of path possibilities evolves. The manoeuvres that make the bicycle unstable are discarded first; after this, manoeuvres that cause the bicycle to come into forbidden cells are discarded. Finally, all but the trajectories that penetrate in the end cells with the lowest value of the cost function are discarded. The method uses a moving window method for the path tracking to reduce the computational power required by moving the end cell(s) through the complex space. This speeds up the computing as branches can be deleted along the way and then the simulation can be restarted. They give interesting examples implementing the method and showing how an optimal path changes with the movement of the end cells. In the given example the method is applied at a high (stable) forward speed of 8 m/s removing the need for a lateral feedback controller. While it is not clear if the method is directly applicable to a human rider as a rider most likely does not compute all possible paths when determining which route to take, it does give interesting insight into possible route choices and is certainly interesting for automatic vehicles.

A similar approach was used by Cook [99] for determining the stabilising path of a bicycle. He did this for an unstable forward speed, beneath the weave speed. At each time step, the effect on the trajectory of a handlebar push to the left, right or no push is calculated. The process is repeated at each time step for each path. Each path is evaluated until the bicycle has fallen over and that path ends. This leads too to a large tree of possible paths. The control applied in the example that Cook gives, however, was unable to stabilise the bicycle over a long distance.

3.3. *Experimental validation*

Very little experimental validation of rider models has been done in bicycling: most of the rider model identification in single-track vehicles has been devoted to motorcycles [6]. While many have developed rider models for bicycling, only a few authors have gone to the expense of actually validating them. Stassen and van Lunteren [62,78] were the first to carry out many experiments with riders (on a bicycle simulator), but they never explicitly validated their models. However, as they used the experiments to identify the model parameters, they thereby implicitly validated them. Nevertheless, a number of authors including Eaton [104], Roland and Lynch [48] and Koenen *et al.* [105] were critical of the work. In particular, the used simulator and the steer angle control as opposed to steer torque control were doubted. In [104], Eaton writes

> While the research represents a pioneering effort in obtaining transfer functions experimentally (with a bicycle simulator), it should be pointed out that van Lunteren's major interest was the performance of the human operator under various conditions (drugs etc.) and not the dynamics of the bicycle. Thus, the accuracy of the simulator

> dynamics with respect to real bicycles and the validity of the assumption of steering angle control (rather than steering torque) are questionable.

Stassen and van Lunteren deemed the simulator sufficient for the intended purpose but the fact that riders had to learn to ride the bicycle simulator is an indication that the control of the simulator was probably not the same as on a real bicycle. Furthermore, recently de Lange [106] discovered some sign errors in the models of Stassen and van Lunteren, and he discovered that the complete model of the bicycle with the identified rider model is unstable, even after having corrected sign errors in the equations.

Doyle [67] too developed bicycle rider models based on experiments. However, his experiments did take place on real bicycles. He compared the state time-series results of a number of control models with measured data and found that it is insufficient to use an average or filtered roll velocity as a feedback signal: the actual roll velocity has to be fed back. Furthermore, he found that continuous feedback of the roll angle gives very different simulations from what is seen in real life; instead, intermittent torque pulses are required to stabilise the bicycle. He observed that impulsive control is applied when the roll angle exceeds the threshold of 1.6°. Doyle also found that

> if an attempt is made to control the system by responding to absolute angle without any velocity feed-back then after one or two reversals the velocity reaches such a high value that excessive lean angles are generated before control takes effect.

This was also found by de Lange [106], who made a simple desktop computer-game-style bicycle simulator. The game player had a gamepad to apply steer torque and pedalling force. He based the dynamics of the bicycle on the benchmarked linearised equations of motion from [7] and for the visual feedback showed a first or third person's view of the bicycle moving on a flat surface. He found that it was impossible to stabilise the bicycle except by applying impulsive steer torque inputs triggered by an extra roll rate indicator on the screen.

Roland and Lynch [48], Roland and Massing [63] and Roland [66] developed a rider control model that actively controls the upper body in a closed-loop manner in which the rider model feeds steering torque and lean torque inputs to the vehicle dynamics model in response to vehicle roll motion information (for stability) and to vehicle path and heading error information (for guidance). Roland, however, did not use the algorithm, but instead he used a very simple guidance control algorithm for a slalom manoeuvre: the sign of the command roll angle (set at 20°) is opposite to the sign of the current steer angle. This gave very similar qualitative results between model and experiment. However, it is unclear if this was a 'lucky shot' that the actual slalom manoeuvre looks similar, or if this really is a good model for the control carried out by a rider.

Recently, Moore [59] and colleagues at the University of California at Davis performed an extensive experimental validation of the rider control model as described by Hess *et al.* [84]. They built an instrumented bicycle and they identified the rider control model in a large number of experiments with various riders riding on a narrow treadmill and riding in a large sports hall. The tasks were either balancing or balancing and tracking. They found that the fundamental, remnant-free, control response of the rider under lateral perturbations can be described reasonably well by the simple five gain sequential loop closure and an eighth-order closed-loop system. No time delays are needed and the continuous formulation is adequate for good prediction. Moreover, the identified gains seem to exhibit linear trends with respect to speed as predicted by theory and the identified neuromuscular frequency seems to be constant with a median around the theoretical prediction of 30 rad/s. The identified parameters show resemblance to the patterns in the theoretical loop closure techniques, especially in that the riders select their gains such that the closed roll rate loop exhibits a 10 dB peak around 10–11 rad/s and that the riders cross over the outer three loops in the predicted order.

Unfortunately, they combined all measured data in the identification process, which makes it impossible to discriminate between riders and between tasks. In a separate set of experiments, they tried to identify the Carvallo–Whipple bicycle model. The coefficient in the linearised equation for the roll motion agreed well with the model but for the coefficients in the steer equation they could not find a good agreement. The authors attribute this to poor understanding of the interaction of the tyre and the ground and the rider's complicated coupling to the bicycle frames in which the rider is free to use more than one actuation for control.

3.3.1. *Rider robots*

A substantial amount of work has been devoted to building and testing autonomous bicycles, i.e. bicycles controlled by rider robots. In general, four main methods have been used to stabilise and control the bicycle: steer control; a moving mass; a gyroscope; and a combination of these first three methods.

Steer control: Stable control of single-track vehicles has been achieved using either steer torque or steer angle control for both bicycles and motorcycles. However, the vehicle states that were used in the feedback loop and the speed-dependent feedback gains were different for the different approaches. The first to develop a robotic motorcycle using only steer actuation were Ruijs and Pacejka [107], who used steer torque control based on Sharp's [28] motorcycle model with a tyre force model and leaning rider, but they did not include a leaning rider in their hardware. Others that have used steer torque control for robotic single-track machines include Saguchi *et al.* [108,109], who based their bicycle rider robot on a Getz style bicycle model [100] with added tyre slip force model. Bicycle robots of Michini and Torrez [110] and Andreo *et al.* [111,112] were based on the benchmark bicycle model [7]. Out of these robots, only Saguchi *et al.* investigated tracking control for straight-ahead running and constant curve motion, while the other three only investigated stabilisation control. Ruijs and Pacejka, however, were able to set the roll angle by a remote link and thereby to make the motorcycle follow a path.

While at least three of the four used velocity-dependent feedback gains (it is unclear if Michini and Torrez calculated feedback gains for multiple speeds or used the same feedback gains for the two speeds that they tested at), each used a different combination of a set of feedback signals and control strategy. Ruijs and Pacejka used pole placement for proportional control on the roll angle, roll rate and steer rate. Similarly, Michini and Torrez used proportional control, but they only used the roll angle and roll rate and it is unclear if they used pole placement in determining the gains or some other method. Andreo *et al.* used linear parameter varying (LPV) state feedback control for which they measured the forward speed, roll rate and steer angle and calculated the roll angle through integration. Saguchi *et al.* implemented roll angle tracking by optimal control on the difference between the desired and the actual roll angle, and implemented stabilising control, using proportional feedback of the roll angle and rate, steer angle and rate, yaw angle (ω) and slip angle (β), where ω and β are estimated by a Kalman filter.

Despite these major differences, all four projects achieved very encouraging results. Ruijs and Pacejka's motorcycle robot was shown theoretically to be stable between 5 and 60 m/s and experimental tests proved that motorcycle was in fact stable from 2.8 m/s up to at least 30.6 m/s. The robotic bicycle by Michini and Torrez was shown to stabilise the uncontrolled motion at both an unstable speed ($\mathrm{Re}(\lambda_{\mathrm{weave}}) > 0$) and a neutrally stable speed (($\mathrm{Re}(\lambda_{\mathrm{weave}}) = 0$) despite the fact that they calculated their feedback gains using bicycle parameter values from Kooijman *et al.* [46], a totally different bicycle with a much lower mass. With their LPV controller Andreo *et al.* showed that their bicycle was able to stabilise at low speeds (from 1.7 down to 1.0 m/s) and to balance despite an external impulsive roll torque disturbance (for speeds from 2.1 to 1.7 m/s). Saguchi *et al.* demonstrated stable behaviour for a vertical roll angle target and for a 10° roll angle target (steady cornering) at around 2.5 m/s. They also compared experimental results with simulations for straight-ahead running with a lateral impulse on the rear frame and found very good agreement.

Two projects have successfully implemented steer angle (position) control for a bicycle: Tanaka and Murakami [113,114] and Yamaguchi [115]. Tanaka and Murakami [113,114] based the control of their robotic bicycle on the dynamics of a theoretical point mass model of a bicycle (Getz-like [100]) with no steering dynamics. They too implemented separate controllers for stabilising and tracking in series. The stabilising controller consisted of a PD controller (again roll angle and rate). Two path tracking controllers were implemented. First, a lateral velocity controller was tried, based on proportional control with respect to the (set) lateral velocity, which they found to destabilise the posture control as a result of unmodelled dynamics in the system. Second, they implemented a more robust proportional controller using the desired rate of change of path curvature per path length as the control variable. This tracking controller in combination with the stabilising controller was found to be stable. Yamaguchi [115] recently applied 'steer into the lean' control to stabilise a scaled-down bicycle by a biped robot that can pedal and steer. The biped robot uses PID control of the roll rate signal from a gyroscope in the robot and uses servos to actuate the joints in the legs and arms. The bicycle is stabilised by the robot, but the general heading is remotely controlled by a human. No information is available in the open literature about this bicycling biped robot as yet and thus it is unclear if it really uses steer angle control.

Moving mass Control: Theoretical results indicate that stabilisation and tracking using only lean torque, by an inverted pendulum or laterally moving mass, is far more difficult than through steer control as far larger gains in the feedback are required. Only two projects have attempted to stabilise a bicycle using lean torque control. The first was van Zytveld [116] in the mid 1970s, who applied lean torque control to an inverted pendulum placed on a bicycle powered by a petrol engine. The project failed to stabilise the bicycle owing to the neglected geared inertia of the used electric motor.

The second, larger project which is still ongoing, by Yamakita and Utano [117], Yamakita *et al.* [118], Murayama and Yamakita [119], Yamakita and Lychek [120] and Lychek *et al.* [121], has taken advantage of the modern, better controllable, electric bicycle as the platform on which they have applied their pendulum control. They are particularly interested in stabilisation at extremely low speed and the possibility of a stable adjustment of the vehicle's vertical orientation, which is not possible with a gyroscope, and to track a desired vehicle orientation path in time, called posture tracking. Therefore, they model the bicycle as a double inverted pendulum, with a roll and a lean angle and no steering, to carry out stabilisation control at stationary and very low speed (<2 m/s). While the balancing of a bicycle by an inverted pendulum model is interesting, it only works at low speed. The faster the vehicle moves the less it looks like a double pendulum due to its ability to steer.

Yamakita *et al.* independently implemented two separate controllers: a non-linear controller for the stabilisation and a linearised input–output controller for posture tracking. They noted that as the two controllers were developed independently and used the same dynamical system they will cause some oscillations and offset to the balance control. Therefore, a shift on the lean angle and rate set point for the balance algorithm should be applied. Interestingly though, they do not need to perform this offset in either their simulations or experiments as the bicycle performs well enough without it. They carried out simulations to show that the bicycle is indeed stable and can track an orientation. The experimental machine confirmed this, but they had to use a modified control algorithm by adding an H_∞ controller in the feedback loop for robustness as the theoretical controller did not perform well on the experimental machine. The robustness of the controller was demonstrated by using the same controller in stationary and low-speed trials [118,119]. But as yet no successful stabilisation and tracking control has been implemented at a wide range of speeds by moving mass control. In later studies, Yamakita *et al.* theoretically implemented trajectory tracking [120] using steer torque control and recently they adapted the moving mass controller and apparatus such that it can also be configured to act as a gyroscope [121].

Control with a gyroscope: Single-track vehicles with some form of gyroscope control applied to them have been successfully implemented by a number of authors including [95,122,123]. Either the gyroscopic precession or the adjustment of the gyro speed supplies the required torque to keep the bicycle upright. The use of fast spinning gyros is mainly credited by the authors to the extreme level of continuous stability that this method can produce when compared with steer control and moving mass control for stationary and slowly moving vehicle. The speed range that the authors generally investigate and apply control to is therefore also generally stationary and at low speed ($<1\,\mathrm{m/s}$). Different types of controllers for controlling the required gyroscopic precession have been implemented such as the ones based on root locus [123] and H_2/H_∞ control [95]. Active stabilisation by the adjustment of the gyro speed has been implemented by Murata [122], who made a robot riding a miniaturised bicycle using a gyroscope inside the robot's torso. The bicycle itself is not controlled, but the robot measures its orientation and calculates its centre of gravity and accelerates the gyroscope such that the centre of gravity comes over the wheel contact line. The resulting bicycle motion is very unnatural as can be expected from such a stabilised system as the rider has to remain in an upright position at all times to prevent the gyro from reaching its 'top' speed and therefore no longer being able to provide the required stabilising torque. Although both precession and spin rate actively controlled gyroscopes have been shown to work, neither though seems to be an ideal candidate for automatic control due to the required power.

Strictly speaking, the passive implementation of a gyroscope to slow down the dynamics of the vehicle is not control in a strict sense; however, it can be used in combination with other forms of control such as steer control. This is the core of the Gyrobike [124] product, where a fast rotating gyroscope inside the bicycle front wheel is used to reduce the level of instability of children's bicycles enabling the child to learn to cycle without the bicycle falling over as quickly as would happen without the gyroscope. By changing the gyroscope's rotational speed, the level of stability is adjusted. The added value of such a stability enhancement tool is questionable though. The gyroscope changes the dynamics of the bicycle significantly, so the user still has to get used to a normal bicycle without a gyroscopic front wheel.

Multiple control methods: In multiple control methods, both steer and lean are used for the stabilisation and tracking. Nagai [125] and Iuchi *et al.* [126] both applied lean torque (position) control through the use of an inverted pendulum. In the experiments, Nagai placed the bicycle on a treadmill and Iuchi on rollers, so the extent to which 'tracking' was taking place is debatable. However, both did carry out stabilisation experiments, while Iuchi's tracking task was simply to keep the bicycle on the rollers, and Nagai performed lane change manoeuvres. To investigate the required control for stabilising and tracking both Nagai and Iuchi developed simplified linearised equations of motion for a bicycle with a leaning upper body. Nagai used a point mass bicycle (massless front frame and wheels) while Iuchi used a very simple double-pendulum model. For the control, Nagai used the lateral deviation from a preview point (as a function of steering angle) and the roll angle as control variables and Iuchi used the roll angle and roll rate of the bicycle as the reference inputs. Nagai found good agreement with his models except for the situation in which only leaning for tracking control was used. This difference between the simulation and experiments he contributed to backlash and large time delays in the experimental system. Basing on his lane change experiments, Nagai concluded the moving mass reduces the time required to carry out a lane change manoeuvre, but it also increases the size of the steer and roll angle response. Iuchi had to implement completely different control gains on the experimental bicycle compared with the model, in order to stabilise the bicycle, and even then he was not able to keep the bicycle on the rollers for long periods of time. This led him to conclude that the used bicycle model does not consider the physics sufficiently. From the above, it appears that it is essential to use a model that describes the motion of the vehicle sufficiently. Even with a relatively simple model (Nagai) good results can be achieved with a very simple controller for both stabilisation and tracking. On the other hand, when the model is probably not sufficient (Iuchi) far more advanced control models are required and still the results can be mediocre.

3.4. *Handling qualities*

The handling qualities of a vehicle are related to its stability and control characteristics. A vehicle's manoeuvrability is related to its ability to perform a specific (set of) manoeuvre(s).

The aircraft industry obviously had most to gain from research on handling qualities. Each airplane has to be controlled precisely in order to be able to land safely, fighter aircraft have to be highly manoeuvrable to avoid being shot down, yet still have to be controllable for the pilot, and many early aircraft suffered from pilot induced oscillations during flight. Thus, it is not surprising that this is also where most of the insight into handling qualities initially was developed. Cooper and Harper [127] were the first to precisely define what they mean by handling qualities of aircraft, namely: 'Those qualities or characteristics of an aircraft that govern the ease and precision with which a pilot is able to perform the tasks required in support of an aircraft role'. Here, they defined 'task' as 'the actual work assigned to a pilot to be performed in completion of or as representative of a designated flight segment' and 'role' as 'the function or purpose that defines the primary use of an aircraft'.

Cooper and Harper stated that both physical and mental workload need to be taken into account when rating a handling quality. They argued that a pilot can perform specific manoeuvres just as well in very differently behaving aircraft and that the measurable physical workload can be identical but that the mental workload can be very different. They, therefore, developed a 10-scale pilot rating system (shown in Figure 6) for determining aircraft handling qualities which became the norm for the industry and beyond. This rating system takes the mental workload into account.

Correlations have been found indicating that handling qualities can be linked directly to control effort. McRuer and Jex [128] and Hess [129] found that the pilot's perception of the task difficulty and therefore of vehicle handling qualities are highly correlated with the 'power' of the pilot's output-rate feedback signal. They, therefore, only looked at the physical workload and used it to define the handling qualities. This changes handling qualities to a control feedback problem. They found that the complete closed-loop system tends to act as a first-order system (20 dB per decade drop-off in a Bode plot) around the cross-over frequency and where the desired bandwidth is achieved by the pilot's control effort.

The most significant difference between an aircraft and a bicycle, with regard to designing for handling qualities and control strategies, is their ratio of pilot/rider to vehicle mass. The mass of a bicycle rider is usually around 80% of the total mass. On the other hand, for a fighter aircraft the pilot mass is typically less than 1% of the maximum take-off weight. The position, orientation and exact mass of a rider on a bicycle or motorcycle have a far greater influence on the open-loop dynamics of the system than they do in an aircraft. Furthermore, any motions executed by an aircraft pilot that do not disturb the control stick or rudder pedals will have little to no effect on the aircraft's trajectory, while for a bicycle or motorcycle the body motions that do not directly disturb the handlebar can still cause a trajectory change of the vehicle as a result of the lean to steer coupling and relatively large mass of the rider.

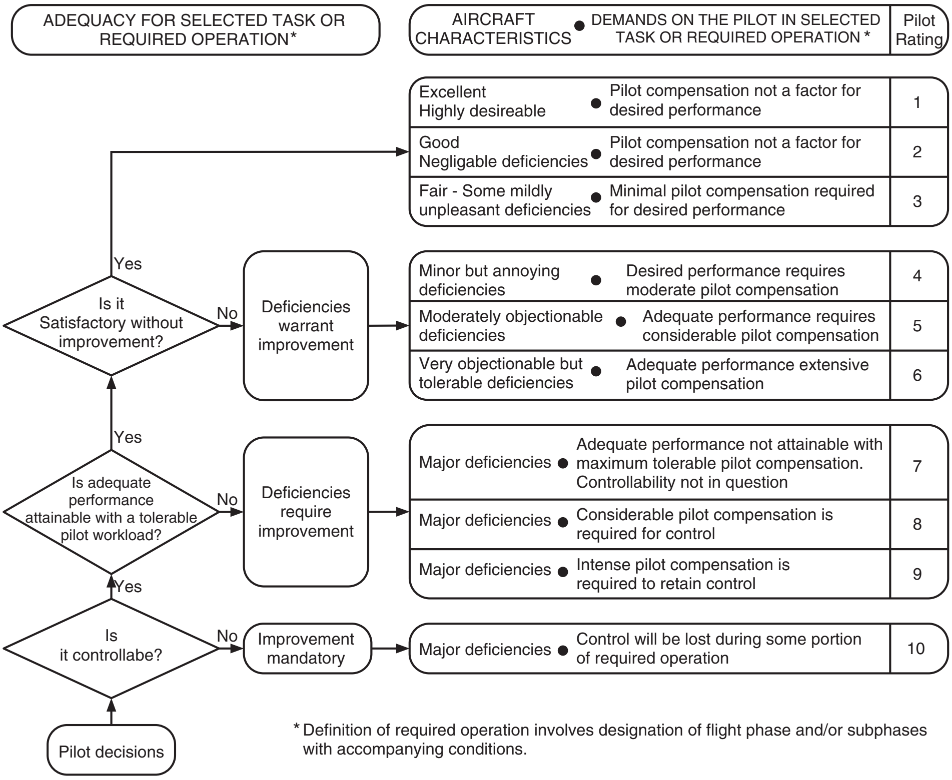

Figure 6. The Cooper–Harper handling qualities rating scale, from [127].

Bicycle designers can only develop the machine part of the complete man–machine system. They design their machines generally for the following roles:

(i) accident avoidance manoeuvres: safety aspects; and
(ii) normal riding: the safe use in and among traffic while obeying the traffic rules.

The first role is more a ‘segment’ of the ‘designated flight plan’ for which the designers would want to achieve maximum performance in all circumstances and this has (logically) received most of the attention. The last two roles generally lead to very differently looking vehicles, largely as a result of ergonomic and aerodynamic aspects. The next sections discuss these three roles in order.

3.4.1. *Handling qualities for accident avoidance: safety*

Most safety related handling quality work has been done experimentally where the complete system, bicycle and rider, was tested as a whole. No standard tests were used, which made direct quantitative comparisons between experiments impossible. The various tests are shown in Table 1. The ‘tasks’ that have to be performed for safety are often categorised under the general terms ‘manoeuvrability’ and ‘stability’. None of the studies actually defined what they exactly meant by these general terms; however, the general gist can be extracted from the experiments they carried out. A number of studies were carried out to investigate the effect of different parameters on the ‘stability’ and ‘manoeuvrability’ of bicycles. Rice, Roland and Lynch [48,134] first investigated the lateral stability and control of two types of bicycle

Table 1. Bicycle manoeuvrability and stability experiments and how the performance is rated.

Term	Authors	Experiment	Performance measure
'Manoeuvrability'	Mortimer *et al.* [130]	Slalom at 5, 8, 10 and 12 mph and maximum speed	Crossing boundary and cones, maximum speed
	Arnberg and Tyden [131]	Block slalom, block pairs, 1-handed curve, 'relay' riding and steady-state circle	Time + interview
	Godthelp and Buist [132]	Complex slalom	Time
	Godthelp and Wouters [133]		
'Performance'	Rice and Roland [134]	Slalom	Minimum time
'Control'	Roland and Lynch [48]	Slalom	Maximum speed
	Mortimer *et al.* [130]	Circle, figure-eight, lane change	Time
		10 mph, 90° corner	Minimum radius
	Arnberg and Tyden [131]	Stationary balance, ride between 2 narrow gates: (a) constant speed, (b) accelerate from rest	Time + interview
High speed 'stability & manoeuvrability'	Godthelp and Buist [132]		
	Godthelp and Wouters [133]	Straight + bend with either left, right or both hands on handlebar	Time
Medium/High speed 'stability'	Mortimer *et al.* [130]	Straight between two lines	Boundary crossings
	Arnberg and Tyden [131]	Looking backwards over shoulder for a number	Boundary crossing, recalling number
	Godthelp and Buist [132]	Straight between two lines	Relative time between lines
	Godthelp and Wouters [133]		
Low speed 'stability'	Rice and Roland [134]	Hands free straight ahead	Minimum speed
	Roland and Lynch [48]	Straight line hands on	Minimum speed
	Mortimer *et al.* [130]	Straight between two lines	Boundary crossings
	Arnberg and Tyden [131]	Straight between two lines	Time + interview
	Godthelp and Buist [132]	Straight between two lines	Relative time between lines
	Godthelp and Wouters [133]		

that were popular in the late 1960s and early 1970s and then investigated the effect of nine parameter changes on the same instrumented bicycle (load on the rear, the rider and the front, increasing the mass moment of inertia of the front wheel and under-inflating the tyres) for four experiments (straight line, obstacle avoidance and a narrow and wide slalom). They concluded that the standard bicycle is the best and that load in the rear placed low is good for 'manoeuvrability' while load on the rear placed high is bad for 'manoeuvrability'.

The effect of different style handlebars (high rise, standard and racing) on the 'manoeuvrability' of the bicycle was experimentally investigated by Mortimer *et al.* [130]. Riders carried out seven experiments and rated each bicycle and each task on a five point scale. They concluded that

> ... since the high rise handlebar configuration allowed good maneuvering performance it should be considered an acceptable design. Standard handlebars offer a good compromise between the characteristics of the racing and high rise types, and provided stable, low-speed tracking which is important for safe riding on streets in the mix of other traffic.

Godthelp and Buist [132] and Godthelp and Wouters [133] developed a bicycle in which they could change geometric parameters such as the wheelbase, trail and moments of inertia of front frame and wheels, and carried out four experiments in each configuration (Table 1). They also carried out these experiments with four different styles of bicycles and four different styles of mopeds) and concluded that all bicycles had the same high-speed stability. For low-speed stability and manoeuvrability they concluded that the rider position was dominant and once again the racing bicycle and the high-rise handlebar bicycle were the worst.

The effect of different riding positions and bicycle styles on a child's ability to control a bicycle safely in traffic was investigated by Arnberg and Tyden [131]. They used the time to complete an experiment as a performance measure in 10 tests to measure the 'stability' and 'manoeuvrability' of six different bicycles when controlled by children for three styles (normal, collapsible and rodeo) of bicycle with two types of handlebar (normal and high rise). They, similar to Mortimer *et al.* [130], concluded that bicycles with extreme handlebars have a poorer manoeuvrability performance than those with standard handlebars and that the race handlebar makes the bicycle the least manoeuvrable while high-rise handlebars are all right. Also, similarly to Roland and Lynch [48], they concluded that the rodeo style bicycles (centre of mass high and to the rear) have the worst manoeuvrability performance out of the three tested models.

Similar safety experiments with young children were carried out by Wierda and Roos [135] and Wierda and Wolf [136] to investigate 'manoeuvrability'. However, they did not measure the time the rider used to complete the experiments; instead, they only recorded the errors made as they view 'safety' completely from the traffic point of view: to safely ride on the roads, the rider should be able to carry out the specified manoeuvre in a specific section of the road as any deviations could result in contact with another road user. They conclude, interestingly enough, that there are no major differences in 'manoeuvrability' between the different bicycle styles for children.

3.4.2. *Handling qualities for normal riding*

Normal riding refers to the bulk of a vehicle's usage on open roads and not under extreme circumstances or at the performance limits.

Bicycle research on this role is surprisingly scarce. However, many have hypothesised that a self-stable bicycle is preferred over an unstable one as the unstable bicycle requires active rider control to be stabilised [9]. Herfkens [27] carried out bicycle model parameter investigations in the late 1940s. He concluded that to increase the low-speed stability of a typical Dutch bicycle the head angle should be increased, the trail decreased, the mass of the front frame decreased and the mass of the front wheel increased.

Jones [68] in a quest to discover what makes a bicycle stable developed a number of what he called 'unridable bicycles'. He reasoned that a bicycle moving slowly is unstable and almost unridable, i.e. the rider cannot keep the bicycle upright, but a bicycle moving at high speed is stable and also easily ridable, and that the stability is therefore connected to a measure for how ridable the bicycle is. To discover more about the stability of a moving bicycle he made examples that should be unstable and therefore unridable. However, he found that the destabilising effect of a counter-rotating gyroscope had very little effect on the rider's ability to stabilise the bicycle, while the inherent stability of the bicycle was affected dramatically. On the other hand, he reasoned and experimentally found that by adjusting the bicycle's trail he could make a bicycle that was both unstable and unridable or uncontrollable for the rider. He attributed this to the trail which has to remain positive and gravitational forces to be overcome to return to the upright straight-ahead orientation. Jones's theories on stability were shown to be incomplete by Meijaard *et al.* [9].

An example of unintentional bad handling qualities is the Itera plastic bicycle from the early 1980s [137]. In an attempt to design and produce a conventional bicycle in fibre composite plastics, the Sweden-based Itera company produced a bicycle which showed a substantially different 'feeling'. This was described as 'Several riders have expressed the opinion that the flexibility of the Itera handlebar, although beneficial for the reason mentioned [shock absorption], does give them a sense of insecurity'. The design turned out to be a complete failure.

The book 'Lords of the Chainring' by Patterson [138] has been used by students in a bicycle design class to develop out of the ordinary bicycles with good handling. The book gives design guidelines based on aircraft handling quality analogies. It also discusses that the steering stiffness is an essential design parameter for bicycles. Design guidelines are given including equations, e.g. for the roll control authority which links the roll rate to the hand movement. These equations only depend on geometrical parameters of the bicycle and not on the masses and moments of inertia, making the validity doubtful. However, good results are claimed to have been achieved with this method by Patterson and Leone [139].

3.5. *Rider control discussion and conclusions*

Models have shown that steering is the dominant control method involved in the stabilising task for bicycles and experiments confirm this. A large number of rider models have been proposed, but unfortunately only one bicycle rider model with force control has been experimentally validated. As both position and force control have been shown to model rider control well, it could be interesting to investigate if controlling for steering impedance is a better approach.

Experiments have shown that riders appear not to use their upper bodies for stabilisation control when they have their hands on the handlebar. LQR optimal control and intuitive control models have also shown that it is highly unlikely that upper body lean will be used for stabilisation at low speed as the gains required are too large. Modelling a rider as rigidly attached to the bicycle and only able to perform steer actions based on roll angle information therefore appears to be a good option for a rider model for stabilising control.

Classical control multiple output models (steer and lean) have been developed for performing tracking and stabilising tasks that compare well with actual rider data. The addition of the tracking task does not significantly alter the roll to steer gains for classical control models capable of stabilising by steering. Stabilisation and tracking have successfully been implemented in parallel in machines that use steering and moving mass control. However, for machines with only steering control (single output), tracking and stabilisation have been successfully implemented in series. An open question is if tracking and stabilisation can be implemented in a single controller.

With a rider model it has been shown that the required preview time in bicycling for tracking capabilities is about 2.5 s, independent of the forward speed. Optimal control models have also shown that for a bicycle the tracking performance improves with increased preview distance, but that there is a limit after which the extra preview distance no longer adds to the performance, as the corresponding optimal gains are almost zero. These theoretical results, however, have not been experimentally validated. The preview time required to control a vehicle safely can be of interest to traffic and road planners as it can have an effect on the design of intersections and tight or blind corners.

Rider upper-body motions in general have been shown only to have a small influence on the overall motion. Furthermore, it is hypothesised that upper-body motions are most likely to be performed to control the orientation of the rider's head for comfort reasons and not the direct control of the vehicle. However, optimal control models have shown that upper-body motion can contribute to manoeuvre performance, particularly for manoeuvres such as a lane change.

Bicycle handling quality research has only been interested in safety, that is, accident avoidance. However, no standard handling quality tests for bicycles have been developed. There is, therefore, no way to compare quantitatively the results of different bicycle handling experiments that have been carried out by the different authors.

In the bicycle research performed, most authors do not accurately define what they mean by the terms handling quality, stability or manoeuvrability, but from the experiments it can be deduced that most mean the same. However, authors interested in normal riding generally measure stability as the ability to remain upright, while authors working on bicycle safety have an additional requirement that the bicycle continues in the same direction, which is defined as directional stability.

For bicycle research, it is also essential to develop a standardised set of tests and handling indices, in a manner similar to those that exist for motorcycles, such that bicycle handling can be compared and quantified both experimentally and in simulations. Another advantage of such a set of handling tests is that a set of handling qualities for normal riding can be determined, such that the designers, who now apply a time-consuming trial and error method to developing new bicycle concepts, can determine a priori what the handling qualities will be.

4. Conclusions

The history on the dynamics and control of bicycles reads like ‘Sleeping Beauty’, the classic fairy tale. After the seminal work of Carvallo and Whipple around 1899, the topic lay more or less dormant for 80 years. The revival, which started in the 1970s, resulted in extensions to the original Carvallo–Whipple model and investigations into rider control. The extensions were deemed necessary to explore beyond the linearised regime and explain unmodelled dynamic behaviour like wobble and shimmy. It is somewhat surprising that experimental validation of these bicycle models only started in the last decade. Nevertheless, the correctness of these riderless or rigid-rider bicycle models seems to be established now, although some work in the experimental determination of tyre parameters and the high-frequency wobble phenomenon needs to be done. Less can be said about rider models.

In trying to address questions on handling and manoeuvrability, most authors have been looking at self-stability of the bicycle, probably due to a lack of valid rider models. However, the link between these two has never been shown. To address questions like, ‘what is good handling and good manoeuvrability?’, validated models of the complete bicycle plus rider systems are needed, together with proper definitions of handling and manoeuvrability. To date, there exists a plethora of rider models of which unfortunately only one has been experimentally validated. Therefore, extensive experimental validation of existing rider models is needed. With that in hand, definitions on handling and manoeuvrability can be made and validated.

With a lot of clever tinkering bicycles evolved to a great design in about 1890. Perhaps, now with careful experiments, and with the help of validated computer simulations, we can move past that nineteenth century bicycle evolution to a twenty-first century bicycle revolution.

Acknowledgements

Thanks to Andy Ruina, Andrew Dressel, Anindya Chatterjee, Atanas Popov, David Limebeer, Jason Moore, Jim Papadopoulos, Jodi Kooijman, Luke Peterson, Mont Hubbard, Pratipa Basu-Mandal, Robin Sharp, Stuart Rowell, and numerous MSc and BSc students, for their enthusiasm, contributions and guidance in this work.

Note

1. This and many other interesting facts about Drais are discussed in Hans-Erhard Lessing’s [12] book.

References

[1] R.S. Sharp, *The dynamics of single track vehicles*, Veh. Syst. Dyn. 5 (1975/1976), pp. 67–77.

[2] R.S. Sharp, *A Review of Motorcycle Steering Behavior and Straight Line Stability Characteristics*, Motorcycle Dynamics and Rider Control, number 780303 in SP-428, 27 February–3 March, SAE Congress Detroit, Michigan, 1978.

[3] R.S. Sharp, *The lateral dynamics of motorcycles and bicycles*, Veh. Syst. Dyn. 14 (1985), pp. 265–283.

[4] R.S. Sharp, *Stability, control and steering responses of motorcycles*, Veh. Syst. Dyn. 35(4–5) (2001), pp. 291–318.

[5] D.J.N. Limebeer and R.S. Sharp, *Bicycles, motorcycles, and models: Single-track vehicle modeling and control*, IEEE Control Syst. Mag. 26(5) (2006), pp. 34–61.

[6] A.A. Popov, S. Rowell, and J.P. Meijaard, *A review on motorcycle and rider modelling for steering control*, Veh. Syst. Dyn. 48 (2010), pp. 775–792.

[7] J.P. Meijaard, J.M. Papadopoulos, A. Ruina, and A.L. Schwab, *Linearized dynamics equations for the balance and steer of a bicycle: A benchmark and review*, Proc. R. Soc. A 463 (2007), pp. 1955–1982, DOI: 10.1098/rspa.2007.1857.

[8] R.S. Sharp, *On the stability and control of the bicycle*, Appl. Mech. Rev. 61 (2008), pp. 060803-1–24, DOI: http://dx.doi.org/10.1115/1.2983014.

[9] J.P. Meijaard, J.P. Papadopoulos, A. Ruina, and A.L. Schwab, *History of thoughts about bicycle self-stability*, 2011. Available at eCommons@Cornell http://hdl.handle.net/1813/22497, http://ecommons.library.cornell.edu/handle/1813/22497.

[10] D.V. Herlihy, *Bicycle: The History*, Yale University Press, New Haven, CT, 2004.

[11] T. Goetz and K.F.C.L. von Drais von Sauerbronn, *Draisinen*, Journal für Literatur, Kunst, Luxus und Mode Jahrgang 35 (1820), pp. 365–377.

[12] H.-E. Lessing, *Automobilität; Karl Drais und die unglaublichen Anfänge*, Maxime, Leipzig, 2003.

[13] P. Lallement, *Improvement in velocipedes*, U.S. Patent 59,915 1866.

[14] J. Means, *Wheeling and flying*, Aeronaut. Ann. 1896(2) (1896), pp. 23–25.

[15] W.J.M. Rankine, *On the dynamical principles of the motion of velocipedes*, Engineer 28 (1869), pp. 79,129,153,175 and 29 (1870), p. 2.

[16] C. Bourlet, *Étude théorique sur la bicyclette*, Bull. Soc. Math. France 27 (1899), pp. 47–67, 76–96.

[17] J. Boussinesq, *Aperçu sur la théorie de la bicyclette*, J. Math. Pures Appl. 5(5) (1899), pp. 117–135.

[18] J. Boussinesq, *Complément à une étude récente concernant la théorie de la bicyclette: Influence, sur l'équilibre, des mouvements latéraux spontanés du cavalier*, J. Math. Pures Appl. 5(5) (1899), pp. 217–232.

[19] H. Bouasse, *Cours De Mécanique Rationnelle Et Expérimentale*, Librairie Ch. Delagrave, Paris, 1910.

[20] S. Timoshenko and D.H. Young, *Advanced Dynamics*, McGraw-Hill Book Company, New York, NY, Toronto and London, 1948.

[21] E. Carvallo, *Théorie Du Mouvement Du Monocycle Et De La Bicyclette*, Gauthier-Villars, Paris, France, 1899.

[22] F.J.W. Whipple, *The stability of the motion of a bicycle*, Q. J. Pure Appl. Math. 30 (1899), pp. 312–348.

[23] F. Klein and A. Sommerfeld, *Über die Theorie des Kreisels*, Teubner, Leipzig, 1910.

[24] J.D.G. Kooijman, J.P. Meijaard, J.P. Papadopoulos, A. Ruina, and A.L. Schwab, *A bicycle can be self-stable without gyroscopic or caster effects*, Science 332 (2011), pp. 339–342.

[25] E. Döhring, *Über die Stabilität und die Lenkkräfte von Einspurfahrzeugen*, Dissertation, Technische Hochschule Carolo-Wilhelmina, Braunschweig, 1953.

[26] E. Döhring, *Die Stabilität von Einspurfahrzeugen*, Forschung auf dem Gebiete des Ingenieurswesen, Ausgabe A 21 (1955), pp. 50–62.

[27] B.D. Herfkens, *De stabiliteit van het rijwiel ("The Stability of the bicycle")*, Tech. Rep. S-98-247-50-10-'49, Instituut voor Rijwielontwikkeling, 's-Gravenhage, September 1949.

[28] R.S. Sharp, *The stability and control of motorcycles*, J. Mech. Eng. Sci. 13(5) (1971), pp. 316–329.

[29] J.P. Meijaard and A.L. Schwab, *Linearized equations for an extended bicycle model*, in *III European Conference on Computational Mechanics Solids, Structures and Coupled Problems in Engineering*, C.A. Mota Soares, J.A.C. Martins, H.C. Rodrigues, and J.A.C. Ambrósio, eds., Lisbon, Portugal, 5–9 June, Springer, Dordrecht, 2006, pp. 1–18.

[30] K.J. Åström, R.E. Klein, and A. Lennartsson, *Bicycle dynamics and control, adapted bicycles for education and research*, IEEE Control Syst. Mag. 25(4) (2005), pp. 26–47.

[31] G. Frosali and F. Ricci, *Kinematics of a bicycle with toroidal wheels*, Commun. Appl. Ind. Math. 3(1) (2012), pp. 1–24, DOI: 10.1685/journal.caim.424.

[32] V. Cossalter, *Motorcycle Dynamics*, 2nd ed., LULU, Raleigh, NC, 2006. ISBN 0-9720514-0-6.

[33] D.L. Peterson and M. Hubbard, *General steady turning of a benchmark bicycle model*, Proceedings of the ASME 2009 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, IDETC/CIE 2009 (August 30–September 2, San Diego, CA), CD-ROM, paper No. DETC2009-86145, American Society of Mechanical Engineers, New York, 2009, pp. 1–10.

[34] R.S. Sharp, *The Stability of Motorcycles in Acceleration and Deceleration*, Braking of Road Vehicles, Institution of Mechanical Engineers Conference Publications 1976-5, Mechanical Engineering Publications, London and New York, 1977, pp. 45–49.

[35] D.J.N. Limebeer, R.S. Sharp, and S. Evangelou, *The stability of motorcycles under acceleration and braking*, Proc. Inst. Mech. Eng. C, J. Mech. Eng. Sci. 215 (2001), pp. 1095–1109.

[36] J.P. Meijaard and A.A. Popov, *Practical stability analysis for transient system dynamics*, Proc. Inst. Mech. Eng. C, J. Mech. Eng. Sci 222(11) (2008), pp. 2123–2135.

[37] D.J.N. Limebeer and A. Sharma, *The Dynamics of the Accelerating Bicycle*, Proceedings of the ISCCSP 2008, Malta, 12–14 March, IEEE, Piscatway, NJ, 2008, pp. 242–273.

[38] A. Sharma, *Stability analysis of bicycles & motorcycles*, Ph.D. thesis, Imperial College London, 2010.

[39] D.J.N. Limebeer and A. Sharma, *Burst oscillations in the accelerating bicycle*, J. Appl. Mech. 77 (2010), pp. 061012-1-20.

[40] G. Franke, W. Suhr, and F. Rieß, *An advanced model of bicycle dynamics*, European J. Phys. 11(2) (1990), pp. 116–121.

[41] A. Lennartsson, *Efficient multibody dynamics*, Doctoral thesis, Department of Mechanics, Royal Institute of Technology, Stockholm, 1999.

[42] P. Basu-Mandal, A. Chatterjee, and J.M. Papadopoulos, *Hands-free circular motions of a benchmark bicycle*, Proc. R. Soc. A 463 (2007), pp. 1983–2003.

[43] P. Basu-Mandal, *Studies on the dynamics and stability of bicycles,* Ph.D. thesis, Department of Mechanical Engineering, Indian Institute of Science, Bangalore, 2007.

[44] S.M. Cain and N.C. Perkins, *Steady-state Handling Characteristics of a Bicycle*, 34th Annual meeting of the American Society of Biomechanics, Wilmington, DE, 2010.

[45] S.M. Cain and N.C. Perkins, *Comparison of experimental data to a model for bicycle steady-state turning*, Veh. Syst. Dyn. 50 (2012), pp. 1341–1364.

[46] J.D.G. Kooijman, A.L. Schwab, and J.P. Meijaard, *Experimental validation of a model of an uncontrolled bicycle*, Multibody Syst. Dyn. 19 (2008), pp. 115–132.

[47] H.B. Pacejka, *Tire and Vehicle Dynamics*, Butterworths, London, 3rd ed., 2012, ISBN : 9780080970165.

[48] R.D. Roland Jr and J.P. Lynch, *Bicycle dynamics, tire characteristics and rider modeling*, Tech. Rep. YA-3063-K-2, Cornell Aeronautical Laboratory, Inc. Buffalo, NY, March 1972.

[49] D.J. Cole and Y.H. Khoo, *Prediction of vehicle stability using a 'back to back' tyre test method*, Int. J. Veh. Des. 26(5) (2001), pp. 573–582.

[50] A. Dressel and A. Rahman, *Measuring sideslip and camber characteristics of bicycle tyres*, Veh. Syst. Dyn. 50(8) (2012), pp. 1365–1378, DOI: 10.1080/00423114.2011.615408.

[51] A. Doria, M. Tognazzo, G. Cusimano, V. Bulsink, A. Cooke, and B. Koopman, *Identification of the mechanical properties of bicycle tyres for modelling of bicycle dynamics*, Veh. Syst. Dyn. 51 (2013), pp. 405–420.

[52] R.S. Sharp, *Research note: The influence of frame flexibility on the lateral stability of motorcycles*, Proc. Inst. Mech. Eng. C, J. Mech. Eng. Sci. 16 (1974), pp. 117–120.

[53] G.E. Roe, W.M. Pickering, and A. Zinober, *The oscillations of a flexible castor, and the effect of front fork flexibilities on the stability of motorcycles*, Tech. Rep. 780307, SAE, Warrendale, PA, 1978.

[54] M. Plöchl, J. Edelmann, B. Angrosch, and C. Ott, *On the wobble of a bicycle*, Veh. Syst. Dyn. 50 (2012), pp. 415–429.

[55] E. Döhring, *Die Stabilität von Einspurfahrzeugen*, ATZ – Automobiltechnische Zeitschrift 56(3) (1954), pp. 68–72.

[56] D. Stevens, *The stability and handling characteristics of bicycles*, Bachelor thesis, The University of New South Wales, School of Mechanical and Manufacturing Engineering, Kensington, 2009.

[57] T.O. Tak, J.S. Won, and G.Y. Baek, *Design Sensitivity Analysis of Bicycle stability and Experimental Validation*, Proceedings, Bicycle and Motorcycle Dynamics 2010 Symposium on the Dynamics and Control of Single Track Vehicles, 20–22 October, Delft, The Netherlands, 2010.

[58] J.D.G. Kooijman and A.L. Schwab, *Experimental Validation of the Lateral Dynamics of a Bicycle on a Treadmill*, Proceedings of the ASME 2009 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, number DETC2009-86965, DETC2009, 30 August–2 September, San Diego, CA, ASME, New York, 2009.

[59] J.K. Moore, *Human control of a bicycle*, Ph.D. thesis, University of California Davis, 2012.

[60] P. Wesemann and W.A.M. Weijermar. *Verkeersveiligheidsverkenning 2020*, Tech. Rep. R-2011-1, SWOV, Leidschendam, 2011.

[61] A. van Lunteren, H.G. Stassen, and M.S.H. Schlemper, *On the Influence of Drugs on the Behavior of a Bicycle Rider*, Proceedings of the 6th Annual Conference on Manual Control, 7–9 April, Wright Patterson AFB, Ohio, 1970, pp. 419–437.

[62] H.G. Stassen and A. van Lunteren, *Progress report January 1970 until January 1973 of the man-machine systems group*, Tech. Rep. 55, Technische Hogeschool Delft, Delft, 1973.

[63] R.D. Roland Jr and D.E. Massing, *A digital computer simulation of bicycle dynamics*, Tech. Rep. YA-3063-K-1, CALSPAN, Buffalo, NY, June 1971.

[64] R.S. Rice, *Accident Avoidance Capabilities of Motorcycles*, Proceedings International Motorcycle Safety Conference, 16–17 December, US Department of Transportation, National Highway Traffic Safety Administration, Washington DC, 1975, pp. 121–134.

[65] R.S. Rice and D.T. Kunkel, *Accident avoidance capabilities of motorcycles – lane change maneuver simulation and full-scale tests*, Tech. Rep. ZN-5899-V-1, Calspan Corporation, Buffalo, NY, 1976.

[66] R.D. Roland Jr, *Computer Simulation of Bicycle Dynamics*, Mechanics and Sport, Detroit, MI, 11–15 November, The Winter Annual Meeting of the American Society of Mechanical Engineers, ASME, New York, 1973, pp. 35–83.

[67] A.J.R. Doyle, *The essential human contribution to bicycle riding*, in *Training, Human Decision Making, and Control*, J. Patrick and K.D. Duncan, eds., North-Holland, Amsterdam, 1988, pp. 351–370.

[68] D.E.H. Jones, *The stability of the bicycle*, Phys. Today 23(4) (1970), pp. 34–40.

[69] J.D.G. Kooijman, A.L. Schwab, and J.K. Moore, *Some Observations on Human Control of a Bicycle*, Proceedings of the ASME 2009 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, IDETC/CIE 2009 (August 30–September 2, San Diego, CA), CD-ROM, paper No. DETC2009-86959, American Society of Mechanical Engineers, New York, 2009, 8 pp.

[70] J.K. Moore, J.D.G. Kooijman, A.L. Schwab, and M. Hubbard, *Rider motion identification during normal bicycling by means of principal component analysis*, Multibody Syst. Dyn. 25(2) (2011), pp. 225–244.

[71] K. Guo and H. Guan, *Modelling of driver/vehicle directional control system*, Veh. Syst. Dyn. 22(3–4) (1993), pp. 141–184, DOI: 10.1080/00423119308969025.

[72] C.C. MacAdam, *Understanding and modeling the human driver*, Veh. Syst. Dyn. 40(1–3) (2003), pp. 101–134.

[73] M. Plöchl and J. Edelmann, *Driver models in automobile dynamics application*, Veh. Syst. Dyn. 45(7–8) (2007), pp. 699–741, DOI: 10.1080/00423110701432482.

[74] D.T. McRuer and E.S. Krendel, *The human operator as a servo system element: Part I*, J. Franklin Inst. 267(5) (1959), pp. 381–403, DOI: 10.1016/0016-0032(59)90091-2.

[75] D.T. McRuer and E.S. Krendel, *The human operator as a servo system element: Part II*, J. Franklin Inst. 267(6) (1959), pp. 511–536, DOI: 10.1016/0016-0032(59)90072-9.

[76] D.T. McRuer, D. Graham, and E.S. Krendel, *Manual control of single-loop systems: Part I*, J. Franklin Inst. 283(1) (1967), pp. 1–29, DOI: 10.1016/0016-0032(67)90112-3.

[77] D.T. McRuer, D. Graham, and E.S. Krendel, *Manual control of single-loop systems: Part II*, J. Franklin Inst. 283(2) (1967), pp. 145–168, DOI: 10.1016/0016-0032(67)90231-1.

[78] H.G. Stassen and A. van Lunteren, *Het stabilisatie proces van een fietser op een rechte weg*, Tech. Rep. 36, Technische Hogeschool Delft, Delft, 1967.

[79] A. van Lunteren and H.G. Stassen, *Investigations on the bicycle simulator, chapter III of the annual report 1969 of the man-machine systems group*, Annual report 1969 of the man-machine systems group WTHD21, Technische Hogeschool Delft, Delft, 1970.

[80] A. van Lunteren and H.G. Stassen, *On the Variance of the Bicycle Rider's Behavior*, Proceedings of the 6th Annual Conference on Manual Control, Wright-Patterson AFB, Dayton, Ohio, 1970, pp. 701–722.

[81] A. van Lunteren and H.G. Stassen, *Parameter Estimation in Linear Models of the Human Operator in a Closed Loop with Application of Deterministic Test Signals*, Proceedings of the 9th Annual Conference on Manual Control, 23–25 May, Massachusetts Institute of Technology, Cambridge, MA, 1973, pp. 503–520.

[82] J.I. Elkind, *Characteristics of simple manual control systems*, Tech. Rep. 111, MIT, Lincoln Laboratory, Cambridge, MA, 1956.

[83] R.S. Rice, *Rider Skill Influences on Motorcycle Maneuvering*, SAE, number 780312 in SP-428, 27 February–3 March, SAE Congress Detroit, Michigan, 1978, SAE, Warrendale.

[84] R. Hess, J.K. Moore, and M. Hubbard, *Modeling the manually controlled bicycle*, IEEE Trans. Syst. Man Cybern. Part A Syst. Humans 42 (2012), pp. 545–557.

[85] A.E. Bryson and Y.C. Ho, *Applied Optimal Control*, Wiley, New York, 1975, ISBN 0470114819.

[86] D.L. Kleinman, S. Baron, and W.H. Levison, *An optimal control model of human response part I: Theory and validation*, Automatica 6(3) (1970), pp. 357–369, DOI: 10.1016/0005-1098(70)90051-8.

[87] S. Baron, D.L. Kleinman, and W.H. Levison, *An optimal control model of human response part II: Prediction of human performance in a complex task*, Automatica 6(3) (1970), pp. 371–383, DOI: 10.1016/0005-1098(70)90052-X.

[88] M. Hubbard, *Lateral dynamics and stability of the skateboard*, J. Appl. Mech. 46 (1979), pp. 931–936.

[89] M. Hubbard, *Human control of the skateboard*, J. Biomech. 13(9) (1980), pp. 745–754.

[90] A.L. Schwab, J.D.G. Kooijman, and J.P. Meijaard, *Some recent developments in bicycle dynamics and control*, in *Fourth European Conference on Structural Control (4ECSC)*, A.K Belyaev and D.A. Indeitsev, eds., Institute of Problems in Mechanical Engineering, Russian Academy of Sciences, Saint-Petersburg, 2008, pp. 695–702.

[91] B. Connors and M. Hubbard, *Modelling and stability analysis of a recumbent bicycle with oscillating leg masses*, in *The Engineering of Sport 7*, M. Estivalet and P. Brisson, eds., Vol. 1, Springer Verlag, Paris, 2008, pp. 677–685, DOI:10.1007/978-2-287-09411-8_79.

[92] R.S. Sharp, *Optimal stabilization and path-following controls for a bicycle*, Proc. Inst. Mech. Eng. C, J. Mech. Eng. Sci. 221 (2007), pp. 415–428.

[93] R.S. Sharp, *Optimal linear time-invariant preview steering control for motorcycles*, Veh. Syst. Dyn. 44(Supplement) (2006), pp. 329–340.

[94] R.S. Sharp, *Motorcycle steering control by road preview*, Trans. ASME, J. Dyn. Syst. Meas. Control 129 (2007), pp. 373–381.

[95] B.T. Thanh and M. Parnichkun, *Balancing control of Bicyrobo by particle swarm optimization-based structure-specified mixed* H_2/H_∞ *control*, Int. J. Adv. Rob. Syst. 5(4) (2008), pp. 187–195.

[96] C.K. Chen and T.S. Dao, *Dynamics and Path-Tracking Control of an Unmanned Bicycle*, Proceedings of IDETC/CIE 2005, number DETC2005-84107, ASME, New York, 2005.

[97] C.K. Chen and T.S. Dao, *Fuzzy control for equilibrium and roll-angle tracking of an unmanned bicycle*, Multibody Syst. Dyn. 15(4) (2006), pp. 325–350.

[98] C.K. Chen and T.S. Dao, *Genetic fuzzy control for path-tracking of an autonomous robotic bicycle*, J. Syst. Design Dyn. 1(3) (2007), pp. 536–547.

[99] M. Cook, *It Takes Two Neurons to Ride a Bicycle*, Proceedings of the 18th Annual Conference on Neural Information Processing, NIPS'04, NIPS Foundation, La Jolla, CA, 2004.

[100] N.H. Getz, *Dynamic inversion of nonlinear maps with applications to nonlinear control and robotics*, Ph.D. thesis, University of California at Berkeley, 1995.

[101] N.H. Getz, *Internal Equilibrium Control of a Bicycle*, in Proceedings of the 34th Conference on Decision & Control, December, New Orleans, LA, 1995, pp. 4285–4287.

[102] N.H. Getz and J.E. Marsden, *Control for an Autonomous Bicycle*, IEEE International Conference on Robotics and Automation, IEEE, Piscataway, NJ, 1995.

[103] D. von Wissel and R. Nikoukhah, *Maneuver-based obstacle-avoiding trajectory optimization: Example of a riderless bicycle*, Mech. Struct. Mach. 23(2) (1995), pp. 223–255.

[104] D.J. Eaton, *An experimental study of the motorcycle roll stabilization task*, Tech. Rep. N75 19145, Highway Safety Research Institute, University of Michigan, Ann Arbor, 1973.

[105] C. Koenen, H.B. Pacejka, D.A.Timan, and J.A. Zwaan, *Bewegingen van motorrijwielen verstoord door wegdek onregelmatigheden*, Tech. Rep. 104, Rijkswaterstaat, Dienst Verkeerskunde, 1977.

[106] P. de Lange, *Rider control identification in bicycling*, Literature review report, Delft University of Technology, November 2011. Available at http://bicycle.tudelft.nl/schwab/Bicycle/lange2011rider.pdf

[107] P.A.J. Ruijs and H.B. Pacejka, *Recent Research in Lateral Dynamics of Motorcycles*, Procedings of 9th IAVSD Symposium on The Dynamics of Vehicles on roads and on tracks, Nordström, ed., Vol. 15, 24–28 June 1985, Supplement to Vehicle System Dynamics, Linköping, Sweden, 1986, pp. 467–480.

[108] T. Saguchi, M. Takahashi, and K. Yoshida, *Stable Running Control of Autonomous Bicycle Robot*, Dynamics and Design Conference, The Japan Society of Mechanical Engineers, Japan, 2006.

[109] T. Saguchi, K. Yoshida, and M. Takahashi, *Stable running control of autonomous bicycle robot*, Trans. Jpn. Soc. Mech. Eng. C 73(731) (2007), pp. 2036–2041.

[110] B. Michini and S. Torrez, *Autonomous stability control of a moving bicycle*, Massachusetts Institute of Technology course 16.622 report, MIT, Cambridge, MA, 2006.

[111] D. Andreo, V. Cerone, D. Dzung, and D. Regruto, *Experimental Results on LPV Stabilization of a Riderless Bicycle*, American Control Conference, 2009. ACC '09, June, IEEE, Piscataway, NJ, 2009, pp. 3124–3129, DOI: 10.1109/ACC.2009.5160397.

[112] V. Cerone, D. Andreo, M. Larsson, and D. Regruto, *Stabilization of a riderless bicycle, a linear-parameter-varying approach*, IEEE Control Syst. Mag. 30(5) (2010), pp. 23–32, DOI: 10.1109/MCS.2010.937745.

[113] Y. Tanaka and T. Murakami, *Self Sustaining Bicycle Robot with Steering Controller*, The 8th IEEE International Workshop on Advanced Motion Control, 2004. AMC '04, IEEE, Piscataway, NJ, 2004, pp. 193–197, DOI: 10.1109/AMC.2004.1297665.

[114] Y. Tanaka and T. Murakami, *A study on straight-line tracking and posture control in electric bicycle*, IEEE Trans. Ind. Electron. 56(1) (2009), pp. 159–168, DOI: 10.1109/TIE.2008.927406.

[115] M. Yamaguchi, 2011. *Two-legged walking robot riding a bike* (in Japanese). Available at http://ai2001.ifdef.jp/

[116] P.J. van Zytveld, *A method for the automatic stabilization of an unmanned bicycle*, Master's thesis, Stanford University, June 1975.

[117] M. Yamakita and A. Utano, *Automatic Control of Bicycles with a Balancer*, Proceedings of the 2005 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, 24–28 July, Monterey, California, 2005, pp. 1245–1250.

[118] M. Yamakita, A. Utano, and K. Sekiguchi, *Experimental Study of Automatic Control of Bicycle with Balancer*, Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems, 9–15 October, Beijing, China, IEEE, Piscataway, NJ, 2006, pp. 5606–5611.

[119] A. Murayama and M. Yamakita, *Development of Autonomous Bike Robot with Balancer*, SICE Annual Conference, 17–20 September, Kagawa University, Japan, 2007, pp. 1048–1052.

[120] M. Yamakita and K. Lychek, *Trajectory Control for an Autonomous Bicycle with Balancer*, IEEE/ASME International Conference on Advanced Intelligent Mechatronics, AIM 2008, number 4601741, IEEE, Piscataway, NJ, 2008, pp. 676–681.

[121] K. Lychek, K. Yoshino, M. Kawaguchi, and M. Yamakita, *Experimental Results for Stabilizing of a Bicycle with a Flywheel Balancer*, 2011 IEEE International Conference on Robotics and Automation, Shanghai International Conference Center, 9–13 May, Shanghai, China, 2011, pp. 6150–6155.

[122] Murataboy, 2006. Available at http://www.murataboy.com/ssk-3/index.html

[123] J. M. Gallaspy, *Gyroscopic stabilization of an unmanned bicycle*, MSc thesis, Electrical Engineering Department, University of Auburn, Auburn, AL., 1999.

[124] Gyrobike, 2012. Available at http://www.thegyrobike.com/

[125] M. Nagai, *Analysis of rider and single-track-vehicle system: its application to computer-controlled bicycles*, Automatica 19(6) (1983), pp. 737–740.

[126] K. Iuchi, H. Niki, and T. Murakami, *Attitude control of bicycle motion by steering angle and variable COG control*, Proceedings of the 32nd IEEE/IECON Annual Conference on Industrial Electronics, IEEE Piscataway, NJ, 2005, pp. 2065–2070.

[127] G.E. Cooper and R.P. Harper, *The use of pilot rating in the evaluation of aircraft handling qualities*, Tech. Rep. NASA TN D-5153, NASA, Washington, DC, 1969.

[128] D.T. McRuer and H.R. Jex, *A review of quasi-linear pilot models*, IEEE Trans. Hum. Factors Electron. 8(3) (1967), pp. 231–249.

[129] R.A. Hess, *Prediction of pilot opinion ratings using an optimal pilot model*, Hum. Factors: J. Hum. Factors Ergonomics Soc. 19(5) (1977), pp. 459–475.

[130] R.G. Mortimer, P.A. Domas, and R.E. Dewar, *The relationship of bicycle maneuverability to handlebar configuration*, Tech. Rep. UM-HSRI-HF-TM-73-5, Highway Safety Research Institute, Univeristy of Michigan, Huron Parkway & Baxter Road, Ann Arbor, Michigan 48105, June 1973.

[131] P.W. Arnberg and T. Tydén, *Stability and manoeuvrability performance of different types of bicycles*, Tech. Rep. 45 A, National Swedish Road and Traffic Research Institute, Linköping, 1974.

[132] J. Godthelp and M. Buist, *Stability and manoeuvrability characteristics of single track vehicles*, Tech. Rep. IZF 1975 0-2, TNO Institute for Perception, The Netherlands, 1975.

[133] J. Godthelp and P.I.J. Wouters, *Koers houden door fietsers en bromfietsers*, Verkeerskunde 11 (1978), pp. 537–543.

[134] R.S. Rice and R.D. Roland Jr, *An evaluation of the performance and handling qualities of bicycles*, Tech. Rep., Cornell Aero. Lab. Rep. VJ-2888-K, Buffalo, NY, April 1970.

[135] M. Wierda and E. Roos, *Gangbare kinderfietsen op comfort, manoeuvreerbaarheid en remweg vergeleken*, Haren, Verkeerskundig Studiecentrum, Rijksuniversiteit Groningen, 1988.

[136] M. Wierda and J. Wolf, *Drie typen kinderfietsen op manoeuvreerbaarheid en remweg vergeleken*, Haren, Verkeerskundig Studiecentrum, Rijksuniversiteit Groningen, 1989.

[137] J. Hult, *The Itera plastic bicycle*, Soc. Stud. Sci. 22 (1992), pp. 373–385, DOI: 10.1177/030631292022002011.

[138] W.B. Patterson, *Lords of the Chainring*, W.B. Patterson, Santa Maria, CA, 2004.

[139] W.B. Patterson and G.L. Leone, *The Application of Handling Qualities to Bicycle Design*, Proceedings, Bicycle and Motorcycle Dynamics 2010 Symposium on the Dynamics and Control of Single Track Vehicles, 20–22 October, Delft, The Netherlands, 2010, pp. 1–10.