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Review Article

A Review on the Control of Second Order Underactuated Mechanical Systems

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This paper describes some important classes of two degrees of freedom of underactuated mechanical system and also surveys review of the recent state-of-the-art concerning the mathematical modeling of these systems, their classification, and all the control strategies (linear, nonlinear, and intelligent) that have been made so far (i.e., from the year 2000 to date) to control these systems. Future research and challenges concerning the improvement, the effectiveness, and robustness of the proposed controllers for underactuated mechanical systems are presented.

1. Introduction

Mechanical systems can be classified into three major classes according to their degree of actuation. A mechanical system can be fully actuated [1–3], in the case that each degree of freedom can be individually controlled because the system has as many actuators as degrees of freedom. When the system has more actuators than degrees of freedom, the system is said to be overactuated [4, 5]. Finally, the last class of mechanical systems is systems with fewer actuators than degrees of freedom that is called underactuated systems [6, 7]. This last class includes a lot of applications in different fields such as in robotics [8, 9], aeronautical [10] and spatial systems, marine and underwater systems [11], and flexible and mobile systems. Underactuated mechanical systems have several advantages: reduction of weight, reduction of the propensity to breakdown or energy cost of the reduced control.

The diversity and complexity of these systems have led researchers in the field to analyze, on a case-by-case, the examples of underactuated mechanical systems of small size (i.e., with few degrees of freedom) such as the pendulum systems, the Acrobot, the Pendubot [12], the TORA, and the ball and beam systems. These systems, although small in size, exhibit a non-zero degree of underactuation and a highly nonlinear dynamic. It is important to emphasize that none of the techniques proposed and developed for fully actuated robots by different authors can be applied directly to any underactuated mechanical system.

Indeed, the control inputs can only control part of the dynamics and the remaining part defines what is called the internal dynamics of the system. However, it is possible, using appropriate techniques, to indirectly control the coordinates of the internal dynamics. An overall stabilization of the system remains possible under certain conditions, which is why the underactuated mechanical systems are concretely used to benefit from their quality, by means of a complexification of the control methods.

Reference [13] has made a survey on controlling the Rotary Inverted Pendulum, starting with the determination of the system model using Newton-Euler, Lagrange-Euler, and Lagrange methods. After that, the authors have defined all the control objectives that are controlling the pendulum from downward stable position to an upward unstable position (swing-up control), regulating the pendulum to remain at the unstable position (stabilization control), switching between these two last (switching control) and controlling the system in such a way that the arm tracks the desired trajectory while the pendulum remains at an unstable position (trajectory tracking). Then, and based on each control objective defined, the authors have explained the control strategies that have been applied to control the system and that comprise the linear (PID, LQR, PP), nonlinear time-invariant (sliding mode, fuzzy logic control, and backstepping), self-learning, and adaptive nonlinear controllers. Finally, and in order to test the effectiveness and robustness of the proposed controllers, the authors have mentioned some other complex system that can be added to the system of Rotary Inverted Pendulum (Two Wheeled Rotary Inverted Pendulum) and have proposed other control strategies to apply in order to control it.

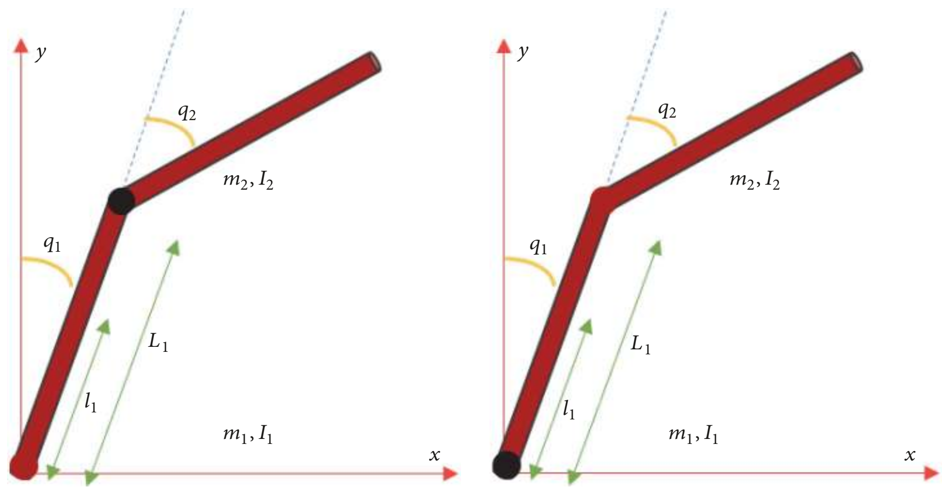
Paper [14] has presented a survey of illustrative academic books, survey and research papers on nonlinear control of the inverted pendulum. Starting with the description of motion of the pendulum using the Newton-Euler approach, after that, the author has mentioned the fact that many standard techniques in control theory fail to control the system of the inverted pendulum and has explained how other controls can give satisfactory results such as PID, LQR, the energy-based methods, the energy-shaping techniques, and the hybrid control approaches. In order to guarantee robustness performances, it is usually desirable to use the sliding mode control approach. For the purpose of reducing the complexity of controllers, it has been mentioned that it is desirable to use hybrid control approaches, such as fuzzy neural control approaches and genetic algorithms. Finally, they have presented possible future trends that can be considered such as delays, unstable internal dynamics, uncertainty conditions, saturation of actuators, and chaos dynamics.

Reference [15] has proposed a book for controlling underactuated mechanical systems. The authors start by describing and formalizing a MATLAB-based identification procedure of two underactuated mechanical systems [16]: the Furuta pendulum and the inertia wheel pendulum. In order to achieve this goal, the system model of the two systems was obtained using Euler-Lagrange form and has been expressed as a linear regression model. They have been then filtered in order to get the discrete form so they can be implemented to the real-time experimental platform where it has been mentioned that it can be easily extended to fully actuated mechanisms of a higher degree of freedom. In the next chapter [17], the authors have introduced a composed control scheme containing the input-output linearization methodology and the energy-based compensation derived from the energy function of the system, which have been applied for the trajectory tracking of the Furuta pendulum. The proposed method has been compared with the tracking controller methods reported in the literature, where it has been shown that the proposed control scheme shows better performance in the trajectory tracking. In the following chapter [18], the authors have proposed a new trajectory tracking controller based on the input-output feedback linearization technique applied to the Furuta pendulum. The proposed control strategy has been compared to two additional controllers, a PID controller and an output tracking controller, where it has been proved that the proposed controller exhibits better performance for both tracking of the arm and regulation of the pendulum than the PID controller and the output tracking controller. In the next chapter [19], the authors have introduced a novel adaptive neural network-based control scheme for the Furuta pendulum. The new control scheme was compared to other control strategies where simulations results of the experimental study have shown that the proposed method is able to guarantee to track a reference signal for the arm while the pendulum remains in the upright position better than the other methods. In the following chapter [20], and using the same methodology given for the Furuta pendulum which is the sum of a feedback-linearization based controller and an energy-based compensation, the authors have made a control scheme for the inertia wheel pendulum, where the control objective is the tracking of a desired trajectory in the actuated joint, while the unactuated link is regulated at the upward position. Finally, the proposed method has been compared with a linear controller for which the proposed algorithms show better performance in the tracking of the desired wheel trajectory at a low energetic cost. In the next chapter [21], a new control strategy has been proposed for the tracking control of the inertia wheel pendulum. The control algorithm is derived from the introduction of a new output function. This last is weighted by positive constants and switched control strategy is employed, in which a passivity-based controller is used in such a way that the wheel tracks a time-varying desired trajectory while the pendulum is regulated at the upward inverted position. The performance of the proposed method was compared to a state feedback controller designed using the linear quadratic regulator design approach based on the linearized model of the system where it shows the superior performance of the new algorithms.

In the chapter that follows the previous one [22], two new robust trajectory tracking controllers were proposed for the inertia wheel pendulum which is neural network-based and regressor based. Both methods have been implemented in an experimental platform where their performance has been compared to the classical linear PID controller. Finally, the last chapter [23] explores control methodologies for controlling underactuated mechanical systems. Among them is the feedback linearization control for linear time-invariant systems that have been applied after that to control a flexible joint robot.

In view of the advantage of impulse control of underactuated systems, which resides in the fact that it can be used to recover the stability of a balance from configurations outside their region of attraction, [24] has proposed an impulsive controller to control the underactuated mechanical system: the inertia wheel pendulum. The use of impulse inputs simplifies system dynamics and an implementation using high gain feedback has been used. The proposed method has been compared to the energy-based controller where the simulation results show similarities between the two methods.

In order to eliminate the phenomenon of the limit cycle which appears in systems under the effect of nonlinearity, [25] has designed a linear feedback regulator to stabilize the Furuta pendulum and the Pendubot. In order to achieve their objective, the authors applied the differential flatness approach to the approximate linear model of pendulums. The resulting systems are subsequently translated into the frequency domain. And a controller has been designed in such a way that the amplitude of the limit cycle is sufficiently reduced. The proposed method was verified by experimental tests.



(a) The Acrobot

(b) The Pendubot

Figure 1: The 2-link planar robots.

The main goal of this article is to gather the various researches carried out in modeling, classifying, and controlling some important classes of two degrees of freedom of the underactuated mechanical system, in order to help the future researchers to detect what problems are studied and what are not. Adding to this survey will give the opportunity for future research and challenges concerning the improvement of the effectiveness and robustness of the proposed controllers for this class of underactuated mechanical systems.

The outline of this article is as follows. In Section 2, examples of two degrees of freedom underactuated mechanical system are presented. In Section 3, the dynamic model of each system is described. In Sections 4 and 5 a classification and control methods that have been made to control underactuated mechanical system are presented, respectively. Finally, a conclusion and a future work are given in Section 6.

2. Examples of Underactuated Mechanical Systems

This section presents some examples of underactuated mechanical systems [26, 27] that represent useful benchmarks for nonlinear control and complexity of their control design which are of great interest by researchers. These examples include the Acrobot, the Pendubot, the cart-pole system, the rotating pendulum, the inertia wheel pendulum, the beam and ball system, and the translational oscillator with rotational actuator (TORA) system. Each example will be treated briefly.

2.1. The Acrobot and the Pendubot. The Acrobot, short for ACRObat robot [12, 28, 29] and the Pendubot [30–32] shown in Figures 1(a) and 1(b), respectively, are 2-link planar robots with a single actuator. They graphically seem to be very similar; however, the difference resides in the location of their single actuator that causes a major difference in their standard representation. Thus, the first link of the Acrobot is attached to a passive joint and, for the Pendubot, it is attached to an active joint with the joint between two links unactuated which allowed it to swing freely.

The inertia matrix for both systems is the same as shown in Table 1, where the control objective for both systems is to stabilize the two-link manipulators to their upright equilibrium point from any initial condition.

2.2. The Inverted Pendulum and the Crane. The cart-pole system shown in Figure 2(a) consists of an inverted pendulum on a cart [33–35] that is considered as one of the most popular laboratory experiments used for illustrating nonlinear control techniques. Its control objective is to swing up the pendulum from any initial condition to the upright unstable equilibrium position, while keeping the cart at its original position.

The convey crane system [36, 37] is presented in Figure 2(b), which is similar to the inverted pendulum on a cart, where its control objective is to move the load to the origin, keeping the oscillations of the suspended mass as small as possible.

2.3. The Rotational Pendulum. The rotational pendulum [38, 39] also known as the Furuta pendulum [40, 41] is an inverted pendulum on a rotating arm. It consists of an unactuated pendulum that is free to rotate in the vertical plane and is attached to the end of a horizontal rotating arm that is driven by a DC motor Figure 3.

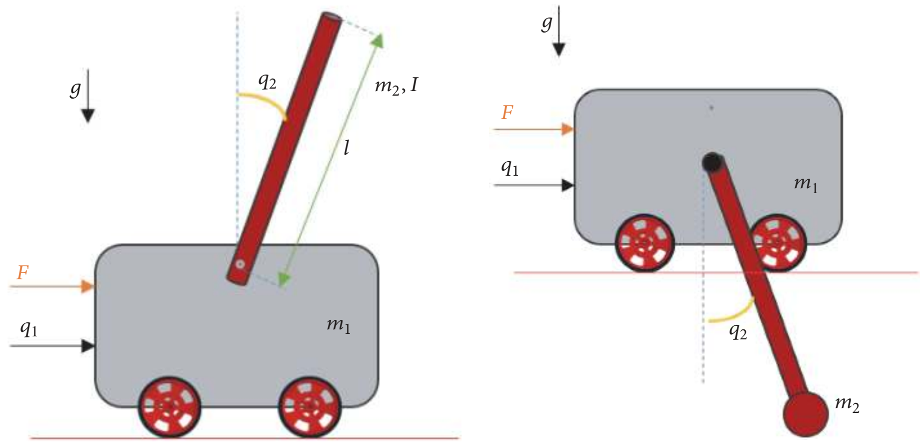
Clearly, the only difference between the inertia matrices of these two mechanical systems is in the first element m_{11}. Another similarity between the cart-pole system and the rotating pendulum is that both have the same form of the potential energy.

2.4. The Inertial Wheel Pendulum. The inertia or inertial wheel pendulum [34, 42–44] is a two-degree of freedom robot as shown in Figure 4. The pendulum constitutes the first link that is unactuated, while the rotating wheel is the second one that is supposed to control the pendulum. The main goal

Table 1: Motion of equations for underactuated mechanical systems.

System	$\mathbf{M}(\mathbf{q})$	$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$	$\mathbf{G}(\mathbf{q})$	$\mathbf{R}(\mathbf{q})$
Acrobot	$\begin{bmatrix} m_1 l_{c1}^2 + m_1(l_1^2 + l_{c2}^2 + 2l_1 l_2 C_2) + I_1 + I_2 & m_2(l_{c2}^2 + l_1 l_{c2} C_2) + I_2 \\ m_2(l_{c2}^2 + l_1 l_{c2} C_2) + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$	$\begin{bmatrix} -m_2 l_1 l_{c2} S_2 \dot{q}_2 & -m_2 l_1 l_{c2} S_2(\dot{q}_1 + \dot{q}_2) \\ m_2 l_1 l_{c2} S_2 \dot{q}_1 & 0 \end{bmatrix}$	$\begin{bmatrix} (m_1 l_{c1} + m_2 l_1) g C_1 + m_2 l_{c2} g C_{12} \\ m_2 l_{c2} g C_{12} \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Pendubot	$\begin{bmatrix} m_1 l_{c1}^2 + m_1(l_1^2 + l_{c2}^2 + 2l_1 l_2 C_2) + I_1 + I_2 & m_2(l_{c2}^2 + l_1 l_{c2} C_2) + I_2 \\ m_2(l_{c2}^2 + l_1 l_{c2} C_2) + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$	$\begin{bmatrix} -m_2 l_1 l_{c2} S_2 \dot{q}_2 & -m_2 l_1 l_{c2} S_2(\dot{q}_1 + \dot{q}_2) \\ m_2 l_1 l_{c2} S_2 \dot{q}_1 & 0 \end{bmatrix}$	$\begin{bmatrix} (m_1 l_{c1} + m_2 l_1) g C_1 + m_2 l_{c2} g C_{12} \\ m_2 l_{c2} g C_{12} \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
Inverted pendulum on cart	$\begin{bmatrix} m_1 + m_2 & m_2 l C_1 \\ m_2 l C_1 & m_2 l^2 + I \end{bmatrix}$	$\begin{bmatrix} 0 & -m_2 l \dot{q}_1 S_1 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -m_2 g l S_1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
The crane system	$\begin{bmatrix} m_1 + m_2 & m_2 l C_1 \\ m_2 l C_1 & m_2 l^2 + I \end{bmatrix}$	$\begin{bmatrix} 0 & -m_2 l \dot{q}_1 S_1 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ m_2 g l S_1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
Furuta Pendulum	$\begin{bmatrix} I_1 + m_1 l_1^2 + m_2(L_1^2 + I_2 S_2^2) & m_2 L_1 l_2 C_2 \\ m_2 L_1 l_2 C_2 & m_2 l_2^2 + I_2 \end{bmatrix}$	$\begin{bmatrix} \frac{1}{2} m_2 l_2^2 \dot{q}_2 S_{22} & -m_2 L_1 l_2 \dot{q}_2 S_2 + \frac{1}{2} m_2 l_2^2 \dot{q}_1 S_{22} \\ -\frac{1}{2} m_2 l_2^2 \dot{q}_1 S_{22} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ -m_2 g l_2 S_2 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
Inertia Wheel Pendulum	$\begin{bmatrix} I_1 + I_2 + m_1 l_1^2 + m_2 L_1^2 & I_2 \\ I_2 & I_2 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -(m_1 l_1 + m_2 L_1) g S_1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Beam and ball	$\begin{bmatrix} I + I_b + m q_2^2 & 0 \\ 0 & \frac{I_b}{r^2} + m \end{bmatrix}$	$\begin{bmatrix} m q_2 \dot{q}_2 & m q_2 \dot{q}_1 \\ -m q_2 \dot{q}_1 & 0 \end{bmatrix}$	$\begin{bmatrix} m g q_2 C_1 \\ m g S_1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
TORA	$\begin{bmatrix} m_1 + m_2 & m_2 r C_2 \\ m_2 r C_2 & I + m_2 r^2 \end{bmatrix}$	$\begin{bmatrix} 0 & -m_2 r \dot{q}_2 S_2 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} k q_1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$S_i \coloneqq \sin q_i$, $C_i \coloneqq \cos q_i$, $S_{ij} \coloneqq \sin q_i + q_j$, $C_{ij} \coloneqq \cos q_i + q_j$.



(a) The cart-pole system (b) The crane system

Figure 2: The 2 underactuated mechanical systems.

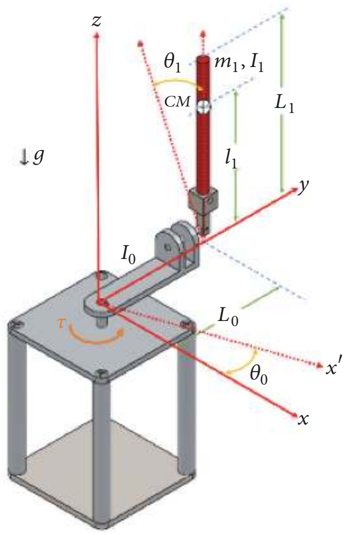


Figure 3: The rotational pendulum.

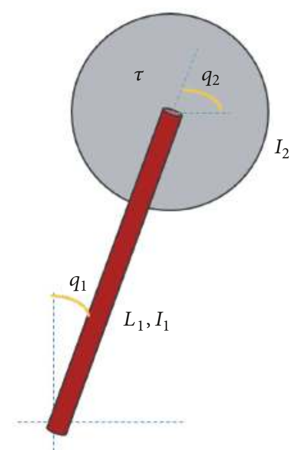


Figure 4: The Inverted wheel pendulum.

is to stabilize the pendulum in its upright equilibrium point while the wheel stops rotating.

2.5. The Beam and Ball. The ball and beam system in Figure 5 [45–49] consists of a beam and a ball on it. It is composed of a beam that can pivot in the vertical plane via a torque τ at the center of rotation and a ball whose aim is to reach the center of the beam.

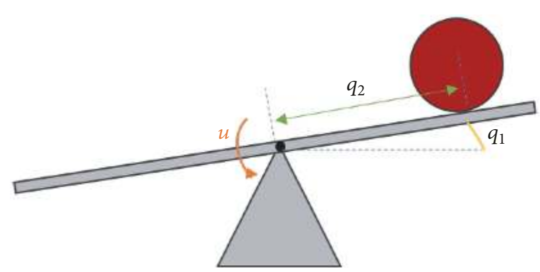


Figure 5: The beam and ball.

2.6. TORA. The TORA (Translational Oscillator with Rotational Actuator) in Figure 6 consists of a translational oscillating platform with mass m_1, that is controlled via a rotational eccentric mass m_2 [50–54].

3. Dynamic Model of Underactuated Mechanical Systems

In order to determine the equation motions of the systems, the Lagrangian of the system is first calculated. The Lagrangian of a mechanical system [55] is the difference between its total kinetic energy and its potential energy. From this Lagrangian, the equations of the mechanical system are derived using the Euler-Lagrange equations below:

$$\frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathscr{L}}{\partial q_i} = \tau_i, \quad \forall i \in \{1, \ldots, n\} \tag{1}$$

In the case of a mechanical system consisting of solids connected by bonds, the kinetic energy is simply calculated

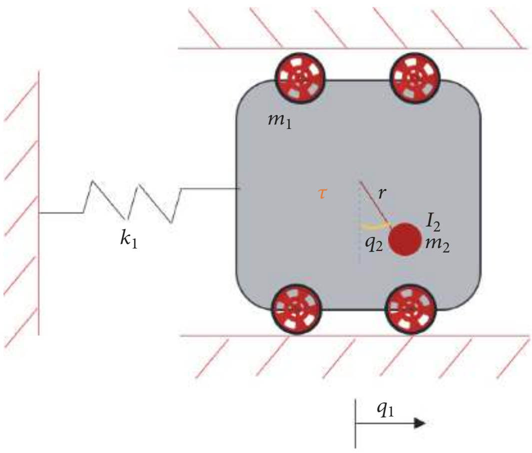


Figure 6: TORA.

as the sum of the kinetic energies of each solid. The kinetic energy decomposes for each solid in two terms, the first resulting from the translational movement of the mass center of the solid and the second resulting from the rotation of the solid around its center of inertia. Potential energy is generally reduced to a term derived from gravity. The latter depends only on the position of the center of mass of the solid. The application of the Euler-Lagrange equations (1) provides the equations describing the evolution of the generalized coordinates over time. For a mechanical system consisting of rigid solids, as is the case of the majority of robotic manipulators, these equations take the following general form [38, 56, 57]:

$$\sum_j m_{kj}(q)\ddot{q}_j + \sum_{i,j} c_{i,j,k}(q,\dot{q})\dot{q}_i\dot{q}_j + \varphi_k(q) = \tau_k, \quad \forall k \in \{1,\ldots,n\} \tag{2}$$

The $m_{kj}(q)$ are the coefficients of the second derivatives of the generalized coordinates. The $c_{i,j,k}$ are those of quadratic terms of the first derivatives of the generalized coordinates. These are divided into two parts: the terms of the form $c_{i,j,k}$ with $i = j$ which are derived from the centrifugal forces, and those of the form $c_{i,j,k}$ with $i \neq j$ which are derived from the Coriolis forces. Finally, the terms $\varphi_k(q)$ depend only on the position q of the generalized coordinates and are derived from the potential energy. These equations are often put in matrix form, becoming

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau = R(q)u \tag{3}$$

The symmetric and positive definite matrix $M(q) \in \mathfrak{R}^{n\times n}$ is called the inertial matrix of the mechanical system. It depends, in the general case, on the configuration q of the mechanical system. The matrix $C(q,\dot{q})\dot{q} \in \mathfrak{R}^{n\times n}$ corresponds to the centrifugal and Coriolis forces, depending on the configuration q but also on the generalized coordinate velocities $\dot{q}$. The vector $G(q) \in \mathfrak{R}^n$ corresponds to gravity and depends only on the configuration q. τ is the vector of actuator torques. The matrix $R(q) \in \mathfrak{R}^{n\times m}$ is the distribution of the forces on the generalized coordinates. And $u \in \mathfrak{R}^m$ is the actuator input vector. A mechanical system is said to be underactuated if $\text{rank}\{R(q)\} < n$, i.e., that it has fewer independent control inputs than degrees of freedom. It is assumed in the following that $[r[1-r]]^T u$, where $r = 1$ or 0 and $u \in R$ is the control action.

The development of the mathematical models for the examples treated in the previous section is given in the Appendices.

4. Classification of Underactuated Mechanical Systems

In the case of underactuated systems with two degrees of freedom, three classes are defined, namely, Class I, Class II, and Class III associated with strict feedback, nontriangular quadratic, and feedforward forms, respectively.

Some efforts of classification of the underactuated mechanical systems were carried out, in particular in [58, 59] where the classification is based on certain characteristics of the model of the studied system.

The general model for UMSs with two degrees of freedom is of the form [60]:

$$\begin{aligned} m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + m'_{11}\dot{q}_1\dot{q}_2 + m'_{12}\dot{q}_2^2 - g_1(q_1,q_2) &= \tau_1 \\ m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 - \frac{1}{2}m'_{11}\dot{q}_1^2 + \frac{1}{2}m'_{22}\dot{q}_2^2 - g_2(q_1,q_2) &= \tau_2 \end{aligned} \tag{4}$$

where ’ denotes d/dq_2.

Class I are those for which q_2 is actuated $\tau_1 = 0$. Class II are those for which q_2 is not actuated $\tau_2 = 0$.

It is shown that every underactuated system of Class I can be transformed into a strict feedback form, Class II can be transformed into nontriangular quadratic form and Class III can be transformed into feedforward forms that are summarized in Figure 7.

The main advantage of the classification of underactuated mechanical systems is that it enables to define an adequate control according to the obtained class. For example, the systems that belong to Class I that can be transformed into a strict feedback form may be controlled by a backstepping controller, while the systems that belong to Class II that can be transformed into nontriangular quadratic form would be controlled via a forwarding scheme. On the other hand, for the systems that belong to Class III that can be transformed into feedforward form, their control problem is still an open issue.

5. Control of Underactuated Mechanical Systems

Once the model of a mechanical system is established, it is possible to study its dynamics and to design a controller that allows controlling it. And because the control of underactuated mechanical systems is an active field of research in robotics and control system engineering, the main goal of this section is to highlight the contributions in controlling underactuated mechanical systems. Among the most recognized

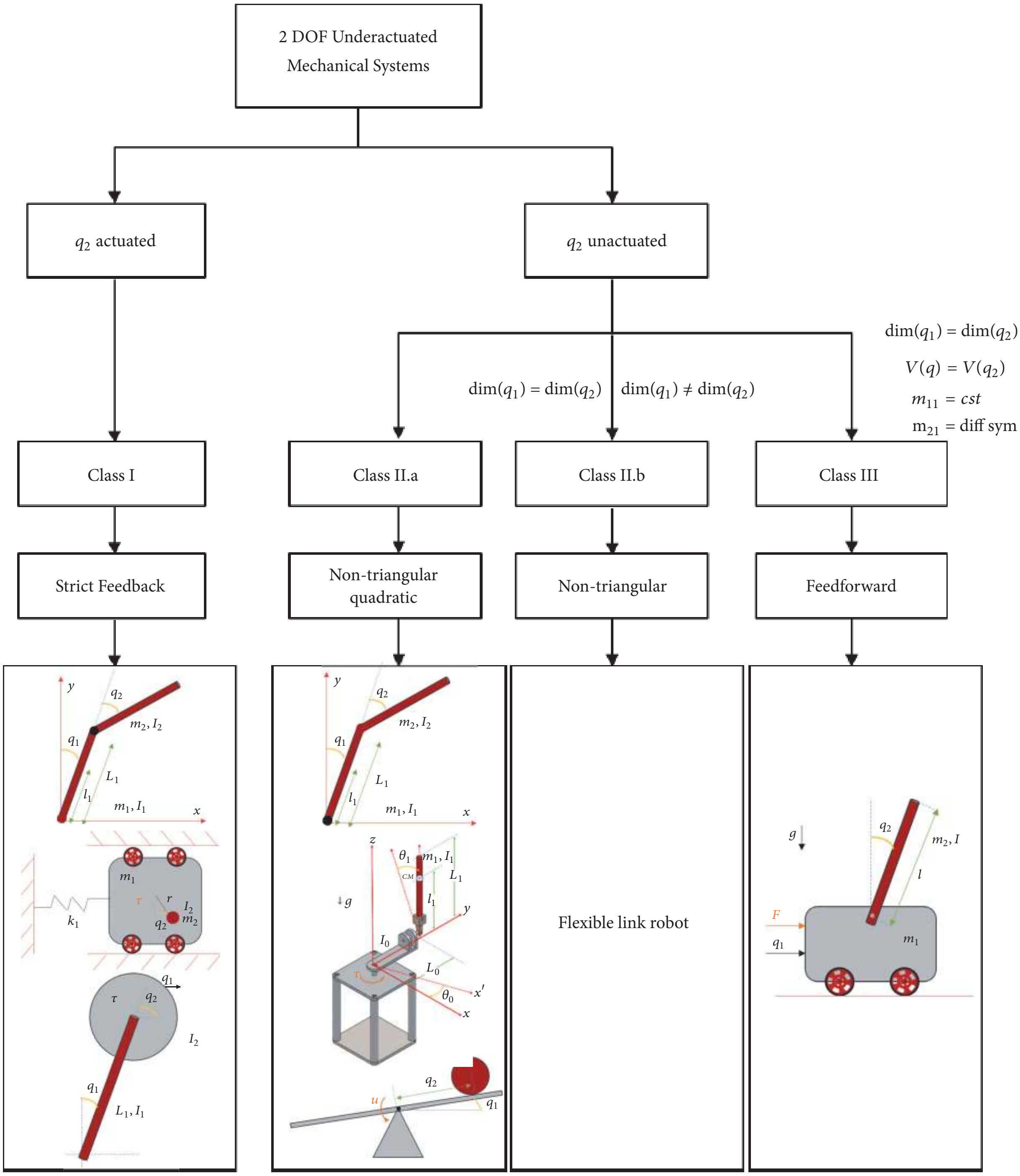


FIGURE 7: Classification of 2 degrees of freedom underactuated mechanical systems.

works, there are some that are based on linear controllers and nonlinear controllers given in following.

5.1. Linear Control. Linear controllers offer a simple control design for real-world systems. In the early days of the research, several linear control design techniques were proposed to solve the problem of control of underactuated systems.

Reference [61] has linearized the equations of motion of the rotary pendulum system about an operation point and used a robust LQR-based ANFIS to control the system. The addition of ANFIS is due to the fact that the LQR lacks the property of robustness. Furthermore, the proposed controller has been compared to LQR and showed that it has better robustness along with satisfactory performance as compared to the LQR controller.

Paper [62] used Jacobian linearization method to linearize the system of the ball and beam around operating point and used a linear quadratic regulator and hybrid PID with LQR as a combination to compare the performance of the proposed controllers.

Reference [63] has proposed a threefold step to control the two-link underactuated planar robot: the Pendubot using energy-based method. The authors started with defining necessary and sufficient conditions to provide parameter for a bigger region of controllability. After that, they demonstrated that the Pendubot can enter the region of attraction for any initial conditions except a set of Lebesgue measure zero. Finally, the simulations results are made to validate the proposed controller and have been compared to feedback linearization.

Reference [64] has proposed an energy-based controller with the combination of the neural network compensation to swing up the Pendubot. The idea behind this proposition is that because the energy-based method can successfully avoid the singularity and the neural network offset the bad effect of friction. It has been shown through the experimental studies that the proposed controller has better performance than other algorithms under the same conditions.

Paper [65] has made a combination of two controllers where one is derived from the input-output linearization and the other is derived from an energy function of the system based to control the well-known underactuated mechanical system with two degrees of freedom, the Rotary Inverted Pendulum or Furuta pendulum. The controller is made in such a way to make it possible to apply this procedure to address the trajectory tracking problem for other systems.

Although linear techniques are capable of providing a plausible solution for a particular application, even then the complicated nonlinear dynamics of such systems severely limits the generalized applications of control laws. In addition, a linear approximation of UMS often results in uncontrollable systems, which cannot be the subject of linear control algorithms for stabilization. In addition, the approximate linearization of a complicated nonlinear system provides only a precise linear approximation of the original system near the equilibrium points. It is also known that the linearization of the nonlinear system model often reduces the speed of response. The thing that motivates many researchers to design several nonlinear control algorithms for underactuated mechanical systems.

5.2. Nonlinear Control

5.2.1. Feedback Linearization. Lots of nonlinear controllers have been evolved in the last few years. Feedback linearization is one of the well-known nonlinear design tools for underactuated mechanical systems. The idea behind this method is that it can cancel nonlinearities through a feedback control and transform the nonlinear system into a (fully or partly) equivalent linear system.

Therefore, a particular form of feedback linearization, called partial feedback linearization, is used to solve the control problem for this class of underactuated mechanical systems.

Papers [18, 66] have derived a controller from the input-output feedback linearization technique to control a Furuta pendulum; the simulation results showed that the arm tracked the desired trajectory while the pendulum remained to the upright position.

A feedback linearization control has been proposed by [21] to control the inertia wheel pendulum and by [67] to control the TORA system.

5.2.2. Sliding Mode. However, the feedback linearization approach and the partial feedback linearization both have the problem of lack of robustness. And in order to get the robustness, another robust method which is based on the sliding mode approach could be considered a reasonable solution for controlling of such systems [68]. The behavior of the sliding mode depends on the switching surface. Thus, the sliding mode controller becomes insensitive to parameter variations and external disturbances. The basic idea of sliding mode design is to modify the dynamics of the system by applying a discontinuous feedback control input that forces the system to slide over a predefined state surface and the system produces the desired behavior by limiting its state on this surface. The sliding mode control finds its wide range of applications on several underactuated systems such as the TORA, the ball and the beam, and the robot and chattering, which in turn reduces the longevity of the actuators due to the wear of mechanical parts. Another disadvantage of the sliding mode is that most of the time the sliding mode controller takes a very high value of related perturbation. Therefore, most of the time, the sliding mode controller produces a too conservative design approach. In order to reduce the phenomenon of chattering, several modifications have already been proposed.

An advanced sliding mode control with integral sliding function was applied in [69] for swing-up and balancing the Pendubot to follow with various trajectories.

Reference [70] has proposed sliding mode controller to drive the Pendubot system towards the sliding surface. And in order to overcome the chattering phenomenon, a Lyapunov function with a sufficient condition was derived in terms of LMI with the sliding mode controller. The proposed method was compared to the classic feedback linearization technique and the LQR method. Simulation results show that the proposed sliding mode is a successful technique for controlling the Pendubot at the upright position, reducing the chattering and improving the robustness, better than feedback linearization technique and the LQR method.

Reference [71] designs a fuzzy-sliding control for this system. The concept of the proposed method is to use a fuzzy algorithm in order to change the sliding mode control parameter in such a way that it eliminates the chattering phenomenon. The results show that the fuzzy-sliding control is better than sliding mode control.

Paper [72] has proposed a control scheme based on the combination of a nonlinear optimal control with sliding modes for a class of nonlinear systems that have been applied to the Pendubot. The nonlinear and optimal controller was proposed in order to define the optimal sliding surface. After that, this last was used for synthesizing a super-twisting controller, which has resulted in a robust controller able to reject parametric uncertainties and external disturbances. The system [73] is presented in a cascade form using strict feedback technique, and a disturbance observer is designed to estimate the unknown external disturbances and model uncertainties of the underactuated system. Moreover, a sliding mode control is developed to control the system. The combination of the disturbance observer and the sliding mode control has been applied to the acrobat system and has proved the ability to compensate the disturbances and obtain more satisfactory control performance.

Paper [74] proposed a state feedback control based on sliding mode control scheme for the inertia wheel pendulum. The state feedback controller is extended to an output feedback control using a high gain observer. The analysis and simulation results indicate that the proposed feedback control technique gives good convergence and may be extended to other underactuated systems of similar class which includes systems like TORA and Acrobot.

Reference [75] proposed a sliding mode control for the inertia wheel pendulum. In order to achieve this goal, the dynamic equations were separated into two parts, i.e., an unactuated quasilinear part and an actuated nonlinear part. An appropriate manifold is then designed as well as a corresponding sliding mode controller that controlled the system.

In [76] a nonlinear disturbance observer was made to estimate the nonlinear terms in the model of Furuta, after that a sliding mode control was designed to control the system using the linear quadratic regulator (LQR) technique for the determination of the sliding coefficient.

Paper [77] has investigated the sliding mode control of the simplified and the full models of the ball on a beam system, where it has been proven that the controllers designed using the full model of the system gave better performance than the controllers designed using the simplified model of the system.

Reference [78] has proposed a fuzzy control and decoupled sliding mode controller for TORA system. The proposed controller employed the expert knowledge of the decoupled sliding mode to guarantee through simulation results a good stability and robustness. In the case of the cart-pole system, a review article has been proposed [79] which reviews all the control strategies that have been applied to control this system.

5.3. Passivity-Based Control. Another nonlinear control method has been proposed to control the underactuated systems like the inertia wheel pendulum, ball and beam system [80], and the cart-pole system [81] which is the passivity-based control approach. The main goal of this method is to passivate the system with a storage function, which has a minimum at the desired balance point.

5.4. Backstepping. Another energy-based method is commonly known as backstepping. Not necessarily using linearization, backstepping allows preserving useful nonlinearities that often help to retain finite values of the state vector. This technique assumes that one is able to find at least for a scalar system a control law u and a control function of Lyapunov which stabilize its origin. It also offers an efficient tool that allows, for nonlinear systems of any order, to build recursively, and in a systematic and direct way, the control law and the function of Lyapunov which ensure the stability of the loop. Although the backstepping theory has a fairly short history, many practical applications can be found in the literature. This fact indicates that the need for a nonlinear design methodology addressing a number of practical problems that used the backstepping controller.

Paper [82] proposed a book that presents a control law based on backstepping controller and had applied it to several classes of underactuated mechanical systems such as the inertia wheel pendulum [83], the TORA [84, 85], the Furuta pendulum [86], the Acrobot [87], the Pendubot [88], and the cart-pole system [89].

6. Intelligent Controller

6.1. Fuzzy Logic. Reference [90] has combined the sliding mode controller with the fuzzy controller (decoupled fuzzy sliding-mode controllers) to balance the ball and beam system. To get a good performance, the control parameters of the fuzzy sliding-mode controllers were optimized using ant colony optimization. Simulation and experimental results all indicate the superiority of the proposed scheme over others.

Paper [91] also used an intelligent controller for the ball and beam system, which is the fuzzy logic controller, where the type of membership functions their parameters and the fuzzy rules were optimized using ant colony optimization. The simulation results were compared to other related works, where it has been shown that the proposed algorithm achieves much better results.

Reference [92] has proposed a T-S fuzzy model-based adaptive dynamic surface controller to be applied to a real ball and beam system. First the system model was formulated as a strict feedback form. After that, an adaptive dynamic surface control was applied to achieve the goal of positioning the ball according to uncertainties about the parameters and the controller is applied in such a way to control the hall system with better performance.

6.2. Neural Network. An algorithm based on the neural network least squares method is applied in [93] to derive the $H\infty$ optimal control with output feedback of discrete-time affine nonlinear systems. The resulting system is used to obtain the dynamic of the output feedback control law that has been applied to the TORA system.

Reference [94] has proposed an energy-based controller incorporated with fuzzy neural network compensation to swing up the Pendubot to the unstable nonequilibrium position. Numerical simulations and experimental results have shown the performance of the proposed controller over other algorithms.

Table 2 summarizes the various control strategies that have been made to control some class of two degrees of freedom underactuated mechanical systems. We can conclude that none of these single controllers presents the best required result. However a good combination of them

Table 2: Different control strategies that have been applied to the second order of underactuated mechanical systems.

		The Acrobot	The Pendubot	The Cart-Pole system	The crane system	The Furuta Pendulum	Inertia Wheel Pendulum	The Ball and Beam	The TORA
Linear	PID					[95]			
	LQR					[96]			
	Pole Placement					[97]			
	Energy		[63, 64]			[65, 98]			
Nonlinear	Feedback linearization	[99]	[100]	[101]		[65, 96]	[42]		[102]
	Lyapunov	[12]			[103, 104]			[105]	
	Sliding mode	[106]		[107–109]	[110, 111]		[107, 112, 113]	[77, 90]	[109]
	Backstepping		[88]	[114]		[86]			[114, 115]
Intelligent	Fuzzy		[94]	[109]		[95]		[90–92, 116]	[109, 117]
	Neural Network		[64, 94]					[118]	[93]

can give fast response, robustness, adaptability, tracking the surface desired, and good rejection of disturbance.

7. Conclusion and Future Work

A mechanical system is underactuated when the number of control inputs is less than the number of degrees of freedom to control. They constitute a rich class of systems both in terms of applications and control problems. This paper examines a state-of-the-art of some important classes of two degrees of freedom of the underactuated mechanical system on modeling, classification, and control.

In a future work, we will try to answer and analyze the following question: given an underactuated mechanical system with n degree of freedom and m input, what is the best number of inputs that can give a good stability performance? The idea behind the question is when the human tries to balance a pen or pendulum on their hand, they actually not only use horizontal but also vertical forces to stabilize the pendulum, not to let the pendulum fall. This idea was proved by the author of [119] who has made a study about controlling and stabilizing the inverted pendulum using the vertical force instead of the horizontal force. And after analyzing the control and stabilization of the two systems, the author has concluded that the vertical force has an excellent and fast stabilization effect than the horizontal one. After this conclusion, the author has proposed to combine both of the horizontal and vertical forces and applied them to the inverted pendulum system. The investigation of the theoretical analysis of this combination has proved the excellent properties of adding the vertical force to the horizontal force as regards the stabilization of the inverted pendulum. The author of [120] has also proposed an X–Z inverted pendulum that can move with the combination of the vertical and horizontal forces and has applied a sliding mode control and PID to compare the performance of the system. The same system has been proposed by [121] using the fuzzy control design methodology to stabilize the inverted pendulum via a vertical force, where it has been proved that the proposed hybrid fuzzy control scheme provides a more flexible and intuitive way to stabilize the system via a vertical force. The excellent stabilization effect of the added force made us think about the necessary and sufficient number of inputs that we can apply to an underactuated mechanical system to get a good performance in stability and a large region of stability.

Appendix

A. Development of the Mathematical Models

A.1. The Pendubot. In order to simplify the calculation, we introduce the following parameters:

$$\begin{aligned}\theta_1 &= m_1 l_{c1}^2 + m_2 l_1^2 + I_1\\ \theta_2 &= m_2 l_{c2}^2 + I_2\\ \theta_3 &= m_2 l_1 l_{c2}\\ \theta_4 &= m_1 l_{c1}^2 + m_2 l_1\\ \theta_5 &= m_2 l_{c2}^2\end{aligned} \tag{A.1}$$

The kinetic and potential energies are given by

$$K_1 = \frac{1}{2}\left(I_1 + m_1 l_{c1}^2\right)\dot{q}_1^2 \tag{A.2}$$

The kinetic energy of link 2 is

$$\begin{aligned}K_2 = \frac{1}{2}&\left(I_2 + m_2 l_1 l_{c2}\cos q_2 + m_2 l_{c2}^2 + m_2 l_1^2\right)\dot{q}_1^2\\ &+ \left(I_2 + m_2 l_1 l_{c2}\cos q_2 + m_2 l_{c2}^2\right)\dot{q}_1\dot{q}_2\\ &+ \frac{1}{2}\left(I_2 + m_2 l_{c2}^2\right)\dot{q}_2^2\end{aligned} \tag{A.3}$$

With the parameters given in (A.1), the total kinetic energy is

$$\begin{aligned}K &= K_1 + K_2\\ K &= \frac{1}{2}\left(\theta_1 + \theta_2 + 2\theta_3\cos q_2\right)\dot{q}_1^2 + \frac{1}{2}\theta_2\dot{q}_2^2\\ &\quad + \left(\theta_2 + \theta_3\cos q_2\right)\dot{q}_1\dot{q}_2\end{aligned} \tag{A.4}$$

The total potential energy is $P = \theta_4 g\sin q_1 + \theta_5 g\sin(q_1 + q_2)$. The Lagrangian function is given by

$$\begin{aligned}L &= K - P\\ L &= \frac{1}{2}\left(\theta_1 + \theta_2 + 2\theta_3\cos q_2\right)\dot{q}_1^2 + \frac{1}{2}\theta_2\dot{q}_2^2 + \left(\theta_2 + \theta_3\cos q_2\right)\dot{q}_1\dot{q}_2\\ &\quad - \theta_4 g\sin q_1 - \theta_5 g\sin\left(q_1 + q_2\right)\end{aligned} \tag{A.5}$$

The corresponding equations of motion are derived using (1):

$$\begin{aligned}&\left(\theta_1 + \theta_2 + 2\theta_3\cos q_2\right)\ddot{q}_1 + \left(\theta_2 + \theta_3\cos q_2\right)\ddot{q}_2\\ &\quad - \theta_3\sin q_2\dot{q}_2^2 - 2\theta_3\sin q_2\dot{q}_1\dot{q}_2 + \theta_4 g\cos q_1\\ &\quad + \theta_5 g\cos q_1 + q_2 = \tau_1\\ &\theta_2\ddot{q}_2 + \left(\theta_2 + \theta_3\cos q_2\right)\ddot{q}_1 + \theta_3\sin q_2\dot{q}_1^2\\ &\quad + \theta_5 g\cos\left(q_1 + q_2\right) = 0\end{aligned} \tag{A.6}$$

A.2. The Acrobot. As it has been mentioned in previous sections, the Acrobot and the Pendubot seem to be very similar graphically. However, the difference is in the location of their single actuator. This is why they share the same motion of equations where the difference is in the input matrix.

The corresponding equations of motion for the Acrobot are given by

$$(\theta_1 + \theta_2 + 2\theta_3 \cos q_2)\ddot{q}_1 + (\theta_2 + \theta_3 \cos q_2)\ddot{q}_2 - \theta_3 \sin q_2 \dot{q}_2^2 - 2\theta_3 \sin q_2 \dot{q}_1 \dot{q}_2 + \theta_4 g \cos q_1 + \theta_5 g \cos q_1 + q_2 = 0$$
$$\theta_2 \ddot{q}_2 + (\theta_2 + \theta_3 \cos q_2)\ddot{q}_1 + \theta_3 \sin q_2 \dot{q}_1^2 + \theta_5 g \cos(q_1 + q_2) = \tau \quad \text{(A.7)}$$

A.3. The Cart-Pole System. The kinetic energy of the system is

$$K = K_1 + K_2 = \frac{1}{2}(M + m)\dot{x}^2 + ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}\left(I + ml^2\right)\dot{\theta}^2 \quad \text{(A.8)}$$

The potential energy is given by

$$P = mgl(\cos\theta - 1) \quad \text{(A.9)}$$

The Lagrangian function in given by

$$L = K - P$$
$$L = \frac{1}{2}(M + m)\dot{x}^2 + ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}\left(I + ml^2\right)\dot{\theta}^2 - mgl(\cos\theta - 1) \quad \text{(A.10)}$$

The corresponding equations of motion are given using (1):

$$(M + m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 \sin\theta = F$$
$$ml\ddot{x}\cos\theta + \left(I + ml^2\right)\ddot{\theta} - 2ml\dot{x}\dot{\theta}\sin\theta - mgl\sin\theta = 0 \quad \text{(A.11)}$$

A.4. The Convey Crane. The system dynamics of the convey crane correspond exactly to the equations of motion of the inverted pendulum on a cart, but now the point of interest is a lower equilibrium point.

$$(M + m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 \sin\theta = F$$
$$ml\ddot{x}\cos\theta + \left(I + ml^2\right)\ddot{\theta} - 2ml\dot{x}\dot{\theta}\sin\theta + mgl\sin\theta = 0 \quad \text{(A.12)}$$

A.5. The Furuta Pendulum. K is the sum of the kinetic energy of the arm and the pendulum, which are, respectively, defined as follows:

$$K_0 = \frac{1}{2} I_0 \dot{\theta}_0^2 \quad \text{(A.13)}$$

$$K_1 = \frac{1}{2} J_1 \dot{\theta}_0^2 + \frac{1}{2} m_1 v_1^T v_1 \quad \text{(A.14)}$$

where v_1 is the linear velocity of the pendulum center of mass. Hence, an analysis of the Furuta pendulum kinematics is required. Then, the location of the pendulum center of mass is determined by

$$x = \left[x_x,\ x_y,\ x_z\right]^T \quad \text{(A.15)}$$

where x_x, x_y, and x_z are defined as follows:

$$x_x = L_0 \cos(\theta_0) - l_1 \sin(\theta_1)\sin(\theta_0)$$
$$x_y = L_0 \sin(\theta_0) + l_1 \sin(\theta_1)\cos(\theta_0) \quad \text{(A.16)}$$
$$x_z = L_0 \cos(\theta_0)$$

Thus, v_1 is given by

$$v_1 = \left[\dot{x}_x,\ \dot{x}_y, \dot{x}_z\right]^T \quad \text{(A.17)}$$

with

$$\dot{x}_x = -\dot{\theta}_0 L_0 \sin(\theta_0) - l_1\left[\dot{\theta}_0 \sin(\theta_1)\cos(\theta_0) + \dot{\theta}_1 \sin(\theta_0)\cos(\theta_1)\right]$$
$$\dot{x}_y = \dot{\theta}_0 L_0 \cos(\theta_0) + l_1\left[\dot{\theta}_1 \cos(\theta_0)\cos(\theta_1) - \dot{\theta}_0 \sin(\theta_0)\sin(\theta_1)\right] \quad \text{(A.18)}$$
$$\dot{x}_z = -\dot{\theta}_1 l_1 \sin(\theta_1)$$

After replacing (A.18) in (A.14) and reducing the resulting expression the following is found:

$$K_1 = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_1 \left[\left(\dot{\theta}_0 L_0\right)^2 + \left(l_1 \dot{\theta}_0 \sin(\theta_1)\right)^2 + \left(l_1 \dot{\theta}_1\right)^2 + 2\dot{\theta}_0 \dot{\theta}_1 L_0 l_1 \cos\theta_1\right] \quad \text{(A.19)}$$

Therefore, the Furuta pendulum kinetic energy, K, is given by

$$K = K_0 + K_1 = \frac{1}{2} I_0 \dot{\theta}_0^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_1 \left[\left(\dot{\theta}_0 L_0\right)^2 + \left(l_1 \dot{\theta}_0 \sin(\theta_1)\right)^2 + \left(l_1 \dot{\theta}_1\right)^2 + 2\dot{\theta}_0 \dot{\theta}_1 L_0 l_1 \cos(\theta_1)\right] \quad \text{(A.20)}$$

On the other hand, V is the sum of the potential energy of the arm and pendulum. Since the arm is moved on the horizontal plane, its potential energy is constant and is considered equal to zero. Hence, the Furuta pendulum potential energy V is reduced to the pendulum potential energy, that is,

$$V = -hm_1 g = m_1 g l_1 (\cos\theta_1 - 1) \quad \text{(A.21)}$$

The dynamics of the Furuta pendulum is found using (1) as follows:

$$\begin{aligned} \alpha\ddot{\theta}_0 + \beta\dot{\theta}_0\dot{\theta}_1 + \gamma\ddot{\theta}_1 - \sigma\dot{\theta}_1^2 &= \tau \\ \gamma\ddot{\theta}_0 + \left(m_1 l_1^2 + J_1\right)\ddot{\theta}_1 \frac{1}{2}\beta\dot{\theta}_0^2 - m_1 g l_1 \sin\theta_1 &= 0 \end{aligned} \quad (A.22)$$

where

$$\begin{aligned} \alpha &= I_0 + m_1 L_0^2 + m_1 l_1^2 \sin\theta_1^2 \\ \gamma &= m_1 L_0 l_1 \cos(\theta_1) \\ \beta &= m_1 l_1^2 \sin 2\theta_1 \\ \sigma &= m_1 L_0 l_1 \sin(\theta_1) \end{aligned} \quad (A.23)$$

A.6. The Reaction Wheel Pendulum. We introduce the parameter $m = m_1 l_{c1} + m_2 l_1$. The kinetic energy of the pendulum is

$$K_1 = \frac{1}{2}\left(m_1 l_{c1}^2 + I_1\right)\dot{q}_1^2 \quad (A.24)$$

and the kinetic energy of the wheel is

$$K_2 = \frac{1}{2} m_2 l_1^2 \dot{q}_1^2 + \frac{1}{2} I_2 \left(\dot{q}_1 + \dot{q}_2\right)^2 \quad (A.25)$$

Therefore, the total kinetic energy is given by

$$\begin{aligned} K &= K_1 + K_2 \\ &= \frac{1}{2}\left(m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + I_2\right)\dot{q}_1^2 + I_2\dot{q}_1\dot{q}_2 \\ &\quad + \frac{1}{2} I_2 \dot{q}_2^2 \end{aligned} \quad (A.26)$$

The potential energy of the system is $P = mg(\cos(q_1 - 1)$. Finally, the Lagrangian is given by

$$\begin{aligned} L &= K - P \\ L &= \frac{1}{2}\left(m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + I_2\right)\dot{q}_1^2 + I_2\dot{q}_1\dot{q}_2 + \frac{1}{2} I_2 \dot{q}_2^2 \\ &\quad - mg\left(\cos(q_1) - 1\right) \end{aligned} \quad (A.27)$$

Using (1), the dynamic equations of the system are given by

$$\begin{aligned} &\left(m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + I_2\right)\ddot{q}_1 + I_2\ddot{q}_2 - mg\sin(q_1) \\ &\quad = 0 \\ &I_2\ddot{q}_1 + I_2\ddot{q}_2 = \tau \end{aligned} \quad (A.28)$$

A.7. The Beam and Ball. The kinetic and the potential energy of the beam and ball system are given by

$$\begin{aligned} K &= \frac{1}{2}\left(\frac{J_b}{r^2} + m\right)\dot{q}_2^2 + \frac{1}{2} m r^2 \dot{q}_1^2 + \frac{1}{2} I \dot{q}_1^2 \\ P &= mgr\sin q_1 \end{aligned} \quad (A.29)$$

The motion equations of the beam and ball system are determined as follows using (1):

$$\begin{aligned} \left(I + I_b + mr^2\right)\ddot{q}_1 + 2mr\dot{r}\dot{q}_1 + mrg\cos q_1 &= \tau \\ \left(m + \frac{J_b}{R^2}\right)\ddot{q}_2 - m\dot{q}_1^2 r + mg\sin q_1 &= 0 \end{aligned} \quad (A.30)$$

A.8. The TORA. The kinetic and the potential energy are given by

$$\begin{aligned} K &= \frac{1}{2}\left(m_1 + m_2\right)\dot{q}_1^2 - m_2 r \dot{q}_1 \dot{q}_2 \cos q_2 \\ &\quad + \left(I + mr^2\right)\dot{q}_1^2 \\ P &= \frac{1}{2} k q_1^2 \end{aligned} \quad (A.31)$$

Using Euler-Lagrange formulation, the motion of equations is given by

$$\begin{aligned} &\left(m_1 + m_2\right)\ddot{q}_1 + m_2 r\cos q_2 \ddot{q}_2 - m_2 r \sin q_2 \dot{q}_2^2 + k q_1 \\ &\quad = 0 \\ &m_2 r \cos q_2 \ddot{q}_1 + \left(m_2 r^2 + I\right)\ddot{q}_2 + m_2 g r \sin q_2 = \tau \end{aligned} \quad (A.32)$$

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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