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A review on the fluid structure interaction of circular plates using numerical methods

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Abstract: Fluid structure interaction is a nonlinear multi physics phenomenon that have wide range of applications in science and engineering fields. This article presents the development of numerical methods to solve the fluid structure interaction problem deals with the vibration analysis of plate structures in contact with fluid. The modeling of fluid and structure are essential to study the fluid structure interaction problems. The development of suitable mathematical models and their validation are discussed herewith.

Keywords: Fluid structure interaction; plates; added virtual mass incremental factor; natural frequency

INTRODUCTION

The interaction between fluid flows and immersed structures comes under nonlinear multi physics phenomena that have a broad range of scientific and engineering applications. Presently, finite element methods are widely used in the analysis of structures owing to the development of modern computing techniques; thus, providing significant benefits in designing safer and economical products. Commercial computing programs such as ANSYS, ADINA, OpenFOAM were commonly used in the fluid analysis and its applications in product design such as in airplane and flow pipes(Lavrov & Guedes Soares, 2016; Liu, , Wang, Waite, & Leslie, 2016; Ruiz-Díez, Hernando-García, Ababneh, Seidel, & Sánchez-Rojas, 2016). Two major application areas of fluid flow analysis at present are aerodynamic compressible flow analysis during airplane design and incompressible and compressible flow analysis in mechanical and civil engineering design.

Numerical simulations in aeronautics and its application in mechanical engineering had started and gained attention since 1950's (Kolsky, 1949; Lighthill, 1953; Resler Jr & Sears, 1958). Although the expense required for flow simulations in mechanical engineering is lower compared to structural analysis, the number of applications in fluid flow analysis is higher. This is largely due to valuable analysis capabilities that are now available for many practical cases of fluid flow in mechanical engineering. Furthermore, the coupling of solutions of fluid flows with structural interactions develops a new field of analysis known as Fluid Structure Interaction (FSI).

Stability and response of aircraft wings (aerospace engineering), the flow of blood through veins (biomedical applications), the response of bridges and tall buildings to winds (civil engineering), and oscillation of heat exchangers and pressure vessels (nuclear industry) (Bathe, 1998)are some of the distinctive realistic examples of multidisciplinary interfacing. Although these interfaces perform a distinguished role in most of the scientific and engineering fields, still an adequate study of FSI remains as a challenge due to its strong nonlinearity and multidisciplinary nature. Furthermore, these problems are often too complicated (cost effectiveness and time-consuming procedure) to resolve analytically and to overcome this limitation, the numerical simulation technique is preferred. The usage of the numerical formulation can reduce the amount of time consumed for experimental techniques to evaluate many alternative designs. An improved understanding of the problem is obtained through a computational approach owing to the increased amount of information gathered during computation. The continuous research progress in the fields of computational fluid dynamics and computational solid mechanics had reached a maturity level in solving large industrial and academic problems that were not accessible in the past.

In general, a fluid structure interaction system is classified as either strongly or weakly coupled.

(a) Weakly coupled fluid structure system: If a structure in the flow field or containing flowing fluid deforms slightly or vibrates with small amplitude, it will affect negligibly the flow field because of the relatively low pressure. Even if significant thermal stresses in the solid may be induced by thermal gradients in the flow field, the flow field may not be greatly affected if the resulting deformation of the solid is too small. These FSI systems are called weakly coupled systems, if a thermo - fluid deforms a structure while the deformed structure hardly alters the flow field.

(b) Strongly coupled fluid – structure system: On the other hand, the fluid structure systems are called strongly coupled systems if the alteration of the flow field is due to large deformation or high amplitude vibration of the structure which cannot be neglected. In such strongly coupled FSI systems in which large structural deformation or displacement results in a significant alteration of the original flow field, both altered and original flow fields cannot be linearly super – imposed upon each other.

The investigation of FSI as in the form known to engineers working in the area of pressure vessels and piping systems is considered to have begun in the 1960's. From the early 1970's to the late 1980's a lot of investigators studied the dynamics interaction between fluid and elastic shell systems including pipes, tubes, vessels and co – axial cylinders. Nowadays, various techniques for simulating the strongly coupled fluid structure systems numerically are under development as Computational Fluid Dynamics (CFD) analysis techniques which evolves rapidly.

FLUID STRUCTURE INTERACTION OF PLATE STRUCTURES

The static and dynamic nature of plate structures under various loading conditions has great in importance in both theoretical and practical point of view. The influence of fluid on the natural frequencies of plates is of interest since the natural frequencies and mode shapes of the fluid is different from the air. These studies help to get useful information for FSI problems in science as well as engineering fields such as aerospace, construction engineering, bioscience etc.

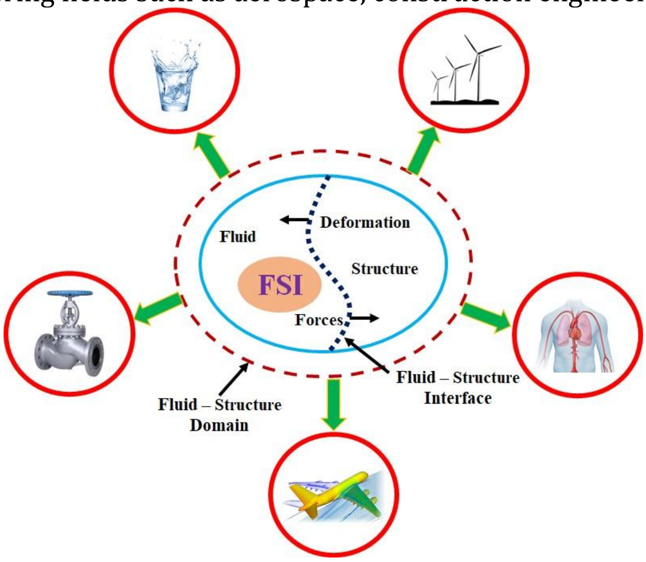

Figure 1*:* Fluid structure interaction and its applications

Many research works have been carried out to explain the interaction behavior of plates in contact with fluid, especially circular plates. The analytical and numerical approaches used to estimate the natural frequencies of circular plates in contact with a liquid on one side and placed into the hole of an infinite rigid wall was studied by Lamb(Amabili & Kwak, 1996). The eigen vectors of free vibration in vacuum was evaluated using Rayleigh-Ritz method as well as the NAVMI factor was calculated using integration method. Using the Rayleigh – Ritz method, it was found that the fundamental mode and frequency, for all the plate boundary conditions considered, is well estimated by the NAVMI factor method. The effect of fluid on the natural frequencies of circular plates vibrating axisymmetrically in contact with fluid was also investigated (Kwak & Kim, 1991). The ratio between the natural frequencies in fluid and the natural frequencies in air is a function of so-called added virtual mass incremental (AVMI) factor, which reflects the increase of inertia due to the presence of fluid. The nondimensional added virtual mass incremental (NAVMI) factor was evaluated for simply supported and clamped plates using integration techniques as well as the natural frequencies of plates in contact with fluid were evaluated. It was shown that the effect of fluid on the natural frequencies decrease with mode orders. Moreover, the effect of fluid on the natural frequencies of annular plates placed on the annular aperture of an infinite rigid wall and in contact with a fluid on one side was studied (Amabili, Frosali, & Kwak, 1996). The fluid domain is assumed to be incompressible, inviscid and unbounded. The Hankel transform is used to solve the fluid plate coupled system; boundary conditions are expressed by integral equations. Eigenfunctions of the plate vibrating in vacuum are assumed as admissible functions and the Rayleigh quotient for coupled vibration is used to obtain a Galerkin equation. The effects of fluid were explained by evaluating NAVMI factors using assumed modes approach. Besides, study on the vibration response of a cantilever cylinder surrounded by an annular fluid, which is known to be the pioneering study of fluid structure interaction for power plants (Fritz & Kiss, 1966).

However, The non-dimensionalized added virtual mass incremental factors for uniform circular plates having simply supported, clamped and free edge boundary conditions were obtained by employing the integral transformation technique in conjunction with the Fourier–Bessel series approach(Kwak, 1997). It was found that the NAVMI factors for circular plates vibrating in an infinite rigid wall with one side exposed to water are larger than those for circular plates which rest on a free surface. Furthermore, vibrations of circular plates resting on a sloshing liquid free surface were studied by solving the fully coupling problem between sloshing modes of the free surface and bulging modes of the plate using Rayleigh Ritz method (Amabili, 2001). It was verified experimentally that only small changes in the wet mode shapes occur under fluid movement which enable us to assume that the wet mode shapes are almost equivalent to the dry mode shapes.

A finite element analysis of the fluid–structure systems considered the coupled effect of elastic structure and fluid (Maity & Bhattacharyya, 2003). The equations of motion of the fluid considered inviscid and compressible were expressed in terms of the pressure variable alone. The hydro-elastic vibration of two identical circular plates coupled with a bounded fluid were investigated using an analytical method based on the finite Fourier–Bessel series expansion and the Rayleigh–Ritz method(Jeong, 2003). The numerical calculations have been carried out by assuming that a rigid cylindrical container is filled with the ideal fluid and the two plates are clamped along the container edges. It was found that the normalized natural frequency of the system monotonically increases with an increase in the number of nodal diameters and circles by virtue of a decrease in relative hydrodynamic mass.

The mathematical model for the vibration analysis of any kind of curved structure subjected to turbulent flow was developed using a combination of the finite element method and Sanders' shell theory(Kerboua, Lakis, Thomas, & Marcouiller, 2008). The transverse displacement function of the plate finite-element is derived from the equation of motion. Then, mass and stiffness matrices required by the finite element method are determined by exact analytical integration. The velocity potential and Bernoulli's equation are adopted to express the fluid pressure acting on the structure. The product of the pressure expression and the developed structural shape function is integrated over the structure-fluid interface to assess the virtual added mass due to the fluid. Variation of fluid level is considered in the calculation of the natural frequencies.

The dynamic response analysis of fluid-structure systems based on the finite element discretization of the complete system assuming pressure to be the nodal unknown for the compressible fluid domain was explored(Sharan & Gladwell, 1985). Some approximations such as reduced 'added equivalent mass' matrix for the structure, diagonalized 'mass' and 'damping' matrices for the fluid were proposed. Besides, the researchers presented finite element frequency domain and time domain methods to investigate the flutter behavior of curved panels at supersonic flow(Ghoman & Azzouz, 2012b, 2012a). The von-Kàrmàn large deflection theory and quasi – steady thermoelectricity was used in the formulation. The Newton – Raphson method was used to determine the panel deflection under static thermos aerodynamic loading and eigen value solution, was used to predict the critical dynamic pressure.

The nonlinear flutter dynamics of a cantilever plate in supersonic flow has been investigated by using simple proper orthogonal decomposition method and semi – analytical proper orthogonal decomposition method and a comparison was carried out in this study(Xie & Xu, 2015). The aero elastic instability of a plate in a gas flow has been discussed using direct time domain numerical simulation by considering three types of plate responses such as stability, static divergence and flutter(Vedeneev, Shishaeva, Kuznetsov, & Aksenov, 2014). Amplitudes and frequencies of flutter oscillations were evaluated. In case of high Mach numbers excellent correlation with classical results based on piston theory has been achieved. Maximum stress amplitude was attained at chaotic oscillations and much higher than for other flutter types because of the higher mode shapes dominating in shape of the plate oscillations. Furthermore, the dependence of amplitude on the frequencies of the nonlinear aero elastic behaviour of isotropic rectangular plates in supersonic gas flow was examined (Baghdasaryan, Mikilyan, Saghoyan, Cestino, Frulla, & Marzocca, 2015). The influence of flow speed and associated aero dynamic loading on the amplitude frequency characteristics of nonlinear aero elastic oscillations of thin and relatively thick plates were presented in this study. It can be seen that frequency increases with increasing amplitude. The aero elastic performance of flexible plate under a uniform axial flow was investigated using lumped vortex panel method and nonlinear Bernoulli beam model respectively(Dessi & Mazzocconi, 2015).

The free vibration of a rectangular isotropic plate in contact with fluid was investigated by calculating the natural frequencies for general boundary conditions(Chang & Liu, 2000). The natural frequencies of the plate in contact with the fluid are determined by calculating the added virtual mass incremental (AVMI) factor which represents the kinetic energy due to the fluid. Also, the nonlinear flutter behavior of an orthotropic composite laminated rectangular plate under aerodynamic pressures and transverse excitation was also presented (Chen & Li, 2016). The air pressures were modeled by applying first-order linear piston theory. The nonlinear governing equations of motion were derived for the plate using Hamilton's principle based on Reddy's third order shear deformation plate theory and von-Kàrmàn type equation for the geometric nonlinearity. The partial differential governing equations were transformed into a set of nonlinear ordinary differential equations by employing Galerkin method. The critical Mach number for occurrence of the flutter of the plate was investigated by solving the eigenvalues problem. The relationship between the limit cycle oscillation and the critical Mach number was analyzed based on the nonlinear equations. The numerical simulation studies the influences of the transverse excitation on the nonlinear dynamics of the composited laminated plate.

Moreover, the effects of the nondimensionalized added virtual mass incremental (NAVMI) factor and various parameters on the natural frequencies and hence the vibration behavior of plates submerged in fluid were investigated using dual integration technique and Galerkin method respectively(Cheung & Zhou, 2002; Kwak, 1991). The hypersonic fluid structure interaction of cantilever plate involving shock impingement has been studied numerically and experimentally (Currao, Neely, Kennell, Gai, & Buttsworth, 2019). The shock induces a pressure differential across the plate thickness that drives its oscillatory behavior as well as the data are used to predict the performance of two- dimensional control surfaces using boundary layer interaction.

Also, fluid structure interaction solver based on finite difference method for compressible flows on plate structures discussed the coupling between the nonlinear dynamics of plate and blast loading (Bailoor, Annangi, Seo, & Bhardwaj, 2017). The effects of material properties as well as length of the plate on the flow induced deformation is studied. The discontinuous Galerkin method is used to solve the FSI problem deals with compressible viscous flow with nonlinear elastic structures (Kosík, Feistauer, Hadrava, & Horáček, 2015). The flow is described by Navier Stoke's equations and Kirchhoff model is applied for structural deformation. Furthermore, the FSI problem of thin hot plate inside an enclosure has been solved by the assumption that the plate is isothermal and fixed at an alterable point(Mehryan, Alsabery, Modir, Izadpanahi, & Ghalambaz, 2020). The finite element method associated with Arbitrary Lagrangian Eulerian (ALE) formulation is used to get steady state contours of isotherms and streamlines for various fixed points. FSI with compressible multiphase flows involving large structural deformation has been studied by using immersed boundary layer method(Wang, Currao, Han, Neely, Young, & Tian, 2017). The mathematical modeling of structure interaction with fluid also depends on its material property in nature. The interaction of circular plates with fluid is important in the industrial application for the proper and safer design of structures. For most FSI problems, analytical solutions to the model equations are impossible to obtain, whereas laboratory experiments are limited in scope; thus, to investigate the fundamental physics involved in the complex interaction between fluids and solids, numerical simulations may be employed to find a solution for the governing equations of the individual problems.

NUMERICAL METHOD TO SOLVE THE FLUID STRUCTURE INTERACTION PROBLEM

A circular plate of isotropic material properties with clamped boundary condition is considered herewith. The non - dimensionalized added virtual mass incremental (NAVMI) factor for plates in contact with the fluid is evaluated using integral transformation method by taking the advantage of the admissible function which satisfies the boundary condition(Nair, Kasim, & Salleh, 2017). Galerkin method is applied to evaluate the natural frequencies and mode shape of the circular plate in the air and are used to evaluate the natural frequencies of the plate in contact with a fluid for practical applications.

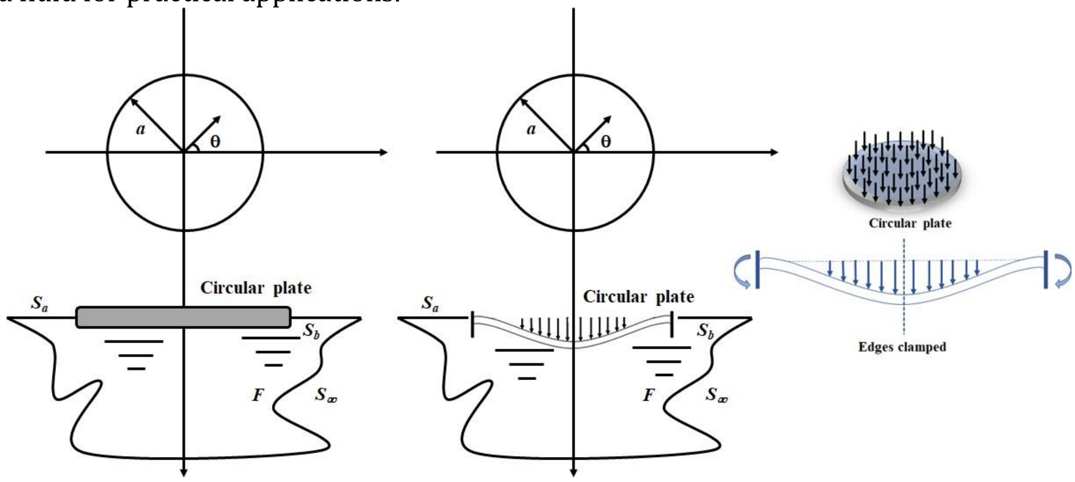

Figure 2: A circular plate in contact with fluid with respect to clamped boundary conditions

The numerical formulation is produced using the hypothesis that the fluid is inviscid, incompressible and irrotational. Since the diameter of the plate is notably greater than the wavelength, the motion will be very small. Due to the strong loading of liquid there are r changes in kinetic as well as potential energies of structures in contact with fluid from the a governing mathematical equation of circular plates in contact with liquid based on the above assumptions can be written as

$$\boldsymbol{D\nabla^4 w - \frac{1-v}{r}\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial w}{\partial r}\right) + \rho_\rho h + \delta \frac{\partial^2 w}{\partial r^2} = 0} \tag{1}$$

where w is the plate's deflection, $D = \frac{Eh^3}{12(1-v^2)}$ is the flexural rigidity, ρ, is the mass density of the plate, $\boldsymbol{\delta}$ is the AVMI factor and ν = 0.3 is the Poisson's ratio.

A suitable admissible function for $W(r)$ is introduced to simplify the calculations and the

Rayleigh-Ritz method is applied to obtain the mode shapes of the circular plate. For various mode orders of the plate, the shapes under wet mode can be supposed as a harmonic function, which can be written as

$$w(r, \theta, t) = \sum_{s=0}^{\infty} \sum_{n=0}^{\infty} W(r) \cos s\theta \sin \omega t \tag{2}$$

The following parameters for plates are also considered

$$a = 1m;\ h_0 = 0.5m;\ \rho_p = \frac{2{,}44x10^3 kg}{m^3};\rho_f = 1000kg/m^3$$

Since the fluid is assumed to be irrotational, the velocity potential and can be indicated as

(3)

$$\tilde{U}(r,\theta,z,t) = \phi(r,\theta,z)f(t) = \phi(r,z)f(t)$$

where ϕ(r z,) is the spatial distribution which satisfies the Laplace equation,

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{1\partial\phi}{r\partial r} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad \text{in F, the fluid domain} \tag{4}$$

and $f(t) = e^{i\omega t}$ is the frequency of the circular plate coupled with the fluid.

The boundary condition of the rigid wall on S_a can be expressed as $\frac{\partial\phi(r,z)}{\partial z} = 0$ at z=0 on S_a, signifies the layer between the fluid and rigid wall. Furthermore, the boundary condition for the interaction amid the fluid and the surface is indicated by $\frac{\partial\phi(r,z)}{\partial n} = -W(r)$ at z=0 on S_b, indicates the layer between the fluid and the plate. Since the distance from the plate becomes very large, ϕ and the velocities $\frac{\partial\phi}{\partial r}$ plus $\frac{\partial\phi}{\partial z}$ tends to zero. ie, $\phi(r,\theta,z), \frac{\partial\phi(r,\theta,z)}{\partial r}, \frac{\partial\phi(r,\theta,z)}{\partial z} \to 0$ for $r, z \to \infty$ on S_∞, represents the infinity surface.

Based on Rayleigh's quotient, we can write

$$f_a^2 \propto \left(\frac{V_p}{T_p^*}\right)_{air} \text{ and } f_a^2 \propto \left(\frac{V_p}{T_p^* + T_l^*}\right)_{fluid} \tag{5}$$

Where f_a is the natural frequency of the plate, f_1 is the natural frequency of the plate in contact with the fluid, T_p^* and V_p are the reference kinetic energy and maximum potential energy of the plate and T_1^* is the reference kinetic energy of the fluid due to the motion of the plate (Nair, Kasim, & Salleh, 2017) Besides, the relation between reference ad maximum kinetic energies can be written as $T_{max} = T^*\omega^2$ where ω is the frequency in radians per second (Meirovitch, 1975). Using the hypothesis that due to the dynamic loading of liquid the kinetic energy and potential energy of the circular plate coupled with fluid have a trivial effect on mode shapes as well as wet mode shape is equal to dry mode shape, eqn. (5) can be reduced to

$$f_1 = \frac{f_a}{\sqrt{1+\delta}} \tag{6}$$

where δ is called AVMI factor which is the ratio of the kinetic energy of the plate to the kinetic energy of the plate itself. Hence, δ can be signified as $\delta = \frac{T_l^*}{T_p^*} = \Gamma \frac{\rho_l}{\rho_p}\left(\frac{a}{h}\right)$ (7)

where Γ is termed of non-dimensionalized NAVMI factor, ρ_l is the density of the fluid and ρ_p is the mass density of the plate. Using the assumption that the fluid is irrotational, the reference kinetic energy of the fluid can be evaluated from its velocity potential as

$$T_l^* = -\frac{1}{2}\rho_l \int_0^{2\pi} \int_{-\infty}^{\infty} \frac{\partial\phi(r,z)}{\partial n} \phi(r,z) dr d\theta \tag{8}$$

Using eqn. (2), the reference kinetic energy of the plate can be represented as

$$T_p^* = \frac{1}{2}\rho_p \int_0^{2\pi} \int_0^{a} h\widetilde{W}(r) r \, dr \, d\theta \tag{9}$$

where $\widetilde{W}(r)$ is the mode shape of the plate in the air.

The reference kinetic energies of the fluid as well as the plate can be evaluated from eqns. (8) and (9) and hence the values of f_l are calculated using eqn. (5). The mode shapes of the plates in the air are evaluated using the differential equation which can be taken as

$$\boldsymbol{D\nabla^4 w - \frac{1-v}{r}\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial w}{\partial r}\right) + \rho_\rho h \frac{\partial^2 w}{\partial r^2} = 0} \tag{10}$$

By considering the following assumptions:

$h(r) = h_0 f(r); f(r) = 1 - \mu r^2$ as well as $w(r,t) = W(r)f(t) = W(r)e^{i\omega t}$ where h_0 denotes the thickness of the centre of the plate whereas μ is the taper parameter of the varying curve leads the equation to the following form

$$\boldsymbol{F\nabla^4 W - \frac{1-v}{r}\left(F_{rr}\frac{\partial^2 w}{\partial r^2} + F_r \frac{\partial w}{\partial r}\right)} + \eta^2 \boldsymbol{fW = 0} \tag{11}$$

Where $F = f^3(r)$ and $\eta^2 = \frac{12(1-v^2)\rho_p\omega^2}{Eh_0^2}$
According to Galerkin method, assume the interpolating function in the algebraic form as given in eqn. (12).

$$\acute{W}_n = \sum_{m=1}^{N} b_m\left(\frac{r^2}{a^2}-1\right)^{m+1} \tag{12}$$

which satisfies the clamped boundary conditions at r = 0, w′ = 0; At r = a, w = 0, w′ = 0.
Then the Galerkin formula can be represented in the integral form as

$$\int_0^{2\pi}\int_0^{a} E_n\left(\frac{r^2}{a^2}-1\right)^{m+1} rdrd\theta = 0 \qquad m = 1,2,\dots,N \tag{13}$$

where E_n is the residual.
The expression for E_n can be attained by substituting eqn. (12) into eqn. (11) and could be illustrated as $E_n = \sum_{n-1}^{2} e_n\left(\frac{d^n w}{dr^n}\right) - \eta^2 fw$ where e_n is the derived coefficient and $n^2 = \frac{12(1-v^2)\rho_p\omega^2}{Eh_0^2}$.

The ordinary differential equation can be reduced from the governing equations by introducing the Hankel transform (Amabili, Frosali, & Kwak, 1996).

$$\overline{\phi}(\xi,z) = \int_0^{\infty} r\,\phi(r,z)J_s(\xi r)dr \tag{14}$$

Therefore, eqn. (4) is reduced to the ordinary differential equation of the form

$$\frac{d^2\overline{\phi}}{dz^2} - \xi^2\overline{\phi} = 0 \tag{15}$$

The inversion formula for Hankel transform is defined by,

$$\phi(r,z) = \int_0^{\infty} \xi\overline{\phi}(\xi,z)J_s(\xi r)d\xi \tag{16}$$

Introduce the nondimensionalized parameters:

$$\rho = \frac{r}{a};\psi = a\xi; A(\psi) = \psi B(\psi) \tag{17}$$

Therefore, the integral equations can be described as the forms given in eqns. (18) and (19) respectively.

$$\int_0^{\infty} \psi A(\psi)\,J_s(\psi\rho)d\psi = 0 \text{ for } \rho > 1 \tag{18}$$

$$\int_0^{\infty} \psi A(\psi)\ [J_s(\psi\rho) + h_0\mu a\rho J_{s+1}(\psi\rho)]d\psi = \sqrt{\mu^2h_0^2a^2p^2+1}\sum_{m=1}^{N}(b_m)^n.a^3.(p^2-1)^{m+1};$$
$$n = 1,2,\dots,N \ for\ 0 \le \rho \le 1 \tag{19}$$

The solution of integral eqns. (18) and (19) can be attained through the properties of Hankel transform as well as the inversion theorem of Hankel transform. Hence, the solution A(ψ) can be evaluated from the following equation

$$A(\psi) = \int_0^{1}\rho\left[\sqrt{\mu^2h_0^2a^2p^2+1}\ \sum_{m=1}^{N}(b_m)^n.a^3.(p^2-1)^{m+1}\right]\times\ [J_s(\psi\rho)+h_0\mu a\rho J_{s+1}(\psi\rho)]d\rho$$

$$= a^3.\sum_{m=1}^{N}(b_m)\left[\sqrt{\mu^2h_0^2a^2p^2+1}\ \sum_{m=1}^{N}(b_m)^n.a^3.(p^2-1)^{m+1}\times[J_s(\psi\rho)+h_0\mu a\rho J_{s+1}(\psi\rho)]d\rho\right] \tag{20}$$

The solution A(ψ) is evaluated using numerical integration techniques by taking the advantage of MAPLE software, and hence, the values of B(ψ) as well as ϕ(r z,) can be calculated using numerical calculations respectively. Therefore, the kinetic energy of the fluid can be computed 11 from eqn. (8) by evaluating the velocity potential function ϕ(r z,) , as well as the reference kinetic energy of the plate, can be measured using eqn. (8) by approximating the mode shapes of the plate. The values NAVMI factor is estimated using these values of kinetic energies of the plate as well as fluid. Hence, the natural frequencies of plate associated with fluid can be determined from eqn. (6) by applying the values of NAVMI factors.

RESULTS AND DISCUSSION

The numerical approximation for the fluid structure interaction of circular plates coupled with fluid is presented as an example. The NAVMI factor and corresponding natural frequencies are formulated using the Galerkin method and as well as integration techniques based on Rayleigh quotient. The results for clamped boundary conditions are compared with the known literature values(M.-F. Liu & Chang, 2004).

Table 1 represent the values of NAVMI factor for clamped circular plates where n, as well as N, indicate the order of the mode number and number of terms used in the interpolating function to estimate the mode shape respectively.

N	n = 1	n = 2	n = 3	n = 4	n = 5
1	0.6823 (0.6826) *	-	-	-	-
2	0.6668 (0.6671) *	0.3150 (0.3153) *	-	-	-
3	0.6665 (0.6668) *	0.2826 (0.2829) *	0.2065 (0.2068) *	-	-
4	0.6665 (0.6668) *	0.2805 (0.2808) *	0.1749 (0.1752) *	0.1527 (0.1530) *	-
5	0.6665 (0.6668) *	0.2797 (0.2800) *	0.1673 (0.1670) *	0.1288 (0.1291) *	0.1189 (0.1192) *

From Table 1, when considering the values of the NAVMI factor, the first mode plays a leading role since the values in the first mode are greater than those of the other modes. Furthermore, the values of NAVMI decreases as the order of mode number increases due to the fluid drive stroke of the lower mode is greater than the higher mode. Besides, it can be noted that the fluid has significant influence on fluid structure interaction of plate structures as the presence of fluid decreases with mode number regardless of boundary conditions.

Table 2 indicates the corresponding natural frequencies of circular plates in contact with fluid with respect to clamped boundary conditions.

	fl		
μ	*n* = 1	*n* = 2	*n* = 3
−0.8	0.0126 (0.0130) *	0.0615 (0.0619) *	0.1592 (0.1596) *
−0.6	0.0113 (0.0117) *	0.0570 (0.0574) *	0.1465 (0.1469) *
−0.4	0.0100 (0.0104) *	0.0522 (0.0526) *	0.1353 (0.1357) *

−0.2	0.0087 (0.0091) *	0.0473 (0.0477) *	0.1240 (0.1244) *
0	0.0074 (0.0078) *	0.0422 (0.0426) *	0.1122 (0.1126) *
0.2	0.0062 (0.0066) *	0.0370 (0.0374) *	0.1005 (0.1009) *
0.4	0.0050 (0.0054) *	0.0316 (0.0320) *	0.0875 (0.0879) *
0.6	0.0038 (0.0042) *	0.0252 (0.0256) *	0.0730 (0.0734) *
0.8	0.0026 (0.0030) *	0.0183 (0.0187) *	0.0575 (0.0579) *

It can be observed from Table 2 that as the value of μ increases the natural frequency of plate decreases. The natural frequencies under clamped conditions is compared with the reference values (M.-F. Liu & Chang, 2004)which shows the accuracy of the hypothetical formulation. The highest value of the taper parameter shows the highest values of the NAVMI factor as well as the smallest values of natural frequencies implies the influence of fluid on the fluid structure interaction of structures. Figure 1 shows the graphical illustration of natural frequencies with respect to different taper parameter and it shows the pattern of how the taper parameter effects on the natural frequencies.

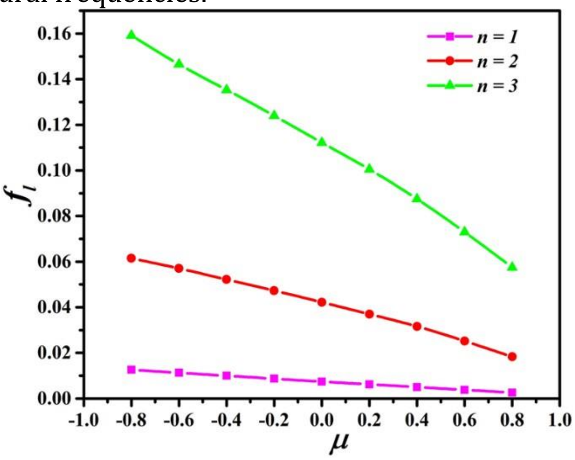

Figure 3: The values of natural frequencies of circular plates in contact with fluid with respect to various taper parameter.

The numerical methods are important because it concerns the circumstance of the coupling is limited only to the fluid interface. Subsequently, these numerical techniques reduce the instability of the fluid-structure problem of different structures and it confirms the stability for fluid motion. Hence, the values of natural frequencies will help the engineers to design the plate structures in contact with fluid safer and more economical.

CONCLUSION

The article reviews the interaction of plate structures in contact with fluid. Owing to the multidisciplinary nature of FSI problems, the numerical procedures used by various methods to solve the interface conditions between fluids and structures. The FSI problems of plates in contact with fluid under clamped boundary conditions are considered. The NAVMI factor as well as the corresponding natural frequencies are evaluated using Galerkin method and integration techniques based on Rayleigh quotient. The comparison of numerical results for clamped circular plates indicates that the numerical hypothetical formulation gives accurate results within the engineering accuracy. Hence these formulations will be useful to the designers in many fields of engineering such as automotive, aircraft, civil engineering and can be adopted as a guide to the design of structures specifically for circular plates in contact with a fluid.

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