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A Review on Tuning of Extended Kalman Filter using Optimization Techniques for State Estimation

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ABSTRACT
State estimation is the common problem in every area of engineering. There are different filters used to overcome the problem of state estimation like Kalman filter, Particle filters etc. Kalman Filter is popular when the system is linear but when the system is highly non-linear then the different derivatives of Kalman Filter are used like Extended Kalman Filter (EKF), Unscented Kalman filter. But these estimation techniques require tuning of process and noise covariance matrices. The different optimization techniques are used to tune the filter parameters of EKF. In this paper, various optimization techniques have been studied for non-linear state estimation based on EKF.

General Terms
Optimization techniques, non-linear.

Keywords
Gravitational Search Algorithm, Extended Kalman Filter, state estimation, Tuning of EKF.

1. INTRODUCTION
There are various problems in every area of engineering which includes the state estimation problem [1]. States of a system are the variables that provides complete status of a system at a given instant of time. This state estimation problem also occurs in data mining applications. When the object changes its state from one place to another there should be addition of some extra parameters. State space model is used to design the state estimator which describes the physical process of a system. The state space model gives the connection between the position and the velocity and these variables are real numbers called continuous variables [2]. To solve this state estimation problem, there are various filters used like Kalman Filter, Extended Kalman Filter, Particle Filters and Unscented Kalman Filter [3]. Kalman Filter is used when the system is linear but, when the system is highly non-linear then the different derivatives of Kalman filter are used. Extended Kalman Filter is used to remove the non-linearity of the system [4]. EKF is popular than other derivatives of Kalman Filter because of its ease of implementation, acceptable performance and computational efficiency.

Extended Kalman Filter suffers from different issues like linearization error, initialization error, covariance estimation and divergence. So, there is a need to correctly tune the filter parameters to improve the performance. These parameters include process (Q) and measurement noise (R) covariance matrices. To find noise covariance matrix, adaptive approach is used but this approach suffers from different problems like lack of convergence and large window requirement [5]. Therefore, different optimization techniques are used to tune the process and noise covariance matrices like Genetic Algorithm, Particle Swarm Optimization, and Human Opinion Dynamics. Genetic Algorithm is based on theory of genetics in which each individual in the population encoded with the chromosome [9]. Particle Swarm Optimization is a transformative algorithm in which social behavior of fish schooling and bird flocking is taken as inspiration [10]. Human Opinion Dynamics is an evolutionary technique that is inspired by physical interaction between the individuals in a group to solve the complex mathematical formulation [11]. Gravitational Search Algorithm is based on Newton's law of gravity in which agents are considered as objects and the performance of objects is measured by their masses [12]

A detailed review of various optimization techniques likes Genetic Algorithm, Particle Swarm Optimization, Human Opinion Dynamics, and Gravitational Search Algorithm (GSA) has been done in this paper for non-linear state estimation.

1.1 Extended Kalman Filter
When the system is highly non-linear then Extended Kalman Filter is used to remove the non-linearity for state estimation. Extended Kalman Filter gives the approximation of the optimal state estimation and using Taylor series expansion the non-linear model is linearized. The estimate of the previous time step is taken as the nominal value for linearization.

By considering the following non-linear equation with continuous process dynamics and discrete measurement noise dynamics is described by:

$$\dot{y}(t) = f\big(y(t)\big) + \Gamma_c u(t)$$
$$z_k = h(y_k) + m_k \qquad (1)$$

In the above equations, $y \epsilon \Re^{n}$ indicates the n-dimensional state vector of the non-linear system, $f(.): N_y \rightarrow \Re^{n}$ represents a mapping of finite nonlinear system states to inputs of system, $z_k \in N_k \subset \Re^{P}$ indicates the system measurement of p-dimensional, $h(.): N_y \subset \Re^{n} \rightarrow \Re^{P}$ denotes nonlinear mapping of system states to output, $\Gamma_c \in \Re^{n \times u}$ indicates the scaling matrix of continuous process noise, $u \in N_u \subset \Re^{u}$ represents the u-dimensional random process noise and $m \in N_m \subset \Re^{m}$ represents the m-dimensional random measurement noise.

The process measurement and noise measurement should be assumed zero mean, band-limited, AWGN processes are given by:

$$E[u(t)u(t-\tau)^T] = Q\delta(T-\tau) = \begin{cases} Q \rightarrow t = \tau \\ 0 \rightarrow t \neq \tau \end{cases}$$
$$E[m_k m_j^T] = R_k\delta_{kj} = \begin{cases} R_k \rightarrow k = j \\ 0 \rightarrow k \neq j \end{cases} \quad (2)$$

where, Q indicates the continuous process noise covariance, R_k indicates the discrete measurement noise covariance, δ(.) indicates the Dirac delta function and δ_{kj} denotes the Kronecker delta function. The uncorrelated random variables u and m_k are represented as u (0 Q) & m(0 R_k) respectively. The initial state of the system which is assumed to be a Gaussian random vector with mean $\bar{y}_0$ and covariance $\bar{S}_0$ and assumed k be the discrete index. Then the EKF form is given as:

Predictor Step:

$$\hat{y}(t) = \hat{y}_{k-1}, S(t) = S_{k-1}$$
$$\dot{\hat{y}}(t) = f(\hat{y}(t))$$
$$\dot{S}(t) = F(t)S(t) + S(t)F^T(t) + \Gamma_c Q \Gamma_c^T$$
$$F(t) = \frac{\partial f}{\partial y(t)} | y(t) = \hat{y}(t)$$

Corrector Step:

$$D_k = S_k^- H_k^T \left(H_K S_k^- H_k^T + R_k\right)^{-1}, S_k^- = S(t) + \dot{S}(t)\Delta t$$
$$\hat{y}_k = \hat{y}_k^- + K_k(z_k - z_k^-),$$
$$\hat{y}_k^- = \hat{y}(t) + \dot{\hat{y}}(t).\Delta t, z_k^- = h(\hat{y}_k^-)$$
$$S_k = (I - K_k H_k)S_k^-$$
$$H_k = \frac{\partial h}{\partial y_k}|_{y_k = \hat{y}_k^-} \quad (3)$$

2. RELATED WORK

Erik Bolviken et al. discussed the Monte Carlo filters for non-linear state estimation and shows that a Monte Carlo method does not demand thousands of samples. These Monte Carlo methods are easy to implement and can be applied generally. The Monte Carlo methods are also compared with Kalman Filters is terms of error in position, standard deviation and speed. But, there is a disadvantage of Monte Carlo methods that it takes many attempts to converge due to its slow numerical process [6]. Arulampalam et al. reviewed the optimal Bayesian Algorithms and suboptimal Bayesian Algorithms for non-linear state estimation problems using Particle Filters. Monte Carlo filters include Particle Filters which are based on point mass and Particle filters approximate the density as a finite number of samples. Various variants of Extended Kalman Filter i.e. RPF, SIR and ASIR are introduced by the author and then these techniques are compared with the standard Extended Kalman Filter. Extended Kalman filter has two stages: prediction stage and updating stage. The prediction stage predicts the next stage and the updating stage modify the prediction [1]. K. Xiong et al. proposed a novel method to design the Robust Extended Kalman Filter to ensure the stability of the filter and then the adaptive approach is applied to tune the error covariance matrix. The sufficient conditions are applied to ensure the stability of the Robust Extended Kalman Filter and the adaptive Robust Extended Kalman Filter is developed to cope the estimation error limitation. Unscented Kalman Filter is accurate than Extended Kalman filter but UKF suffers from divergence problem. Therefore, adaptive Extended Kalman Filter is used for online estimation of covariance matrices but it is useful in some cases. Then, the Robust Extended Kalman Filter is used because it is easy to use and there should be no any complicated computation procedures [7]. Kandepu et al. uses the Unscented Kalman Filter for non-linear state estimation and discuss the difference between the Extended Kalman Filter and Unscented Kalman Filter. Unscented Kalman Filter is basically used for process control systems. Extended Kalman Filter is based on linearization but Unscented Kalman Filter is not and the author also proposed the simple method to integrate state constraints. Four different examples are used to compare the Unscented Kalman Filter and Extended Kalman Filter. The first example is of Van der Pol in which behavior and robustness are compared. In Second, Induction machine is used to evaluate the performance. In third, reversible reaction is studied and in fourth, Solid Oxide Fuel Cell is considered to calculate the performance [8].

In [9], Genetic Algorithm has been proposed to solve non-linear models. Genetic Algorithm based on the theory of genetics in which each individual in the population encoded with chromosomes and this encoding represents the objective function parameters. There are different techniques used to encode the parameters by performing the selection and adaptation stages of the system. The adaptation stage is divided into two parts: Crossover and Mutation. Using Genetic Algorithm, the results can be found out without converting the non-linear equations into linear equations. Then, the obtained results are compared with the results obtained using numerical methods. The results show that the Genetic Algorithm is effective to solve the non-linear equations. Then, Gravitational Search Algorithm has been proposed in [12] which is based on Newton's law of gravity. In this algorithm where agents are considered as the objects and the performance is measured by the masses of that objects. The objects may attract each other by the force of gravity. For a high dimensional search space, the classical optimization techniques do not provide the sufficient solution to solve optimization problems. Therefore, various heuristic optimization algorithms have been proposed to solve this problem. Different optimization techniques are used to solve different optimization problems. But, there is no any technique which is suitable for all the optimization problems. Therefore, Gravitational Search Algorithm has been proposed by Rashedi et al. (2009). And, then the results are compared with Central Force Optimization, Particle Swarm optimization and Real Genetic Algorithm. A new method Particle Swarm Optimization- Particle Filter (PSO-PF) has been presented in [13] to distribute the particles in which particles will imitate the posteriori density accurately for vision tracking. Firstly, Particle Swarm Optimization technique is used to utilize some particles to search the area and then the particles are distributed in this area. Convergence and diversity are combined using distribution method. The motion model helps to distribute the particle in right area but sometimes the motion model of the objects cannot be identified. The resampling scheme set the particles according to their weights. Then again Rashedi et al. proposed another variant of GSA called Binary GSA (BGSA) which is used to solve discrete optimization problems. In BGSA, probability function is employed to update the binary bits string which is based on the absolute velocity value. Another variant of Gravitational Search Algorithm called Fast Discrete Gravitational Search Algorithm (FDGSA) has been proposed in [15] to solve discrete optimization problems using integer values. Unimodal test functions are used to calculate the performance of the Discrete Gravitational Search Algorithm.

Based on the velocity and position, the integer values are updated and then randomly the selection is done.

Mallick et al. proposed two novel evolutionary techniques i.e. Gravitational Search Algorithm and Improved Particle Swarm Optimization to solve static state estimation problem. And the results are compared with the Particle Swarm Optimization, Least Square State Estimation Technique, and Hybrid Particle Swarm Optimization Gravitational Search Algorithm. And the proposed algorithm is used to improve the error performance also. To get the better success ratio and to achieve faster convergence, Gravitational Search Algorithm is used. Gravitational search algorithm is used in various fields by the researchers [16]. [17] modified the Gravitational Search Algorithm because GSA has no memory ability and compare the results with Gravitational Search Algorithm using standard 12 benchmark functions. Therefore, the idea of local and global optimum solution is taken from the Particle Swarm Optimization into GSA. Hence, Modified Gravitational Search algorithm enhances the particle memory ability and improves the search ability. The classical optimization techniques cannot provide a suitable solution for non-linear problems. Therefore, swarm based algorithms are used to solve the non-linear systems. There are different swarm based algorithms used and the new algorithm used is Gravitational Search algorithm. In Modified Gravitational search Algorithm the velocity updation is modified and therefore, MGSA can be used in other areas also. [18] presented a new optimization method Artificial Bee Colony Algorithm to solve parameter identification problems. Artificial Bee Colony Algorithm is a swarm based algorithm which is inspired by honey bee foraging. The parameter identification problem is solved for Bouc-Wen model for the first time. Then, the results are compared with techniques that are used in the literature. There are two types of approaches used for solving optimizations problems- mathematical approaches and metaheuristic approaches. Metaheuristic approaches are widely used to solve the parameter identification problems. The most challenging problem is due to highly non-linear nature of Bouc-Wen system. The different methods are used to solve this problem therefore, Artificial Bee Colony algorithm is proposed by the author to solve this problem.

Mirjalili et al. proposed hybrid PSO-GSA i.e. Particle Swarm Optimization- Gravitational Search Algorithmto solve binary optimization problems. The results show the better performance than Binary Particle Swarm Optimization, Binary Gravitational Search Algorithm and Genetic Algorithm in terms of convergence rate and avoiding local minima. To evaluate the efficiencies of the proposed algorithm 22 benchmark are used which are divided in three categories: Unimodal; Multimodal and Composite [19]. In [20], the Unscented Kalman Filter and Particle Filter are compared with Extended Kalman Filter to estimate the state for various non-linear systems. Unscented Kalman Filter uses the deterministic sampling approach and Particle Filter is based on statistical signal processing. Extended Kalman filter uses linearization with standard Kalman Filter but Unscented Kalman Filter use only one linearization method. The author used Gaussian Probability Distribution function to control the disturbing functions. These Disturbing factors include measurement and model uncertainties. Kalman Filter is used when the system is linear, but when the system is non-linear then the commonly used technique is Extended Kalman Filter. For some systems EKF may show errors in the estimated states therefore, Unscented Kalman Filter is used which is a derivative free method. The Particle filter and Unscented Kalman filter are simple to use as compared to extended Kalman filter. A hybrid optimization technique i.e. Genetic Algorithm- Particle Swarm Optimization (GA-PSO) has been proposed in [21] to tune the Unscented Kalman Filter (UKF) parameters. UKF uses deterministic approach to tune the process and noise covariance matrices. Kalman filter is used to estimate the position, velocity and acceleration of a target in tracking applications. Extended Kalman Filter is used when the system is non-linear which uses linearization but sometimes linearization shows errors. Therefore, the author used the Unscented Kalman Filter to overcome this problem. By using evolutionary computing tools, which are based on biological inspired approach, the process and noise covariance matrices are tuned.

In [22], Human Opinion Dynamics (HOD) based optimization technique has been proposed in which the process and measurement noise covariance matrices of Extended Kalman are tuned for state estimation and compare the results with Particle Swarm Optimization. The Extended Kalman Filter simulations are carried out by using permanent magnet synchronous model system. The state estimates obtained from this model using Human Opinion Dynamics based optimization are compared with the state estimates acquired from Particle swarm optimization. There are adaptive and non-adaptive methods used earlier to tune the filter parameters of the Extended Kalman filter but these methods suffers from some problems like large window requirement and lack of convergence. So, optimization techniques are used to tune these filter parameters of Extended Kalman Filter. Human Opinion Dynamics is inspired from physical interaction between various individuals in a group to solve the complex mathematical problems.

In Table 1, different optimization techniques have been studied with the problem formulated and the performance metrics that are used to compare the results.

Table 1. Different optimization techniques

Author	Algorithm	Problem	Perform--ance Metrics
Ibrahiem M.M. El-Emary et al. [9] (2008)	Genetic Algorithm	To find solution of a system of non-linear equations	Number of quadratic nodes and associated weights
Rashedi et al. [12] (2009)	Gravitationa l Search Algorithm	To solve various non-linear functions	23 Non-linear unimodal and multimodal Benchmark Functions
Jing Zhao, Zhiyuan Li [13] (2010)	Particle Filter based on PSO	Vision tracking problem	Tracking error
Rashedi et al. [14] (2011)	GSA	To solve parameter estimation problem	Unimodal and multimodal functions, complexity
Shahdan Sudin et al. [15]	Fast Discrete GSA	To solve discrete optimization problems	Convergence, mean, median, S.D and Fitness

(2012)		using integer values	value
Binjie Gu and Feng Pan [17] (2013)	Modified GSA	To enhance particle memory ability of GSA	12 standard benchmark functions
S. Talatahari et al. [18] (2014)	Artificial Bee Colony Algorithm	Parameter identification problem of Bouc-Wen model	Parameters of Bouc-Wen models
Ravi Kumar Jatoth and T. Kishore Kumar [21] (2014)	Hybrid GA-PSO	To tune Unscented Kalman filter parameters	Filter estimations, RMS error and mean error
Sajjad Hussain et al. [22] (2015)	Human Opinion Dynamics	To tune Extended Kalman filter parameters	Error value, S.D and convergence

3. GAPS IN STUDY

- There have been various approaches to deal with this issue of finding optimized noise co-variance matrix, and are broadly classified under two different categories, i.e., Adaptive and Non- adaptive. First type of the methods, varies the noise covariance values adaptively after each instance, and is online in its essence, whereas the second method tunes these values offline, and does not change for a specific run of the system.
- Furthermore, there have been several advancements to these approaches employing artificial intelligence techniques, like Fuzzy logic and Neural network. But, these Adaptive Kalman filtering techniques add on to the computational requirement, and might not be feasible for generic real time applications.
- As a result, non-adaptive offline tuning of filter parameters seems to be a better choice, which gives sufficient results for most of the applications. Most of the time, tuning is left to the engineering intuition, however, owing to the high dimensionality of the problem, it is not only a cumbersome task but might as well not lead to an optimized value.

4. CONCLUSION

In this paper, different optimization techniques have been studied with the performance metrics for non-linear state estimation based Extended Kalman Filter. In future more optimization techniques will be proposed for the non-linear state estimation problem by doing some more adjustments to get better results.

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