A Ring-Type Digital Spiking Neural Network and Spike-Train Approximation

Hiroaki Uchida and Toshimichi Saito

Hosei University, Koganei, Tokyo, 184-8584 Japan, tsaito@hosei.ac.jp

Abstract. This paper studies ring-type digital spiking neural networks that can exhibit multi-phase synchronization phenomena of various periodic spike-trains. First, in order to realize approximation of a class of spike-trains, a winner-take-all switching is applied to the network. Second, in order to design efficient networks, relationship between approximation error and the network size is investigated. Executing Verilog simulation, approximation function is confirmed experimentally.

Keywords: spiking neural networks, multi-phase synchronization, spike-train approximation.

1 introduction

This paper presents a network of digital spiking neurons (DSN [1][2]) and considers its application to time-series approximation. The DSN can be regarded as a digital version of analog spiking neurons that have been studied from both fundamental and application viewpoints [3][4]. Repeating integrate-and-fire behavior between a periodic base signal and constant threshold, the DSN can output a variety of periodic spike-trains (PSTs). Applying delayed ring connection to multiple DSNs, a ring-type digital spiking neuron (RDSNN) is constructed. The RDSNN can realize multi-phase synchronization of various PSTs [5][6]. Adjusting the base signal, stability of the synchronization can be reinforced. The PSTs and synchronization of them are applicable to various engineering systems including spike-based encoding communication [7], central pattern generators [8], and time series approximation [9][10].

In order to realize time-series approximation, winner-take-all (WTA) switching [4] is applied in the RDSNN where an objective time series is represented by a PST. This paper gives two main results. First, the WTA switching enables RDSNN to approximate a target PST automatically if the number of spikes in the PST satisfies some condition. Second, relationship between approximation error and the number of DSNs is investigated for typical examples of target PSTs. Executing Verilog simulation, typical synchronization phenomena and basic approximation performance are confirmed experimentally.

The results of the paper will be developed into systematic analysis of nonlinear dynamics in RDSNNs, optimal design of RDSNN for approximation of target PSTs, and its application to various systems including reservoir computing systems for time series approximation. As novelty of this paper, it should be noted that existing papers discuss neither the WTA-based switching nor the relationship between approximation error and the number of DSNs.

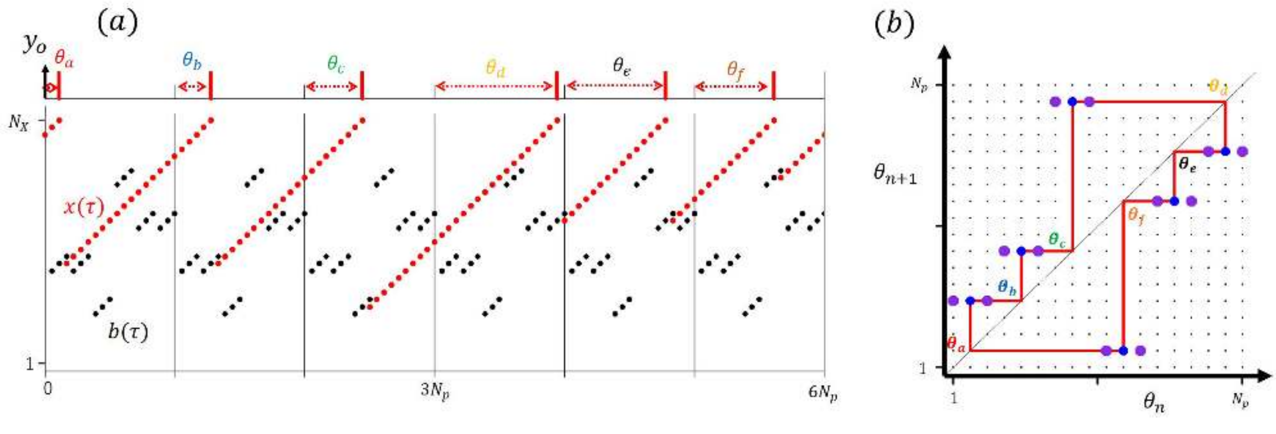

Fig. 1. DSN and digital spike map. (a) PST with period $6N_p$. (b) Periodic orbit with period 6 for $\delta = (5,5,5,8,8,8,17,17,17,2,2,2,11,11,11,14,14,14)$.

2 Digital Spiking Neurons

We introduce the DSN that is a building block of the RDSNN. Let $x(\tau)$ denote a discrete state variable at discrete time τ. Repeating integrate-and-fire behavior between a base signal $b(\tau)$ with period N_p and a constant threshold N_x, the DSN outputs a spike-train $y(\tau)$.

$$\begin{array}{ll} \text{Integrating: } x(\tau+1) = x(\tau)+1, & y(\tau) = 0 \text{ if } x(\tau) < N_x \\ \text{Self-firing: } \ x(\tau+1) = b(\tau), & y(\tau) = 1 \text{ if } x(\tau) = N_x \end{array} \quad (1)$$

where $x(\tau) \in \{0, 1, \cdots, N_x\}$ and $b(\tau + N_p) = b(\tau)$. Fig. 1 (a) illustrates the dynamics. For simplicity, we assume the following condition.

$$\tau - 2N_p + 1 \leq b(\tau) - N_x \leq \tau - N_p \text{ for } \tau \in \{0, \cdots, N_p - 1\},\ N_x \leq 2N_p - 1. \quad (2)$$

In this case, the DSN outputs one spike per one period of $b(\tau)$ and outputs a spike-train

$$y_o(\tau) = \begin{cases} 1 & \text{for } \tau = \tau_n \\ 0 & \text{for } \tau \neq \tau_n \end{cases} \quad \tau_n \in I_n = [(n-1)N_p, nN_p) \quad (3)$$

where τ_n denote the n-th spike-position. Let $\theta_n = \tau_n \bmod N_p$ ($\theta_n \in \{1, \cdots, N_p\}$) be the n-th spike-phase. A spike-position is given by $\tau_n = \theta_n + N_p(n-1)$ and a spike-train $y(\tau)$ is governed by the digital spike map F.

$$\theta_{n+1} = F(\theta_n) = f(\theta_n) \bmod N_p,\ f(\theta_n) = \theta_n - b(\theta_n) + N_x + 1 \quad (4)$$

The digital spike map is represented by a characteristic vector δ of integers:

$$\delta \equiv (\delta_1, \cdots, \delta_{N_p}),\ F(i) = \delta_i,\ \delta_i \in \{1, \cdots, N_p\},\ i \in \{1, \cdots, N_p\} \quad (5)$$

Fig. 1 (b) shows an example of digital spike map with periodic orbit with period 6. This periodic orbit is stable and corresponds to PST with period $6N_p$ ($y_o(\tau + 6N_p) = y_o(\tau)$) in Fig. 1 (a). Adjusting the base signal, the DSN can generate various stable PSTs. More detailed discussion of periodic orbits and their stability can be found in [2] [6].

3 Ring-coupled Digital Spiking Neural Networks

Connecting M pieces of DSNs with a common base signal $b(\tau)$ in ring topology, the RDSNN is constructed (see Fig. 2). The dynamics is described by

$$\begin{array}{ll} \text{Integrating: } x_i(\tau+1) = x_i(\tau)+1, & y_i(\tau) = 0 \text{ if } x_i(\tau) < N_x \\ \text{Self-firing: } \ x_i(\tau+1) = b(\tau), & y_i(\tau) = 1 \text{ if } x_i(\tau) = N_x \\ \text{Cross-firing: } x_{j+1}(\tau+1) = N_x - N_p + 1, & z_j(\tau) = 1 \text{ if } x_j(\tau) = N_x \text{ and} \\ & \qquad x_{j+1}(\tau) \leq N_x - N_p \end{array} \quad (6)$$

$$\text{Connection signal: } z_i(\tau) = \begin{cases} 1 \text{ if } x_i(\tau) = N_x \text{ and } x_{i+1}(\tau) \leq N_x - N_p \\ 0 \text{ otherwise} \end{cases} \quad (7)$$

The integrating and self-firing are the same as the single DSN. Fig. 3 shows the cross-firing that connects DSNs in ring topology. The cross-firing is characterized by the connection signal z_i. For simplicity, we consider the case where each DSN outputs a PST with period MN_p and the RDSNN consists of M pieces of DSNs. We define the M-phase synchronization (M-SYN) with period MN_p.

$$\begin{array}{l} x_i(\tau) = x_i(\tau + MN_p), y_i(\tau) = y_i(\tau + MN_p), i \in \{1, \cdots, M\} \\ x_j(\tau) = x_{j+1}(\tau + N_p), y_j(\tau) = y_{j+1}(\tau + N_p), j \in \{1, \cdots, M\} \\ z_i(\tau) = 1 \text{ for some } \tau \in \{1, \cdots, MN_p\} \end{array} \quad (8)$$

where $x_{M+1} \equiv x_1$ and $y_{M+1} \equiv y_1$. Note that non-zero connection signal $z_i(\tau)$ is required for the M-SYN because all the DSNs are isolated if $z_i(\tau) = 0$ for all τ. Fig. 4 illustrates an M-SYN of PSTs with period MN_p for $M = 6$ and corresponding output spike-train of the RDSNN. The existence and stability of M-SYN is discussed in [6].

The output spike-train y is given through time dependent selection switches S_i operation of which is represented by the connection matrix $\boldsymbol{W} = (w_{ij})$:

$$y(\tau) = \sum_{i=1}^{M} w_{ij} y_i(\tau) \text{ for } \tau \in I_j,\ j \in \{1, \cdots, M\},\ y(\tau + T) = y(\tau) \quad (9)$$

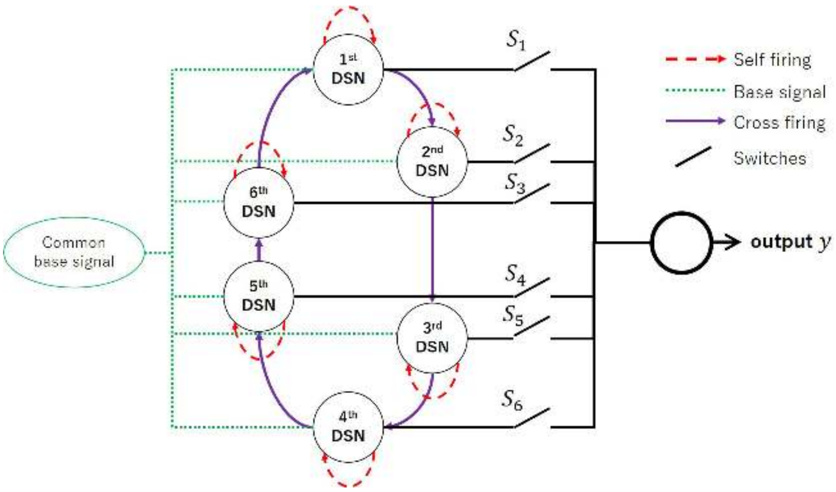

Fig. 2. RDSNN for $M = 6$.

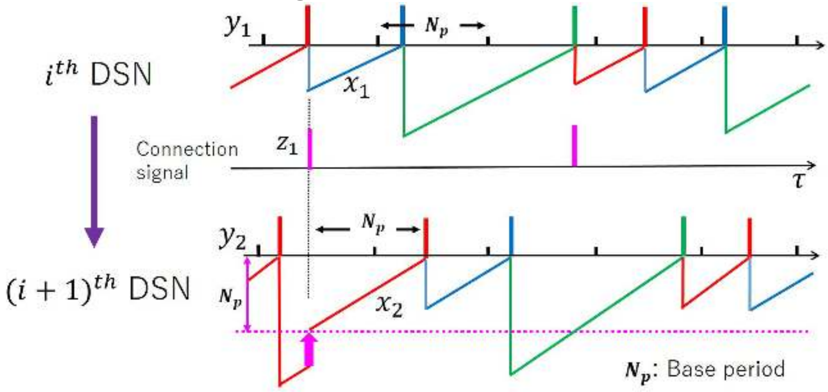

Fig. 3. Cross-firung of the RDSNN

$$w_{ij} = \begin{cases} 1 & \text{if the } i\text{-th DSN is selected } (S_i = \text{on}) \text{ for } \tau \in I_j \\ 0 & \text{if the } i\text{-th DSN is not selected } (S_i = \text{off}) \text{ for } \tau \in I_j \end{cases} \tag{10}$$

where one period of PST ($I \equiv [0, MN_p)$) is divided into M subintervals

$$I_1 = [0, N_p), I_2 = [N_p, 2N_p), \cdots, I_M = [(M-1)N_p, MN_p).$$

and w_{ij} is constant in each subinterval. If the i-th DSN is selected in the j-th subinterval I_j ($w_{ij} = 1$) then I_j is said to be activated by S_i.

Since possible connection number of DSNs is zero to M in each subinterval and since each DSN outputs one spike in each subinterval, the RDSNN can output zero to M spikes in each subinterval. Fig. 4 shows an example of output y for $M = 6$ where the selection is given by the following selection matrix and the PST of the first DSN is given by Fig. 1.

$$W = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{11}$$

The output PST is characterized by six spike-phases: $\{\theta_a, \theta_b, \theta_c, \theta_d, \theta_e, \theta_f\}$. Adjusting the selection matrix, the RDSNN can output various PSTs consisting of any combination of 6 spike-phases $\{\theta_a, \theta_b, \theta_c, \theta_d, \theta_e, \theta_f\}$. In general, if a DSN outputs a PST with period MN_p consisting of M spike-phases then the RDSNN can output various PSTs consisting of any combination of the M spike-phases. It goes without saying that such an output is impossible in the single DSN [5].

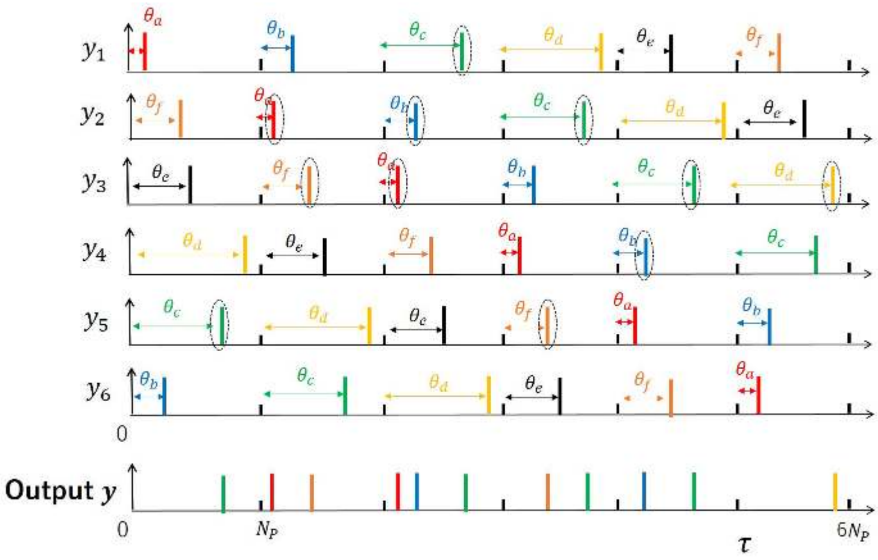

Fig. 4. M-phase synchronization of PSTs and output spike-train for $M = 6$.

4 Spike-train Approximation

We apply the RDSNN to spike-train approximation. First, we define a target PST $y_t(\tau)$ with period $T = MN_p$:

$$y_t(\tau) = \begin{cases} 1 & \text{for } \tau = \tau_k \\ 0 & \text{otherwise} \end{cases} \quad k \in \{1, \cdots, Q\},\ 0 < \tau_1 < \cdots < \tau_Q < T \tag{12}$$

where τ_n denotes the n-th spike-position and $y_t(\tau + T) = y_t(\tau)$. The target PST consists of Q spikes per one period. For convenience, let the target spike-train be represented by inter-spike intervals (ISIs)

$$D = (d_1, d_2, \cdots, d_{Q-1}),\ d_l = \tau_{l+1} - \tau_l,\ l \in \{1, \cdots, Q-1\} \tag{13}$$

where d_l is the l-th ISI and D is the target ISI sequence.

Here we consider approximate of a target PST by the output of the RDSNN. In order to realize the approximation, suitable operation of the selection switches S_i is necessary. We present the WTA switching to determine the selection matrix for the suitable operation of S_i:

$$w_{ij} = \begin{cases} 1 \text{ if } y_t(\tau) = 1 \text{ and } x_i(\tau) \text{ is the maximum at time } \tau (x_i(\tau) > x_k(\tau), k \neq j) \\ 0 \text{ otherwise} \end{cases} \tag{14}$$

where $\tau \in I_j$. Fig. 5 illustrate the WTA switching. This switching tries to select a DSN having the highest approximation potential when the target spike arrives.

If the WTA switching selects the i-th DSN at the j-th subinterval I_j ($w_{ij} = 1$) then the I_j is said to be activated by S_i. As an target PST $y_t(\tau)$ is applied, the RDSNN with the WTA switching outputs an

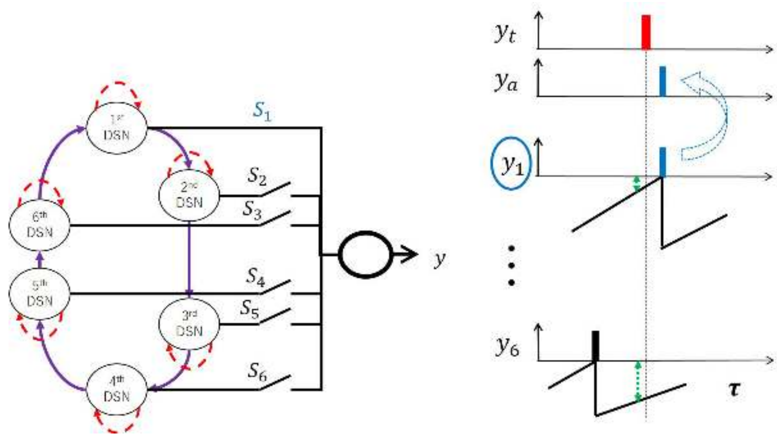

Fig. 5. WTA switching rule.

approximated PST $y_a(\tau)$ where $0 \leq \tau < T = MN_p$. For simplicity, we consider the case where $y_t(\tau)$ and $y_a(\tau)$ consist of the same number of spikes. The approximated PST y_a is characterized by an ISI sequence $D' = (d'_1, d'_2, \cdots, d'_{Q-1})$. The approximation accuracy is evaluated by the matric of ISI error:

$$\varepsilon_p = \frac{1}{Q-1} \sum_{i=1}^{Q-1} |d_i - d'_i|. \tag{15}$$

If the WTA misses some target spike then the ISI error is calculated after removing the missing spike(s). The number of the missing spikes is used as the other evaluation matric.

We have investigated the approximation function in Verilog simulation. The Verilog simulation is a first step to realize a utility hardware. Fig. 6 shows the target PST characterized by $D = (21, 11, 9, 7, 7, 6, 6, 7, 6, 9)$ and approximated PST from the RDSNN with the WTA switching. In the RDSNN, the selection matrix is given by

$$W = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{16}$$

The target PST is given by discrete exponential distribution discussed afterward. In the Verilog simulation, the RDSNN is constructed by shift registers, D flip-flops and several switching elements. The basic circuit design can be found in [6]. The approximation is evaluated by $\varepsilon_p = 0.5$.

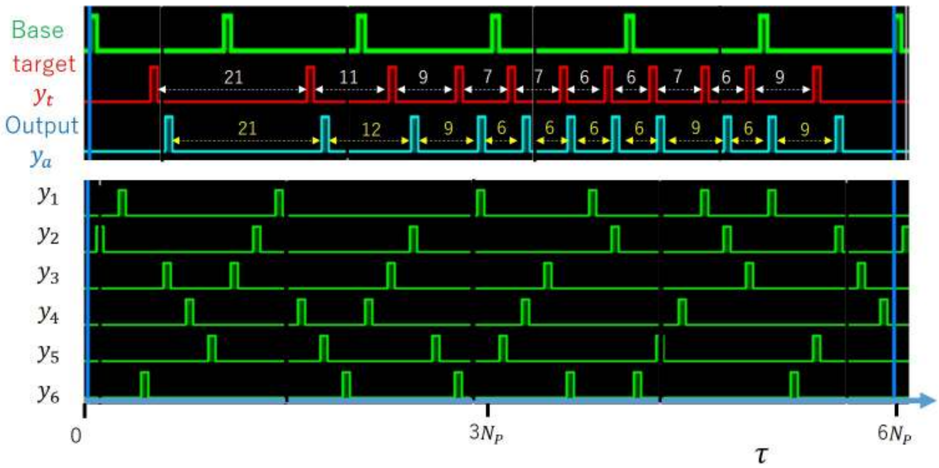

Fig. 6. Target spike-train and approximated spike-train in Verilog similation.

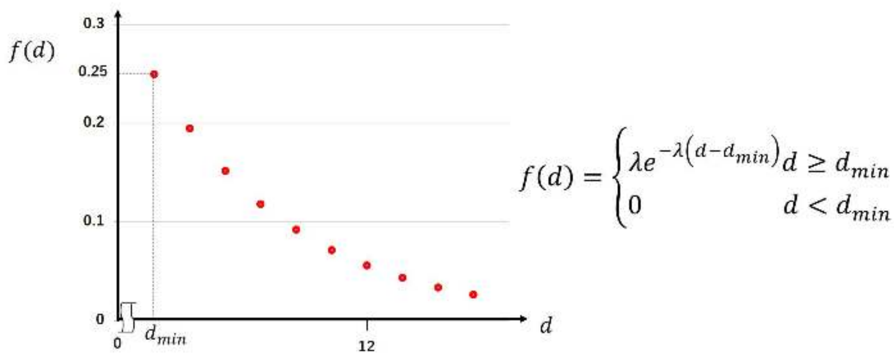

Fig. 7. Discrete exponential distribution ($d_{min} = 6$, $\lambda = 0.25$).

In order to consider the approximation function in more detail, we have performed fundamental numerical experiments. For simplicity, we consider the approximation of target PST with period $T = 6N_p$ by RDSNN

of 6 DSNs ($M = 6$). The target PST is given by discrete exponential distribution $f(d)$ as shown in Fig. 7 where d is a random variable corresponding to ISI. If the firing frequency of spikes per subinterval follows the Poisson distribution, the firing interval of spikes (ISI) follows the exponential distribution. In the exponential distribution, we have fixed parameters as $d_{min} = 6$ and $\lambda = 0.25$: in some subinterval I_n, 0 to 3 spikes can appear. In order not to missing spikes, the number of DSNs must be 3 or more ($M \geq 3$). We have investigated influence of the number of DSNs in the following two cases.
Case 1: The number of DSN is reduced in the order of activation frequency in the subinterval selection. For example, if the j-th DSN has the lowest activation frequency then the j-th DSN is disconnected (S_j=off) from the output.
Case 2: The number of DSN is reduced randomly.

We have executed 10 trials and the results are measured by average of ISI error (Avg of ε_p), standard deviation of ISI error (SD of ε_p), and spike missing rate (SMR). In Tables 1 and 2, we can see that if the number of DSNs is 6, the approximation property of almost the same in the Case 1 and Case 2. As the number of DSNs decreases, the Case 1 exhibits better approximation performance than Case 2 in both ISI error and spike missing rate. Especially, in the case of three DSNs, the Case 1 exhibits much better approximation performance than the Case 2. These results suggest that existence of an optimal combination of DSNs for efficient approximation and the WTA switching is effective to determine the selection matrix.

Table 1. Results in Case 1.

#DSN	AVG of ε_p	SD of ε_p	SMR[%]
6	0.96	0.18	0.00
5	1.08	0.22	0.00
4	1.20	0.28	0.94
3	1.99	0.69	0.94

Table 2. Results in Case 2.

#DSN	AVG of ε_p	SD of ε_p	SMR[%]
6	0.96	0.18	0.00
5	2.01	0.79	0.00
4	2.21	0.60	7.55
3	2.88	0.78	11.3

5 Conclusions

The RDSNN is presented and its application to spike-train approximation is considered in this paper. The RDSNN can realize stable multi-phase synchronization of various PSTs. Applying the WTA switching to selection of suitable DSNs, the RDSNN can approximate target PSTs if the number of spikes per period of base signal does not exceed the number of DSNs. The relation between approximation error and the number of DSNs are investigated and basic information for efficient network design is given. Presenting a Verilog simulation, basic approximation function is confirmed experimentally.

Future problems include analysis of the optimal combination of DSNs for spike-train approximation, implementation of RDSNN on an FPGA board and development into large scale spike-based digital reservoir computing system.

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