A risk-qualified approach to calculate locally varying herbicide application rates

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Received 21 September 2001
Revised version accepted 30 August 2002

Summary

Weed competition can decrease crop yield and profit. Herbicides are applied to reduce weed populations, minimize crop loss and maximize profit. Traditional practice is to apply herbicides at a uniform rate over an entire field. Complete knowledge of the weed distribution and appropriate instrumentation on the spraying equipment would allow the farm manager to apply the 'correct' locally varying herbicide application rate. The locally variable rate would be greater in areas of high weed density and less where there are few weeds. A locally varying treatment would have both economic and environmental advantages. A major challenge facing farm managers is the unavoidable uncertainty in the spatial distribution of weeds in any particular field. This uncertainty in weed distribution influences the optimal locally varying herbicide rate. A mathematical model is presented to calculate the optimal herbicide application rate using geostatistical models of uncertainty in weed density combined with principles from decision making. Weed data from a 34-ha field near Saskatoon, Saskatchewan, Canada, illustrate the application of these tools. Weed control was achieved with a significant reduction in total herbicide use.

Keywords: weed density, uncertainty, variogram, geostatistics.

Introduction

Weeds reduce crop yield and profit (Thomas *et al.*, 1998). Herbicides are important in controlling weeds and increasing yield. In western Canada, herbicides represent up to 30% of the cost of crop production and are applied to more than 60% of the cropped area. Herbicides control weeds but are expensive and can also adversely affect the environment. Spatially selective application of herbicides would increase profitability and reduce environmental impact.

The prospect of increased profit and reduced environmental impact has sparked interest in precision farming techniques. Advances in technologies such as global positioning systems (GPS), computer-integrated farm equipment and numerical modelling, including geostatistics, offer the potential for site-specific and locally varying weed management. The work presented here illustrates a method for risk-qualified and optimal locally varying herbicide application rates. The proposed method has the following steps: (1) sample the field for weed density; (2) create maps of weed density and related uncertainty over the entire field; (3) generate optimal application rate maps; and (4) download the optimal application rate maps to computer-integrated farm equipment. The computer-integrated farm equipment would apply the optimal herbicide rate throughout the field using GPS.

Weeds do not have a homogeneous spatial distribution; they are often said to be 'patchy' (Mortensen *et al.*, 1993, 1998; Dieleman & Mortensen, 1998; Clay *et al.*, 1999) and are thus amenable to site-specific and locally varying herbicide application rates.

Many farm managers apply herbicides at a 'uniform' rate over an entire field; locations with low weed density receive the same amount of herbicide as those with high weed density. A uniform application rate is often based on a visual assessment of weed density before application, but there is no procedure to balance the risks associated with under- and overspraying. The result is a subjective assessment of weed density and uncertainty in both weed density and optimal application rate.

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Crop yield losses increase with increasing weed density (Carlson *et al*., 1981; Cousens, 1985; O'Donovan *et al*., 1985; Cousens *et al*., 1987; Cousens & Mortimer, 1995). Examples are common in the literature describing how the competitive effects of weeds increase with increasing density. This response tends to be hyperbolic in shape, as discussed by Cousens (1985), with an initial steep slope for increasing weed density that declines as the yield loss reaches an asymptotic maximum yield loss. The parameters, I and A, from the yield loss response represent initial slope and asymptote respectively. For low weed densities, the response is linear, whereas at higher densities, weeds compete intraspecifically, and yield loss approaches an asymptote.

Herbicide application rate can be adjusted to account for variations in the spatial distribution of weed density. This assumes that weed control is complete, no crop damage occurs and weed density is a satisfactory measure of weed competition (Auld *et al*., 1987). A critical assumption in this research is that the optimal application rate is proportional to weed density, that is areas with high weed density should receive more herbicide and areas with low weed density should receive less herbicide for effective control (Holm *et al*., 2000; Zhang *et al*., 2000).

The herbicide dose per plant is directly proportional to the application rate. Two rates of barban, recommended and 50% of recommended, and difenzoquat at recommended and 33% of recommended were applied in spring barley (*Hordeum vulgare* L.) to control *Avena fatua* L. (wild-oat) (Cussans & Taylor, 1976). Both rates resulted in an 80% or higher reduction in *A. fatua* seed, suggesting that low doses of herbicide can provide adequate control. Number of spray drops cm^{-2} was calculated for these two rates of herbicide. The rate did not affect *A. fatua* control, indicating that a sufficient number of drops contacted *A. fatua* leaves at both doses.

The application rate can be reduced for low weed densities; however, a higher herbicide dose is required for a high-density patch. The crop's competitive position relative to the weeds must be enhanced. A larger crop loss is anticipated with a high initial weed density. For example, a low weed density may reduce crop yield to 75% of the weed-free yield, whereas a high weed density may result in the production of only 5% of the weed-free yield. When herbicide is applied, weed density is reduced as a result of weed kill causing an increase in crop yield. This increase will be greater at high weed densities. The economic benefit of this change in crop yield will be small at low weed densities and may not exceed the costs of herbicide and application. However, at high weed density, the change in crop yield will be substantial, and it will probably be economically beneficial to apply herbicide. The crop yield response will dictate the herbicide rate that is economically beneficial to apply and, in areas of high weed density, high rates of herbicide will ensure that the crop is competitive and has the potential to yield more.

Varying the herbicide application rate from the recommended rate is not supported by manufacturers because there is a guaranteed response only at the label rate. Herbicide performance testing is conducted on a range of crop cultivars, weed densities and species, soil types and weather conditions. The recommended application rate is established for a wide range of conditions. A central premise to our work is that deviations from the recommended rate could be optimal for local conditions. The optimal application rate strikes a balance between cost, control and crop yield loss.

The local optimal herbicide application rate scheme requires that the weed density is known before application. It is unrealistic to sample a field exhaustively to establish the unique true weed density over the entire field. A more efficient method is to sample the field strategically and construct numerical models of weed density that are based on the sample data and reflect significant physical and biological features of the weed species. Strategic sampling needs to account for scale and provide a complete picture of the spatial and biological relationships of weeds. Then, a model of uncertainty in weed density addresses the limitations of sparse sampling. Thus, a numerical model's predictive ability is significantly enhanced.

Geostatistics, a branch of applied statistics, has tools for mapping an attribute value and characterizing the uncertainty in weed density at unsampled locations. These techniques are applied systematically in other disciplines, such as petroleum reservoirs (Deutsch & Journel, 1998), mining (Journel & Huijbregts, 1978; Isaaks & Srivastava, 1989) and natural resources (Goovaerts, 1997). Major decisions are made in the presence of unavoidable uncertainty in these related disciplines. Deciding on locally varying herbicide application rate can be done in the light of uncertainty in spatial weed density.

The objective of this paper was to develop a method for mapping locally varying herbicide application rate in the presence of uncertainty in weed density. An increased rate will be recommended in areas of high weed density to achieve optimal control and increased yield; a decreased rate will be recommended in areas of low weed density. The optimal locally varying herbicide rate can be determined mathematically with a satisfactory model.

Details of model

The optimal locally varying herbicide application rate is the rate that leads to maximum profit. The following development assumes knowledge of: (1) herbicide efficacy at a given application rate; (2) the competitive relationship between weeds and crops; and (3) the cost of applying herbicide. The notion of optimal locally varying herbicide rates will be derived for known weed density, and then uncertainty in weed density will be introduced. Finally, the optimal risk-qualified rate is calculated in the presence of uncertainty in weed density. A risk-qualified approach requires multiple realizations of weed density in order to define a space of uncertainty. This uncertainty allows an assessment of risk for decision-making.

Optimal rate with deterministic parameters

To begin, geographic location is denoted by the vector variable u that consists of east and north co-ordinates. The herbicide application rate for location u is denoted by $a(u)$, which is measured in L ha^{-1}. The optimal local herbicide application rate for location u is denoted $a_{opt}(u)$.

Weed density $w(u)$ is defined as the number of weeds m^{-2} at location u. This variable is nominally categorical, taking values from 0 to some maximum number of weeds that could grow simultaneously in a square metre. In general, there are numerous weed species present in a field, but each set of calculations only considers the critical species that are responsible for the weed application decision.

The farm manager cannot spray a different rate on each square metre of the field. We must consider a selective spraying area (SSA) denoted v. This area is probably 20–35 m wide (depending on spray boom length and electronic controls built into the sprayer) and 1–2 m deep because of the potential drift of herbicide. The SSA can be customized for site-specific conditions given the fact that sprayer boom sections are being developed that apply herbicide over smaller SSAs. The weed density must be averaged from the sampling area, m^{-2}, to the SSA:

$$w_v(u) = \frac{1}{v(u)} \int_v w(u')\mathrm{d}u' \tag{1}$$

Weed density is informed by: (1) samples of weed density, perhaps over a small area with relatively great spatial detail; and (2) scouting or remotely sensed data, probably over a large area with less spatial detail. The numerical tools of geostatistics are used to model the weed density at the correct SSA areal size (Journel & Huijbregts, 1978; Isaaks & Srivastava, 1989; Goovaerts, 1997; Deutsch & Journel, 1998). Next, weed density is averaged from a sampling area to correspond to an SSA that is relevant to the limitations of the application equipment.

The first required input is the maximum attainable weed-free yield or $y_0(u)$. This maximum attainable yield $y_0(u)$ is in units of tonnes ha^{-1}, and $y_0(u)$ depends on location u in the field. Historical information and recent environmental and weather conditions will provide an approximation of $y_0(u)$ over the entire field.

The second required input is the fractional yield loss resulting from non-zero weed density or $L(w)$. This fractional yield loss is a function of weed density. When weed density is high, high yield loss can be expected, whereas low yield loss is expected when weed density is low. Yield loss starts at zero, that is $L(w) = 0$ at $w = 0$, and may increase to its maximum value, 100%, depending on the competitive ability of the weed as its density increases. Experimental data are required to establish this function. Yield loss values resulting from different weed densities were fitted from values provided in the literature (Carlson *et al.*, 1981; Cousens, 1985; Cousens *et al.*, 1987). A family of fitted curves from a look-up table of a hyperbolic type is used to model $L(w)$. The curves represent fractional yield losses due to weeds under different cropping, weed and environmental conditions.

The third critical piece of information is the fractional weed control as a function of the herbicide application rate or $Wc(a)$, where a is the herbicide application rate in litres or kg ha^{-1}. Herbicide manufacturers probably have data on this function $Wc(a)$; however, these data may not be publicly available. Model parameters and bounds of this function were based on values from the literature (Cousens, 1985; Cousens & Mortimer, 1995; Anonymous, 1998). Where these were unavailable, parameters were hypothesized from our understanding of weed control. Nevertheless, much is known about this function: (1) it is bounded between 0 and 100%; (2) there is zero weed control at zero application rate; (3) there will be 80% or more control at the recommended application rate (Anonymous, 1998); and (4) full control, $Wc = 100\%$, will be reached asymptotically as a increases (Cousens, 1985; Cousens & Mortimer, 1995). Experimental data or a fitted hyperbolic or exponential-type function could be used to model $Wc(a)$ (Cousens & Mortimer, 1995; Swanton *et al.*, 1999). There may be a different $Wc(a)$ curve for herbicides with different formulations, as illustrated by weed response for parallel dose–response curves (Streibig, 1984, 1988).

Other price and cost inputs are required. The net price or net value of the crop, np, in financial units such as dollars tonne^{-1}, must be known. The cost of the herbicide, c, in for example dollars L^{-1}, must also be known.

Using the input variables described above, it is possible to calculate the incremental revenue for a specific application rate, a:

$$r(a;u) =< L[W_v(u)] - L\{W_v(u)\cdot[1 - W_C(a)]\} > y_0(u)\cdot np \tag{2}$$

where the units of $r(a)$ are in financial units (e.g. dollars ha^{-1}). The variable $r(a;\ u)$ represents a non-decreasing function of the herbicide application rate a at location u; see the top figure in Fig. 1. Of course, the cost of applying herbicide must also be included.

Yield loss and herbicide application each have associated costs. Increasing herbicide application rate costs money because of increased product consumption. Decreasing herbicide application decreases this consumption but increases yield loss. This is shown in the bottom figure of Fig. 1 where the three curves represent the sum of the cost of applying herbicide and the cost of crop loss.

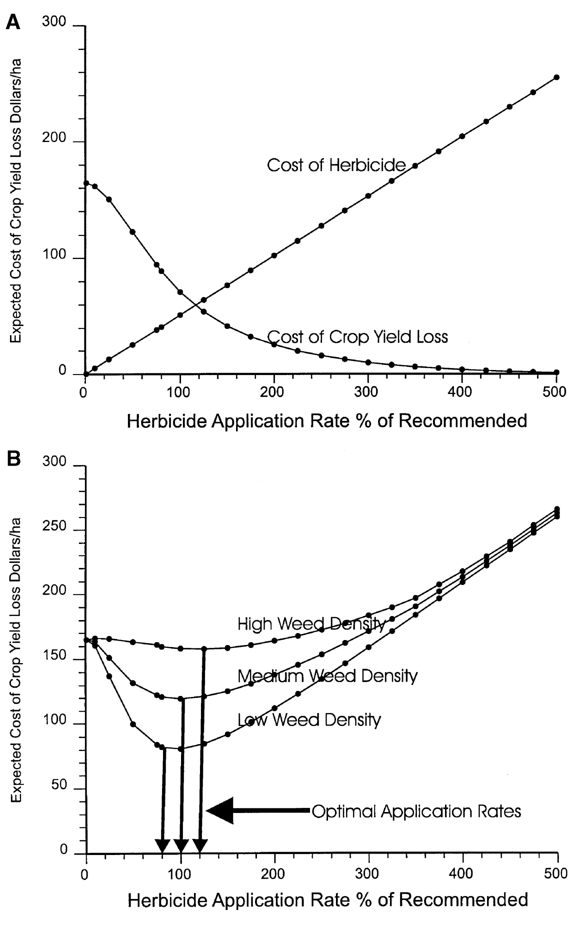

Fig. 1 Examples of the economic consequences of changing herbicide application rate (expressed as a percentage of the manufacturer's recommended rate). (A) Response of herbicide cost and crop yield loss. (B) Effects of three weed densities on the costs of crop yield loss, identifying optimal application rates.

There are fixed costs for equipment ownership, depreciation, interest, insurance and so on. These fixed costs are not considered in the equation below, as it is assumed that it is economical to spray; the goal is to determine the optimal application rate. Clearly, there are cases of low weed densities where the fixed costs exceed the total revenues and the correct decision is not to spray at all. Given that spraying will occur, the cost of applying herbicide at rate a is given by:

$$c(a) = -c\cdot a \tag{3}$$

where $c(a)$ is in financial units (e.g. dollars ha^{-1}). A typical approach for optimal application rate is to determine a value function for each decision, then choose the maximum. For a loss function, the idea is to determine the optimal application rate in the presence of uncertainty for which the loss is minimized (Goovaerts, 1997). This is the reason for the negative c. The incremental profit of spraying at rate a is simply the sum of $r(a)$ and $c(a)$:

$$p(a;u) = r(a;u) + c(a) \tag{4}$$

The optimal rate $a_{opt}(u)$ maximizes this incremental profit.

The optimal application rate and profit for a given location, u, will be affected by several factors. Areas with low weed density will have a low optimal herbicide application rate, whereas areas with high weed density will have high application rates (see Fig. 1). Thus, fields with a patchy weed distribution will be the most amenable to locally varying herbicide application rates. Two additional comments need to be made on $p(a;\ u)$ and the determination of the optimal rate $a_{opt}(u)$:

- The incremental revenue $r(a;u)$ curve flattens as a increases because the weed control, $Wc(a)$, and fractional yield loss response, $L(w)$, curves flatten off. The cost of herbicide $c(a)$, on the other hand, continues to decrease linearly because a constant per litre cost is used and, as herbicide rate increases, so does its cost. Thus, the optimal application rate $a_{opt}(u)$ is always finite.
- The optimal rate will be zero if the herbicide is very expensive (c large), there are few weeds [$w_v(u)$ low] and the weeds are poor competitors, and there is moderate response to the herbicide [$Wc(a)$ rises slowly].

The function $p(a;\ u)$ may be maximized by any classical technique. The $p(a;\ u)$ function is well behaved, and evaluation of $p(a;\ u)$ is extremely fast; therefore, almost any optimization technique can be considered. For example, simple Newton iterations, which is a well-known method of optimizing a non-linear function, are suitable (Householder, 1970).

There are two parameters that depend on location: the weed density $w_v(u)$ and the maximum attainable weed-free yield $y_0(u)$. Knowledge of these two parameters permits calculation of the optimal rate for each location.

An important feature of field-scale weed treatment is that the weed density is not known precisely at each location. There is uncertainty in the weed density for each SSA to be sprayed (see Fig. 2). The optimal herbicide application rate must account for this uncertainty.

Accounting for uncertainty

The consequence of uncertainty is that we have to calculate an expected profit instead of the actual profit. In the presence of uncertainty, we must calculate the expected profit:

$$\overline{p(a;u)} = E\{[L(w_v(u)) - L(w_v(u)\cdot(1 - W_C(a)))] \times y_0(u)\cdot np - c\cdot a\} \quad (5)$$

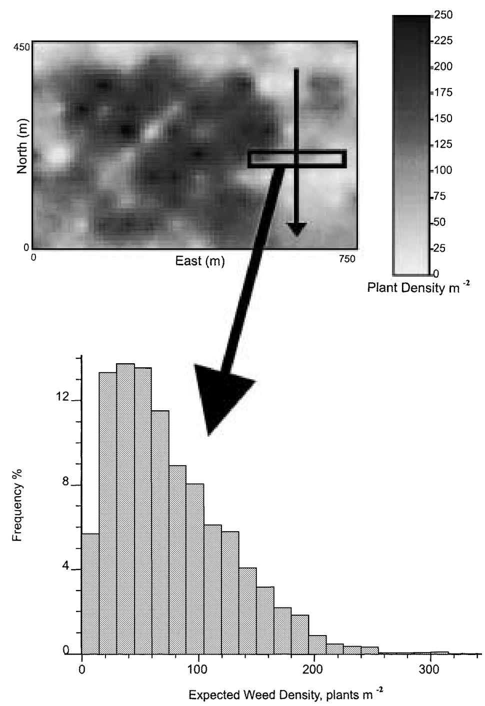

Fig. 2 Weed density variation in a 34-ha field sampled on a 50-m × 50-m grid. The distribution of weed densities (and the uncertainty in the mean density) in a sampled selective spraying area (2 m × 35 m) is shown in the lower histogram.

The optimal rate, $a_{opt}(u)$, maximizes the expected incremental profit at location u, that is $\max\{\overline{p(a;u)}\}$. The expected value operator is a probability weighted average $\bar{p} = \int_{-\infty}^{\infty} pf(p)dp$. In practice, this continuous integral is solved by creating a large number, N, of equal probability values. In the context of expected profit, there are N pairs for weed density and maximum weed-free yield $\{w_v^{(i)}(u), y_0(u)^{(i)}, i = 1,\ldots,N\}$ and $\{L(w_v^{(i)}(u))\}$ respectively. The expected value is then approximated as:

$$\overline{p(a;u)} \approx \frac{1}{N}\sum_{i=1}^{N}\left\{\left[L(w_v(u)^{(i)}) - L(w_v(u)^{(i)}\cdot(1 - W_C(a)))\right] \times y_0(u)^{(i)}\cdot np - c\cdot a\right\} \quad (6)$$

The amount of computer work for this added calculation is reasonable. The result is the same: a map of optimal locally varying herbicide application rate for use in computer integrated, GPS-guided, herbicide application equipment.

Model validation

Weed density data used in this research are taken from a 34-ha field near Saskatoon, Saskatchewan, Canada, which was seeded to spring wheat (*Triticum aestivum* L.) in 1995 and oilseed rape (*Brassica napus* L.) in 1996. All weed species were identified and counted at the three- to four-leaf stage in both years with a 50-m by 50-m grid. In 1996, two 100-point sampling grids with a 10-m by 10-m spacing were established in areas of high weed density. Weeds were counted by species in four (1995) and nine (1996) 50-cm by 50-cm quadrats at each sampling point in the fields before post-emergence herbicide application. The various weed species were categorized as either broad-leaved or grass weeds. Fourteen broad-leaved species were recorded in 1995, and 15 were identified in 1996. The frequency of occurrence, which represents the percentage of total sampling points for which a species appeared for the three most abundant weeds, was 51–99% for *Fallopia convolvulus* (L.) A. Löve (black bindweed), 54–91% for *A. fatua* and 99–100% for *Thlapsi arvense* L. (field penny-cress). Other weed species identified at this site with a frequency of occurrence of > 25% included *Cirsium arvense* (L.) Scop. (creeping thistle) and *Taraxacum officinale* Weber (dandelion). The results analysed here are for broad-leaved weeds only, which were present at 100% and 93% of the sampling sites in 1995 and 1996 respectively. A histogram for the 137 sample locations in 1995 indicated a broad-leaved weed density of 1–408 broad-leaved weeds m^{-2} with a mean of 70.8 m^{-2} (Fig. 3).

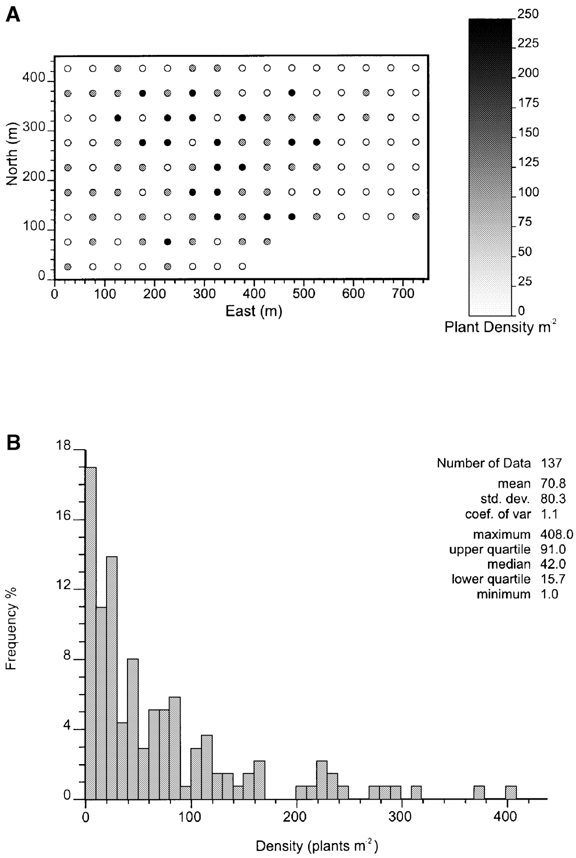

Fig. 3 Measurement of weed distribution in the 34-ha field. (A) Location and mean weed densities in each of the 137 sampling points. (B) Histogram of the distribution of broad-leaved weed densities in the field in 1995.

A map showing the sampling points for the weed data in 1995 and 1996 is displayed in Fig. 3. Increasing grey-scale indicates increasing weed density, so white is no weeds present whereas black represents over 250 broad-leaved weeds m^{-2}. Note the significant variability in weed density throughout the field.

The directional, spherical variograms shown in Fig. 4 quantify semi-variance vs. distance for the 1995 broad-leaved data in Fig. 3. The dashed lines show the experimental variogram calculated from normal score-transformed data, whereas the solid lines are the spherical model fitted to the experimental variogram. The total variability explained by these spherical models was calculated with GSLIB software (Deutsch & Journel, 1998) using the nugget and three-nested structures for 1995 and 1996 data. The top experimental variogram and model are for the north–south direction (N 0° E), whereas the bottom variogram and model are for the east–west direction (N 90° E). Owing to limited short-scale data from 1995, short-scale data from 1996 were used to infer the nugget effect. This assumes that weed density does not change

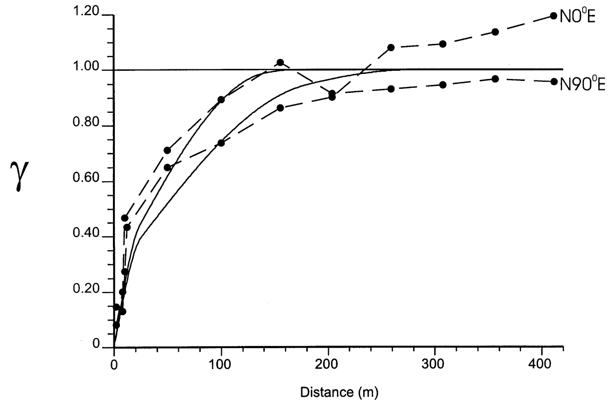

Fig. 4 A semi-variogram using the 1995 broad-leaved weed density data from the 34-ha experimental field. The dashed lines represent the experimental variograms, whereas the solid lines are the modelled variograms. The top experimental and model variogram are for the(N 0° E) direction, whereas the bottom variogram and model are for the (N 90° E) direction.

over time at the short scale. This indicates that the variogram is applicable over the entire experimental field over which it has been calculated. The model variogram has a moderate nugget effect of 0.05 and a range of 275 m in the direction of maximal continuity (N 90° E) and 160 m in the direction of minimal continuity (N 0° E). A waterway crosses the (S 45° E) corner of the field (see bottom right corner of the sampling point map in Fig. 3), and it may have influenced the anisotropy of the broad-leaved weed distribution.

The variogram shown in Fig. 4 was used for kriging a 1 by 1-m^{-2} grid. Kriging is a classical geostatistical technique for estimation at unsampled locations (Journel & Huijbregts, 1978; Isaaks & Srivastava, 1989; Goovaerts, 1997; Deutsch & Journel, 1998). A known limitation of kriging is 'smoothing'; low values are typically overestimated, and high values are typically

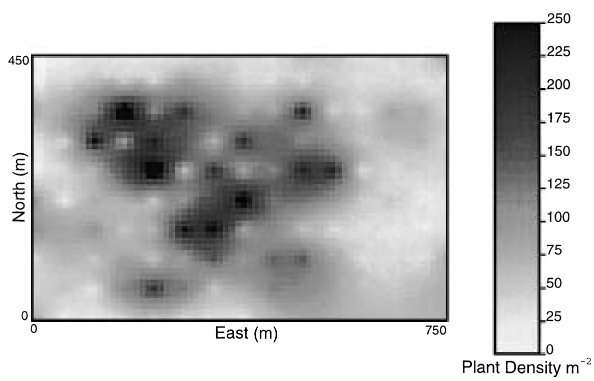

Fig. 5 A kriged map of weed density, plants m^{-2}, from the 34-ha experimental field.

underestimated. A kriged map of weed density data is shown in Fig. 5.

Conditional simulation was initially developed to correct the smoothing effect of kriging by creating maps that reproduce the histogram and variogram. It involves creating multiple, equally probable realizations that are conditional if the realization represents the data at their location. Each realization should reproduce the local data at correct scale, the global histogram and variogram. Many techniques can be used to draw these realizations; however, sequential Gaussian simulation has gained widespread popularity because of its simplicity and flexibility (Journel & Huijbregts, 1978; Deutsch & Journel, 1998).

Multiple simulated realizations are used to quantify uncertainty in this data using GSLIB software (Deutsch & Journel, 1998). One hundred and one realizations were created using sequential Gaussian simulation. Three realizations and the average map of all 101 realizations are shown in Fig. 6. Note that the three realizations are 'noisier' than the kriged map. This is a reflection of the true variability in the weed distribution at small scale using the 1996 small-grid data. Despite the variation, the simulated maps reflect the histogram and variogram. Also note that the average map of all 101 realizations is nearly identical to the kriged map in Fig. 5.

Fractional yield loss as a function of weed density was derived from fitted curves of a hyperbolic or exponential type for $L(w)$ such as:

$$Y^{(l)}(u) = \frac{L \cdot w(u) \cdot W^{(l)}(u)}{1 + \frac{L \cdot w(u) \cdot W^{(l)}(u)}{A}} \tag{7}$$

where $Y^{(l)}$ is the yield loss as a percentage for realization l at location u, L is the percentage yield loss per unit weed density as density approaches zero, w is the weed density at location u, $W^{(l)}$ is the fraction of weeds controlled by herbicide at location u, and A is the maximum crop loss due to weed competition as weed density approaches infinity. Crop yield loss in a mixed stand of weeds can be represented by a family of curves. The experimental field had broad-leaved weeds in which two species were dominant in each year. Fractional yield loss was determined for each realization and averaged to the SSA over the 101 realizations at a location, whereas the kriged fractional yield loss map was averaged to the SSA for one map. This relationship is an example from the literature (Cousens, 1985).

Other assumptions were made when preparing a map of optimal locally varying herbicide application rates:

- $y_0(u)$ grain yield $= 3.0$ t ha^{-1},
- (np) net selling price of grain $= \$100$ t^{-1}

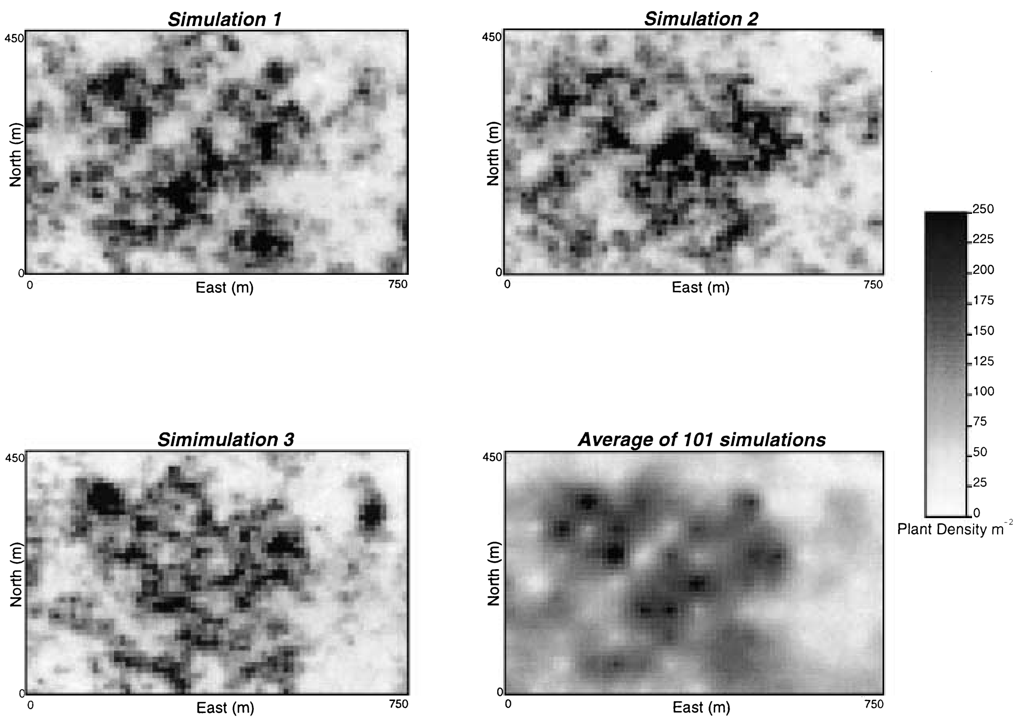

Fig. 6 Three individual simulated maps of weed density and a map of the average for 101 simulated maps of weed density (plants m^{-2}).

- $Wc(a)$ herbicide costs \$50 ha^{-1}, which includes herbicide product at \$40 ha^{-1} and \$10 ha^{-1} for application cost at the recommended application rate,
- A maximum crop yield loss = 40%, and
- maximum permissible application rate is 200% of the manufacturer's recommended herbicide application rate. The maximum permissible application rate exceeds the manufacturer's recommended rate, which is illegal. However, it was allowed in the model to determine whether there are areas in the field where the weed density warrants additional control measures at a future time.

Environmental and soil variability influence weeds, and this variability is characterized by the uncertainty of weed density at each location. A map of optimal locally varying herbicide application rates is shown in Fig. 7. Increasing grey-scale indicates increasing herbicide application rate. The distribution of optimal application rates is shown in the bottom figure of Fig. 7. We predict more than 100% of the manufacturer's recommended application rate at some locations, whereas at other locations, the optimal application rate is predicted to be zero. The average optimal rate is 50.4% per SSA of the recommended rate, with a minimum and maximum rate of 0% and 116% respectively. In this case, the overall optimal rate is not the manufacturer's recommended application rate, and some areas (< 1%) will need extra control treatments. The herbicide cost for this optimal application rate is \$793 for the whole field. A uniform application at the manufacturer's recommended rate would cost \$1575.

A map of the expected cost of crop yield loss using our calculated optimal locally varying herbicide application rate is compared with the expected cost when a uniform herbicide application rate of 50% is applied to the experimental field in Fig. 8. As the average optimal herbicide rate was 50.4%, a uniform rate of 50% was chosen for comparison. The cost histogram for the 50% application rate illustrates a wider, flatter distribution of costs compared with the optimal cost histogram. Cost of herbicide consumption is the same for either application rate; however, some areas receive too much and others too little with the 50% rate. Expected cost of the yield loss is more than 4% greater with a uniform application rate compared with the optimal herbicide application rate.

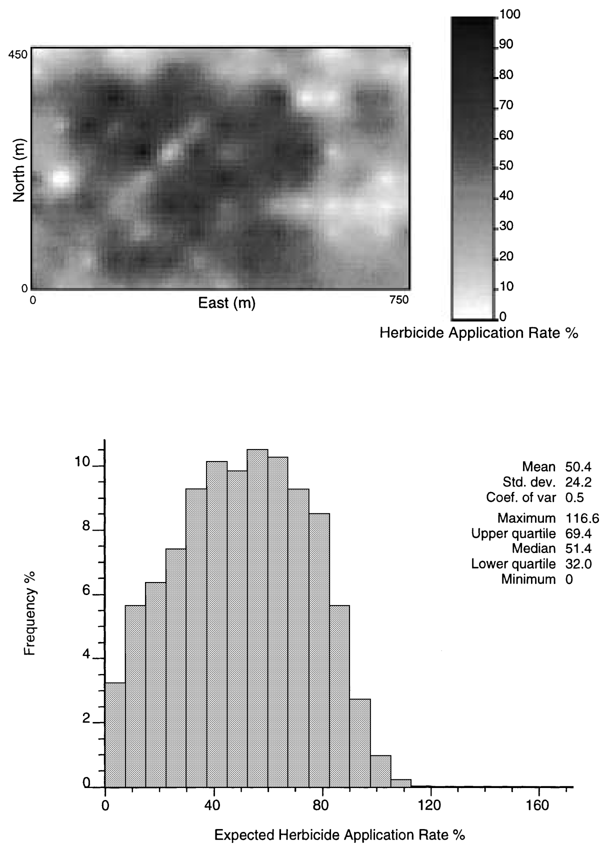

Fig. 7 Locally varying optimal herbicide application rate map compared with manufacturer's recommended application rate at 100% with a histogram of the locally varying optimal herbicide application rates. The locally optimal herbicide rate mean is 50.4%.

Conclusions

We have described a method for establishing optimal locally varying herbicide application rates. The method requires geostatistical models of uncertainty in weed density and a model of weed response for different application rates. This method has the potential to reduce weed control costs. Practical application requires calibration to a particular crop, weed and herbicide.

Our example considered studies published in the literature for the required weed response to herbicide rate. This must be verified under specific environmental and cropping conditions. Such variable rate information is limited.

Spatial statistics are useful to characterize the heterogeneity of weed distributions as well as to quantify the uncertainty caused by incomplete data. The proposed methodology accounts for risk along with uncertainty in the spatial distribution of weeds. Such local precision and optimality is a worthy goal in view of economic and environmental concerns related to herbicide application.

The spatial distribution and uncertainty in weed density can be characterized using geostatistics and weed density data. Additional data will reduce uncertainty, but at increased cost. The optimal sample spacing balances the additional sampling costs with the benefits of improved decisions. Supplementary data could come from weed surveying using an all-terrain vehicle.

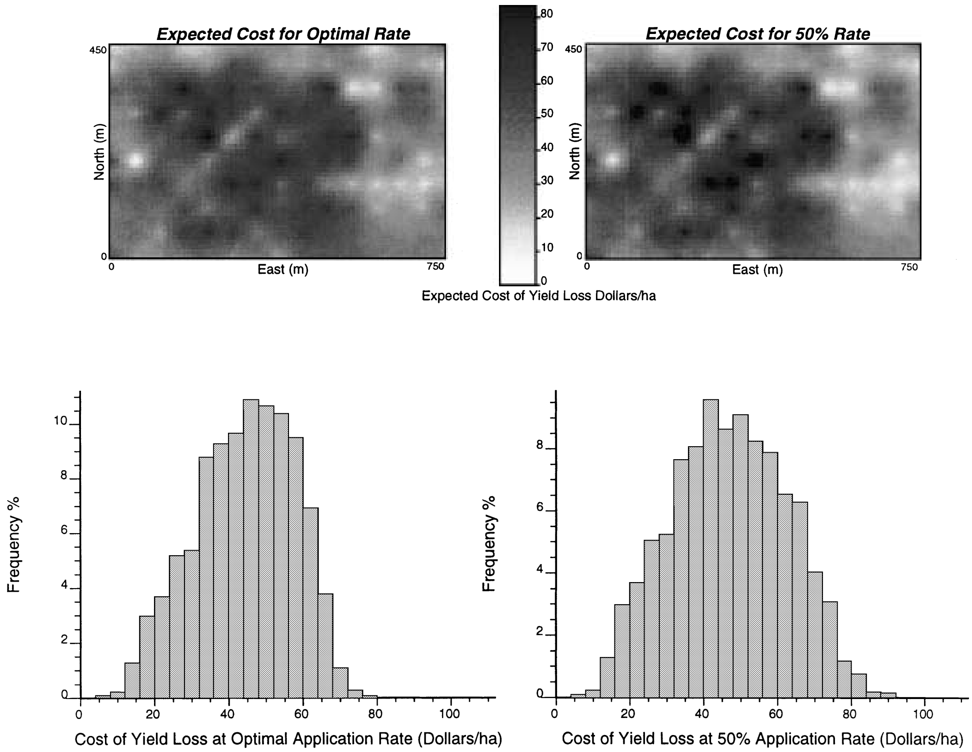

Fig. 8 Map and histogram (left) of the expected cost of crop yield loss in dollars ha^{-1} for the optimal herbicide rate applied to the experimental field. Map and histogram (right) of the expected cost of crop yield loss in dollars ha^{-1} when 50% of the manufacturer's recommended herbicide rate is applied to the experimental field.

A visual rating of weed numbers using an all-terrain vehicle provides a qualitative description for analysis (Hall & Faechner, 1999). Digital elevation maps may also improve weed density mapping because weeds favour specific regions (Faechner *et al.*, 2000). A review of sampling strategies for arable crops highlights some of the challenges facing weed scientists in site-specific weed management (Rew & Cousens, 2001).

The sample data are scaled up to a selective spraying area (SSA). These areas will decrease as spraying equipment becomes more advanced. This will result in further optimization of herbicide application. Wallinga *et al.* (1998) found that herbicide use could be reduced by 26% when changing spatial resolution from 4 m to 2 m.

The effects of a mixed weed species infestation on crop yield must be incorporated into crop yield loss equations. Two species models have been developed (Doyle, 1991). A competitive index has been established for multiple weed types in soyabean (Wilkerson *et al.*, 1991). Additional research is required to generalize such models to practice.

Dose–response curves that quantify a herbicide's effect on weeds and crop have been described by Streibig (1988). Optimizing herbicide doses depends on knowledge of these response curves. There are few studies from the literature that provide data for grass and broad-leaved herbicides. Additional research is required to increase our understanding of how weeds react to different herbicide rates.

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Appendix

Definition of symbols

$w_v(u)$ = number of weeds or weed density at location u, plants m^{-2}

$a(u)$ = herbicide application rate, L ha^{-1}

$a_{opt}(u)$ = optimal local herbicide application rate at location u

y_0(u) = maximum attainable weed-free yield in tonnes ha^{-1} at location u

SSA = selective spraying area denoted as v, m^{-3}

$L(w)$ = fractional yield loss resulting from non-zero weed density, %

$Wc(a)$ = fractional weed control as a function of the herbicide application rate, %

np = net price of grain yield, dollars $tonne^{-1}$

c = cost of the herbicide, dollars L^{-1}

$r(a; u)$ = revenue, dollars ha^{-1}, for a herbicide application rate a at location u

$c(a)$ = cost of application for a herbicide rate, dollars ha^{-1}

$p(a; u)$ = incremental profit, dollars ha^{-1}

$\overline{p(a; u)}$ = expected profit, dollars ha^{-1}

NB. All costs calculated in Canadian dollars.

Variogram parameters

Variance contribution	Variogram model	Maximum continuity, m (N 90° E)	Minimum continuity, m (N 0° E)
0.05	Nugget		
0.35	Spherical	16	15
0.35	Spherical	170	130
0.25	Spherical	275	160