Online Appendix for "A Road Map for Efficiently Taxing Heterogeneous Agents"

Marios Karabarbounis
Federal Reserve Bank of Richmond

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A Alternative models: Aggregate Effects

This section reports the aggregate effects of the optimal labor-income policy in the "Exogenous Human Capital" model and the "Constant Elasticity Model."

Table 1 reports the effects and welfare gains for the model with exogenous human capital. Overall, the aggregate effects are in the same range. In addition, the presence of endogenous human capital seems to add 0.1% to our welfare gains. Table 2 compares the effects of the the age- and assets- dependent tax reform for both models. Changes in macro variables and welfare gains are close to the ones found in the benchmark economy. Table 2 also reports the effects in macro aggregates and welfare gains for different values of labor supply elasticity.[1] A larger value of labor supply elasticity can increase labor supply (and capital) by a larger amount. However, welfare gains seem robust across specifications.

[1] In the benchmark model the (heterogeneity in) labor supply elasticity depends mostly on the distribution of reservation wages and not on the value θ (see Hansen (1985), Rogerson (1988), and Chang and Kim (2006)). In contrast, varying parameter θ_c in the CEM will affect uniformly all agents.

Table 1: Aggregate Effects of Tax Reform: Benchmark and Exogenous Human Capital

Model	Benchmark	Exogenous Human Capital
K	+19.7%	+20.3%
L	+2.7%	+2.8%
C	+6.4%	+6.0%
w	+5.6%	+5.7%
r	-1.0%	-1.0%
Cons. Gini	-0.2%	-0.5%
CEV	+1.5%	+1.4%

Note: The Table reports the percentage change in aggregates due to the tax reform. The tax reform includes age, household assets, and filing status as tax tags. Results are presented for the benchmark case and two alternative specifications. A model with exogenous human capital accumulation and a model with constant elasticity of labor supply.

Table 2: Aggregate Effects of Tax Reform: Benchmark and CEM

Model	Benchmark	CEM		
		$\theta_c = 0.4$	$\theta_c = 0.73$	$\theta_c = 1.5$
K	+17.7%	+18.9%	+18.4%	+22.2%
L	+0.6%	+0.6%	+0.9%	+1.7%
C	+3.7%	+3.0%	+3.1%	+4.2%
w	+5.8%	+6.1%	+5.9%	+6.8%
r	-1.1%	-0.3%	-0.3%	-0.4%
Cons. Gini	+0.3%	+0.2%	+0.2%	+0.2%
CEV	+1.0%	+1.1%	+1.1%	+1.0%

Note: The Table reports the percentage change in aggregates due to the tax reform. The tax reform includes age and household assets as tax tags. Results are presented for the benchmark case and a model with constant elasticity of labor supply. For the latter case, I present the effects of the optimal tax system for a variety of labor supply elasticities: $\theta_c = 0.4$,$\theta_c = 0.73$, $\theta_c = 1.5$.

B Simpler Policies

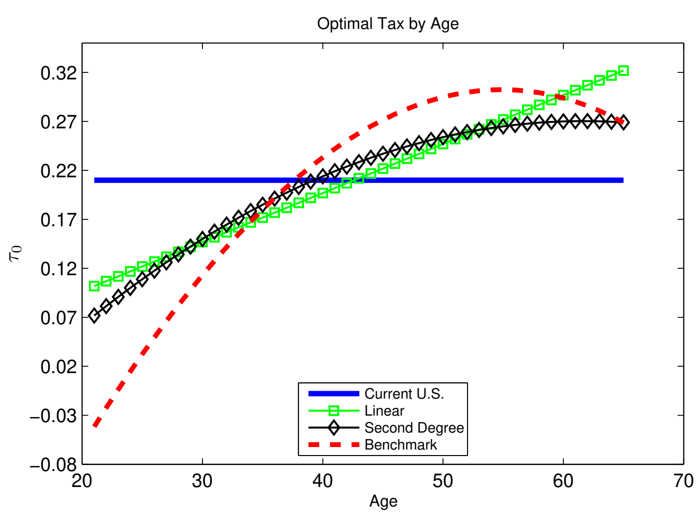
In this section, I restrict the age-dependent tax system to simpler forms. In particular, I analyze a linear form, $\tau_{00} + \tau_{01} j$, a second-degree polynomial, $\tau_{00} + \tau_{01} j + \tau_{02} j^2$, and compare them to our benchmark specification: $\tau_{00} + \tau_{01} j + \tau_{02} j^2 + \tau_{03} j^3$. The aggregate effects and optimal tax properties are shown in Table 3 and Figure 1, respectively.

Table 3: Age-dependent Taxes: Simpler Functional Forms

	Linear	Second-Degree	Benchmark
Functional Form	$\tau_{00} + \tau_{01}j$	$\tau_{00} + \tau_{01}j + \tau_{02}j^2$	$\tau_{00} + \tau_{01}j + \tau_{02}j^2 + \tau_{03}j^3$
K	+5.3%	+5.0%	+9.5%
L	-1.0%	-0.7%	+0.8%
C	+0.1%	+0.4%	+0.9%
w	+2.2%	+2.0%	+4.1%
r	-0.4%	-0.4%	+0.6%
Cons. Gini	-1.1%	-1.3%	-1.7%
CEV	+0.2%	+0.3%	+0.4%

Note: The table reports the percentage change in aggregates due to an age-dependent tax reform. Results are presented for a linear tax function, a second-degree polynomial and the benchmark specification.

Figure 1: Optimal Age-dependent Taxes: Simpler Functional Forms



Note: The figure plots the properties of optimal age-dependent taxes for different functional forms: linear, second-degree polynomial and benchmark case (third-degree polynomial).

A linear increasing function increases capital by a smaller amount compared to the benchmark. Moreover, labor supply decreases as the tax distortion induces older households to retire earlier. If the tax function is a second-degree polynomial then the tax distortion also increases but a smaller rate for older households. This makes the employment reduction smaller. Using the benchmark specification (third-degree polynomial), we can increase capital by a larger amount but also increase employment. The flexibility allows to increase tax distortions steeply up to age 50 and decrease taxes after that age. Although relatively small, welfare gains double compared to a linear tax function.

C Household's Problem: Value Functions

In this section, I write the value function for a household employing the female worker $V^{\{NE,E\}}$ and a household whose members are not employed $V^{\{NE,NE\}}$. The value function for a household employing the female worker is:

$$
\begin{aligned}
V_{zj}^{\{NE,E\}}(a, \mathbf{x}, \boldsymbol{\kappa}, \mathbf{E}_{-1}) = \max_{c,a',h^f} & \Bigg\{ \log(c) + \psi_j^m \frac{(1-h^m)^{1-\theta}}{1-\theta} + \psi_j^f \frac{(1-h^f)^{1-\theta}}{1-\theta} - \zeta(E_{-1}^f) \\
& + \beta s_{j+1} \sum_{x_m'} \sum_{x_f'} \Gamma_{x_m x_m'} \Gamma_{x_f x_f'} \ * \\
& [\frac{(1-\lambda^m)}{1-p} \sum_{s=\{2,3\}} \boldsymbol{p}^s V_{z(j+1)}^1(a', \mathbf{x}', \boldsymbol{\kappa}^s, \mathbf{E}) + \frac{\lambda^m}{1-p} \sum_{s=\{2,3\}} \boldsymbol{p}^s V_{z(j+1)}^{\{NE,NE\}}(a', \mathbf{x}', \boldsymbol{\kappa}^s, \mathbf{E})] \Bigg\}
\end{aligned} \tag{1}
$$

$$
\textbf{s.t.} \qquad h^f = 0 \tag{2}
$$

$$
(1+\tau_c)c + a' = (1-\tau_{ss})W - T_L(W; \mathrm{S}) + (1 + r(1-\tau_k))(a + Tr) \tag{3}
$$

$$
\mathbf{E} = \{u, e\} \tag{4}
$$

The value function for a household with no earners is:

$$V_{zj}^{\{NE,NE\}}(a,\mathbf{x},\boldsymbol{\kappa},\mathbf{E}_{-1}) = \max_{c,a'} \left\{ \log(c) + \psi_j^m \frac{(1-h^m)^{1-\theta}}{1-\theta} + \psi_j^f \frac{(1-h^f)^{1-\theta}}{1-\theta} \right.$$

$$\left. +\beta s_{j+1} \sum_{x_m'} \sum_{x_f'} \Gamma_{x_m x_m'} \Gamma_{x_f x_f'} \ \ V_{z(j+1)}^2(a',\mathbf{x}',\boldsymbol{\kappa}^0,\mathbf{E})] \right\} \quad (5)$$

$$\textbf{s.t.} \qquad h^m = 0, \quad h^f = 0 \quad (6)$$

$$(1+\tau_c)c + a' = (1+r(1-\tau_k))(a+Tr) \quad (7)$$

$$\mathbf{E} = \{u,u\} \quad (8)$$

References

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