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A ROBUST AND EFFICIENT UNCERTAINTY QUANTIFICATION METHOD FOR COUPLED FLUID-STRUCTURE INTERACTION PROBLEMS

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Abstract. A robust and efficient uncertainty quantification method is presented for resolving the effect of uncertainty on the behavior of multi-physics systems. The extrema diminishing method in probability space maintains a bounded error due to the interpolation of deterministic samples at constant phase in a transonic airfoil flutter problem.

1 INTRODUCTION

Numerical errors in multi-physics simulations start to reach acceptable engineering accuracy levels due to the increasing availability of computational resources. Nowadays, uncertainties in multi-scale models, physical parameter variations, and lack of knowledge of initial and boundary conditions have a larger effect on computational predictions than discretization errors. It is, therefore, vital to take these uncertainties into account in coupled problems to obtain reliable computational predictions for reducing safety factors by robust design optimization.

Classical Monte Carlo uncertainty quantification for modeling random parameters is computationally intensive compared to the more efficient Polynomial Chaos method. The global polynomial approximation employed in the Polynomial Chaos formulation can, however, give unreliable results for discontinuous responses. Polynomial Chaos methods also require a fast increasing number of samples with time to maintain a constant accuracy in unsteady problems.

In this paper, a robust extrema diminishing Polynomial Chaos method is presented based on Newton-Cotes quadrature in a simplex elements discretization of probability space [1, 2]. The method results in a bounded error in time-dependent problems by performing the uncertainty quantification interpolation of oscillatory samples efficiently at constant phase. The application to a transonic airfoil flutter problem shows that the asymptotic pitch angle standard deviation is 16 times larger than the initial condition.

2 MATHEMATICAL UNCERTAINTY QUANTIFICATION PROBLEM

Consider a dynamical system subject to n_{a} uncorrelated second-order random input parameters $\mathbf{a}(\omega) = \{a_1(\omega), .., a_{n_{\mathrm{a}}}(\omega)\} \in A$ with parameter space $A \in \mathbb{R}^{n_{\mathrm{a}}}$, which governs an oscillatory response $u(\mathbf{x}, t, \mathbf{a})$

$$\mathcal{L}(\mathbf{x}, t, \mathbf{a}; u(\mathbf{x}, t, \mathbf{a})) = S(\mathbf{x}, t, \mathbf{a}), \tag{1}$$

with operator $\mathcal{L}$ and source term S defined on domain $D \times T \times A$. The spatial and temporal dimensions are defined as $\mathbf{x} \in D$ and $t \in T$, with $D \subset \mathbb{R}^d$, $d = \{1, 2, 3\}$, and $T = [0, t_{\max}]$. A realization of the set of outcomes Ω of the probability space $(\Omega, \mathcal{F}, P)$ is denoted by $\omega \in \Omega$, with $\mathcal{F} \subset 2^{\Omega}$ the σ-algebra of events and P a probability measure.

Here we consider a non-intrusive uncertainty quantification method l which reuses an existing deterministic solver for fluid-structure interaction simulation of (1). Non-intrusive uncertainty quantification method l is a combination of a sampling method g and an interpolation method h. Sampling method g defines the n_{s} sampling points $\mathbf{a}_k \equiv \mathbf{a}(\omega_k)$ and returns the deterministic samples $\mathbf{v}(\mathbf{x}, t) = \{v_1(\mathbf{x}, t), .., v_{n_{\mathrm{s}}}(\mathbf{x}, t)\}$ with $v_k(\mathbf{x}, t) \equiv u(\mathbf{x}, t, \mathbf{a}_k)$. Interpolation method h constructs an interpolation surface $w(\mathbf{x}, t, \mathbf{a})$ through the n_{s} samples $\mathbf{v}(\mathbf{x}, t)$ as a weighted approximation of $u(\mathbf{x}, t, \mathbf{a})$.

3 ROBUST EXTREMA DIMINISHING METHOD

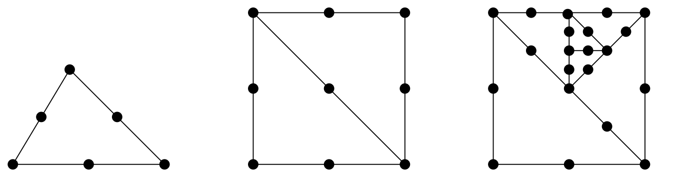
The multi-element Polynomial Chaos method l based on Newton-Cotes quadrature points in simplex elements [1] evaluates the statistical moment integral by dividing parameter space A into n_{e} non-overlapping simplex elements $A_j \subset A$. A piecewise polynomial approximation $w(\mathbf{x}, t, \mathbf{a})$ is then constructed based on n_{s} deterministic solutions $v_{j,k}(\mathbf{x}, t) = u(\mathbf{x}, t, \mathbf{a}_{j,k})$ for the values of the random parameters $\mathbf{a}_{j,k}$ that correspond to the $\tilde{n}_{\mathrm{s}}$ Newton-Cotes quadrature points of degree d in the elements A_j

$$\mu_{\mathrm{w}_i}(\mathbf{x}, t) = \sum_{j=1}^{n_{\mathrm{e}}} \int_{A_j} w(\mathbf{x}, t, \mathbf{a})^i f_{\mathbf{a}}(\mathbf{a}) \mathrm{d}\mathbf{a} = \sum_{j=1}^{n_{\mathrm{e}}} \sum_{k=1}^{\tilde{n}_{\mathrm{s}}} c_{j,k} v_{j,k}(\mathbf{x}, t)^i, \tag{2}$$

where $c_{j,k}$ are Polynomial Chaos Newton-Cotes quadrature weights. Here, second degree Newton-Cotes quadrature is considered in combination with adaptive mesh refinement in probability space, see Figure 1. It is proven in [2] that the resulting approach satisfies the extrema diminishing (ED) robustness concept in probability space

$$\min_A(w(\mathbf{a})) \geq \min_A(u(\mathbf{a})) \;\wedge\; \max_A(w(\mathbf{a})) \leq \max_A(u(\mathbf{a})) \quad \forall u(\mathbf{a}). \tag{3}$$

The ED property leads to the advantage that no non-zero probabilities of unphysical realizations can be predicted due to overshoots at discontinuities in the response surface.



(a) Element (b) Initial grid (c) Adapted grid

Figure 1: Discretization of two-dimensional parameter space A using 2-simplex elements and second-degree Newton-Cotes quadrature points given by the dots.

4 EFFICIENT INTERPOLATION AT CONSTANT PHASE

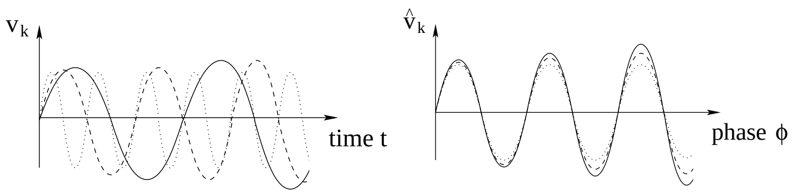
Assume that solving equation (1) for realizations of the random parameters $\mathbf{a}_k$ results in oscillatory samples $v_k(t) = u(\mathbf{a}_k)$, of which the phase $v_{\phi_k}(t) = \phi(t, \mathbf{a}_k)$ is a well-defined monotonically increasing function of time t for $k = 1, .., n_\mathrm{s}$. In order to interpolate the samples $\mathbf{v}(t) = \{v_1(t), .., v_{n_\mathrm{a}}(t)\}$ at constant phase [1], they are transformed into functions of their phase $\hat{\mathbf{v}}(\mathbf{v}_\phi(t))$ according to $\hat{v}_k(v_{\phi_k}(t)) = v_k(t)$ for $k = 1, .., n_\mathrm{s}$, see Figure 2. The interpolation $\hat{w}(w_\phi(t, \mathbf{a}), \mathbf{a})$ of the samples $\hat{\mathbf{v}}(\mathbf{v}_\phi(t))$ is transformed back to an approximation in the time domain $w(t, \mathbf{a}) = \hat{w}(w_\phi(t, \mathbf{a}), \mathbf{a})$. This uncertainty quantification formulation for oscillatory responses is proven to achieve a bounded error $\hat{\varepsilon}(\varphi, \mathbf{a}) = \hat{w}(\varphi, \mathbf{a}) - \hat{u}(\varphi, \mathbf{a})$ as function of phase φ for periodic responses according to

$$\hat{\varepsilon}(\varphi, \mathbf{a}) < \delta \quad \forall \varphi \in \mathbb{R}, \mathbf{a} \in A, \tag{4}$$

where δ is defined by

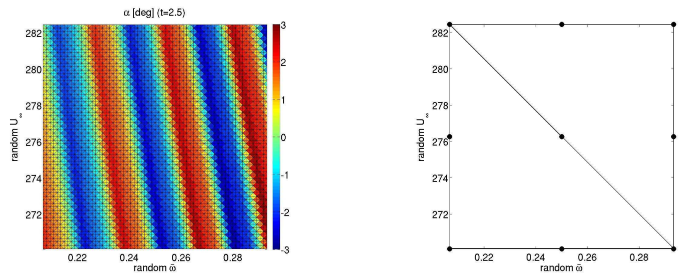
$$\hat{\varepsilon}(\varphi, \mathbf{a}) < \delta, \quad \forall \varphi \in [0, 1], \mathbf{a} \in A. \tag{5}$$

The error $\varepsilon(t, \mathbf{a}) = w(t, \mathbf{a}) - u(t, \mathbf{a})$ is also bounded in time under certain conditions, see [2]. The phases $\mathbf{v}_\phi(t)$ are extracted from the samples using a trial-and-error algorithm based on the local extrema of the time series $\mathbf{v}(t)$.



(a) samples $v_k(t)$ (b) samples $\hat{v}_k(\phi)$

Figure 2: Oscillatory samples as function of time and phase.



(a) $t = 2.5$ (b) Stochastic grid

Figure 3: Response surface of angle of attack $\alpha(\omega)$ as function of random natural frequency ratio $\bar{\omega}(\omega)$ and free stream velocity $U_\infty(\omega)$ for the transonic airfoil flutter problem.

5 STOCHASTIC TRANSONIC AIRFOIL FLUTTER

The stochastic post flutter behavior of an elastically mounted airfoil in Euler flow is analyzed at a bifurcation parameter value U^* of 130% of the deterministic linear bifurcation point. The response of a structural model of a pitch-plunge airfoil with cubic nonlinear spring stiffness at a Mach number of $M_\infty = 0.8$ is considered. The randomness in the ratio of natural frequencies $\bar{\omega}(\omega)$ and free stream velocity $U_\infty(\omega)$ is given by a uniform and beta distribution with a coefficient of variation of 10% and 1%, respectively.

The response surface approximation of the angle of attack $\alpha(t, \omega)$ as function of the random parameters $\bar{\omega}(\omega)$ and $U_\infty(\omega)$ given in Figure 3a shows a highly oscillatory response surface at $t = 2.5$. The resulting asymptotic standard deviation of $\sigma_\alpha = 1.6^\mathrm{o}$ is a factor 16 larger than the initial angle of attack $\alpha(0) = 0.1^\mathrm{o}$. This result is obtained using the time-independent grid with $n_\mathrm{s} = 9$ samples and $n_\mathrm{e} = 2$ elements in probability space shown in Figure 3b.

6 CONCLUSIONS

The presented extrema diminishing uncertainty quantification method with bounded error predicts a 16 times larger asymptotic standard deviation compared to the initial condition due to the effect of physical uncertainties in a transonic airfoil flutter problem.

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