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A Robust Fault Detection and Isolation Scheme for a Class of Uncertain Input-output Discrete-time Nonlinear Systems

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Abstract— **This paper extends very recent results on discrete-time nonlinear fault detection and isolation to the case of discrete-time nonlinear systems with unstructured modeling uncertainty and partial state measurement. The fault diagnosis architecture consists of a fault detection and approximation estimator and a bank of fault isolation estimators, each corresponding to a particular type of fault. A time-varying threshold that guarantees no false-positive alarms and fault detectability conditions are derived analytically. For the fault isolation scheme, we design adaptive residual thresholds associated with each isolation estimator and obtain sufficient conditions for fault isolability. To illustrate the theoretical results, a simulation example based on a input-output discrete-time version of the three-tank benchmark problem is presented.**

I. INTRODUCTION

The design and analysis of fault detection and isolation (FDI) architectures for linear and nonlinear systems using the model-based analytical redundancy approach have received significant attention in the literature [6], [7], [10], [2], [8], [3]. Recently there has been a lot of research activity on fault detection and isolation of uncertain nonlinear systems. Several of the techniques developed for nonlinear systems are extension of methods that were originally developed for linear systems, such as the unknown input observer approach, parity relations, etc. Another significant approach is based on adaptive approximation techniques for nonlinear fault diagnosis [15], [16], [18].

In the very recent paper [5], the FDI methodology considered in [18] has been tailored to address the discrete-time case. In the present paper, the results reported in [5] will be extended to a class of input-output nonlinear uncertain discrete-time systems (see [19] for the input-output continuous-time case). More specifically, the unstructured uncertainty may affect either the discrete-time state or output equation and the considered class of faults is allowed to have a nonlinear structure with respect to the state (or the output) and input, and includes both abrupt and incipient faults.

Analogously with [5], the FDI scheme consists of a bank of nonlinear discrete-time adaptive estimators. One of them is the fault detection and approximation estimator, whereas the others are used for fault isolation (each associated with a specific type of fault). Under normal operating conditions, only the detection and approximation estimator is used to monitor the process for any fault. Once a fault is detected, the fault isolation estimators are activated, and the fault detection and approximation estimator adopts the mode of approximating the fault, by using online approximation methods.

The main contributions of this paper are the design of a fault isolation scheme in a input-output discrete-time framework and the derivation of rigorous analytical results for the detectability and isolability properties. The residual of each fault isolation estimator is associated with an adaptive threshold, which can be implemented in real-time.

The paper is organized as follows. Section II formulates the problem that will be addressed. A fault detection and isolation architecture is presented in Section III. In the same section, we also present the analytical results regarding the fault detectability and fault isolability. Finally, the FDI scheme design and the analytical results are illustrated by a simulation example in Section IV, and Section V contains some concluding remarks.

II. PROBLEM FORMULATION

In this paper a class of multi–input, multi–output uncertain nonlinear systems described by the following discrete–time dynamic equations will be considered:

$$\begin{cases} x(k+1) = A\,x(k) + f(x(k), u(k)) + \beta(k-k_0)\times \\ \qquad\qquad \phi(y(k), u(k)) + \eta_x(x(k), u(k), k) \\ \quad y(k) = C\,x(k) + \eta_y(x(k), u(k), k) \end{cases} \quad (1)$$

for $k = 0, 1, \ldots$, where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ denote the state, the control input and the measured output vectors, respectively; the matrix $A \in \mathbb{R}^{n\times n}$ and the vector field $f : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ represent the nominal healthy dynamics[1], $C \in \mathbb{R}^{p\times n}$ represents the nominal output equation while $\eta_x : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{N} \mapsto \mathbb{R}^n$ and $\eta_y : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{N} \mapsto \mathbb{R}^p$ are the uncertainties in the state and output equations which may be caused by several factors such as, for instance, external disturbances, the possible discretization error and so on.

The term $\beta(k-k_0)\phi(y(k), u(k))$ denotes the changes in the system dynamics due to the occurrence of a fault. More specifically, the vector $\phi(y(k), u(k))$ represents the functional structure of the deviation in the state equation due to the fault and the function $\beta(k-k_0)$ characterizes the time profile of the fault, where k_0 is the unknown index of the

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[1] Of course, given a nonlinear system there are infinite ways of decomposing its dynamic equation into a linear and a nonlinear term.

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fault occurrence time. In this paper, we shall consider either *abrupt* faults characterized by a "step-like" time-profile

$$\beta(k-k_0)=\begin{cases}0 & \text{if} \quad k<k_0\\ 1 & \text{if} \quad k\geq k_0\end{cases}, \tag{2}$$

or *incipient* faults characterized by an "exponential-like" one

$$\beta(k-k_0)=\begin{cases}0 & \text{if} \quad k<k_0\\ 1-b^{-(k-k_0)} & \text{if} \quad k\geq k_0\end{cases}. \tag{3}$$

where $b>1$ denotes the unknown fault-evolution rate.

For isolation purposes, we assume that there are N types of possible nonlinear fault functions; specifically, $\phi(y,u)$ belongs to a finite set of functions given by

$$\mathscr{F} \triangleq \{\phi^1(y,u),\dots,\phi^N(y,u)\}\,.$$

Each fault function in $\mathscr{F}$ is assumed to be in the form

$$\phi^l(y(k),u(k))=[(\vartheta_1^l)^\top g_1^l(y(k),u(k)),\dots,\\ (\vartheta_n^l)^\top g_n^l(y(k),u(k))]^\top, \tag{4}$$

where, for $i\in\{1,\dots,n\}$, $l\in\{1,\dots,N\}$, the *known* functions $g_i^l(y(k),u(k)):\mathbb{R}^p\times\mathbb{R}^m\mapsto\mathbb{R}^{q_i^l}$ describe the "structure" of the fault, and the *unknown* parameter vectors $\vartheta_i^l\in\Theta_i^l\subset\mathbb{R}^{q_i^l}$ provide its "magnitude". In this paper, for the sake of simplicity and without much loss of generality, the parameter domains Θ_i^l are assumed to be origin–centered hyper–spheres. The following assumptions are needed.

Assumption 1: At time $k=0$ no faults act on the system. Moreover, A is chosen so that (A,C) is an observable pair, and the state variables $x(k)$ and control variables $u(k)$ remain bounded before and after the occurrence of a fault, i.e., there exist some stability regions $\mathcal{R}=\mathcal{R}^x\times\mathcal{R}^u\subset\mathbb{R}^n\times\mathbb{R}^m$, such that $(x(k),u(k))\in\mathcal{R}^x\times\mathcal{R}^u$, $\forall\,k$. □

Assumption 2: The modeling uncertainty represented by the vectors η_x and η_y in (1) are unstructured and possibly unknown nonlinear functions of x, u, and k, but are bounded by some known functions $\bar\eta_x$ and $\bar\eta_y$, i.e.,

$$\|\eta_x(x(k),u(k),k)\|\leq\bar\eta_x(x(k),u(k),k),\\ |\eta_{y,(i)}(x(k),u(k),k)|\leq\bar\eta_{y,(i)}(x(k),u(k),k)\,,$$

where by $x_{(i)}$ we mean the i–th component of a vector and, for each $i=1,\dots,n$, the bounding functions $\bar\eta_{x,(i)}$ and $\bar\eta_{y,(i)}$ are positive, known and bounded for all $(x,u)\in\mathcal{R}$ and for all k. □

Assumption 3: The time profile parameter b is unknown but it is lower bounded by a known constant $\bar b$. □

As this paper considers only the fault diagnosis problem and not the fault accommodation one, Ass. 1 is required for well–posedness. Ass. 2 and 3 are required for the analysis but, in practical situations, if some a-priori knowledge on healthy and faulty modes of behavior is available, are not a significant limitation.

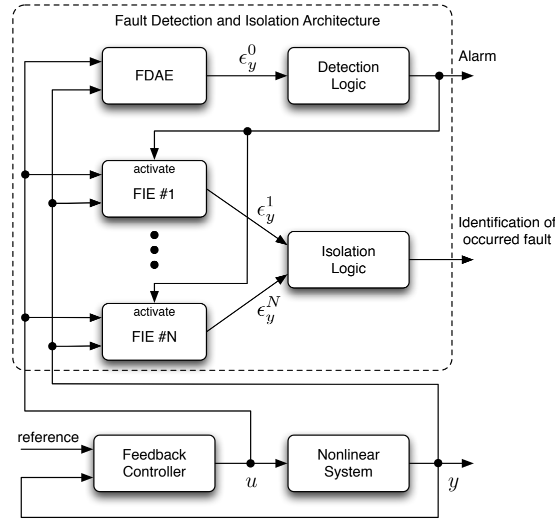

Fig. 1. A scheme of the proposed FDI architecture.

III. Fault Detection and Isolation Architecture

In this section a discrete–time *Fault Detection and Isolation* (FDI) architecture, to some extent analogous to the continuous-time ones in [18], [19], will be proposed. A bank of $N+1$ nonlinear adaptive estimators is employed (Fig. 1), each one yielding an output estimate $\hat y^j\in\mathbb{R}^p$, where N is the number of nonlinear faults of the fault class $\mathscr{F}$. The *fault detection and approximation estimator* (FDAE) detects faults and approximate possibly unknown faults, while the *fault isolation estimators* (FIEs), corresponding to the faults in $\mathscr{F}$, are used for isolation purposes only after detection.

The output estimation error

$$\epsilon_y^j(k)\triangleq y(k)-\hat y^j(k)\,,\quad j=0,\dots,N$$

can be defined for each estimator and is associated to a given *fault hypothesis*. A fault hypothesis will be rejected by the detection and isolation logic if the absolute value of at least one component $\epsilon_{y,(i)}^j(k)$ of the corresponding error will cross a suitable time-varying threshold $\bar\epsilon_{y,(i)}^j(k)$.

The FDAE error is associated to the healthy mode of behavior, which is rejected at the detection time k_d:

Definition 3.1: The *fault detection time* k_d is defined as $k_d\triangleq\min\{k:\exists\,i,\,i\in\{1,\dots,n\},\,|\epsilon_{y,(i)}^0(k)|>\bar\epsilon_{y,(i)}^0(k)\}$. □

At time $k=k_d$, the N FIEs are activated to implement a kind of *Generalized Observer Scheme* [13], [18]. The l–th FIE is associated to the "l–th fault has occurred" hypothesis and its threshold $\bar\epsilon_y^l$ is designed in order not to be crossed if that fault actually occurred. The isolation logic is based on excluding every but one hypothesis, as defined here:

Definition 3.2: The *l–th fault exclusion time* k_e^l is defined as $k_e^l\triangleq\min\{k:\exists\,i,\,i\in\{1,\dots,n\},\,|\epsilon_{y,(i)}^l(k)|>\bar\epsilon_{y,(i)}^l(k)\}$. □

Definition 3.3: A fault $\phi^s\in\mathscr{F}$ is *isolated* at time k iff $\forall l,\,l\in\{1,\dots,N\}\setminus s$, $k_e^l\leq k$ and $\nexists\,k_e^s$. Furthermore $k_{isol}^s\triangleq\max\{k_e^l,\,l\in\{1,\dots,N\}\setminus s\}$ is the *fault isolation time.* □

Remark 3.1: To conclude that an isolated fault actually occurred, it must be assumed that only faults in $\mathscr{F}$ are possible. Otherwise, it must be said that it is not impossible that it occurred. If every fault in $\mathscr{F}$ is rejected, then it will be said that the proposed FDI architecture has isolated an unknown fault. In order to gain knowledge about it and possibly extend $\mathscr{F}$, an on–line approximator capable of learning any fault that can reasonably occur is embedded in the FDAE and started at $k = k_d$.

A. FDAE Estimator and Fault Detection

Before the detection of a fault, for $0 \leq k < k_d$, the dynamics of the FDAE estimator can be written as

$$\begin{cases} \hat{x}^0(k+1) = A\,\hat{x}^0(k) + f(\hat{x}^0(k), u(k)) + L\,(y(k) - \hat{y}^0(k)) \\ \quad\hat{y}^0(k) = C\,\hat{x}^0(k) \end{cases} \tag{5}$$

where the output error gain matrix $L \in \mathbb{R}^{n\times p}$ is chosen such that $A^0 \triangleq A - LC$ is a Hurwitz matrix[2]. Before the occurrence of a fault, for $0 \leq k \leq k_0$, the dynamics of the state estimation error $\epsilon_x^0(k) \triangleq x(k) - \hat{x}^0(k)$ are[3]

$$\epsilon_x^0(k{+}1) = A^0\epsilon_x^0(k) + f(x(k), u(k)) - f(\hat{x}^0(k), u(k)) + \eta_x(k)\,, \tag{6}$$

so that the output estimation error i–th component is

$$\epsilon^0_{y,(i)}(k) = C_i\left\{\sum_{h=0}^{k-1}(A^0)^{k-1-h}[\Delta f(x(h), \hat{x}^0(h), u(h)) + \eta_x(h)] + (A^0)^k\epsilon_x^0(0)\right\} + \eta_{y,(i)}(k)\,,$$

where C_i is the i–th row of C and

$$\Delta f(x(k), \hat{x}^0(k), u(k)) \triangleq f(x(k), u(k)) - f(\hat{x}^0(k), u(k))\,.$$

Recalling Assumption 2 and by defining

$$\bar{\Delta} f(\hat{x}^0(k), u(k)) \triangleq \max_{x\in\mathcal{R}^x}\{\|\Delta f(x, \hat{x}^0(k), u(k))\|\}$$

we can bound $|\epsilon^0_{y,(i)}(k)|$ by the following threshold

$$\bar{\epsilon}^0_{y,(i)}(k) \triangleq \sum_{h=0}^{k-1}\alpha_i\delta_i^{k-1-h}[\bar{\Delta} f(h) + \bar{\eta}_x(h)] + \alpha_i\delta_i^k\bar{\epsilon}_x^0(0) + \bar{\eta}_{y,(i)}(k)\,, \tag{7}$$

where α_i and δ_i, analogously to [19], are two constants such that $\|C_i(A^0)^k\| \leq \alpha_i\delta_i^k \leq \|C_i\|\,\|A^0\|^k$, $\alpha_i > 0$, $0 < \delta_i \leq 1$. Furthermore,

$$\bar{\epsilon}_x^0(k) \triangleq \max_{x\in\mathcal{R}^x}\{\|x - \hat{x}^0(k)\|\}\,,\ i = 1, \ldots, n\,.$$

The threshold in (7) guarantees that no false–positive alarms will be issued until k_0 because of the uncertainties η_x and η_y. This, of course, comes at the cost of the impossibility of detecting faults "hidden by the uncertanties in the system dynamics". This is formalized by the following

Theorem 3.1 (Fault Detectability): If there exists a time index $k_2 > k_0$ such that the fault ϕ fulfills the following inequality for at least one component $i \in \{1, \ldots, n\}$

$$\left|\sum_{h=k_0}^{k_2-1} C_i(A^0)^{k_2-1-h}(1 - b^{-(h-k_0)})\phi(h)\right| > 2\bar{\eta}_{y,(i)}(k_2) + \delta_i^{k_2-k_0}[\alpha_i\bar{\epsilon}_x^0(k_0) + \bar{\epsilon}^0_{y,(i)}(k_0)] + \sum_{h=k_0}^{k_2-1} 2\alpha_i\delta_i^{k_2-1-h}[\bar{\Delta} f(h) + \bar{\eta}_x(h)] \tag{8}$$

then it will be detected at time $k = k_2$, that is $|\epsilon^0_{y,(i)}(k_2)| > \bar{\epsilon}^0_{y,(i)}(k_2)$. □

Proof: At the time index $k_2 > k_0$ the i–th component of the output estimation error is

$$\epsilon^0_{y,(i)}(k_2) = C_i\big\{(A^0)^{k_2-k_0}\epsilon_x^0(k_0) + \sum_{h=k_0}^{k_2-1}(A^0)^{k_2-1-h}\times [\eta_x(h) + \Delta f(h) + (1 - b^{-(h-k_0)})\phi(h)]\big\} + \eta_{y,(i)}(k_2)$$

By the triangle inequality it follows that

$$|\epsilon^0_{y,(i)}(k_2)| \geq -\sum_{h=k_0}^{k_2-1}\|C_i(A^0)^{k_2-1-h}\|\,\|\eta_x(h) + \Delta f(h)\| + \left|\sum_{h=k_0}^{k_2-1} C_i(A^0)^{k_2-1-h}(1 - b^{-(h-k_0)})\phi(h)\right| - \|C_i(A^0)^{k_2-k_0}\|\,\|\epsilon_x^0(k_0)\| - |\eta_{y,(i)}(k_2)|\,.$$

By recalling Assumption 2 and the definition of $\bar{\epsilon}_x^0$, it holds

$$|\epsilon^0_{y,(i)}(k_2)| \geq -\sum_{h=k_0}^{k_2-1}\|C_i(A^0)^{k_2-1-h}\|[\bar{\eta}_x(h) + \bar{\Delta} f(h)] + \left|\sum_{h=k_0}^{k_2-1} C_i(A^0)^{k_2-1-h}(1 - b^{-(h-k_0)})\phi(h)\right| - \|C_i(A^0)^{k_2-k_0}\|\bar{\epsilon}_x^0(k_0) - \bar{\eta}_{y,(i)}(k_2)\,.$$

Therefore, a sufficient condition for detecting the fault is

$$\left|\sum_{h=k_0}^{k_2-1} C_i(A^0)^{k_2-1-h}(1 - b^{-(h-k_0)})\phi(h)\right| > \bar{\epsilon}^0_{y,(i)}(k_2) + \sum_{h=k_0}^{k_2-1}\|C_i(A^0)^{k_2-1-h}\|[\bar{\eta}_x(h) + \bar{\Delta} f(h)] + \|C_i(A^0)^{k_2-k_0}\|\bar{\epsilon}_x^0(k_0) + \bar{\eta}_{y,(i)}(k_2)\,.$$

As we can write

$$\bar{\epsilon}^0_{y,(i)}(k_2) = \sum_{h=k_0}^{k_2-1}\alpha_i\delta_i^{k_2-1-h}[\bar{\Delta} f(h) + \bar{\eta}_x(h)] + \delta_i^{k_2-k_0}\bar{\epsilon}^0_{y,(i)}(k_0) + \bar{\eta}_{y,(i)}(k_2)\,,$$

[2]This condition can alway be satisfied when Assumption 1 holds or when A is a Hurwitz matrix itself.

[3]In the paper, when there is no risk of ambiguity and for the sake of simplicity, a compact notation like, for instance, $\eta(k) \equiv \eta(x(k), u(k), k)$, will be used.

if ϕ is such that the inequality in the hypothesis holds, then $|\epsilon^0_{y,(i)}(k_2)| > \bar{\epsilon}^0_{y,(i)}(k_2)$ and a fault will be detected. ■

Remark 3.2: We assumed in equation (1) that the fault function ϕ depends upon y and u, in order to make possible the approximation and the isolation and estimation tasks. Anyway, the detectability theorem given here will apply also to fault functions ϕ that may depend upon the full state x.

After the detection of a fault at time $k = k_d$, the FDAE approximator is turned on and the dynamics (5) becomes

$$\begin{cases} \hat{x}^0(k+1) = A\,\hat{x}^0(k) + f(\hat{x}^0(k), u(k)) + \\ \qquad L\,(y(k) - \hat{y}^0(k)) + \hat{\phi}^0(y(k), u(k), \hat{\vartheta}^0(k)) \\ \hat{y}^0(k) = C\,\hat{x}^0(k) \end{cases} \tag{9}$$

where $\hat{\phi}^0$ is an adaptive approximator, which can be any nonlinear multivariable approximation model with adjustable parameters, contained in the vector $\hat{\vartheta}^0(k) \in \hat{\Theta}^0 \subset \mathbb{R}^{q^0}$. Again, for the sake of simplicity, $\hat{\Theta}^0$ is assumed to be an origin–centered hyper–sphere with radius $M_{\hat{\Theta}^0}$.

In order for $\hat{\phi}^0$ to learn the fault function ϕ, its parameters vector is updated according to the following learning law:

$$\hat{\vartheta}^0(k+1) = \mathcal{P}_{\hat{\Theta}^0}(\hat{\vartheta}^0(k) + \gamma^0(k) Z^\top(k) C^\top \epsilon^0_y(k+1)),$$

where $Z(k) \triangleq \partial\hat{\phi}^0(y(k), u(k), \hat{\vartheta}^0(k))/\partial\hat{\vartheta}^0 \in \mathbb{R}^{n\times q^0}$ is the gradient matrix of the on–line approximator wrt its adjustable parameters and $P_{\hat{\Theta}^0}$ is a projection operator [14]

$$\mathcal{P}_{\hat{\Theta}^0}(\hat{\vartheta}^0) \triangleq \begin{cases} \hat{\vartheta}^0 & \text{if } |\hat{\vartheta}^0| \le M_{\hat{\Theta}^0} \\ \frac{M_{\hat{\Theta}^0}}{|\hat{\vartheta}^0|}\hat{\vartheta}^0 & \text{if } |\hat{\vartheta}^0| > M_{\hat{\Theta}^0} \end{cases},$$

The learning rate $\gamma^0(k)$ is computed at each time-step as

$$\gamma^0(k) \triangleq \frac{\mu^0}{\varepsilon^0 + \|Z^\top(k) C^\top\|_F^2},\ \varepsilon^0 > 0,\ 0 < \mu^0 < 2$$

where $\|\cdot\|_F$ is the Frobenius norm and ε^0, μ^0 guarantee the stability of the learning law [14], [11], [1], [12], [9].

B. FIE Estimators and Isolation Scheme

After a fault has been detected at time $k = k_d$, the bank of N FIEs is activated in order to possibly isolate it. The dynamics of the l–th FIE estimator, with $l \in \{1, \dots, N\}$, is

$$\begin{cases} \hat{x}^l(k+1) = A\,\hat{x}^l(k) + f(\hat{x}^l(k), u(k)) + \\ \qquad L\,(y(k) - \hat{y}^l(k)) + \hat{\phi}^l(y(k), u(k), \hat{\vartheta}^l(k)) \\ \hat{y}^l(k) = C\,\hat{x}^l(k) \end{cases} \tag{10}$$

where $\hat{\phi}^l_{(i)}(y(k), u(k), \hat{\vartheta}^l(k)) \triangleq (\hat{\vartheta}^l_i)^\top g^l_i(y(k), u(k))$ is a linearly-parameterized function matching the structure of $\phi^l_{(i)}$, with $\hat{\vartheta}^l_i \in \Theta^l_i$ and $\hat{\vartheta}^l \triangleq \mathrm{col}(\hat{\vartheta}^l_i,\ i = 1, \dots, n)$.

The learning law for $\hat{\vartheta}^l_i$ is analogous to the FDAE one:

$$\hat{\vartheta}^l_i(k+1) = \mathcal{P}_{\Theta^l_i}(\hat{\vartheta}^l_i(k) + \gamma^l_i(k) g^l_i(k) C_i^\top \epsilon^l_y(k+1)),$$

where $C_i^\top$ is the i–th row of $C^\top$, $P_{\Theta^l_i}$ is the projection operator on Θ^l_i and $\gamma^l_i(k)$ is

$$\gamma^l_i(k) \triangleq \frac{\mu^l_i}{\varepsilon^l_i + \|g^l_i(k) C_i^\top\|_F^2},\ \varepsilon^l_i > 0,\ 0 < \mu^l_i < 2.$$

Remark 3.3: In spite of their similarity, the FDAE on–line approximator must be complex enough to approximate any reasonable unknown fault, while the FIE ones are designed to match a single fault function in $\mathscr{F}$. And although it is possible for a FIE to exactly match a fault function ϕ^l if $\hat{\vartheta}^l_i(k) = \vartheta^l_i$, $\forall i \in \{1, \dots, n\}$, there is no guarantee that $\hat{\vartheta}^l_i(k)$ will converge to the true value ϑ^l_i, as *persistency of excitation is not assumed in this paper*.

Assuming a matched fault, that is $\phi = \phi^l$, the state estimation error dynamics equation is

$$\epsilon^l_x(k+1) = A^0 \epsilon^l_x(k) + \Delta f(x(k), \hat{x}^l(k), u(k)) +$$
$$(1 - b^{-(k-k_0)}) g^l(k) \tilde{\vartheta}^l(k) - b^{-(k-k_0)} g^l(k) \hat{\vartheta}^l(k) + \eta_x(k),$$

where g^l is the following diagonal block matrix

$$g^l \triangleq \begin{bmatrix} (g^l_1)^\top & \emptyset & & & \\ \emptyset & (g^l_2)^\top & \emptyset & & \\ & & \ddots & & \\ & & \emptyset & (g^l_{n-1})^\top & \emptyset \\ & & & \emptyset & (g^l_n)^\top \end{bmatrix} \in \mathbb{R}^{n\times q^l},$$

$q^l \triangleq \sum_{i=1}^n q^l_i$ and $\tilde{\vartheta}^l(k) \triangleq \vartheta^l - \hat{\vartheta}^l(k)$ is the parameter estimation error, with $\vartheta^l \triangleq \mathrm{col}(\vartheta^l_i, i = 1, \dots, n) \in \Theta^l$ and $\Theta^l \triangleq \prod_{i=1}^n \Theta^l_i \subset \mathbb{R}^{q^l}$. Then the i–th component of the output estimation error is

$$\epsilon^l_{y,(i)}(k) = C_i \Bigg\{ \sum_{h=k_d}^{k-1} (A^0)^{k-1-h} [\Delta f(h) + \eta_x(h)] + g^l(h) \times$$
$$[\tilde{\vartheta}^l(h)(1 - b^{-(h-k_0)}) + \hat{\vartheta}^l(h) b^{-(h-k_0)}] + (A^0)^{k-k_d} \epsilon^0_x(k_d) \Bigg\},$$

Owing to Ass. 2 and Ass. 3, and by using the same reasoning as in Section (III-A), the output estimation error absolute value for a matched fault can be upper bounded by

$$\bar{\epsilon}^l_{y,(i)}(k) = \bar{\eta}_{y,(i)}(k) + \alpha_i \Bigg\{ \sum_{h=k_d}^{k-1} \delta_i^{k-1-h} [\bar{\Delta} f(h) + \bar{\eta}_x(h)] +$$
$$\|g^l(h)\| [\kappa^l(h) + \|\hat{\vartheta}^l(h)\| \bar{b}^{-(h-k_d)}] + \delta_i^{k-k_d} \bar{\epsilon}^l_x(k_d) \Bigg\}, \tag{11}$$

where we let

$$\bar{\epsilon}^l_x(k) \triangleq \max_{x\in\mathcal{R}^x} \{\|x - \hat{x}^l(k)\|\}.$$

In order to make $\bar{\epsilon}^l_x$ computable, we introduced the following function that depends on the geometry of Θ^l and bounds the norm of the parameter estimation error

$$\kappa^l(k) \triangleq \max_{\vartheta^l\in\Theta^l} \{\|\vartheta^l - \hat{\vartheta}^l(k)\|\}.$$

The threshold in (11) guarantees that if the fault $\phi^l \in \mathscr{F}$ occurs it will not be rejected by its FIE. But, because of the uncertainties η_x and η_y and of the parameter estimation error, there is no assurance that others FIEs will reject the fault ϕ^l so that it may be isolated. A sufficient condition for a succesfull isolation decision is given in the following:

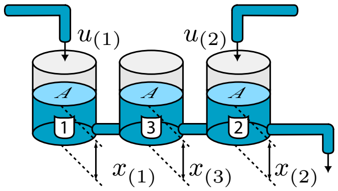

Fig. 2. Structure of the three–tanks system under consideration.

Theorem 3.2 (Fault Isolability): Given a fault $\phi^s \in \mathscr{F}$, if for each $l \in \{1,\ldots,N\} \setminus s$ there exists some time instant $k_l > k_d$ and some $i_l \in \{1,\ldots,n\}$ such that

$$\left| C_{i_l} \sum_{h=k_d}^{k_l-1} (A^0)^{k_l-1-h} \xi^{s,l}(h) \right| > \alpha_{i_l} \Big\{ 2\delta_{i_l}^{k_l-k_d} \bar{\epsilon}_x^l(k_d) + \sum_{h=k_d}^{k_l-1} \delta_{i_l}^{k_l-1-h} \Big[2\bar{\Delta} f(\hat{x}^l(h), u(h)) + \|g^l(h)\| \Big(\kappa^l(h) + \bar{b}^{-(k_l-k_d)} \|\hat{\vartheta}^l(h)\| \Big) + 2\bar{\eta}_x(h) \Big] \Big\} + 2\bar{\eta}_{y,(i_l)}(k_l) \quad (12)$$

where

$$\xi^{s,l}(k) \triangleq (1 - b^{-(k-k_0)}) g^s(k) \vartheta^s - g^l(k) \hat{\vartheta}^l(k), \quad \forall l, s \in \{1,\ldots,N\},\, l \neq s$$

is the *fault mismatch function*. Then, the s–th fault will be isolated at time $\max_{l \in \{1,\ldots,N\} \setminus s} (k_l)$. □

Proof: Omitted because of space constraints. ■

Remark 3.4: It is worth noting that Theorems 3.1 and 3.2 provide a sufficient conditions that may result to be quite conservative in practice, depending on the accuracy of the a-priori available knowledge. For a given fault, its detectability and isolability can be checked by conditions (8) and (12), so that the classes of detectable and isolable faults can be approximately determined by a suitable numerical algorithm.

IV. Simulation results

A simple example is presented to illustrate the effectiveness of the proposed FDI scheme, based on the well-known three-tank problem (see Fig. 2). Nominally, the tanks cross-section are $A_i = 1$ m^2, and the pipes have cross-section $A_j^p = 0.01$ m^2 and unitary outflow coefficient c_j, $i, j = 1, \ldots, 3$. These values will be used to compute the nominal term $f(k)$ in the FDAE and FIEs' estimators, but when simulating the actual system we will add an uncertainty term η_x accounting for a 7%, 10% and 15% inaccuracy in A, A^p and c, respectively. The measurement uncertainty η_y will be a random noise bounded by $\bar{\eta}_{y,(i)} = 0.01$, $i = 1, \ldots, 3$.

The tank discrete-time model will be obtained from the continuous-time version [18] by employing a simple forward Euler discretization with $T_s = 0.1$, so that A turns out to be a 3×3 identity matrix. The output error gain matrix is

$$L = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 0 & 0 \end{bmatrix},$$

while the FDAE on–line approximator $\hat{\phi}^0$ will consist of a 5–input, 3–output RBF neural network with one hidden layer of 3^5 fixed neurons equally spaced in the hyper–rectangle $[0, 10]^3 \times [0, 1]^2 \subset \mathbb{R}^5$, covering all the admissible values for state and input variables. $\hat{\vartheta}^0$ will have $3 \cdot 3^5$ components containing the weights by which the hidden layer outputs are linearly combined in order to compute the network output.

Three FIEs will be employed, in order to match the following faults:

1) **Actuator fault in pump 1**
2) **Leakage in tank 1**
3) **Actuator fault in pump 2**

For both the FDAE and the FIEs the learning rate is $\mu_i^j = 0.01$. After suitable offline simulations all the parameter domains were chosen to be hyper–spheres with unitary radius. The bound on the state uncertainty function was set to the constant value $\bar{\eta}_i = T_s \cdot 0.05$, $i = 1, \ldots, 3$, while the bound on the time profile parameter was set to $\bar{b} = 1.01$.

Fig. 3 shows the results of a simulation where at $T_0 = 15$ s a leak of section 0.2 m^2 is introduced into the first tank, with a time profile described by $b = 1.05$. In Fig. 3(a) it can be seen that the fault is detected about 1.5 s later, and then it takes about 1.5 seconds more to isolate it (Fig. 3(b)-(d)). Because of space constraints the behaviour of the estimation error $\epsilon_{y,(2)}^2$ is not reported, but anyway it is clear that it do not cross its corresponding thresholds as the fault function considered do not affect the dynamics of $x_{(2)}$ and, thus, of $y_{(2)}$. In Fig. 3(e)-(f) the approximated fault function $\hat{\phi}_{(1)}^2$ and the corresponding parameter $\hat{\vartheta}_1^2$ are plotted: it can be seen that the second FIE correctly estimate the actual fault function $\phi_{(1)}^2$.

The inflows were $u_1(k) = 0.3 - 0.25 \cdot \cos(0.05 \cdot kT_s)$ and $u_2(k) = 0.1 \cdot \cos(0.5 \cdot kT_s) + 0.15$; the nominal tank initial levels were 1, 2.5 and 2 m and were used for initialising the FDAE estimator, while the actual one were 20% off.

V. Concluding remarks

In this paper a robust FDI scheme for a class of input–output non–linear discrete-time uncertain was proposed, that relies on a bank of non–linear estimators based on a nominal model of the healthy system dynamics. Both abrupt and incipient kinds of faults were addressed and theoretical results characterizing the ability of the FDI scheme to detect and to isolate a given fault were derived. Furthermore, simulations results about the well-known three-tanks benchmark illustrated in practice its effectiveness.

Future developments will include a larger class of discrete-time nonlinear systems, the effects of delays in the measurements, and possibly a more general fault model. Another important issue is the combination of the proposed FDI scheme with fault tolerant control design, therefore providing a unified architecture for fault detection, isolation and accommodation (as done in [17] for continuous time). Finally, the FDI scheme may be extended to accomodate recent advances about the FDI problem for large–scale, distributed systems.

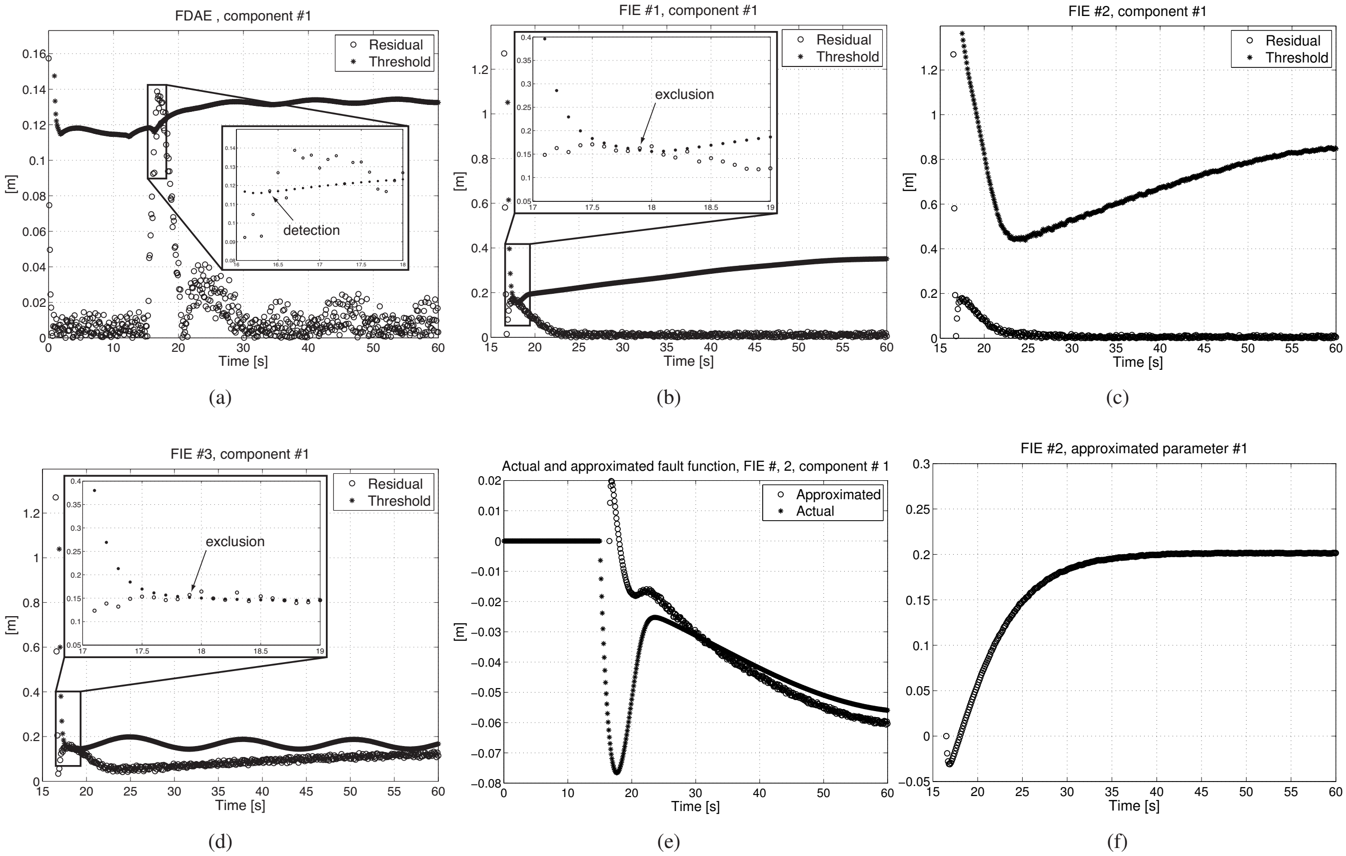

Fig. 3. Time-behaviours of signals related to tank n.1 when an incipient leak in tank 1 is introduced at time 15 s.

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