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A Robust Formulation of the v2-f Model

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Abstract. The elliptic relaxation approach of Durbin (Durbin, P.A., *J. Theor. Comput. Fluid. Dyn.* **3** (1991) 1–13), which accounts for wall blocking effects on the Reynolds stresses, is analysed herein from the numerical stability point of view, in the form of the $\bar{v}^2 - f$. This model has been shown to perform very well on many challenging test cases such as separated, impinging and bluff-body flows, and including heat transfer. However, numerical convergence of the original model suggested by Durbin is quite difficult due to the boundary conditions requiring a coupling of variables at walls. A 'code-friendly' version of the model was suggested by Lien and Durbin (Lien, F.S. and Durbin, P.A., Non linear $k-\varepsilon-v^2$ modelling with application to high-lift. In: *Proceedings of the Summer Program 1996*, Stanford University (1996), pp. 5–22) which removes the need of this coupling to allow a segregated numerical procedure, but with somewhat less accurate predictions. A robust modification of the model is developed to obtain homogeneous boundary conditions at a wall for both $\bar{v}^2$ and f. The modification is based on both a change of variables and alteration of the governing equations. The new version is tested on a channel, a diffuser flow and flow over periodic hills and shown to reproduce the better results of the original model, while retaining the easier convergence properties of the 'code-friendly' version.

Key words: turbulence, $\bar{v}^2 - f$ model, robust modification, near-wall flow.

1. Introduction

In the near wall region of a turbulent flow, the Reynolds stresses are highly anisotropic. The wall normal and shear stresses ($\bar{v}^2$, $\overline{uv}$) are severely reduced. At variance with wall functions which bypass the problem, 'low-Reynolds' models attempt to accurately reproduce this buffer layer and viscous sub-layer below the log-layer. Simple low Re eddy-viscosity models (EVM) rely on damping functions to mimic the attenuation of the turbulence near the wall. These damping functions depend on the local turbulent Re number or wall distance. However, very near the wall it is not so much the reduction of the kinetic energy k , but the fact that $\overline{uv}/k$ and $\bar{v}^2/k$ tend to zero at the wall that is difficult to model in the EVM framework. In second moment closures where all stress components are computed, the damping is obtained by a blocking of the energy redistribution by the pressure fluctuations, in accordance with DNS data. In particular the wall normal component, $\bar{v}^2$, a key

contributor to the mixing process, is only sustained by pressure-strain terms which are severely damped as one approaches the wall.

In the $\bar{v}^2 - f$ model suggested by Durbin [1], the variable $\bar{v}^2$, and its source term f , are retained as variables in addition to the traditional k and ε (energy and dissipation) parameters of the $k-\varepsilon$ EVM. This enables to account for wall blocking effects as in second moment closures. The main idea is to approximate directly the two-point correlation in the integral equation of the pressure redistribution term by an isotropic exponential function. Then, the redistribution term is defined by a relaxation equation which is the modified Helmholtz equation (the solution of which is close to an exponential decay as the wall is approached). The non-local character of the redistribution term is preserved by the elliptic nature of the equation. The $\bar{v}^2 - f$ model is based on the elliptic relaxation equation for f and three transport equations for k, ε and $\bar{v}^2$ which near a wall is similar to the second moment closure of the wall normal Re stress. Away from the wall $\bar{v}^2$ is not tied to a particular direction but $\bar{v}^2/k$ provides some measure of the Re stress anisotropy.

The $\bar{v}^2 - f$ model has been used in several CFD problems as an alternative for the conventional $k - \varepsilon$ models. Its usefulness has been successfully shown, among others, in aerospace configurations by Kalitzin [2]; both in subsonic and transonic flows around aerofoils, flows with adverse pressure gradient and around bluff bodies by Durbin [3]; three-dimensional boundary layers by Parneix and Durbin [4]; aerodynamics by Lien et al. [5] and heat transfer by Behnia et al. [6] or Manceau et al. [7].

In spite of encouraging results in the application of the model its use is very complicated because of a removable singularity in the boundary conditions. A robust version of the model to avoid the singularity, in a manner similar to the $\widetilde{\varepsilon}$ change of variable used by Jones and Launder [8] for the $k - \varepsilon$ model, has been suggested earlier by Lien and Durbin [9]. However, the validity of this modification is problematic as it neglects some non-vanishing terms. We suggest a different change of variable to avoid the singularity in the boundary conditions.

2. Description of the Problem

The transport equations for the Reynolds stresses in the Reynolds Stress Transport Model are as follows:

$$\frac{\partial(\overline{u_i u_j})}{\partial t} + U_k \nabla^k \overline{u_i u_j} = D_{ij}^{\nu} + D_{ij}^{T} + \phi_{ij} + P_{ij} + \varepsilon_{ij}, \tag{1}$$

where D_{ij}^{ν} is the viscous diffusion, D_{ij}^{T} is the turbulent diffusion, ϕ_{ij} is the redistribution term, P_{ij} is the production term and ε_{ij} is the dissipation term.

These terms are:

$$D_{ij}^{v} = \nu \nabla_k \nabla^k \overline{u_i u_j}, \; D_{ij}^{T} = -\nabla^k \overline{u_i u_j u_k}, \quad \phi_{ij} = -\frac{1}{\rho}\overline{u_i \nabla^j p} - \frac{1}{\rho}\overline{u^j \nabla_i p},$$

$$P_{ij} = -\overline{u_i u_k}\nabla^k U_j - \overline{u_j u_k}\nabla^k U_i, \qquad \varepsilon_{ij} = -2\nu\overline{\nabla_k u_i \nabla^k u_j}.$$

The redistribution term ϕ_{ij} that arises from pressure fluctuations is usually modelled by low-order algebraic expressions of the mean flow quantities, but this is not sufficient to reproduce the strong decay of wall-normal fluctuations as the wall is approached.

In free flows, the pressure fluctuations redistribute the kinetic energy on all components of the Reynolds stress tensor; this is the 'return to isotropy' effect. The presence of a wall produces the 'wall-blocking effects.' The transfer of energy from stream-wise to wall-normal velocity fluctuations is suppressed by distant interaction of pressure fluctuations with the solid wall and the turbulence is made highly anisotropic.

The wall normal component $\bar{v}^2$ and shear stress one $\overline{uv}$ are strongly damped in the inner near-wall region. Detailed analysis of DNS data done by Manceau et al. [10] shows that this kinematic blockage effect is much stronger than the viscous, or 'low Reynolds number,' effect.

Most models rely on empirical damping functions to account for the wall effect. According to the standard $k - \varepsilon$ model, it is assumed that the eddy-viscosity ν_t is defined by the Prandtl–Kolmogorov formula:

$$\nu_t = C_\mu \frac{k^2}{\varepsilon},$$

where C_μ is a constant.

The correlation between ν_t and k^2/ε near a wall (i.e. C_μ) as a function of the normal coordinate y is far from constant. The profile is corrected to fit with an expected curve near the wall by introducing a 'damping function' f_μ:

$$\nu_t = f_\mu C_\mu \frac{k^2}{\varepsilon}.$$

Nevertheless, the approach based on *ad hoc* tuning of f_μ is not universal and has little justification. The tuning is based on channel flow data and is questionable in the presence of complex geometries and sophisticated problem statements. Also, damping functions are often non-linear and produce a numerical stiffness.

Durbin [1] proposed an elliptic relaxation approach to modelling the pressure–velocity fluctuations term ϕ_{ij} in order to take into account the wall effects in the form of:

$$\phi_{ij} - L^2 \nabla^2 \phi_{ij} = \phi_{ij}^h, \tag{2}$$

where ϕ_{ij}^h corresponds to the quasi-homogeneous solution.

In the framework of the full Reynolds stress model, Durbin solves an equation for $f_{ij} = \phi_{ij}/k$ for each ij-component of the tensor. Thus, the total number of equations is increased by six.

3. Durbin's Original $\bar{v}^2 - f$ Turbulence Model

In the case of the $\bar{v}^2 - f$ model the system of equations for the Reynolds stress tensor components is replaced by a transport equation for the value $\bar{v}^2$ and an elliptic equation for a scalar function f related to the energy distribution in the equation for $\bar{v}^2$. The main far-wall-flow is supposed to be isotropic and the k-ε model can be used.

The transport equations for k and ε are:

$$\frac{Dk}{Dt} = \frac{\partial k}{\partial t} + U_j \nabla^j k = P_k - \varepsilon + \nabla^j \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \nabla_j k \right] \tag{3}$$

$$\frac{D\varepsilon}{Dt} = \frac{\partial \varepsilon}{\partial t} + U_j \nabla^j \varepsilon = \frac{C_{\varepsilon 1} P_k - C_{\varepsilon 2} \varepsilon}{T} + \nabla^j \left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \nabla_j \varepsilon \right], \tag{4}$$

where constants $C_{\varepsilon 1}$and $C_{\varepsilon 2}$ correspond to the creation and the destruction of dissipation accordingly, P_k is the production of the turbulent energy.

The transport equation for $\bar{v}^2$ received on the base of second-moment closure is as follows:

$$\frac{D\bar{v}^2}{Dt} = \frac{\partial \bar{v}^2}{\partial t} + U_j \nabla^j \bar{v}^2 = kf - \bar{v}^2 \frac{\varepsilon}{k} + \nabla^j \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \nabla_j \bar{v}^2 \right] \tag{5}$$

In (5) the associated pressure-strain term kf is defined as

$$kf = \phi_{22} - \varepsilon_{22} + \frac{\bar{v}^2}{k} \varepsilon,$$

where ϕ_{22} and ε_{22} are the normal components of the pressure-strain and dissipation tensors to the wall, accordingly. In parallel shear flow, kf represents the redistribution of the turbulent energy from the stream-wise component to the wall-normal one.

The following auxiliary elliptic relaxation equation is solved for f:

$$L^2 \nabla^2 f - f = \frac{1}{T}(C_1 - 1) \left[\frac{\bar{v}^2}{k} - \frac{2}{3} \right] - C_2 \frac{P_k}{k} \tag{6}$$

In a region of homogeneous flow ($\nabla^2 f = 0$), the classical 'return to isotropy' and 'isotropization of production' model for ϕ_{22} is recovered. The turbulent time and length scales are determined as:

$$T = \max \left[\frac{k}{\varepsilon}, 6\sqrt{\frac{\nu}{\varepsilon}} \right], L = C_L \max \left[\frac{k^{3/2}}{\varepsilon}, C_\eta \frac{\nu^{3/4}}{\varepsilon^{1/4}} \right]. \tag{7}$$

The strain-rate magnitude S is defined as $S = \sqrt{2S^{ij}S_{ij}}$, where the strain velocity tensor is

$$S_{ij} = 0.5(\nabla_i U_j + \nabla_j U_i).$$

Accordingly, in the framework of the Boussinesq hypothesis, the turbulent energy production P_k is $P_k = \nu_t S^2$.

The turbulent viscosity is defined now as

$$\nu_t = C_\mu \bar{v}^2 T.$$

In the last equation, as $\bar{v}^2$ decreases faster than k, the wall-damping effects are taken into account by including $\bar{v}^2$ instead of a damping function f_μ.

The coefficients of the model are the following [11]

$$C_\mu = 0.19, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3$$
$$C_{\varepsilon 1} = 1.4\left[1 + 0.045\sqrt{\frac{k}{\bar{v}^2}}\right], \quad C_{\varepsilon 2} = 1.9$$
$$C_1 = 1.4, \quad C_2 = 0.3, \quad C_L = 0.3, \quad C_\eta = 70.$$

The boundary conditions at the wall for k and $\bar{v}^2$ are uniform: $k = 0$; $\bar{v}^2 = 0$

As it is well known, ε satisfies the following asymptotic: $\varepsilon \to 2\nu\frac{k}{y^2}$ [8], where y is the wall normal distance, and the limit results from the balance between dissipation and molecular diffusion of k at the wall.

The boundary condition for f follows from its definition. As it has been shown by Mansour et al. [12], near the wall the right-hand-side terms are:

$$\phi_{22} = -2\frac{\bar{v}^2}{k}\varepsilon, \quad \varepsilon_{22} = 4\frac{\bar{v}^2}{k}\varepsilon.$$

It gives the following asymptotic behaviour for f:

$$f(0) \to -\frac{5\bar{v}^2}{k^2}\varepsilon \tag{8}$$

or:

$$f(0) \to -\frac{20\nu^2\bar{v}^2}{\varepsilon(0)y^4}.$$

As well as the boundary condition for ε, boundary condition (8) is represented by a removable singularity. Although the limit exists, it is not *a priori* known. It results in substantial difficulties in numerical solution.

In the case of a segregated solver, when the f-equation is solved separately, the boundary value problem for this equation becomes ill-conditioned due to the asymptotic y^4 at the wall. It means that small pertubations of input data (either the numerator or denominator) can lead to big changes in the solution. In an exact solution both the numerator and denominator asymptotically have the same order but in the numerical solution procedure they can have different orders and it causes substantial numerical problems. The similar problem takes place for ε but in the case

of the f-equation the problem is more severe. Numerically it leads to oscillations or divergence of the solution mainly encountered in segregated numerical procedures for solving the $\bar{v}^2$ and f equations. It is well known among users of the $\bar{v}^2 - f$ model that stability problems appear when the near-wall cells are too small, typically $y^+ < 1$.

To avoid such a problem, there is a well-known approach by Jones and Launder [8], where ε is changed by the so-called isotropic dissipation $\widetilde{\varepsilon} = \varepsilon - 2\nu(\frac{\partial\sqrt{k}}{\partial y})^2$. In this case the boundary condition for $\widetilde{\varepsilon}$ at a wall is homogeneous. It allows one to solve the equations for k and $\widetilde{\varepsilon}$ separately [8] since the coupling of the variables becomes weaker. A similar fix was suggested by Lien and Durbin [9] in the case of the $\bar{v}^2 - f$ model.

4. 'Code Friendly' Version by Lien and Durbin

Following Lien and Durbin [9], the change of the variable f is as follows:

$$f = \bar{f} - 5\varepsilon\frac{\bar{v}^2}{k^2} \tag{9}$$

The boundary condition for $\bar{f}$ becomes zero at the wall, hence it reduces the dependence between variables. Finally, it allows us to solve the two equations separately. The new sets of equations are:

$$\frac{D\bar{v}^2}{Dt} = k\bar{f} - 6\bar{v}^2\frac{\varepsilon}{k} + \nabla^j\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\nabla_j\bar{v}^2\right] \tag{10}$$

$$L^2\nabla^2\bar{f} - \bar{f} = \frac{1}{T}(C_1 - 1)\left[\frac{\bar{v}^2}{k} - \frac{2}{3}\right] - C_2\frac{P_k}{k} - 5\varepsilon\frac{\bar{v}^2}{k^2} \tag{11}$$

The equation for $\bar{f}$ is obtained by substituting (9) in (6) and neglecting the term $5L^2\nabla^2(\bar{v}^2\varepsilon/k^2)$. This modification of the model has the advantage of a well-posed boundary value problem for the equation for $\bar{f}$ but it neglects a term which may be important even outside the near-wall region. The change in the definition of the coefficient $C_{\varepsilon 1}$ is not only unrelated to the change of variable but also introduced in [9] is:

$$C_{\varepsilon 1} = 1.4\left(1 + 0.05\sqrt{\frac{k}{\bar{v}^2}}\right) \tag{12}$$

It creates a beneficial increase of ε near the edge of the sublayer without the explicit reference to the distance to a wall used in the previous version.

The constants of the model are the following [13]:

$$\begin{aligned} &C_\mu = 0.22, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3, \\ &C_{\varepsilon 1} = 1.4\left[1 + 0.05\sqrt{\frac{k}{\bar{v}^2}}\right], \quad C_{\varepsilon 2} = 1.9, \\ &C_1 = 1.4, \quad C_2 = 0.3, \quad C_L = 0.23, \quad C_\eta = 70. \end{aligned} \tag{13}$$

As the $\bar{v}^2 - f$ model is fairly recent, and in particular the code friendly version, the constants are being optimized over time. Chronologically, the first paper that introduces the Code Friendly modification is by Lien and Durbin in 1996 [9]. Coefficients were later modified in Lien et al. 1998 [14] then again in 1999 by Kalitzin [2]. The constants used by Wu and Durbin [15] are again slightly modified. Finally, the set of constants given in Lien and Kalitzin 2001, [13] is the one used herein. This version will subsequently be referred to as the Lien-Durbin Model (LDM). In any case the point of this paper is to stress that the LDM or the new code friendly version introduced below are both much more robust than the original [1] model, independently from the sets of constants used.

Since the boundary conditions for both $\bar{v}^2$ and f are homogeneous at the wall now, it is possible to decouple the $\bar{v}^2$ and f equations in a numerical solution. As we mentioned above, the shortcoming of this model is in neglecting the term $5L^2\nabla^2(\bar{v}^2\varepsilon/k^2)$. There is no theoretical justification for this approximation. With the usual log-layer assumptions for a channel flow ($\bar{v}^2$ and k constant, $L \sim y$, $\varepsilon \sim 1/y$) this term is proportional to $1/y$, hence just as large as the other terms in the f equation.

5. A New φ Code Friendly Version

First, we introduce a new variable, φ , in place of $\bar{v}^2$, as:

$$\varphi = \frac{\bar{v}^2}{k} \tag{14}$$

In an isotropic flow $\varphi \to const = 2/3$. A clear benefit is in the boundary condition for f:

$$f(0) \to -\frac{5\varphi}{k}\varepsilon \tag{15}$$

The singularity near the wall is now second order only (ratio of two *discretized* variables with a y^2 limit instead of y^4). It is essential because the lower the order the less the 'stiffness' of the boundary condition and the less is the sensitivity of the boundary condition to truncation error.

By reformulating the limit of f at the wall, it is possible to suggest a new change in variable that will lead to the zero boundary condition:

$$f = \bar{f} - \frac{2\nu(\nabla\varphi\nabla k)}{k} - \nu\nabla^2\varphi \tag{16}$$

Considering the limit $y \to 0$, it is possible to show that $\bar{f} \to 0$. There are different possible substitutions to reach the homogeneous boundary condition for $\bar{f}$. Some of them, e.g. $f = \bar{f} - 5\nu\nabla^2\varphi$, can lead to an ill-posed problem for φ since in the right-hand side we obtain a negative dissipative term: $-4\nu\nabla^2\varphi$.

Substitution (16) allows us to remove viscous terms in the φ-equation.

Applying these new definitions to Equations (5) and (6), one obtains:

$$\frac{D\varphi}{Dt} = \bar{f} - P_k\frac{\varphi}{k} + \frac{2}{k}\frac{\nu_t}{\sigma_k}\nabla^j\varphi\nabla_j k + \nabla^j\left[\frac{\nu_t}{\sigma_k}\nabla_j\varphi\right] \tag{17}$$

$$L^2\nabla^2\bar{f} - \bar{f} = \frac{1}{T}(C_1 - 1)\left[\varphi - \frac{2}{3}\right] - C_2\frac{P_k}{k} - 2\frac{\nu}{k}\nabla^j\varphi\nabla_j k - \nu\nabla^2\varphi \tag{18}$$

Now the equation for φ no longer depends on ε explicitly. In the second equation, we neglected the term $\nu L^2\nabla^2[\frac{2(\nabla\varphi\nabla k)}{k} + \nabla^2\varphi]$ that comes from introducing the change of variable (16) in the elliptic operator. The form of the elliptic operator is exactly derived from the Poisson equation only when considering the pressure-strain term as a primitive variable. Subsequently Durbin divided this quantity by the kinetic energy as a convenience for obtaining the desired near-wall asymptotic behaviour. From there on it makes no difference, from the theoretical point of view, whether f or $\bar{f}$ is inserted in the elliptic operator. In other words, in this paper the expression 'neglected term' means 'neglected' in comparison to Durbin's original model, but not in terms of an exact equation. The 'neclected term' is perhaps better described as the difference between the original (which is known to perform well) and 'code friendly' modelling procedures. In any case it seems a less drastic simplification in comparison to the LDM assumption as its effect is limited to the viscous sublayer because of the coefficient ν. It is difficult to estimate and compare the neglected terms in the both code-friendly models analytically in a general case. At least it is easy to see that in the isotropic flow the term neglected in the $\varphi-$model equals 0 while in the LDM model it is $10/3L^2\nabla^2(\varepsilon/k)$.

The boundary conditions are as follows:

$$k(0) = 0, \quad \varepsilon(0) \to \frac{2\nu k}{y^2},$$
$$\varphi(0) = 0, \quad \bar{f}(0) = 0.$$

The boundary conditions for both $\bar{f}$ and φ are zero in the wall, which makes it possible to solve the system uncoupled.

It is useful to note that because $\bar{f}$ is a non-negative function now, all source term in (17) (18) are non-negative functions. It simplifies constructing positive-type schemes.

The term $C_{\varepsilon 1}$ is changed to:

$$C_{\varepsilon 1} = 1.4\left(1 + 0.05\sqrt{\frac{1}{\varphi}}\right) \tag{19}$$

The coefficients used in this formulation have been tuned to match the DNS data for a channel flow at the Reynolds number $\mathrm{Re}_\tau = 395$ and they are:

$$C_\mu = 0.22, \quad \sigma_k = 1, \quad \sigma_\varepsilon = 1.3,$$
$$C_{\varepsilon 2} = 1.9, \quad C_\eta = 110,$$
$$C_1 = 1.4, \quad C_2 = 0.3, \quad C_L = 0.25. \tag{20}$$

6. Test Cases and Results

We considered a number of 1D-and 2D-test cases to show the advantages of our modification. As the focus is on numerical stability, a general purpose industrial unstructured finite volume code is used for testing (namely *code_Saturne*, Archambeau et. al. [16]). The 1D-and 2D-cases were actually run as thin 3D cases with symmetry or periodicity. Hereafter, comparison with experimental results on turbulent flow in a channel, an asymmetric diffuser and flow over hills are shown.

The channel flow results are compared with the DNS data of Kim et al. [17]. The Reynolds number based on the friction velocity u_τ and a half of the channel height h is $Re_\tau \equiv u_\tau h/\nu = 395$.

In the calculations, we have used a mesh of 100 nodes in the wall-normal direction, imposing periodic boundary conditions in the stream-wise direction. The expansion ratio of the cells in the wall-normal direction is 1.1 and the nearest cell from the wall is $y^+ \equiv u_\tau y/\nu = 0.5$. The mesh used was fine enough to guarantee mesh independence of the solutions represented in figures below.

The profiles of $U^+ = U/u_\tau$, $k^+ = k/u_\tau^2$, $\varepsilon^+ = \nu\varepsilon/u_\tau^4$, $\bar{v}^{2^+} = \bar{v}^2/u_\tau^2$, φ and $\nu_t^+ = \nu_t/\nu$ are given in Figure 1. The dotted line is the DNS data of Kim et al. [17]; the solid line is the profiles obtained by Durbin's original model; the line marked by dots corresponds to the φ-model while the dashed one corresponds to the LDM. As we can see, the φ-model produces better results than the LDM and quite close to the original model, especially for the turbulent viscosity profile ν_t which is the most important parameter, and quite severely overestimated by the LDM at the centre of the channel.

Figure 2 shows a comparison of the terms neglected at the LDM and φ-models and obtained from the solution of Durbin's original model. The dashed line is the distribution of the term neglected in the LDM, the dot-and-dash line is the term neglected in the φ-model. At the wall both terms are finite and similar. Outside the viscous sublayer, the negleted term in the present model is nearly two orders of magnitude smaller than the one in the LDM model. The terms in the f equation can be seen in Figure 3. The so-called homogenous f is the right-hand side of Equation (6), i.e. the pressure-strain obtained without elliptic relaxation. The figure is split in two for better visualization, beyond $y^+ = 100$ the scale is magnified as the scale of all terms decreases rapidly. The left-hand side (Figure 3a) shows the near wall region where it can be seen that the neglected term in the φ model goes to zero as of $y^+ = 30$ whereas the neglected term in the LDM only goes to zero around $y^+ = 80$. On the right-hand side of Figure 3b the term neglected in the LDM increases near the centre of the channel and is actually larger than f itself. On the contrary the term neglected in the φ-model is almost zero, as expected for a purely viscous term. Near the wall it is also seen that the elliptic term $L^2\nabla^2 f$ is strongly damping the homogenous part of f [11], mimicking the blockage that the wall is exerting on pressure-strain redistribution, as discussed in the introduction. As discussed by Manceau et al. [18] this term should actually vanish in the central

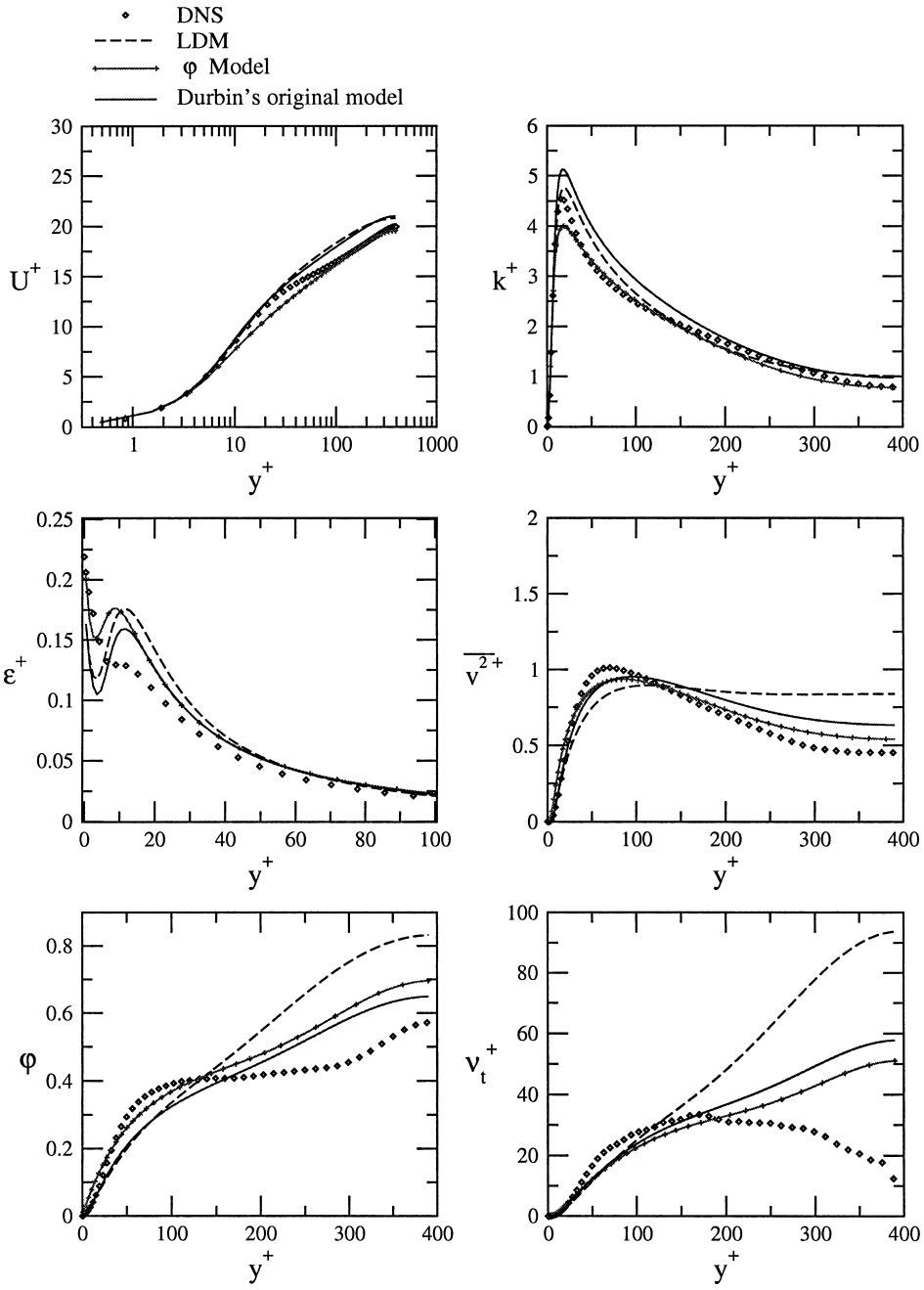

Figure 1. Channel flow $Re_\tau = 395$.

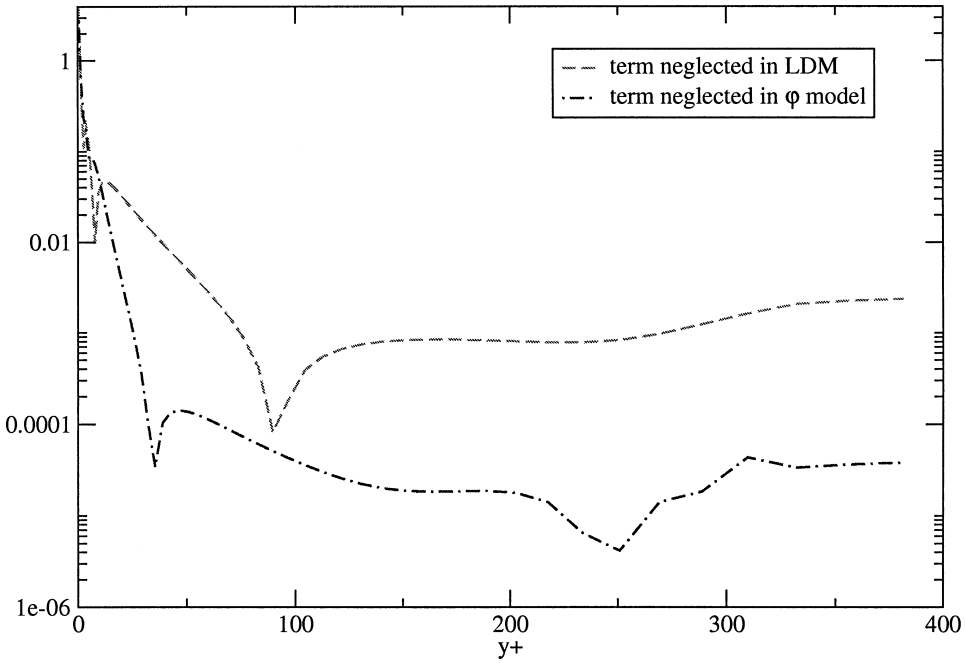

Figure 2. Comparison of the neglected terms in logarithmic scale (absolute values).

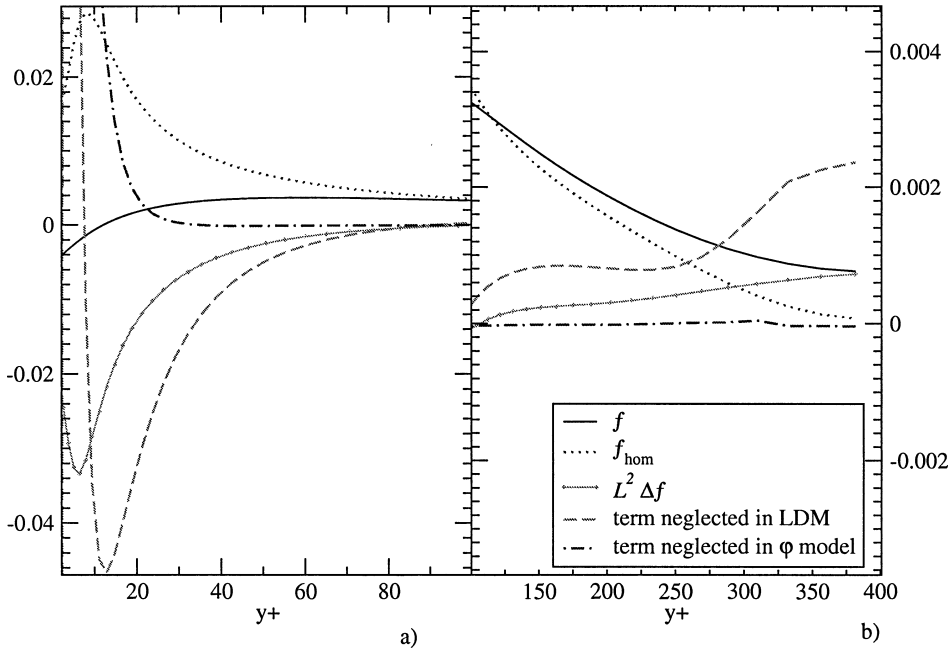

Figure 3. Terms on the f equation.

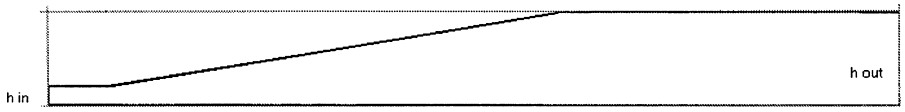

Figure 4. Asymetric Diffuser. Computational domain.

part of the flow rather than increase the pressure-strain term. Versions [18] and [10], of the elliptic relaxation approach resolve this issue but were not considered at this stage.

The plane diffuser test case has been studied by Obi et al. [19] and later by Buice and Eaton [20], Kaltenbach et al. [21]. LES calculation and comparison with experiments have been done in the latter paper. The geometric characteristics of the diffuser based on the inlet height h_{in}, are as follows: the outlet height: h_{out} is $4.7\,h_{\text{in}}$; the expansion length is $21\,h_{\text{in}}$; the recovery length is $40\,h_{\text{in}}$; the opening angle equals to 10^0. The description of the geometry is given in Figure 4.

The inlet flow for the diffuser is taken from a calculation of the fully turbulent channel flow problem considered above. The Reynolds number based on the centerline velocity equals 2×10^4 in this test case .

This test case has been shown to be particularly challenging for RANS models by Apsley and Leschziner [22], as well as one issue of ERCOFTAC workshops (Hellsten et al. 1999 [23]). The mesh provided by Apsley and used herein consists of 96×292 cells and can be considered sufficiently fine based on this previous experience. The maximum non-dimensional distance from the first cell to the wall is $y^+ = 0.62$.

The velocity profiles predicted by the φ model and LDM (dashed and solid lines, accordingly) are shown in Figure 5. The computational results are compared against the experimental data marked by triangles.

The LDM under-predicts the recirculation length whereas the φ-model gives a larger recirculation zone, closer to the experiment. Overall, the φ-model performs better than the LDM. It should be noted that the pressure field is very sensitive to the recirculation bubble and affects the bulk of the flow as can be seen concerning the mean velocity in the region of the straight wall. The LDM model underestimates the velocity in this region, similarly to more standard $k - \varepsilon$ models.

In Figure 6 the computational pressure coefficient along the inclined wall is compared against the experimental results marked by squares. The notation of the computational results are the same as on the previous figure; additionally the dotted line corresponds to the $k - \omega$ model [24]. Here again, the φ-model gives better prediction than the LDM; while the both models are more accurate than the $k - \omega$ one. In Figure 7 the skin friction coefficient is shown. The doted line represents the $k - \omega$ model. In general, the LDM gives a more accurate skin friction coefficient prediction than both the φ-model and the $k - \omega$ model in this test case, but this may be by chance because as seen on Figure 5 the velocity profiles in the

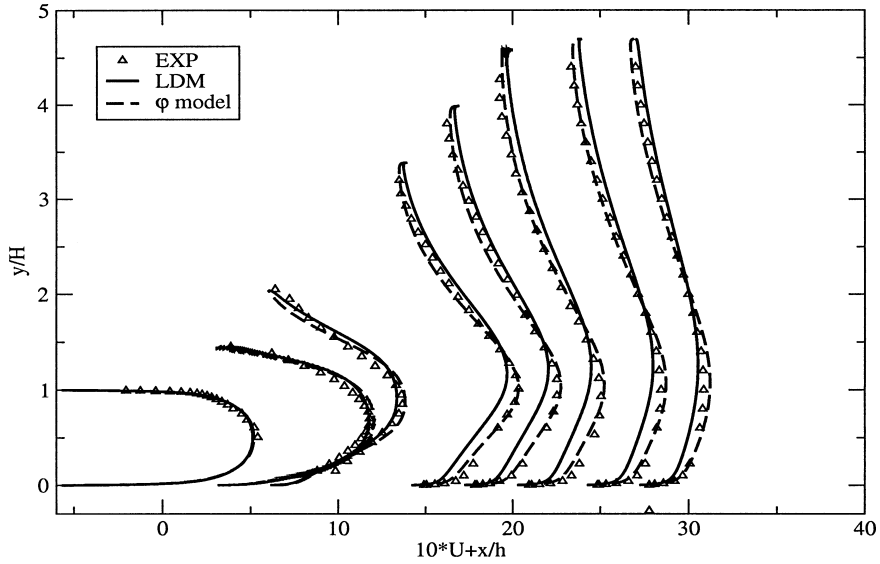

Figure 5. Velocity profiles on the expansion section. LDM and φ-model.

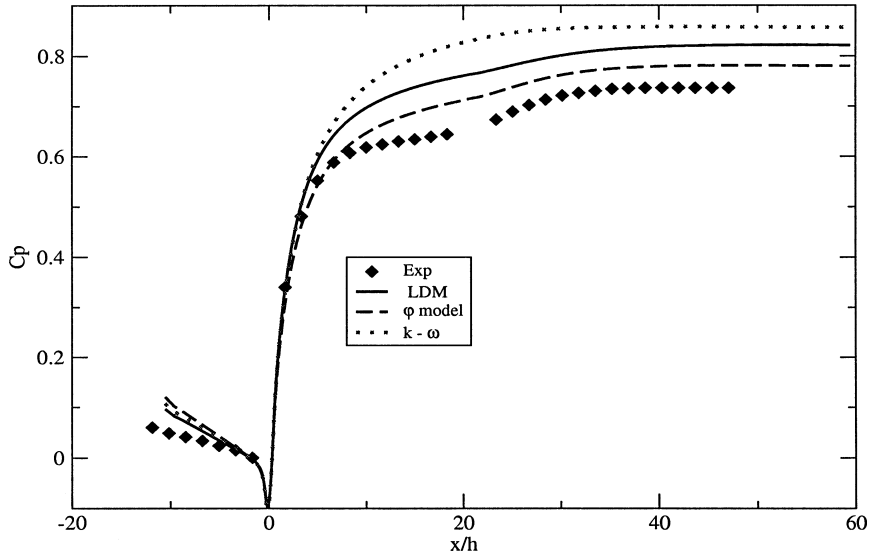

Figure 6. Pressure coefficient. LDM, $\varphi-$ and $k-\omega$ models.

core of the flow are erroneous. This is due to the large values of the neglected term and eddy viscosity away from the walls, as seen previously in the channel flow central region.

Finally, the flow over a series of periodic hills has been studied. The case was also investigated at the 9th ERCOFTAC/IAHR/COST Workshop [25]. The description of

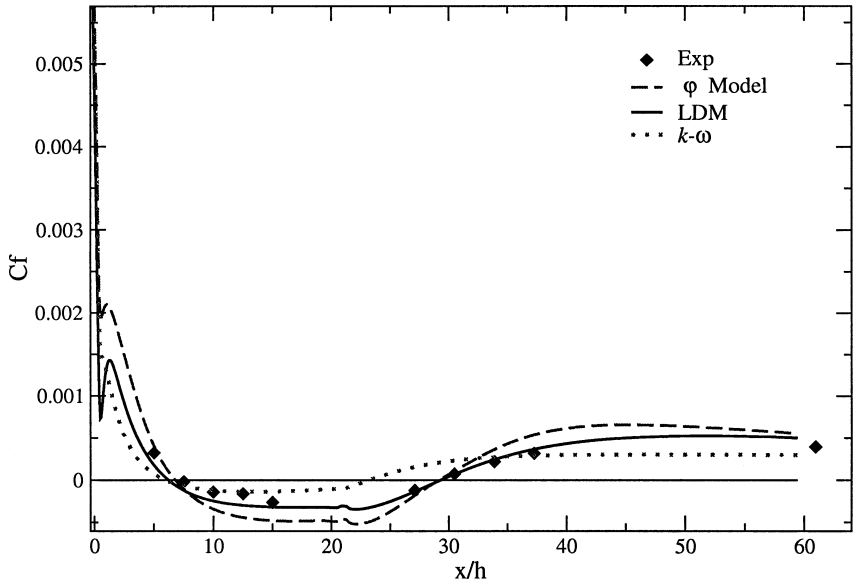

Figure 7. Skin friction coefficient. LDM φ and $k-\omega$ models.

the geometry can be seen in Figure 8, the Reynolds number based on the height of the hill h is $Re = 10.595$ and the domain dimensions are: $L_x = 9\,h, L_y = 3.036\,h$. The flow is periodic in the stream-wise direction. LES data is available for comparison [26].

The three versions have been tested in this case: the original Durbin model, the LDM and the φ model, and the velocity profiles can be seen in Figure 9. From the velocity profiles it can be seen that in the major part of the flow, the φ model and the original now have a very similar behaviour (the curves almost coincide) whereas the LDM is different. At the same time, the difference between the results obtained by all these models is much less than the discrepancy between the results that has been recently observed at the 9th ERCOFTAC/IAHR/COST Workshop [25]. In fact considering that the periodic inflow–outflow conditions usually amplify differences in models, the differences between the three $\bar{v}^2 - f$ versions here are

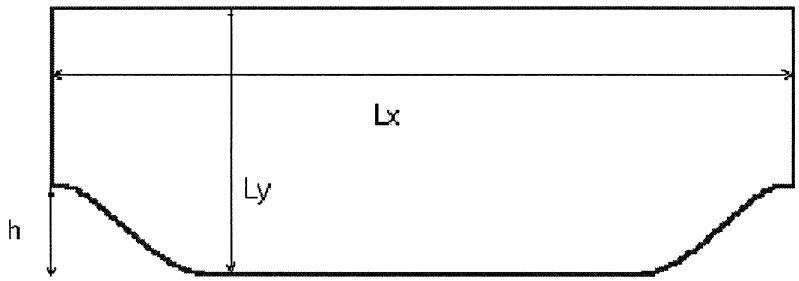

Figure 8. Periodic hills test case. Computational domain.

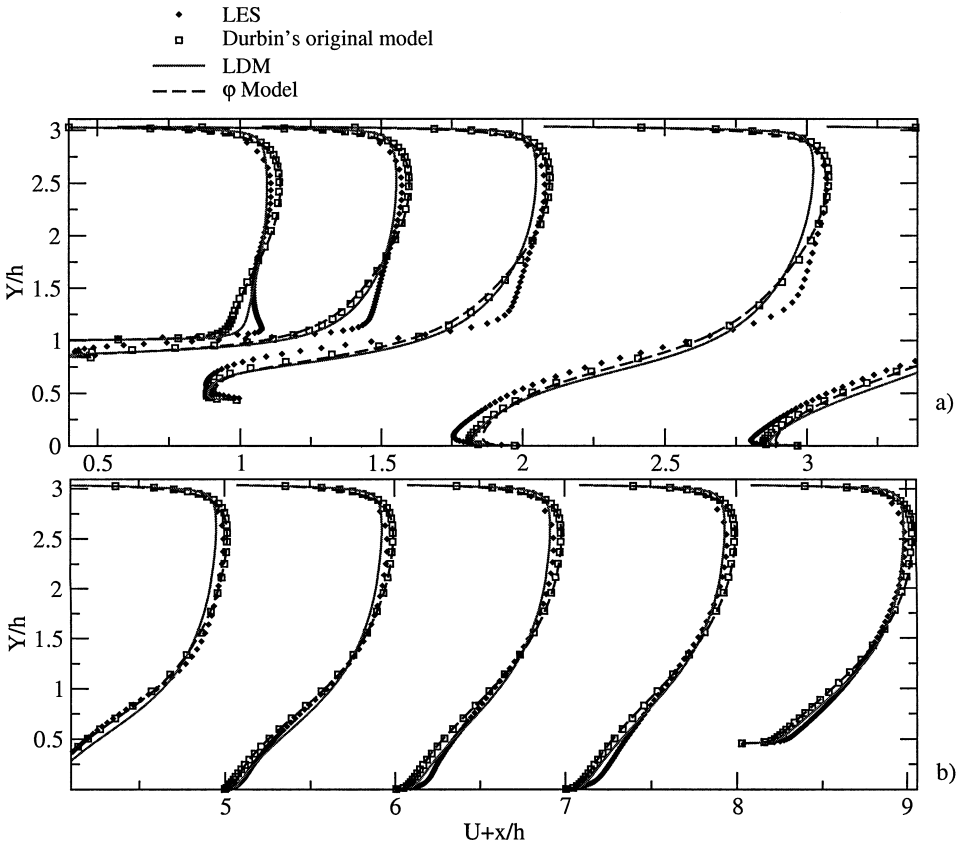

Figure 9. Velocity profiles: (a) first half of domain, (b) second half.

fairly small. The main advantages of the code friendly methods are robustness and a much weaker time step limit. The original model needed a time step thousand times smaller than both the LDM and φ models in order to converge.

7. Conclusion

A new robust modification of the $\bar{v}^2 - f$ model has been suggested. The modification is based on a change of variable from $\bar{v}^2$ to $\varphi = \bar{v}^2/k$ and leads to a boundary value problem with homogeneous boundary conditions and fixed-sign source terms. It allows one to solve the governing equations by an uncoupled and much more robust way than in the case of the original model. The objective is similar to that of the Lien and Durbin 'code friendly' variant, however that earlier version is obtained by neglecting a term which may affect the flow throughout the domain and is shown herein to lead to degraded results.

A channel, diffuser and hill flow test cases were used to assess the capability of the different versions of the $\bar{v}^2 - f$ model in predicting turbulent flows. The new

formulation proposed presents an overall good performance, similar to the original model, but convergences much more easily when implemented in a general purpose industrial finite volume code.

Although the new model was successfully implemented in a general purpose unstructured finite volume solver similar to commercial codes, before recommending a final version for widespread industrial two issues need to be considered. First, the rescaling of f itself as suggested by Manceau et al. [18] which has the advantage of eliminating undesirable effects of the elliptic relaxation in the log layer and further away form the wall. Second, revisiting the change of variable in the dissipation as in the well-known Launder-Sharma model, as this follows the same logic in decoupling k and ε variables, as done herein for the $\bar{v}^2$ and f quantities.

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