J. Opl Res. Soc. Vol. 45, No. 3, pp. 261–275
Printed in Great Britain. All rights reserved

A Robust Replacement Model with Applications to Medical Equipment

A. H. CHRISTER and P. A. SCARF
Centre for OR and Applied Statistics, University of Salford

Perceived shortcomings in the applicability of capital equipment replacement modelling, identified in a 1987 survey within the UK are addressed and a robust replacement model formulated. First, however, a comparison between the 1987 survey and a similar 1988 survey undertaken within the USA is made and explanations for apparent differences in conclusions are presented. A replacement model is then developed in the context of medical equipment where factors such as service and risk play a role in replacement decision-making. A mechanism for quantitatively allowing for qualitative and for political type factors within a short time horizon replacement model is introduced by means of a penalty factor. A case example is presented for medical ventilator equipment.

Key words: replacement models, penalty costs, medical equipment

INTRODUCTION

In discussing a recent survey of procedures for industrial equipment replacement decision-making,[1] the authors, Christer and Waller, were critical of both the general nature and the role of the modelling they found. Sources of concern were the perceived relevance of the observed modelling to the situation being modelled, and the lack of influence modelling appeared to have on replacement decisions. For the more costly equipment surveyed, modelling often had little actual influence upon decision-making because replacement issues were dominated by factors omitted from the analysis. Where modelling was influential, usually for the lower unit value items, the modelling was considered to be lacking in relevance to the actual decision situation. These points are further commented on in the next section.

In this paper a model is presented which attempts to resolve some of the concerns highlighted by Christer and Waller[1] by attempting to identify and address the actual issues of a replacement decision. At the same time, the model will take into account subjective considerations which, while difficult to quantify, nevertheless influence replacement decision-making. The model will be robust in that it can cope with changing discount rate, technological development of plant, and the more emotional issues that may influence replacement.

Although the prototype model presented is a general model, it is developed here specifically in the context of medical equipment. There are several reasons for this choice. First, the replacement of medical equipment is a particularly complex problem to model since it embraces, on occasions, emotive and behavioural considerations along with value judgements. Secondly, equipment within a hospital group is often of a 'high-tech' and rapidly developing nature representing a multi-million pound investment, with the annual maintenance cost being of the order of 10% of replacement value. Thirdly, a recent study undertaken for the Department of Health provided a source of suitable case material.

RECENT SURVEY

Much of the motivation for the replacement modelling research reported here was provided by the results of a survey by Christer and Waller[1]. It is therefore interesting to compare the conclusion of their survey to the report by Hsu[2] on the results of a questionnaire survey of 200 randomly selected Fortune 500 industrial firms. This survey was designed to investigate firms'

Correspondence: A. H. Christer, Centre for OR and Applied Statistics, University of Salford, Salford, Manchester M5 4WT, UK

equipment replacement policy. Of particular interest was whether or not a firm had a definite policy for equipment replacement, and the methods adopted for measuring and determining equipment life.

The paper gives a reasonably positive picture of the state of replacement decision-making as far as modelling is concerned, with 89% of firms reporting definite policies for equipment replacement. Frequently, several methods for determining equipment life would be employed within the same firm, but with the particular choice depending upon the equipment concerned. The most commonly reported methods for determining replacement age are given in Table 1.

TABLE 1. *Service life calculation*

Method	% of firms
Discounted rate of return	83
Payback	63
Net present value	50
Maintenance expense limit	20
Company's own formulae	17

At the same time, the various factors reportedly used in the replacement decision process are given in Table 2.

TABLE 2. *Factors influencing life calculation*

Factors	% of firms
Maintenance expenditure	97
Downtime cost	80
Depreciation	80
Taxes	73
Cost of parts inventory	64
Salvage value	60
Market expansion and contraction	53
Inflation	30
Cost of deferring replacement	27
Cost of operator training	10

In both these tables, multiple entries by a firm means the total percentage is greater than 100%.

These findings contrast, in their upbeat tone, to those of Christer and Waller[1] which motivated the modelling work reported below. To explain these two apparently conflicting views, it is necessary to compare the methodology employed. The Christer and Waller[1] paper is based upon in-depth surveys of specific pieces of equipment within 19 organizations. In common with Hsu, this survey was concerned with identifying whether a definite policy for equipment replacement existed, and what measured inputs and models were used. However, unlike Hsu's survey, the inquiry went on to investigate the extent to which any modelling actually influenced replacement decision-making. The conclusion was that for major items of equipment, various forms of modelling were undertaken—often of a discounted cash flow type, but they had very little influence on the resulting decision. Such decisions were dominated by factors which were not included within the modelling. For minor items such as vehicles, modelling in some form did have a recognized influence upon the decision-making process, but the modelling was considered in the main to be inadequate to its task. Operating costs and other factors were often modelled and presented to the decision-maker as an information base which influenced decision-making, but there was no actual model of the replacement problem itself.

Both these surveys are of common vintage, with one conducted in the USA and the other in the UK. It is suspected that the apparent different views presented could be attributed to the different research methodologies adopted. In the case of Hsu[2], an 'at distance' survey was undertaken. It is unclear to what extent the survey was validated and the results verified in the sense of Belson[3], although without this process one has very little by way of conclusions. For example, 30% of firms reportedly take inflation into account in replacement modelling. Similar statements have been encountered within the UK, but with further investigation, it transpired that inflation was not so much taken into account in the modelling as worried about and considered as an external factor. As such, inflation is a factor in the decision process, but not in the decision model.

In the UK survey of Christer and Waller[1], most organizations would have qualified as having a policy for replacement decision making in the sense of the 89% firms in the USA survey of Hsu[2]. It appears, therefore, that where the two surveys can be compared (measuring the perception of firms) they produce comparable results. However, it is an open question as to how many of these 89% of firms surveyed have replacement processes in which modelling actually influences the replacement decision. To address such issues, an in-depth study is required.

PROTOTYPE REPLACEMENT MODEL

A pilot study on medical equipment replacement policy undertaken for the Department of Health gave the authors and their colleagues a valued opportunity to study the replacement of medical equipment. Although the study also embraced issues of management structure, data capture, services provided, maintenance, decision-making and information flow, this paper addresses only the modelling element of the study.

Considerable time was spent contemplating an appropriate format for a prototype model of medical equipment replacement. The model was required to be sufficiently general to accommodate considerable variability in equipment types, to incorporate parameters and variables thought important to the end objective of provision of medical service, to highlight economic performance, and yet be required to be sufficiently simple in concept to be understood to the extent that it would be accepted. The first task is therefore to be clear as to the objective of the model and the factors which might influence replacement decision-making.

The prime aim of the model is to aid replacement decision-making by identifying a 'good' replacement decision and the consequences of alternative decisions. For the model to be an aid to management, it needs to address the actual decision problem being encountered. Characteristic features of the replacement decision problem for medical equipment are the following.

1. The existing item being considered for replacement is not new but, say N years old. The decision is therefore whether or not to replace it now, in one year, . . ., in K years, or only when forced by technical obsolescence. Technical obsolescence is reached when spare parts are no longer available or when it is unacceptable to use the equipment.
2. Equipment is unlikely to be replaced with like equipment. It is possible that the replacement decision is being driven by technical obsolescence, or by changing medical requirements, or by technological developments. In all these cases, the like-with-like option does not exist.
3. The replacement age of equipment should be related to its usage. If there are two items of equipment, one used once a week and the other twice a day, a straight age-based replacement decision based upon average costs may not be appropriate. Also, new equipment may have a higher usage by choice or by design. In this case, although the observed cost per unit time may be greater for the newer equipment, the corresponding cost per unit of usage could be less than that of the existing item. Since we are concerned with equipment usage our objective will be to seek to optimize the total cost per unit of usage.
4. The influence on patient care of equipment's proper operation must be considered. Failure of, or a lack of availability of, medical equipment is possibly associated with consequences to patient care, requiring perhaps different treatment, or greater medical resources, or worse. During the study there was no shortage of people pointing out that failures should not be measured simply by their costs. The patient care consequences need to be considered and allowed for in some way. The same 'care' element arises when considering alternative equipment options. The summation of all adverse effects to the patient and to the medical staff associated with equipment failure, and its related unavailability, we call penalty.
5. The purchase of new equipment subsequent to a replacement decision does not imply equipment is replaced. Equipment is often not a unique item but one of a pool of equipment. In this case, there is evidence that old equipment may not be scrapped, but be retained in use. Indeed, in some cases old equipment appears to be subject to a major overhaul before rejoining the pool

of equipment or retiring as 'spare'. The real implication here is that what are sometimes viewed as replacement decisions are really purchase decisions. It will be seen that a purchase decision which is presented as, modelled as, and decided as, a replacement decision could prove very expensive.

6. Equipment of one model type or another will be required for the foreseeable future. This continual demand for equipment does not imply we need to attempt the conventional replacement cost modelling over an infinite time horizon. It is recognized that the decision relates only to when the current equipment should be replaced, and this particular replacement decision does not realistically have operational consequences over all time. It has already been observed that equipment types will change, and therefore cost forecasts will be required. Such forecasting is particularly speculative if technological forecasting is also concerned, and it serves our interest to confine the period over which a forecast is required.

The proposal here is to formulate the total expected cost of retaining the existing plant for a further K years, replacing it with a new model, retaining this for a period L and replacing again with the new model. The criterion function would be the total expected discounted cost per unit of usage over $(K + L)$, and this would be minimized with respect to K and L. Only K, of course, is a decision parameter which would influence action. The L cycle and the replacement at the end of the L cycle are introduced to influence the optimal K by the ongoing need to retain operating equipment.

Before the replacement model is developed in detail, a further discussion of both the concepts of usage and of penalty is provided both to clarify and underpin the proposed format of the prototype replacement model.

USAGE MEASURE

Usage is the measure of time that equipment is actually being used to serve patients. For some equipment such as fire extinguishers and defibrillators, usage will be the actual time the equipment is available in working conditions since it is 'insurance', this type of equipment serving through its presence. Other equipment, such as dialysis machines, are in use only when connected to a patient, in which case usage might be measured by the cumulative number of patient hours of dialysis. Usage data is not likely to be available in the normal course of events, but would need to be collected by a sample survey.

Operating costs, failure rates and equipment downtime can be expected to depend upon the level of equipment usage. Usage of equipment will, in turn, depend upon the demand for the equipment, which is generated by the number of patients, their medical problems, patient's expectation of treatment, and consultant's choice. It will also be influenced, of course, by the quantity of equipment as measured by the size of the pool of equipment available at the point of supply and by the operational method of selection.

Such considerations of usage pattern are of importance in replacement modelling if certain pitfalls are to be avoided. For instance, if ageing equipment is subjected to increasing cost per usage, the increase could be due solely to a bias towards using newer equipment when a choice is available. In such a case, an argument to replace older equipment based upon increased unit costs could be quite bogus. To clarify such modelling issues for specific equipment, the following need to be considered.

(a) It is important to establish whether or not individual items in a pool of equipment are equally likely to be used (i.e. age at same rate) or if some priority system operates, be it by design or by default. If there is evidence that equipment usage varies with age, it is necessary to know if the variation is a consequence of age, or due to some other factor such as changing demand or simply user choice.

(b) Where the level of equipment usage has changed over time, it is necessary to establish the expected usage for the immediate future.

(c) The requirement to establish an appropriate measure of usage over time is related to (a) and (b) above. If usage is highly correlated with time (or age) then time could be taken as an appropriate measure. If usage is not highly correlated with time, then care must be taken in establishing a valid usage measure. This situation is likely to arise with pooled equipment where there are both 'favoured' items and a float, and also where the demand for equipment is variable over time.

Once a usage measure is established, it should be possible to decide after appropriate analysis whether or not parameters such as the rate of occurrence of failure, operating costs and maintenance costs are better measured by usage as opposed to time.

PENALTY MEASURE

The notion of penalty appears to be readily recognized and accepted, but difficult to quantify. Its existence can be influential in replacement decision processes, though not in an objective or quantitative fashion. Fears of litigation and adverse media coverage all have their place here. Much of the contribution to penalty measure, though not all, is very subjective. To what extent a patient suffers through being treated by one type of machine rather than another, or through having the treatment delayed, is not easily quantified for a particular case, and is even more complex in general. For this reason, the modelling procedure does not propose to attempt it. Neither does it attempt to avoid it.

Although penalty may not be known, it can be modelled as a cost parameter p, the sensitivity of a decision to this parameter explored for a particular equipment, and the extent to which penalty should influence a decision may be established. The possibility of penalty arises mostly when equipment fails. The proposal is, therefore, to model the cumulative penalty to date for an equipment by the cumulative number of failures to date times p, the penalty associated with a single failure. One variant of this is to assume that, say, only one in 20 failures are associated with penalty and to have the penalty incurred at random as each multiple of 20 failures arises. However, for the main part, cumulative penalty is taken as $p \times n$, where n is the cumulative number of failures to date.

The proposal is to utilize penalty in the decision-making process in a procedural yet quantified fashion. For instance, if the penalty measure associated with a single failure of an ECG recorder needs to be as much as £10 000, say, before penalty influences the replacement decision, it is doubtful that penalty needs to be seriously considered as a parameter. Again, if an ECG recorder is associated with a penalty cost of, say, £50, if a consultant's personal preference for a replacement decision is to be optimal, a more informed choice may be made. The main point is that where penalty is considered and used to influence a decision, the cost consequences of different options become apparent. Introducing a penalty parameter does not negate the responsibility to make a value judgement related to quality of treatment. What it will do, however, is to make the nature and cost consequences of the judgement clearer.

MATHEMATICAL FORM OF PROTOTYPE MODEL

Having outlined the general philosophy of the replacement model, the task is now to formulate an appropriate objective function. Terms to be used are first defined:

$c(i)$	equipment expected operating cost in its ith year of operation;
$m(i)$	expected maintenance cost per year for equipment in its ith year;
$C(i)$	total expected maintenance and operating cost per year for equipment in its ith year of operation, $(m(i) + c(i))$;
$u(i)$	usage of an equipment in year i relative to usage in year 1;
$p(i)$	penalty measure for equipment in its ith year;
$S(i)$	scrap value or resale value of equipment i years old;
'o', 'n'	suffix 'o' denotes old equipment, suffix 'n' denotes new equipment;
R	purchase cost of equipment—R_n is the purchase cost of the new item;
r	discount factor;
K	remaining life of existing equipment, expressed in months or years;

L	economic life of new replacement equipment measured in years;
N	age of existing equipment in years;
$C(N; K, L)$	total expected discounted cost incurred over time $K + L$ for equipment currently N years old.

SOME GENERAL OBSERVATIONS

All costs are based upon current-day values. That is, $m_o(3)$ and $m_n(1)$ represent the maintenance cost today of operating an item of old equipment aged two years for one year, and a new item of replacement equipment for one year. The total cost of operating and maintaining new replacement equipment for one year in, say, four years time will, of course, be larger than $C_n(1)$ because of inflation. This being so, an underestimate of the discounted maintenance costs actually incurred for a new item of equipment in its ith year of operation is given by

$$C_n(i)r^{i-1/2}. \tag{1}$$

Here, the costs have been modelled as being incurred mid year. If inflation was constant at $q\%$, the actual discounted sum necessary to meet the total maintenance and operation cost in its ith year of operation is

$$C_n(i)\left[r\left(1+\frac{q}{100}\right)\right]^{i-1/2}.$$

If the discount factor is $r = \left(\dfrac{1}{1+\dfrac{j}{100}}\right)$, when j is the internal rate of return (or an equivalent measure), the total discounted cost allowing for inflation and discounting becomes once again $C_n(i)\, r^{i-1/2}$ where now $r = (100 + q)/(100 + j)$. This means that in the subsequent model, any given value of the discount factor r does, in fact, correspond to a multitude of different inflation and discounting scenarios.

Although the above parameters have been written as functions of the year of operation i, it is accepted that, if appropriate, C, m, p and S could all be functions of usage, either the usage in the year i or the cumulative usage to date. The nature of any such relationships would need to be clarified in the analysis prior to modelling.

Figure 1 depicts the replacement decision situation being modelled where the time unit is taken as a year. The existing equipment is N years old and will be operated for a further K years before being replaced. The new equipment is assumed to be of a different type, which will itself be replaced with like equipment after L years.

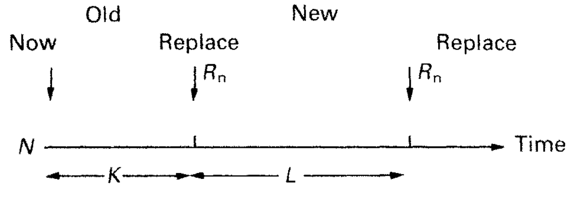

FIG. 1. *Model horizon.*

Measuring i in years and restricting K and L to an integer number of years, the total discounted cost incurred by replacing the existing equipment or age N after K years and again after a further L years is denoted by $C(N; K, L)$. Our task is to formulate the (expected) discounted cost per unit of usage over period $(K + L)$ on the supposition that this is an appropriate measure to optimize to obtain an efficient replacement solution.

We first note that

$$C(N;K,L) = \left[\sum_{i=1}^{K} (C_0(N+i) + p_0(N+i))r^{i-1/2} + \right.$$
$$\left. r^K \left\{ R_n - S_0(N+K) + \sum_{i=1}^{L} (C_n(i) + p(i))r^{i-1/2} + (R_n - S_n(L))r^L \right\} \right]. \qquad (2)$$

In the event that $u(i) = 1$; that is, usage is assumed constant, the objective function becomes

$$\left\{ \frac{C(N;K,L)}{K+L} \right\}, \qquad (3)$$

and the objective is to find K and L which minimize this function.

Figure 2 displays the significance of this criterion measure. For any value of $(K + L)$, when K and L are integers, there is a set of values of $\{K$ and $L\}$ each of which is expected to produce a different value $C(N;K,L)$. The columns of Figure 2 represents these value sets. The tangent of the angle between the origin and any cross representing a $\{K,L\}$ pair and the horizontal axis has the value of the objective function (3) for the $\{K,L\}$ pair.

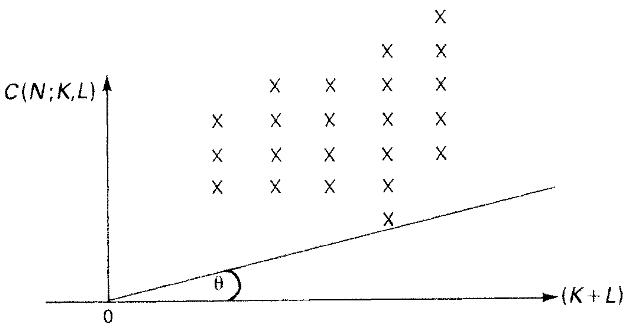

FIG. 2. *Discounted cost per unit time criteria.*

Optimizing function (3) with respect to K and L is equivalent to picking from the set of possible $\{K,L\}$ pairs that which makes the minimum tangent angle θ. When it cannot be assumed that $u(i) = 1$, the criterion becomes

$$\left\{ \frac{c(N;K,L)}{\sum_{i=1}^{K} u_0(N-i) + \sum_{i=1}^{L} u_n(i)} \right\}. \qquad (4)$$

The significance of (4) can still be deduced from Figure 2, though with a change of interpretation. On the time axis, $(K + L)$ now changes to total usage over $(K + L)$ whilst the discounted cost axis remains as before. Each cross now represents, for a specific (K,L) pair, the resulting discounted cost and total usage level. The criterion selects one $\{K,L\}$ pair from the set by again selecting the minimum tangent option.

So far, both these formulations have assumed the old item is scrapped. Should it be retained and fully used for the remaining L years, the problem becomes a purchasing option and the above objective function would be modified to

$$\left\{ \frac{C(N;K;L) + \left\{ \sum_{i=1}^{L} (C_0(N+K+i) - p_0(N+K+i))r^{1-1/2} + S_0(N+K) \right\} r^K}{\sum_{i=1}^{K+L} u_0(N+i) + \sum_{i=1}^{L} u_n(i)} \right\}. \qquad (5)$$

Tests of these objective functions reveal that they behave as one would expect. As penalty measures increase, K and L values decrease; as equipment cost R_n increases, the remaining life K

increases; and as maintenance costs m_n increase in relation to m_0, K increases. The actual behaviour will be demonstrated below in an example.

The envisaged procedure for using this criteria function is as follows. If the penalty p is known, the objective function say, (3), is optimized with respect to K and L, with only K being regarded as a decision variable of the replacement problem which will guide action. In the more likely event of penalty p not being known, the optimal choice K^* is first determined in the absence of penalty, $p = 0$, and the appropriate penalty measure p that justifies any alternative choice of K determined. If, for whatever reasons, the decision maker decided to opt for replacement period K rather than K^*, the decision will now be made in the knowledge of the range of penalty cost associated with an equipment failure that justifies the choice K according to the criterion function. As already stated, the proposed decision-making procedure does not remove the requirement for valued judgements, but it does provide a cost measure to be associated with the judgements.

COST OF DELAYED OPTIMAL REPLACEMENT

For reasons of budgeting constraints, recommended replacements after a period K may not be possible. In such cases, an indication is required of the extra cost to be incurred in revenue expenditure because of a lack of capital expenditure; that is, the marginal extra cost of delayed action.

If, using criterion function (3), the optimal replacement strategy for an N year old piece of equipment was (K^*, L^*), the resulting total discounted cost is $C(N; K^*, L^*)$. Assuming for convenience that usage is measured by time with $u(i) = 1, K^*$ and L^* are given by

$$\min_{K,L} \left\{ \frac{C(N; K, L)}{K + L} \right\} .$$

If the first replacement were delayed one year from K^* to $K^* + 1$, the discounted cost for the extra year beyond the optimal strategy is given by

$$[C(N; K^* + 1, L^*) - C(N; K^*, L^*)] .$$

Since the averaged discounted cost for a year under the optimal strategy is

$$\frac{C(N; K^*, L^*)}{K^* + L^*}$$

a measure of the marginal discounted cost of delaying the replacement decision by one year is given by

$$\left\{ [C(N; K^* + 1, L^*) - C(N; K^*, L^*)] - \frac{C(N; K^*, L^*)}{K^* + L^*} \right\} . \tag{6}$$

If only one of a choice of replacement options was possible, all other things being equal, the choice could presumably be based upon minimizing the above measure of additional cost. If the replacement purchases were delayed a further period k years beyond the optimal K^*, a measure of the additional discounted cost would be the increased discounted cost over the additional period k, less k times the unit discounted cost under the optimal policy, that is

$$\left\{ [C(N; K^* + k, L^*) - C(N; K^*, L^*)] - \frac{kC(N; K^*, L^*)}{(K^* + L^*)} \right\} . \tag{7}$$

Such marginal cost considerations could, it is suggested, be of assistance when having to decide a portfolio of replacement decisions across different types of equipment from a constrained budget.

CASE EXAMPLE

Here we consider the optimum replacement cycle for a ventilator of the type used in operating theatres at Liverpool Royal Hospital (LRH). Data were collected on six such ventilators, during a pilot study on a medical equipment replacement policy undertaken for the Department of Health. These ventilators had been in use since the hospital was opened in 1978. At the time of the study, the current replacement cost of these machines was £2700. The machines are generally 'low-tech' and although most of the machines were in their 12th year of operation, they were not immediate candidates for replacement due to technical obsolescence. Inventory information, such as purchase cost and current asset value, is held on the computerized equipment database MEMS (medical equipment management system), which holds records for all itemized medical equipment in the Liverpool Health District. This database is used principally for strategic asset management and for the budgeting of maintenance expenditure, the latter being of the order of 10% per annum of current replacement value.

During the pilot study, the failure history of a sample of Servovent ventilators at LRH was investigated. This failure history for the machines sampled is summarized in Table 3. In order to consider a homogeneous group of machines for replacement modelling, the subset of six Servovents was chosen for further study.

TABLE 3. *Number of failures per year for sample of 12 ventilators at LRH*

Item/Inv. No.	Year											
	78	79	80	81	82	83	84	85	86	87	88	89
ERICA R1137						5	1	0	2	1	2	2
ERICA R1144						1	2	3	3	2	0	0
Servovent R1061	1	0	0	0	2	1	0	1	0	0	0	0
Servovent R1103	0	0	0	0	0	1	2	1	0	0	0	0
PC3P R1115			0	0	0	0	1	0	0	0	0	0
Servovent R1099		0	0	0	0	1	2	2	0	0	0	0
BIR MK8 R1077			0	0	0	0	0	0	0	1	0	0
BIR MK8 R1075			1	0	0	0	0	0	1	0	0	0
Servovent R1063	0	0	0	0	0	1	0	0	0	1	0	0
Servovent R1062	0	0	1	0	1	3	0	0	0	0	0	0
Blease MP2 R1057	0	0	0	0	0	0	0	0	0	0	0	0
Servovent R1060	0	0	1	0	0	1	1	1	0	0	0	1

The components of costs for failures and for preventive maintenance were clarified after discussion with the head of the Department of Bio-engineering. Preventive maintenance took the form of routine services which were carried out either half yearly or quarterly. The main component in the cost of these services was labour. Failures, on the other hand, were felt to incur costs in addition to labour and spares due to downtime and inconvenience for the users. As a result, maintenance costs were taken as follows:

1. service cost of £180 per year per ventilator based on quarterly services, each taking an average of three hours labour;
2. failure cost of £165 per failure based on an average labour time per failure of five hours; a downtime of two days costed at £20; the cost of spare parts at £40 and additional nursing/anaesthetist time of two hours costed at £20.

Operating costs due to the replacement of disposable parts, cleaning and sterilizing costs and electricity costs associated with operating the ventilators was considered to be negligible.

Usage data were not directly available since the ventilators did not contain an internal counter. However, the investigation of operating theatre records, which held information on the duration of all operations performed in all theatres within the department since the hospital opened, indicated that the number of operations performed per year was roughly constant over the life of the machines. Accepting this point and the absence of evidence indicating other than a reasonably uniform use across ventilators (which could require a usage survey to establish), it is assumed for the current purpose that age is an appropriate measure of usage, that is $u(i) = 1$, all i.

The penalty cost p associated with failure was formulated as follows. The probability that a serious failure occurs (perhaps occurring during an operation) is p^1. If the penalty cost due to serious failure is taken as, say, £10 000 the average penalty cost per failure is £10 000 × p^1. Since, of a total of 27 failures for six ventilators during their lifetime, only one serious failure occurred, an average penalty cost of approximately p = £370 per failure is implied.

The cumulative failure curve, that is the cumulative number of failures per machine per year, for these six ventilators is shown in Figure 3. This shows an increasing rate of occurrence of failures[4] (ROCOF), over the first six or seven years and then a decreasing failure rate over later life. The reason for this apparent improvement in reliability can only be postulated. It might be the result of the settling down of the machine in terms of its reliability, but this is unlikely since the change seems to come about when the machines are already quite old. Alternatively, it could be the result of changing maintenance practice, for example, due to a discontinuity in the quality of record keeping or due to a change from in-house maintenance to contracted-out maintenance. One other possible explanation for this apparent improvement in reliability could be as a result of a decrease in usage due to, for example, the 'retirement' of the machines as spares. On the basis of the operating department records and the fact that the machines were, at the time of the study, still in full-time use in the department, this latter explanation was ruled out. Consequently, no convincing explanation for the shape of the cumulative failure curve could be found. What could be concluded, however, was that the quality of the failure data could not be guaranteed. For this reason, it was proposed to take the cautious option and consider a failure model estimated using only the first eight years of failure data.

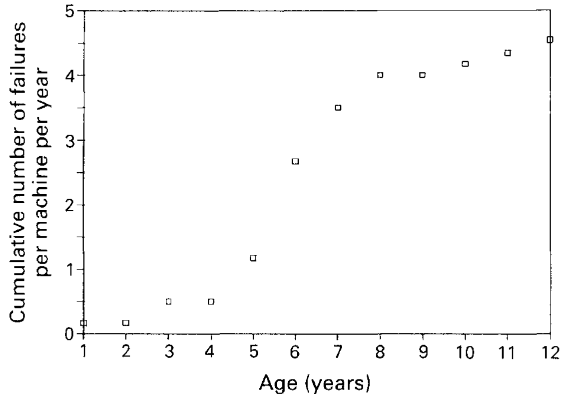

FIG. 3. *Cumulative failure curve for the sample of six ventilators (Servovents) at Liverpool Royal Hospital (purchased 1978, current age 12 years).*

We do not attempt to parameterize the hazard function, as is customary in the reliability literature, but instead propose simply to model the ROCOF, $E\{\mathrm{d}/\mathrm{d}t\,N(t)\} = \mu(t)$, as a function of age t, where $N(t)$ is the cumulative number of failures up to age t (see Reference 4). A log-linear model, with age as covariate and assuming a Poisson number of failures, was fitted using 8 years of failure history on each machine. This model, which is an example of a generalized linear model described by McCullagh and Nelder[5], was fitted using the statistical package GLIM. The mean number of failures per machine per year (ROCOF) was then given by

$$\log_e\{\mu(t)\} = -2.234 + 0.304t$$

where t is the machine age in years.

Costs of the second cycle, that is, the operating and maintenance costs for the new machine, were calculated assuming replacement of like with like. This was done on the basis that the ventilators concerned served the 'low-tech' end of the market, i.e. routine operations, and consequently it was felt that there was unlikely to be a significant change in the sophistication of the machines performing this particular function. For ventilators used in intensive care units, such as the ERICA in Table 3, a like-with-like assumption would be suspect because of the level of technological development in such medical equipment. For replacement with unlike, one possible way of proceeding would be to model the cost of the new machine as a multiplicative factor in the operating and maintenance costs of the old machine.[6] Such an assumption could be validated, for example, by relating operating and maintenance costs to replacements costs for a cross-section of ventilator types, or failing this, for a cross-section of 'similar' machines.

Results of the replacement decision modelling using the criterion function (3) are summarized in Table 4. This analysis has been carried out assuming a range of current machine ages and a discount factor $r = 0.97$. Since the machines studied were all currently 12 years old (approximately), this has been done to illustrate typical replacement decisions that could arise from the modelling. For the 12-year old machines themselves, the modelling suggests that the machines should be replaced within the next year. For a machine assumed to be currently aged 6 years, a plot of total discounted cost per unit use (where use is measured by time) over the two cycles against length of the two cycles is given in Figure 4. This corresponds to the theoretical Figure 2. Here, the minimum tangent angle, $\tan\theta$, occurs at $K^* = 3$, $L = 10$. Similar plots for machines assumed to be currently aged 8 years and 10 years, drawn on the same scale, are given in Figs 5 and 6 respectively. The penalty cost was set at £370 in these instances. Similar plots could be obtained for other penalty costs.

TABLE 4. *Replacement decisions by current age (penalty cost £0)*

Current age	K^*	L^*	$\tan\theta^*$ (£)
4	8	12	473
6	6	12	528
8	4	13	586
10	2	13	641
12	1	13	678

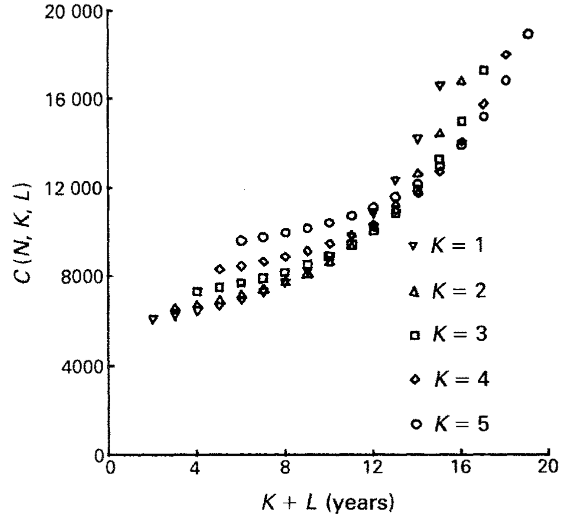

FIG. 4. *Total discounted cost against length of cycles for machine currently aged 6 years.*

The sensitivity of the replacement decision to the penalty cost was investigated and the results are summarized in Table 5. The relationship between time to replacement of the current machine, K^*, and penalty cost seems to follow a similar pattern, regardless of the current machine age. The fact that a penalty cost of £37 leads to a decrease in K^* for a six-year old machine, indicates that the penalty cost needs to be seriously considered as a parameter in the replacement modelling in this case. On the other hand, increasing the penalty cost beyond £37 has no influence on K^* for a machine currently aged 10 years, and so the penalty cost appears to have a much less critical role to play here. What is happening, of course, is that $K^* = 1$ really means initiate the replacement now, and as the penalty cost increases, this decision does not change.

Marginal increased discounted cost can be calculated for delayed replacement decisions. As might be expected these costs will depend on the current age of the machine and also the penalty

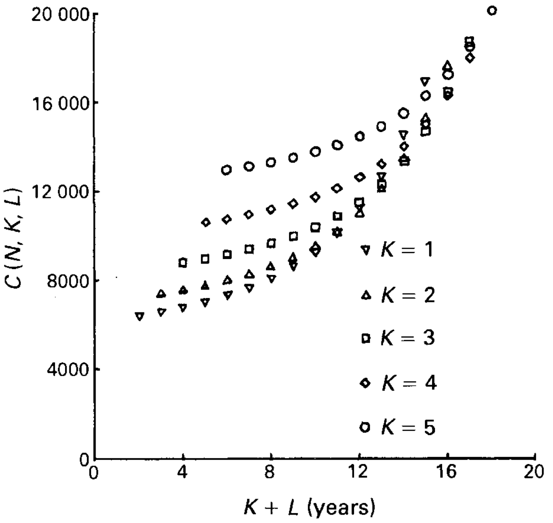

FIG. 5. *Total discounted cost against length of cycles for machine currently aged 8 years.*

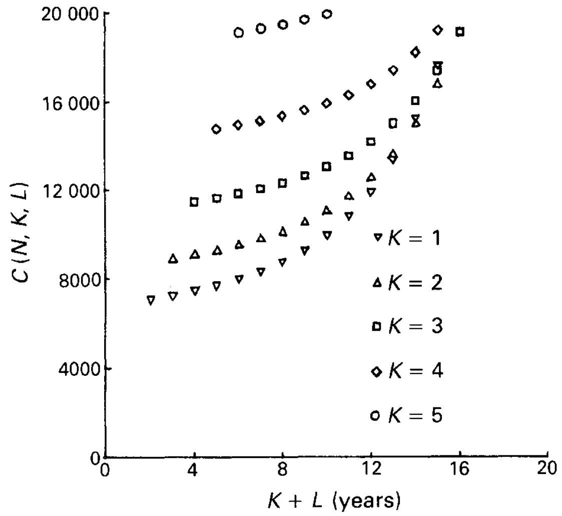

FIG. 6. *Total discounted cost against length of cycle for machine currently aged 10 years.*

TABLE 5. *Sensitivity of replacement decision to penalty cost*

Penalty cost £	Current age of machine					
	6 years		8 years		10 years	
	K^*	L^*	K^*	L^*	K^*	L^*
0	6	12	4	13	2	13
37	5	12	4	12	1	12
74	5	11	3	12	1	12
185	4	11	2	11	1	11
370	3	10	1	10	1	10
740	2	8				
1480	1	7				

cost associated with failures. Since this latter quantity cannot be predetermined, the marginal increased costs are best considered for a range of penalty costs. For machines assumed to be currently aged 6 years these costs, calculated from (7), are given in Table 6. For machines assumed to be currently aged 10 years the costs are given in Table 7. Notice that for a machine currently aged 10 years and a penalty cost of £370, the increased cost of delaying replacement by two years is of the order of the replacement cost of the machine itself, which was £2700 at the time of the study.

TABLE 6. *Marginal increased discounted cost for replacement delay for machine currently 6 years old*

Penalty cost (£)	Marginal increased discounted cost (£)	
	1 year delay	2 years delay
0	95	420
74	85	414
185	48	374
370	42	405

TABLE 7. *Marginal increased discounted cost for replacement delay for machine currently 10 years old*

Penalty cost (£)	Marginal increased discounted cost (£)	
	1 year delay	2 years delay
0	64	357
74	28	330
185	325	1050
370	828	2249

The last point is attributable to the fact that maintenance cost modelling has been based upon an exponential form. Also, with such a penalty measure, the recommended replacement period would be 8 or 9 years, and not 12 years. Had all 12 years of data been used equally in the cost modelling process and a linear model attempted, the conclusion would have been very different, with the model advocating keeping the equipment until technically obsolete. This once again emphasizes the importance of, and necessity for, complete and reliable cost data.

Finally, the possible use of this replacement model can be demonstrated with Table 5. In the absence of consideration of penalty factors, that is $p = 0$, for the current equipment age $N = 6$, the recommended replacement age is $N + K^* = 12$. However, if $N = 6$ and a consultant wished to purchase replacement equipment within a year, based on the costings, he needs to accept the existence of an equivalent penalty cost of the order of £1480 per failure. This contrasts with 10-year old equipment where to replace within one year instead of the recommended two years requires acceptance of only an additional £37 per failure.

DISCUSSION

It is argued here that the basic formula of the model, equation (3) or (4), is very simple. Any complication is inherent in the specific problem situation, the need to model equipment cost and use, and to accept that equipment is generally pooled for use. This can make the task of establishing valid models of both cost and use complex.

Maintenance costs feature as a component of the total operating cost and, as shown in the case example, such costs can be influential and even dominant in replacement decision-making. It is very possible that an alternative maintenance policy could substantially reduce the operating costs or incidence of failure. For this reason, maintenance modelling should be considered as a prime component of equipment management decision-making. The potential for such maintenance modelling in the context of medical equipment has been discussed recently by Baker and Wang[7] who use the maximum likelihood technique to establish a parametric fit to a delay time model and thereby deduce the cost consequences of alternative maintenance practices for infusion pumps.

The expectation of technological development is accepted. What is not always clear is exactly what it will be or how it might be modelled. In the past, technological modelling has assumed, say, a linear growth of a performance parameter such as output or cost, and the consequence on a uniform replacement strategy over all times considered, see Elton and Gruber[8]. Here, the approach is both pragmatic and simpler. Not only is the time horizon over which forecasting is required of limited duration, but the forecasting task itself simplified because the nature of the replacement plant is likely already known. Cost modelling will still be required, but there are various ways of approaching the task under these circumstances.[6]

It was interesting to note in the survey of Hsu[2] that 73% of firms considered tax implications in replacement decision-making. A study of the implications of tax to the optimal replacement decision based upon models such as model (3) of this paper has been made by Christer and Waller[9]. They found that subsequent to the 1984 Finance Act, although tax considerations did influence the total replacement cost, tax did not influence the optimality of replacement decision. For this reason, tax allowances have not been incorporated into the models of this paper. As tax law changes, the situation needs to be kept under review since, in the past, there have been periods where the tax allowance system did influence optimal replacement.[10]

Finally, the discount factor r has been assumed constant for the purposes of developing this model. This is not a necessary assumption, but only convenient for the current purpose. Clearly the discount factor r will be influenced by inflation and other factors and will change with time. It has been shown that there is no serious problem of principle in incorporating a time varying discount factor r in a problem, or of modelling the replacement problem against various discount factor scenarios.[11]

CONCLUSIONS

The model reported here is an attempt to rectify perceived weaknesses in the current practice of modelling replacement decisions. There is now the option of readily allowing for technological development if appropriate, and more importantly, the option of obtaining a penalty cost measure of making a preference-based replacement decision as opposed to an otherwise economic one.

It is recognized that the nature of use of equipment can influence economic replacement decision. This is especially true when dealing with pooled equipment where seemingly age-related costs can be simply a manifestation of usage patterns. Without due care here, replacing old equipment because it has greater cost per unit of use could prove an expensive error.

It is claimed that the objective of building a prototype model that is robust yet simple enough to be readily used has been met. The model is straightforward, responsive to parameters, and appears to behave appropriately to parameter changes. There is, of course, the need to model costs, and this requires the existence of, or survey-based collection of, reliable cost data. Extension of the model to incorporate variable discounting or tax considerations presents no problem in principle.

Acknowledgements—The research reported here has been sponsored in part by the Department of Health under Contract 'Economic Life Cycles of Medical Equipment', 1990. The authors would like to express their gratitude and thanks to the medical equipment staff at Liverpool Royal Hospital, to colleagues in the Centre for OR and Applied Statistics at Salford University, and particularly to the staff in the Department of Health, for their assistance and constructive comments.

REFERENCES

1. A. H. Christer and W. M. Waller (1987) A descriptive model of capital plant replacement. *J. Opl Res. Soc.* **38**, 473–477.
2. J. S. Hsu (1988) Equipment replacement policy—a survey. *Production and Inventory Mgmt J.* **29**(4), 23–27.
3. W. A. Belson (1986) *Validity in Surveying Research.* Gower, London.
4. H. Ascher (1991) Basic probabilistic and statistical concepts for maintenance of parts and systems. *IMA J. Maths Applied in Business and Industry* **3**, 153–168.
5. P. McCullagh and J. A. Nelder (1990) *Generalised Linear Models.* 2nd Edn, Chapman & Hall, London.
6. A. H. Christer (1988) Determining economic replacement ages of equipments incorporating technological developments. In *Operational Research '87.* (G. K. Rand, Ed.) pp. 343–354. North-Holland, Amsterdam.

7. R. Baker and W. Wang (1992) Estimating the delay-time distribution of faults in repairable machinery from failure data. *IMA. J. Math. in Business and Industry* **3**, 259–282.
8. D. J. Elton and M. J. Gruber (1976) On the optimality of an equal life policy for equipment subject to technological improvements. *Opl Res. Q.* **27**, 93–99.
9. A. H. Christer and W. M. Waller (1987) Tax adjusted replacement models. *J. Opl Res. Soc.* **38**, 993–1006.
10. S. Eilon, J. R. King and D. E. Hutchinson (1966) A study of equipment replacement. *Opl Res. Q.* **17**, 59–71.
11. A. H. Christer (1984) Operational research applied to industrial maintenance and replacement. In *Developments in Operational Research.* (R. W. Eglese and G. K. Rand, Eds) pp. 31–58. Operational Research Society, Birmingham.