A Robust Waveform Design for targets tacking in Cognitive MIMO Radars

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Abstract: An adaptive waveform is optimized in order to maximize the information returned from the targets, then the targets informaion is approximated by using a particle filter. This study propose a method to due with the uncertainties due to the dynamics of the targets, e.g., when the number of moving targets is unknown and changing over time. Thus, A decay constant is added to the estimated prior target information before optimizing the waveform by minimizing Cramér-Rao Lower Bound. Jeffreys prior is used to weight the parameters of each targets. Furthermore, the dynamic state space of the targets is estimated by a particle filter. Finally, the simulation results demonstrate the capability of the system to track targets.

Key–Words: cognitive system, adaptive waveform, particles filter

1 Introduction

Adding the ability to adapt the waveform according to environment by using a cognitive scheme is the key of a smart radar system. This technology overtook the traditional radar system in many applications. The accuracy is improved by optimizing a waveform to minimizes the probability of error called theoretical bound. Nevertheless, when the number of parameters are changing, the traditional beam-forming radar system might fail to discover a new target. This study proposed a robust solution to due with the dynamical change of state space.

In [1], the method to generate a beam pattern for the MIMO system was proposed, and it was shown that to improve the estimation accuracy, the power is focused to the targets location. The waveform optimization to minimize the Cramér-Rao Bound (CRB) for the MIMO radar was proposed in [2, 3] for single target and multiple targets, respectively. Based on the information theoretic approach, maximizing mutual information between the target impulse response and the environment also increases the performance, described in [4].

For a cognitive radar [5], N. A. Goodman et al. proposed sequential hypothesis testing for active sensors in target recognition, and it is shown that the waveform based on eigen solution outperformed the waveform based on the water-filling-based approach. For the adaptive method, W. Huleihel et al. [6], proposed technique to determine the transmitted waveform by minimizing the Cramér-Rao bound (CRB) and Reuven-Messer bound (RMB) from prior information which were derived. This achieved higher estimation accuracy than the other existing signal transmission methods.

The measurement of a state vector for multivariate data and non-linear/non-Gaussian process was described in [7]. Since a dynamic state vector can be estimated by using a Bayesian approach, the posterior probability density function (pdf) was constructed from collected data. The recursive filtering approach simultaneously estimates the state vector which consists of two steps: predicting the next stage pdf by using a system model and updating the posterior pdf by using the Bayes theorem. If every posterior pdf is Gaussian, there are optimum algorithms called Kalman filter [8] and extended Kalman filter [9] for linear and non-linear system models, respectively. For non-linear/non-Gaussian, sub-optimum algorithm based on Importance Sampling (IS) [10] was used. In [11], a number of different particle filters were described.

The idea of cognitive is the system that operating in a feedback loop, S. Haykin [12]. After transmit a signal, receivers pick up the reflected signal combining with prior information about the environment to model the environmental parameter and uses a Bayesian filter to track a target. In this step, memory is used to store and to read the prior knowledge of a target. Then, the transmitter uses the modeled environment and the state of the target to design a transmitted waveform signal.

Multiple Input Multiple Output (MIMO) radar is a system that has multiple receivers and multiple transmitters which emit an independent waveform. The receivers can know where the signal come from the received signal of all receivers. The advantage of MIMO over single-input multi-output (SIMO) system is the better parameter identifiability [13]. Increasing parameter identifiability offers more degree of freedom, lower power side-lobe at the antennas, and a variety of the transmitted waveform. We consider on the coherent system where the antennas is closely located. The advantage of this is beamforming, power can be focused to the target location. Therefore, it can be operated in the lower signal-to-noise ratio (SNR) environment.

In this study, we consider a cognitive MIMO waveform optimization and a target's state space estimation in the general case of multiple targets. The main problem of this study is when the uncertainties due to the dynamics of the targets occur in the system, e.g. when the number of targets is changing by time, or when the targets are suddenly moving. Every existing methods are unstable with this uncertainties. First, the waveform is designed by minimizing the mean-square-error (MSE) of the estimated parameters or CRB with the degeneration of the prior information. Second, during the waveform optimization, Jeffreys prior is included as the weights for each target parameter which represents the probability of existence. To estimate the state of the targets, we include a particle filter into the system. This is a Bayesian approach to estimate the posterior probability density of target's parameters. Furthermore, the number of targets is also described by a probability density function.

2 Cognitive MIMO System

The transmission equation of a MIMO system with N_T transmitter and N_R receiver is described by Equation (1). Each pulse step is denoted by k. The simplified matrix form can be written in Equation (2). H is the system transfer function that is assumed to be known. The vector (θ) contains the properties described targets, such as a complex attenuation, angle of arrival, Doppler shift or propagation delay. The noise of the system (n) is assumed to be the multivariate normal distribution with zero mean and covariance R_N.

$$x_{k,l} = H_k(\theta)s_{k,l} + n_{k,l} \tag{1}$$

where

$x_{k,l}$	is received signal at time index l ($x_l \in C^{N_R}$)
$H_k(\theta)$	is system transfer function ($H(\theta) \in C^{N_R \times N_T}$)
θ	is targets parameter vector ($\theta \in C^{5M}$)
M	is the number of targets
$s_{k,l}$	is transmitted signal with code length L at time index l ($s_l \in C^{N_T}$)
$n_{k,l}$	is interference ($n_l \in C^{N_R}$)

$$X_k = H_k(\theta)S_k + N_k \tag{2}$$

where X : is $[x_1, ..., x_l]$
S : is $[s_1, ..., s_l]$
N : is $[n_1, ..., n_l]$

Equation (1) also can be described in Equation (3) [13], whereas the target parameter (θ) consists of five parameters: complex attenuation (α), azimuth angle (φ_{az}), polar angle (φ_{po}), propagation delay (τ), and Doppler shift (D).

$$x_{k,l} = \Sigma_{m=1}^{M} \alpha_m e^{-j\omega D_m t}\mathbf{a}_{Rx}\mathbf{a}_{Tx}^T s_{k,l}(t-\tau_m) + n_{k,l} \tag{3}$$

where $\mathbf{a}_R$: is $[e^{-j\omega k\cdot r_1}, ..., e^{-j\omega k \cdot r_{N_R}}]^T$
$\mathbf{a}_T$: is $[e^{-j\omega k\cdot r_1}, ..., e^{-j\omega k \cdot r_{N_T}}]^T$
k : is $(\sin\varphi_{az}\cos\varphi_{po}, \sin\varphi_{az}\sin\varphi_{po}, \cos\varphi_{az})$
r : is position of the antenna.
ω : is carrier frequency.

A transmit signal (S) can be generated from its covariance matrix (R_S) by giving $S = U\mathcal{N}(0, I)_{N_T \times L}$, where U is an upper triangular matrix from Cholesky factorization [14] of R_S which is the square root of R_S. The generated waveform S is independent of each transmitter, therefore the rank of S is equal to the number of transmitters N_{Tx}.

The process flow of cognitive MIMO radar is shown in Figure 1. This is operated in a feed back loop with a waveform optimizer, parameter estimator, and memory. The waveform optimizer finds the waveform that minimize a statistical performance bound, the estimator usually use a likelihood estimator to estimate a target location, and all prior information is memorized into the system along the time. We can achieve the highest ability of the MIMO system in terms of parameter identifiability and waveform design by using this cognitive scheme.

3 Waveform Optimization

In this section, the input covariance R_S is optimized from the prior. In the presence of uncertainties due to the dynamics of the targets, the theoretical limited is derived, and find the optimum.

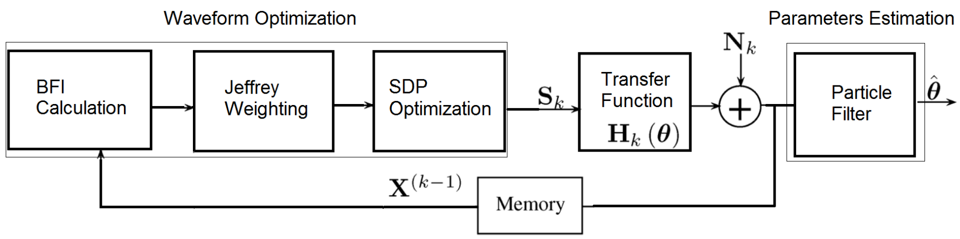

Figure 1: Cognitive Scheme.

3.1 Fisher Information Calculation

Fisher information indicates the amount of information of a parameter. The system finds the waveform that expected to achieve the highest Fisher information.

The MSE of θ is shown in Equation (4). For any unbiased estimator, the MSE always dose not drop below the inverse of its Fisher information [?].

$$\text{MSE}_k = E_{X_k,\theta}[(\hat{\theta}-\theta)(\hat{\theta}-\theta)^T] \tag{4}$$

$$\text{MSE}_k \geq J_k^{-1} \tag{5}$$

By using the law of total expectation, (shown in Equation (6)), the conditional MSE given the history output X_{k-1} is the mean square of the difference between real and estimated parameters when X_{k-1} is known. $\hat{\theta}$ denotes the measurement of θ. Where the random variable θ is a function of $X_k, \theta|X_{k-1}$. Equation (7) shows that the conditional MSE is still greater than the inverse of conditional Bayesian Fisher information (BFI).

$$\text{MSE}_k|X_{k-1} = E_{X_k,\theta|X_{k-1}}[(\hat{\theta}-\theta)(\hat{\theta}-\theta)^T|X_{k-1}] \tag{6}$$

$$\text{MSE}_k|X_{k-1} \geq J_k^{-1}|X_{k-1} \tag{7}$$

By using Bayes theorem [6], $p(X_k,\theta|X_{k-1})$ is equal to $p(X_k|X_{k-1},\theta)p(\theta|X_{k-1})$, then the BFI is separated into statistical (J_S) and incremental (J_D) BFI. The statistical BFI refers to the prior information of a target while the incremental BFI is the prediction of the BFI when the input covariance is equal to R_{S_k}. Hence, the conditional BFI $M \times M$ matrix is rewritten in Equation (8) [6], where i, j are row and column of the matrix. Where M denotes the maximum number of targets which system can detect.

$$\begin{aligned}
J_{k,i,j}|X_{k-1} =& -E_{X_k,\theta|X_{k-1}}[\frac{\partial^2 lnp(X_k,\theta|X_{k-1})}{\partial\theta_i\partial\theta_j}|X_{k-1}] \\
=& J_S + J_D \\
J_{D,k,i,j}|X_{k-1} =& -E_{X_k,\theta|X_{k-1}}[\frac{\partial^2 lnp(X_k|X_{k-1},\theta)}{\partial\theta_i\partial\theta_j}|X_{k-1}] \\
J_{S,k,i,j}|X_{k-1} =& -E_{X_k,\theta|X_{k-1}}[\frac{\partial^2 lnp(\theta|X_{k-1})}{\partial\theta_i\partial\theta_j}|X_{k-1}]
\end{aligned} \tag{8}$$

In order to calculate the posterior from prior when the target parameters are dynamically changing, using too much prior information might lose some sensing ability and some prior information has to be forgotten. Thus, the information is degenerated by a decay constant k_d to take into account the uncertainties. Decay constant denotes the rate of forgetting information. Therefore, Chapman-Kolmogorov equation [?] (9) is modified and shown in Equation (9).

$$p(\theta|X_{k-1}) = \int p(\theta|\theta_{k-1})p(\theta_{k-1}|X_{k-1})^{k_d}d\theta_{k-1} \tag{9}$$

The derivation of BFI is based on the Slepian-Bangs formula [?]. We can write the multivariate Gaussian distribution BFI into Equation (10) and (11). The incremental BFI, Equation (10), depends on the input covariance (R_S). On the contrary, in Equation (11), the statistical BFI is independent of the input covariance.

$$J_{D,k,i,j}|X_{k-1} = E_{\theta|X_{k-1}}[Re[tr(\dot{H}_i^H(\theta)R_N^{-1}\dot{H}_j(\theta)R_{S_k})|X_{k-1}]] \tag{10}$$

$$\begin{aligned}
J_{S,k,i,j}|X_{k-1} =& \Sigma_{\iota=1}^{k-1}\{k_d^{k-1-\iota}(E_{\theta|X_{k-1}}[Re[tr(\dot{H}^H(\theta)_i R_N^{-1}\dot{H}(\theta)_j R_{S_\iota})|X_{k-1}]] \\
&- E_{\theta|X_{k-1}}[Re[tr((X_\iota - H(\theta)S_\iota)^H \ddot{H}_{i,j}(\theta)S_\iota|X_{k-1}]])\}
\end{aligned} \tag{11}$$

3.2 Approximation of the number of target by Jeffreys Prior Weighting

To minimize MSE, we search for the input covariance matrix (R_{S_k}) that minimize the CRB. There are several criteria to simplify the vector to a scalar of multivariate CRB matrix such as using trace operation, det operation or eigenvalue. From [**?**], using trace operation is the best suited for minimizing CRB. The optimization problem by using trace operation is shown in Equation (12) [6], where W is a diagonal weight matrix. The method to invert a multivariate BFI matrix for CRB calculation is shown in [**?**]. The dimension of the BFI matrix is equal to the maximum number of parameters that the system can detect.

$$\begin{aligned} R_S &= \arg\min_{R_S}\{tr\{\frac{W}{J_k|X_{k-1}}\}\} \\ \text{s.t. } R_{S,i,i} &= \frac{P_S}{N_T} \text{ for } i = 1, 2..., N_T \\ R_S &\succeq \mathbf{0} \end{aligned} \tag{12}$$

Furthermore, due to the dynamics of the targets, we cannot determine the number of unknown targets in the system. If the actual number of targets is less than the expected one, there is a probability that the optimized waveform (Equation 12) will not be subject to any targets. Therefore, the uncertainties are overcome by considering the probability of existence as the weight in the optimization.

Jeffreys prior is a generalization of a uninformative prior parametrization. We want to find a prior that can manipulate the likelihood function and invariant to the change of variable. The solution is the square root of Fisher information, as shown in Equation (13) [**?**]. Finally, the square root of Fisher information is added to the weight of each target parameter. The constraints of the problem refer to the power limitation.

$$p(\theta) = J(\theta)^{\frac{1}{2}} \tag{13}$$

Then, the optimization problem is written in Equation (14), and it is convex. This can be cast as a semidefinite program [3, 15] which is solved by an interior-point algorithm and guaranteed in polynomial time.

$$\begin{aligned} R_S &= \arg\min_{R_S}\{\Sigma_{i=1}^{5M}\frac{\sqrt{J_{S,k,i,i}|X_{k-1}}}{J_{S,k,i,i}|X_{k-1} + J_{D,k,i,i}|X_{k-1}}\} \\ \text{s.t. } R_{S,i,i} &= \frac{P_S}{N_T} \text{ for } i = 1, 2..., N_T \\ R_S &\succeq \mathbf{0} \end{aligned} \tag{14}$$

3.3 Optimization by Semidefinite Programming

This problem can be solved by a convex optimization which is a special case of non linear optimization when the objective function is a convex function over positive definite constraint. Hence, every convex optimization can be cast as semidefinite programming and solved by an interior-point algorithm.

The optimization function in Equation (14) has to be manipulated into semidefinite programming form. An auxiliary variable (ξ) is introduced to change objective function to be linear function, thus the optimization problem (14) can be written in Equation (15).

$$\begin{aligned} R_S &= \arg\min_{\xi_i, R_S}\{\Sigma_{i=1}^{5M}\xi_i\sqrt{J_{S,k,i,i}|X_{k-1}}\} \\ \text{s.t. } R_{S,i,i} &= \frac{P_S}{N_T} \text{ for } i = 1, 2..., N_T \\ R_S &\succeq \mathbf{0} \\ \xi &\geq \frac{1}{J_{S,k,i,i}|X_{k-1} + J_{D,k,i,i}|X_{k-1}} \text{ for } i = 1, 2..., 5M \end{aligned} \tag{15}$$

Finally, the optimization problem in semidefinite programming form is written in Equation (16) [3]. This equation is the input to the interior-point algorithm.

$$\begin{aligned} R_S &= \arg\min_{\xi_i, R_S}\{\Sigma_{i=1}^{5M}\xi_i\sqrt{J_{S,k,i,i}|X_{k-1}}\} \\ \text{s.t. } R_{S,i,i} &= \frac{P_S}{N_T} \text{ for } i = 1, 2..., N_T \\ R_S &\succeq \mathbf{0} \\ \begin{bmatrix} J_{S,k}|X_{k-1} + J_{D,k}|X_{k-1}eye(i) \\ eye(i)^T\xi_i \end{bmatrix} &\succeq \mathbf{0} \text{ for } i = 1, 2..., 5M \end{aligned} \tag{16}$$

where $eye(i)$ is ith column of identity matrix.

4 State-Space Estimation

A Bayesian approach to calculate probability distributions is described to estimate the state of a target. A non-parametric distribution is represented by a group of particles. Furthermore, tangent speed (proportional movement to the radar) cannot be extracted by using only a single received signal.

4.1 State-Space

In the presence of moving targets, there is a process of changing the state with time (speed). This can be described by state-space equation (Equation (17) or (18)), where the state transition (f_k) and measurement function (h_k) are possibly a non-linear function with noise (n_k). In addition, we recursively estimate θ from measurement to track the target parameters.

$$\begin{aligned} \theta_k &= f_k(\theta_{k-1}) \\ X_k &= h_k(\theta_k, n_k) \end{aligned} \tag{17}$$

$$\begin{bmatrix} \varphi_{az,k} \\ \varphi_{po,k} \\ \dot{\varphi}_{az,k} \\ \dot{\varphi}_{po,k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_{az,k-1} \\ \varphi_{po,k-1} \\ \dot{\varphi}_{az,k-1} \\ \dot{\varphi}_{po,k-1} \end{bmatrix} \tag{18}$$
$$X_k = H_k(\theta_k) S_k + n_k$$

The first step of the recursive predicting is the prediction. In Equation (9), Chapman-Kolmogorov equation [?] predicts the probability of the parameters given prior knowledge. The second step uses Bayes theorem to calculate the probability of the parameters from current knowledge, where the denumerator is the normalization, as shown in Equation (19) [11]. The Likelihood function in Equation (19) can be written by multivariate Gaussian distribution in Equation (20).

$$p(\theta_k|X_k) = \frac{p(X_k|\theta_k)p(\theta_k|X_{k-1})}{\int p(X_k|\theta_k)p(\theta_k|X_{k-1})d\theta_k} \tag{19}$$

$$p(X_k|\theta_k) = \frac{1}{\sqrt{(2\pi)^{N_R}|R_N|}} e^{-\frac{1}{2}(X_k - H_k S_k)^H R_N^{-1}(X_k - H_k S_k)} \tag{20}$$

4.2 Particle Filter

Since the measurement function is non-linear function and the posterior pdf is non-Gaussian distribution, a sampling method called particle filter is used to calculate a posterior pdf. The particle filter approximates the posterior pdf by using particles, in Equation (21). Let $\{\theta^i, w^i\}_{i=1}^{N_s}$ be a sampling data, where θ^i is the sampling point, w^i is the weight, and N_s is the number of samples. The weight of the particles with the decay rate (k_d) is updated by Equation (22).

$$p(\theta_k|X_k) \approx \Sigma_{i=1}^{N_s} w_k^i \delta(\theta_k - \theta_k^i) \tag{21}$$

$$w_k^i = (w_{k-1}^i)^{k_d} p(X_k|\theta_k^i) p(\theta_k^i|\theta_{k-1}^i) \tag{22}$$

4.3 Parameter Permutation Problem

The parameter θ is infinitely exchangeable (De Finetti's theorem [?]). Therefore, the weights in the optimization (14) is not independent. In order to maximize searching ability the weights have to be independent. Thus, the intersection area between parameter's pdf has to be eliminated. This problem occurs when the information of the particles is exchanging with other targets.

Theorem 1 *Let $\theta_1, \theta_2, ..., \theta_m$ be an infinite sequence of random variable, then θ is exchangeable of all value of m if:*

$$\theta_1, \theta_2, ..., \theta_m = \theta_{s(1)}, \theta_{s(2)}, ..., \theta_{s(m)} \text{ for all } s \in S(m)$$

where $S(m)$ is a permutation function of $\{1, 2, ..., m\}$.

To eliminate this problem, we permute the parameters back. The algorithm minimizes the distance of the parameters between a particle and the highest probability particle, according to the parameter's information. Algorithm 1 shows the rearranging function.

Algorithm 1 Rearranging Algorithm

$(\{\theta_k^i, w_k^j\}_{i=1}^{N_s}) = Rearrange(\{\theta_{k-1}^i, w_{k-1}^i\}_{i=1}^{N_s}, J_{s,k-1})$
$order = DescendingDiagonalSorting(J_{s,k-1})$
for $i = 1 : N_s$
for $j = 1 : m$
    $\theta^i[j] = \theta^i[min(\theta^i[order[j]] - \theta^{max(w)})]$;
end for
end for

4.4 Resampling

One of the major problems of a particle filter is when the algorithm runs for a while, all but one particle will have a zero weight [11]. The estimated effective sample denotes the efficiency of the algorithm, written in Equation (23) [11]. The estimated effective sample is high when the sum of the square weight is high.

$$N_{eff} = \frac{1}{\Sigma_{i=1}^{N_s}(w_k^i)^2} \tag{23}$$

To reduce the effect of degeneracy, the estimated effective sample has to be maximized by using the resampling algorithm (Algorithm 2). After resampling, all particles will share equal weight.

Algorithm 2 Resampling Algorithm [11]

$(\{\theta_k^{j*}, w_k^j, i^j\}_{j=1}^{N_s}) = Resample(\{\theta_k^i, w_k^i\}_{i=1}^{N_s})$
initialization CDF: $c_1 = 0$;
for $i = 2 : N_s$
  $c_i = c_{i-1} + w_k^i$;
end for
set: $i = 1$;
initialization point:$u_1 \sim U[0, N_s^{-1}]$;
for $j = 1 : N_s$
  $u_j = u_1 + N_s^{-1}(j-1)$;
  while $u_j > c_i$
    $i = i + 1$;
  end while
  Assign sample: $\theta_k^{j*} = \theta_k^i$;
  Assign weight: $w_k^j = N_s^{-1}$;
  Assign parent: $i^j = i$;
end for

4.5 Algorithm Summary

The particle filter is shown in Algorithm 3. Thus, the overall cognitive system is concluded in Algorithm 4. First, it transmits an orthogonal waveform with uniform power for any direction and receives the signal back. Second, it calculates BFI and finds the optimum input covariance by using semidefinite programming. Third, it transmits and receives a signal from the targets. Finally, it uses the rearranging function and particle filter to estimate the parameters of the targets.

Algorithm 3 Likelihood Particle Filter Algorithm

$(\{\theta_k^i, w_k^j\}_{i=1}^{N_s}) = Filter(\{\theta_{k-1}^i, w_{k-1}^i\}_{i=1}^{N_s}, X_k)$
for $i = 1 : N_s$
  calculate: $w_k^i = (w_{k-1}^i)^{k_d}$;
end for
normalize w_k;
if N_{eff} is less than some threshold (1 to ∞)
  $(\{\theta_k^i, w_k^i, i^j\}_{i=1}^{N_s}) = Resample(\{\theta_k^i, w_k^i\}_{i=1}^{N_s})$;
end if
for $j = 1 : N_s$
  draw $\theta_k^j \sim p(\theta_k|\theta_{k-1}^{i^j})$;
  calculate: $w_k^i = w_k^i p(X_k|\theta_k^i)p(\theta_k^i|\theta_{k-1}^i)$;
end for
normalize w_k;

Algorithm 4 The adaptive waveform design method.

draw $\theta_0 \sim \mathcal{U}[-\frac{\pi}{3}, \frac{\pi}{3}]$;
$w_0 = N_s^{-1}$;
set $k = 1$;
set $J = \mathbf{0}$;
while user constraints are not satisfied
  $k = k + 1$;
  transmit a waveform;
  collect X_k from system;
  $(\{\theta_k^i, w_k^j\}_{i=1}^{N_s}) = Rearrange(\{\theta_{k-1}^i, w_{k-1}^i\}_{i=1}^{N_s}, J_{s,k-1})$
  $(\{\theta_k^i, w_k^j\}_{i=1}^{N_s}) = Filter(\{\theta_{k-1}^i, w_{k-1}^i\}_{i=1}^{N_s}, X_k)$
  calculate J from Equation (8);
  find optimum R_S from Equation (14);
end while

5 Simulation Results

Consider the MIMO radar system where the antennas are designed for the best parameter identifiability. By half-wavelength spacing, the transmit antennas and receive antennas are located at $[r_1, r_2, r_3, r_4] = \frac{2\pi}{\omega}[-8, -4, 0, 4]$ and $[r_1, r_2, r_3, r_4] = \frac{2\pi}{\omega}[0, 1, 2, 3]$, respectively. To provide the orthogonality to the waveform, we set code length (L) to 8. The algorithm is simulated in the -3 dB SNR environment, where the noise is an i.i.d. Gaussian random variables. In particle filter, the number of particles (N_s) is set to 100,000.

For simplicity, while the robustness of the system is observable, some computation can be reduced. Therefore some parameters are assumed to be known. As the parameters namely complex attenuation (α), azimuth angle (φ_{az}), and azimuth speed ($\dot{\varphi_{az}}$) are considered, we assume that polar angle (φ_{po}), propagation delay (τ), and Doppler shift (D) are assumed to be known.

5.1 Changing of number of targets

This section we concern the uncertainties due to the dynamic of targets in term of the change of number of targets. The result of Jeffreys prior reveals the probability that each M targets are existed. In this case, the maximum number of possible targets (M) is to 4. In Figure 2, the black lines denote the posterior pdf of each target (4 lines in each pulse step), while the green lines denote the corresponding waveform from first to sixth pulse step. The real targets are existed with the stationary speed as indicated by dot lines. The target locates at -50 degree for every pulse step. At the fourth to sixth step, the target is appearing at -10 degree. At the fifth step, a target at -45 degree is appearing, also. From the graph, the number of targets is increasing from 1 to 3, thus the posterior pdf rise at the area of targets. While the pulse step is increasing the posterior pdf obviously distinct for each target. Nonetheless, during fifth to sixth step, only 3 existing targets from maximum of 4, there is a smooth line that dose not depends on any target.

For example, the performance of angle estimation versus SNR at fifth pulse step is shown in Figure 3. Without forgetting some prior information ($k_d = 1$), non-adaptive waveform (orthogonal waveform) performs better results for every SNR, because the system with $k_d = 1$ knows that no new target appear and dose not put an energy to sense it while the orthogonal equally searches through every space. However, when some prior is forgotten, the speed for sensing the new target of the system with $k_d = 0.7$ is faster than $k_d = 1$.

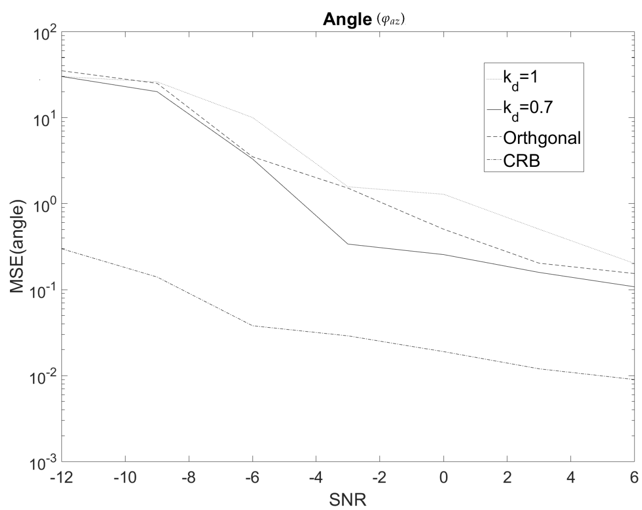

Figure 3: Relation between MSE of angle estimation and SNR at fifth pulse step.

5.2 Behavior of the adaptive waveform

In this part, we examine the beam pattern of the designed waveform when 2 targets are closely located at $\varphi_{az} = -1°, 1°$ with a speed of 0 ($\dot{\varphi_{az}} = 0°/step$) and the complex attenuation of both targets is $0.33+0.33i$.

Figure 5 shows the posterior pdf and the waveform with Jeffreys weight at the first to tenth pulse steps when $k_d = 0.7$. Thus, the MSE and CRB are plotted in Figure 4. With the increasing pulse step, the mean square error of every parameter declines to some limited values. Most of the power focus on the target area. Nevertheless, at the fifth and eighth step the CRB of the angle parameter is significantly higher than the CRB of the attenuation parameter. Therefore, at this step, the waveform is designed to achieve the best angle parameter, as we can see the sharp edges of beam pattern in Figure 5.

As the result from sharp edges beam pattern, we get higher angle information, though we get lower attenuation information. This process is automatically done by the system. Therefore, if the attenuation information and power focusing at the target area are needed, though if want angle information, the sharp edges beam pattern is created. The optimization adapt the waveform in order to get much more information for both angles and attenuation equally.

5.3 Multiple Moving Targets Case

This part shows performance and robustness of the system when two moving targets are considered. Moreover, the importance of using Jeffreys prior as weight in optimization is shown in this part.

The immediately change of targets location might disrupt the system stability, therefore the level of decay constant k_d effects sensibility and stability of the system. Between the first to the fifth pulse step, there are two targets. First target is located at $\varphi_{az} = 50°$ with speed $\dot{\varphi_{az}} = 0°/step$, and the other is located at $\varphi_{az} = -2°$ with speed $\dot{\varphi_{az}} = -2°/step$. At the sixth pulse step, targets are relocated, the first target is located at $\varphi_{az} = -25°$ with speed $\dot{\varphi_{az}} = 0°/step$, the second is located at $\varphi_{az} = 2°$ with speed $\dot{\varphi_{az}} = 2°/step$.

Because the estimation of speed perpendicular to the radar (not related to Doppler shift) is directly obtained from particle filter, the calculation of CRB of speed estimation has no effect on waveform design.

In this example of the posterior pdf and corresponding waveform with Jeffreys weight, information decay rate is set to ($k_d = 0.7$). The average performance obtained by 100 Monte Carlo simulations shown in Figure 6. We compare the system to the system without prior modification ($k_d = 0.1$) and non adaptive waveform (orthogonal waveform). From the graph (Figure 6), angle of arrive during first to fourth pulse step, MSE when $k_d = 1$ decreases with the fastest rate follow by $k_d = 0.7$ system and non adaptive system. At the fifth step, the MSE when $k_d = 0.7$ suddenly drops to the same value of $k_d = 1$ system, while $k_d = 1$ and non adaptive system remain constant. At the sixth step, MSE of all systems rise to the same level as the first pulse step. Then, after sixth pulse step, MSE of $k_d = 0.7$ system decreases with huge slope, while $k_d = 1$ and non adaptive system is insignificant decreasing.

The orthogonal waveform shows the average result for both before and after relocating. Before relo-

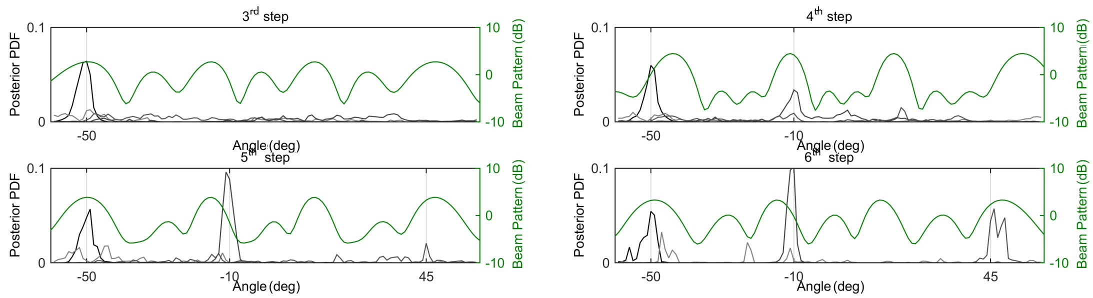

Figure 2: Posterior pdf and corresponding beam pattern when the targets are appearing at first, fourth, and fifth pulse step.

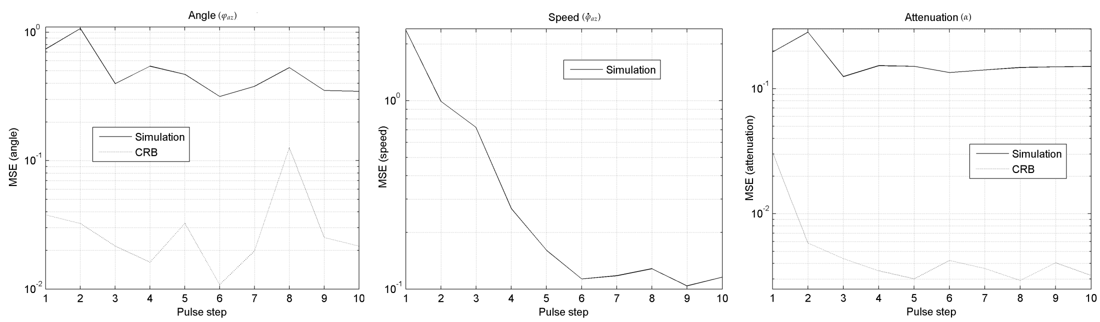

Figure 4: MSE and CRB of angle, speed, and attenuation versus pulse step, where two targets are located at -1,1 deg.

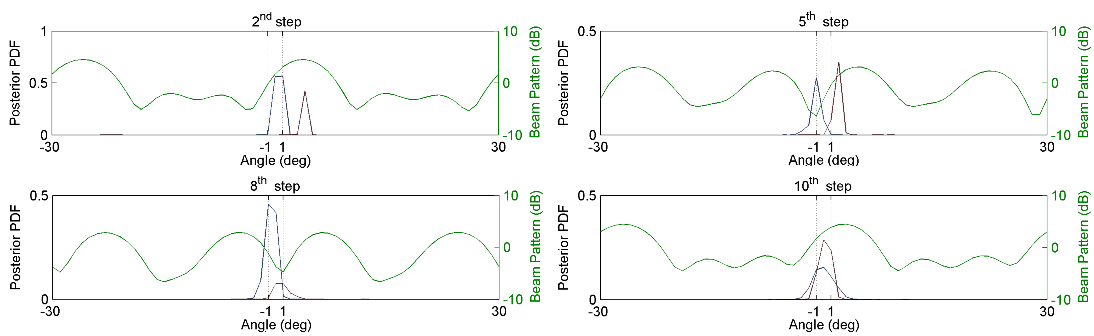

Figure 5: The posterior pdf and the corresponding beam pattern of each pulse step, where two targets are located at -1,1 deg.

cating, the waveform with $k_d = 1$ gives the best performance However, after relocating it gives very poor performance. However, the waveform with $k_d = 0.7$ achieves a good performance for both before and after relocating.

With the same parameters setup, we show the effect of parameter k_d to MSE. A hundred Monte Carlo creates the simulation results in Figure 7. The graphs show the result from the fifth pulse step and seventh pulse step. The result from the fifth step shows the performance of the system when the target is not changing or moving with a constant speed, while the result from the seventh step indicates the capability of the system to manipulate the uncertainties.

In Figure 8, we show the difference between using Jeffreys prior as a weight and not using. We use 100 Monte Carlo simulations to achieve the results. Giving higher attention to the target that has higher existence probability gains stability for the system. From the graph, for every parameters, during first to sixth pulse step, the MSE of the system with and without Jeffreys prior as a weight are reducing with a similar rate. Nevertheless, MSE of the system with Jeffreys prior after sixth pulse step declines significantly faster

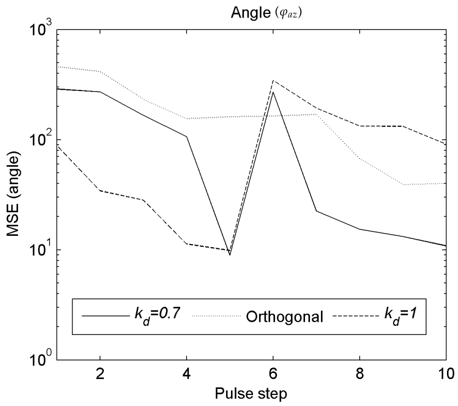

Figure 6: MSE of angle of arrives versus pulse step of three different systems, where the two targets are moving with a speed. The targets are relocated at the 6^{th} pulse step.

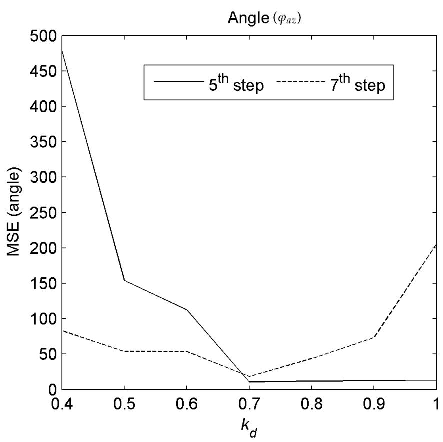

Figure 7: The effect of k_d on the MSE of angle of arrive.

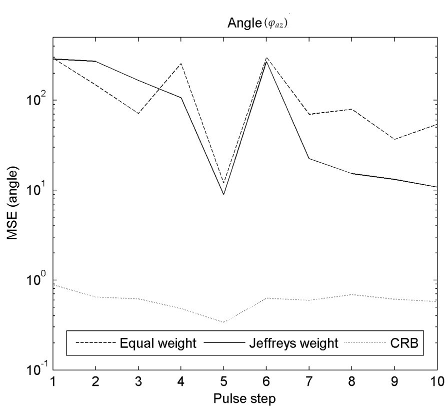

Figure 8: The comparison between equal and Jeffreys prior weight of the waveform optimization. The MSE of angle of arrive.

than the system without Jeffreys prior. Therefore, using Jeffreys prior as a weight in the optimization performs better than not using on every count.

6 Conclusion

The dynamic targets tracking for a cognitive radar system is revisited with the present of the uncertainties due to the dynamics of targets. This study adds the robustness to the system by decaying the prior before the calculation of the posterior in the waveform optimization which controlled by a constant, using Jeffreys prior as the weight for multiple targets optimization, and estimating posterior pdf by particle filter algorithm.

As the result form the simulations, we know the probability that the targets are existing, the possibility of every possible angle of arrive, and the speed of a moving target. Therefore, the proposed techniques gained the ability to cope with the uncertainties due to the dynamics of the targets.

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