A SAMPLING INSPECTION PLAN FOR CONTINUOUS PRODUCTION[1]

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I. Introduction

1. Purpose. This paper presents a plan of sampling inspection for a product consisting of individual units (parts, subassemblies, finished articles, etc.) manufactured in quantity by an essentially continuous process.

The plan, applicable only to characteristics subject to nondestructive inspection on a Go-NoGo basis, is intended primarily for use in process inspection of parts or final inspection of finished articles within a manufacturing plant, where it is desired to have assurance that the percentage of defective units in accepted product will be held down to some prescribed low figure. It differs from others which have been published[2,3] in that it presumes a *continuous flow* of *consecutive articles* or *consecutive lots* of articles offered to the inspector for acceptance in the order of their production. It is accordingly of particular interest for products manufactured by conveyor or other straight line continuous processes.

In operation, the plan provides a corrective inspection, serving as a partial screen for defective[4] units. Normally, a chosen percentage or fraction f of the units are inspected, but when a defective unit is disclosed by the inspection it is required that an additional number of units be inspected, the additional number depending on how many more defective units are found. The result of such inspections is to remove some of the defective units, and the poorer the quality submitted to the inspector, as measured in terms of per cent defective, the greater will be the corrective or screening effect. The object of the plan is the same as that incorporated in some of the sampling tables already published[5], namely, to establish a limiting value of "average outgoing quality" expressed in per cent

[1] Presented at the Joint Meeting of the American Society of Mechanical Engineers and the Institute of Mathematical Statistics, May 29, 1943, by H. F. Dodge, Quality Results Engineer, Bell Telephone Laboratories, New York.

[2] H. F. Dodge and H. G. Romig, "Single Sampling and Double Sampling Inspection Tables", *Bell Sys. Tech. Jour.*, Vol. XX (1941) pp. 1–61. An unpublished paper by Prof. Walter Bartky (developed when he was associated with the Western Electric Co., 1927) provides a continuous multiple sampling plan involving two factors—f, as used here, and i, the number of units in a "compensating sample" required to be inspected for each defective unit found.

[3] Lt. R. J. Saunders, "Standardized Inspection", *Army Ordnance*, Vol. XXIV (1943) pp. 290–292; G. Rupert Gause, "Quality Through Inspection", *Army Ordnance*, Vol. XXIV (1943) pp. 117–120.

[4] A unit of product that fails to meet the requirement for a characteristic is classed as nonconforming with respect to that characteristic, and for convenience is referred to as "defective". Thus, a deviation from a specified requirement or from accepted standards of good workmanship is termed a "defect"

[5] H. F. Dodge and H. G. Romig, loc. cit.

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defective which will not be exceeded no matter what quality is submitted to the inspector. This limiting value of per cent defective is termed the "average outgoing quality limit (AOQL)".

The theoretical solution treats the case of inspecting a continuous flow of individual units and is based on the distribution of *random-order* spacing of defective units in product whose quality is statistically controlled.[6] Part III of the paper extends the application of the method to a continuous flow of individual lots or sub-lots of articles.

II. Inspection of a Flow of Individual Units

2. Inspection of one characteristic. Consider first the inspection of a flow of individual units, offered consecutively in the order of their production. Assume that inspection is to be made for only one quality characteristic, so that interest will be centered on one kind of defect. Subsequently (Section 13), consideration will be given to the procedures when inspection is made simultaneously for several kinds of defects.

3. Procedure A. The procedure is as follows:

(a) At the outset, inspect 100% of the units consecutively as produced and continue such inspection until i units in succession are found clear of defects.

(b) When i units in succession are found clear of defects, discontinue 100% inspection, and inspect only a fraction f of the units, selecting individual sample units one at a time from the flow of product, in such a manner as to assure an unbiased sample.

(c) If a sample unit is found defective, revert immediately to a 100% inspection of succeeding units and continue until again i units in succession are found clear of defects, as in paragraph (a).

(d) Correct or replace with good units, all defective units found.

4. Protection provided by the plan. The inspection plan is defined by the two constants, f and i, which can be altered at will. For given values of f, i, and p (incoming fraction defective), there will result for product of statistically controlled quality a definite average outgoing fraction defective (average outgoing quality, AOQ). For given values of f and i, the AOQ will have a maximum for some particular fraction defective p_1 of incoming quality. As noted above, this maximum is referred to as the average outgoing quality limit (AOQL). For all other values of incoming fraction defective p greater or less than p_1, the AOQ will be less than AOQL. Many combinations of f and i will result in the same AOQL.

The protection offered by the plan discussed here can thus be expressed in terms of the AOQL, in per cent defective.

[6] "Statistical control" as defined in the literature; see W. A. Shewhart, *Statistical Method from the Viewpoint of Quality Control*, The Graduate School, U. S. Dept. of Agriculture, 1939.

5. Theoretical framework. We are concerned with the spacing between defective units when the individual units are arrayed in the order of their production, as shown in Fig. 1. If the manufacturing process is statistically controlled so that the probability of producing a defective unit is constant and equal to p, then defective units will have an order spacing of a random character which is expressible in terms of certain probability laws. Product turned out by such a process will be referred to as having a process average fraction defective p. The "event" of particular interest is a "terminal-defect sequence" of $i + 1$ successive units following the observance of a defect, comprising a succession of i nondefective units followed by a defective unit, as shown in Fig. 1. The totality of all possible such sequences, where i varies from 0 to ∞, constitutes the universe of events under consideration.

Each such sequence of $i + 1$ units, comprising i successive nondefective units followed by a defective one, has a definite probability of occurrence, for a process average fraction defective, p. The complete set of such probabilities for all possible sequences, having respectively $i = 0, 1, 2, 3, \cdots \infty$, defines a probability distribution[7] of random-order spacing of defects in uniform product. This is

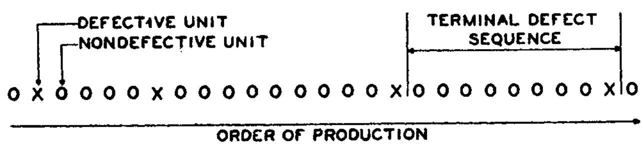

Fig. 1. Spacing of defective units

shown in the table below in which 0 represents a nondefective unit, X represents a defective one, p is the fraction defective, and $q = 1 - p$.

Sequence	*Defect Spacing (No. of units in sequence)*	*No. of Non-defective Units before Finding the Next Defect*	*Probability of Occurrence*	*No. of Term in the Power Series*
X	1	0	p	1st
$0X$	2	1	pq	2nd
$00X$	3	2	pq^2	3rd
$000X$	4	3	pq^3	4th
$0000X$	5	4	pq^4	5th
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
$000 \cdots 0X$	$i + 1$	i	pq^i	$(i + 1)st$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

[7] Romanovsky, V., "Due Nuovi Criteri di Controllo Sull 'andamento Casuale di Una Successione di Valori", *Giornale dell 'Instituto Italiano degli Attuari* (1932) discusses this

These probabilities are the successive terms in the infinite power series

$$p + pq + pq^2 + pq^3 + \cdots$$

(1) $$\text{or} \quad p(1 + q + q^2 + q^3 + \cdots).$$

The sum of this series is $p\left(\frac{1}{1-q}\right) = 1$, i.e., the total probability for all possible sequences is unity (as it should be).

The sum of the first $i + 1$ terms of the series is the probability of occurrence of a "terminal-defect sequence" (defect spacing) of $i + 1$ units or less. The sum of the first i terms is the probability, P_1, of failing to find the next i units clear of defects, which is

(2) $$P_1 = \sum_{j=0}^{j=i-1} pq^j = 1 - q^i.$$

In turn, the sum of all terms beyond the ith term is the probability of finding 0 defects in the next i units, which is

(3) $$Q_1 = 1 - P_1 = q^i.$$

These results and the power series (1) enter into subsequent portions of the discussion. The curves of Fig. 2 give values of $1 - q^i$.

6. Average outgoing quality. Suppose a plan is selected, choosing specific values of f and i.

For given values of i and p, there will be an expected average number of units, u, inspected following the finding of a defect. Likewise, for given values of f and p there will be an expected average number of units, v, that will be passed under the sampling procedure before a defect is found. The latter average number includes the sampling units actually inspected as well as the uninspected units produced between successive sample units.

The average fraction of the total product units inspected in the long run is

(4) $$F = \frac{u + fv}{u + v}.$$

It is now assumed for purposes of solution that the inspection operation itself never overlooks a defect and that all defective units found during the inspection of f and i will be corrected or replaced by good units.[8]

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probability distribution of spacing of events, referring to the spacing as the "length of a partial series". Our term "terminal-defect sequence" has the same significance as his term "partial series". See also P. S. Olmstead, "Note on theoretical and observed distributions of repetitive occurrences", *Annals of Math. Stat.* Vol. XI (1940) pp. 363–366; A. M. Mood, "The distribution theory of runs", *Annals of Math. Stat.*, Vol. XI (1940) pp. 367–392.

[8] The assumption that the inspection operation is perfect cannot be made without reservation. Machine inspection devices have their margins of error. Also, inspection fatigue prevents 100% manual and visual inspections from insuring perfection, particularly if such inspections continue over a considerable period of time. But the efficiency of the latter

As a result of the screening effect of the inspection, the average outgoing quality, AOQ, designated p_A, is related as follows to the incoming quality p:

$$p_A = p(1 - F) = p\left(1 - \frac{u + fv}{u + v}\right). \tag{5}$$

7. Determination of u. The average number of units, u, inspected on a 100% inspection basis following the finding of a defect is a function of i and p, and may be determined from a considration of two power series, one limited and the other infinite.

Once the 100% inspection starts, there are several things that can happen before i units are found clear of defects. The first i may be found clear; or 1, 2, 3, or more defects may be found before finally a run of i units is found clear.

One of the quantities to be determined is the *average number of units* inspected in a "failure sequence," that is, one terminating in a defect and comprising i or less units. This average number, designated as h, is the average of the distribution made up of the first i terms of the power series (1). The average is

$$h = \frac{p}{1 - q^i}(1 + 2q + 3q^2 + 4q^3 + \cdots + iq^{i-1}), \tag{6}$$

where the denominator is the sum of the probabilities for the first i terms. This may be evaluated as follows:

$$h = \frac{p}{1 - q^i}\frac{d}{dq}(1 + q + q^2 + q^3 + \cdots + q^i)$$

$$= \frac{p}{1 - q^i}\frac{d}{dq}\left[\frac{1 - q^{i+1}}{1 - q}\right]$$

$$= \frac{1}{p(1 - q^i)}[1 - q^i(1 + pi)]. \tag{7}$$

Note that if pi is small compared with unity, h is approximately $1/p$.

The next step is to determine the *average number of failure sequences* that will be encountered before finding i units clear of defeets. This average number, designated as G, may be found from the probability distribution of all possible numbers of failure sequences, expressed by the infinite series

$$Q_1(1 + P_1 + P_1^2 + P_1^3 + \cdots) \tag{8}$$

where P_1 is given by equation (2), $Q_1 = 1 - P_1$, as given by equation (3), and the successive terms are the probabilities of occurrence of 0, 1, 2, 3, etc. failure

inspections is generally higher when an interest incentive is provided as is usually the case in sampling inspection plans where the extent of such inspections hinges on their findings.

The solution given assumes correction or replacement of defective units. Where it is expedient to reject such units and not replace them, equations (19) to (22) inclusive, should be modified by replacing i by $i - 1$.

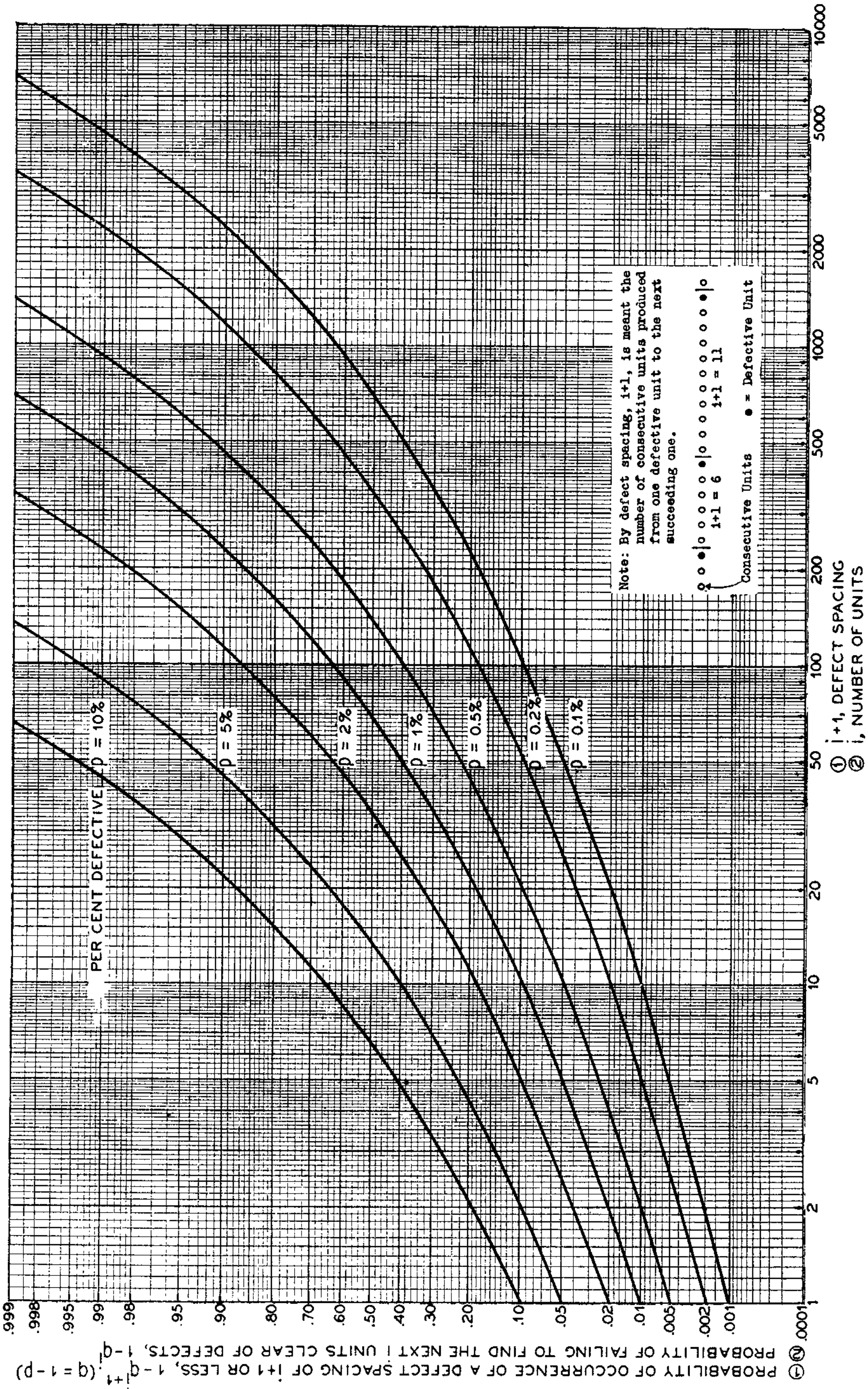

FIG. 2. Curves defining distribution of random order spacing of defects in uniform product

sequences before finding i units clear of defects. The average number of failure sequences, G, is given by the sum of the infinite series

$$G = Q_1(0 + 1P_1 + 2P_1^2 + 3P_1^3 + \cdots)$$
$$= Q_1P_1(1 + 2P_1 + 3P_1^2 + 4P_1^3 + \cdots). \tag{9}$$

Summing the series, we have

$$G = Q_1P_1\frac{1}{(1 - P_1)^2} = \frac{P_1}{Q_1} = \frac{1 - q^i}{q^i}. \tag{10}$$

Now u, the average number of pieces inspected following the finding of a defect, is made up of a number of failure sequences followed by a run of i units clear of defects. Using the average values of G and h just found, we have

$$u = Gh + i = \frac{1 - q^i}{pq^i}. \tag{11}$$

8. Determination of v. The average number of units, v, that will be passed in a period of sampling inspection will be $1/f$ times the average number of individual sample units inspected in such periods. Here again the solution will depend on the random order spacing of defects in uniform product. Whether the individual units selected during the sampling inspection procedure are selected by a random spacing device, or by any other means which will prevent known bias in the sample, we may assume that defects will be found to occur in accordance with the distribution of random order spacing defined by the terms of the series given in (1). The average number of sample units inspected in a period of sampling inspection will thus be the average defect spacing for product having fraction defective, p, which is given by the infinite series.

$$H = p(1 + 2q + 3q^2 + 4q^3 + \cdots). \tag{12}$$

Summing the series, we have

$$H = \frac{p}{(1 - q)^2} = \frac{1}{p}, \tag{13}$$

and the value of v is found to be

$$v = \frac{H}{f} = \frac{1}{fp}. \tag{14}$$

9. Determination of f and i for a given value of AOQL. From the considerations given above, the average fraction of the product inspected, F, and the value of average outgoing quality, p_A, can be determined for any given values of $p, f,$ and i. Substituting in (5), the values of u and v given in (11) and (14), we have

$$p_A = p\left[1 - \frac{f}{f + (1 - f)(1 - p)^i}\right]. \tag{15}$$

The average outgoing quality limit, AOQL, (p_L) is the maximum value of p_A that will result for any given values of f and i, considering all possible values of p in the submitted product. The value of p for which this maximum value of p_A occurs is designated by p_1, hence

$$p_L = p_1\left[1 - \frac{f}{f + (1 - f)(1 - p)^i}\right]. \tag{16}$$

The value of p_1 for which $p_A = p_L$ is determined by differentiating (15) with respect to p, equating to 0, and solving for p, that is

$$\frac{dp_A}{dp} = 1 - \frac{f^2 + f(1 - f)(1 - p)^i + pfi(1 - f)(1 - p)^{i-1}}{[f + (1 - f)(1 - p)^i]^2}. \tag{17}$$

Simplifying, and using the designation p_1 for the maximizing value of p, gives

$$(i + 1)p_1 - 1 = \frac{1 - f}{f}(1 - p_1)^{i+1}, \quad \text{or}$$

$$(1 - p_1)^i = \frac{f[(i + 1)p_1 - 1]}{(1 - f)(1 - p_1)}. \tag{18}$$

Substituting in (16) this value of $(1 - p_1)^i$, we have

$$p_L = \frac{(i + 1)p_1 - 1}{i}, \quad \text{hence} \tag{19}$$

$$p_1 = \frac{1 + ip_L}{i + 1}. \tag{20}$$

From (18) and (19), we have

$$p_L = \frac{1 - f}{fi}(1 - p_1)^{i+1}, \quad \text{hence} \tag{21}$$

$$f = \frac{(1 - p_1)^{i+1}}{ip_L + (1 - p_1)^{i+1}}. \tag{22}$$

The curves given in Fig. 3 were calculated by choosing values of i for given values of AOQL (p_L) and calculating p_1 from equation (20) and f from equation (22). Thus for a given AOQL value, an i value may be found for a chosen f value and vice versa. It will be noted that for a given value of f, i varies inversely with the AOQL value, to a close degree of approximation.

10. Operating characteristics of the plan. Figs. 4(a) and 4(b) give a picture of the operating characteristics of the general plan as f and i are varied. They indicate for example that for a moderate range of f values the factor i has a stronger influence than f in determining the discrimination that the method affords between high and low levels of incoming per cent defective. For the values of f and i shown, Fig. 4(b) indicates just what level of incoming per cent

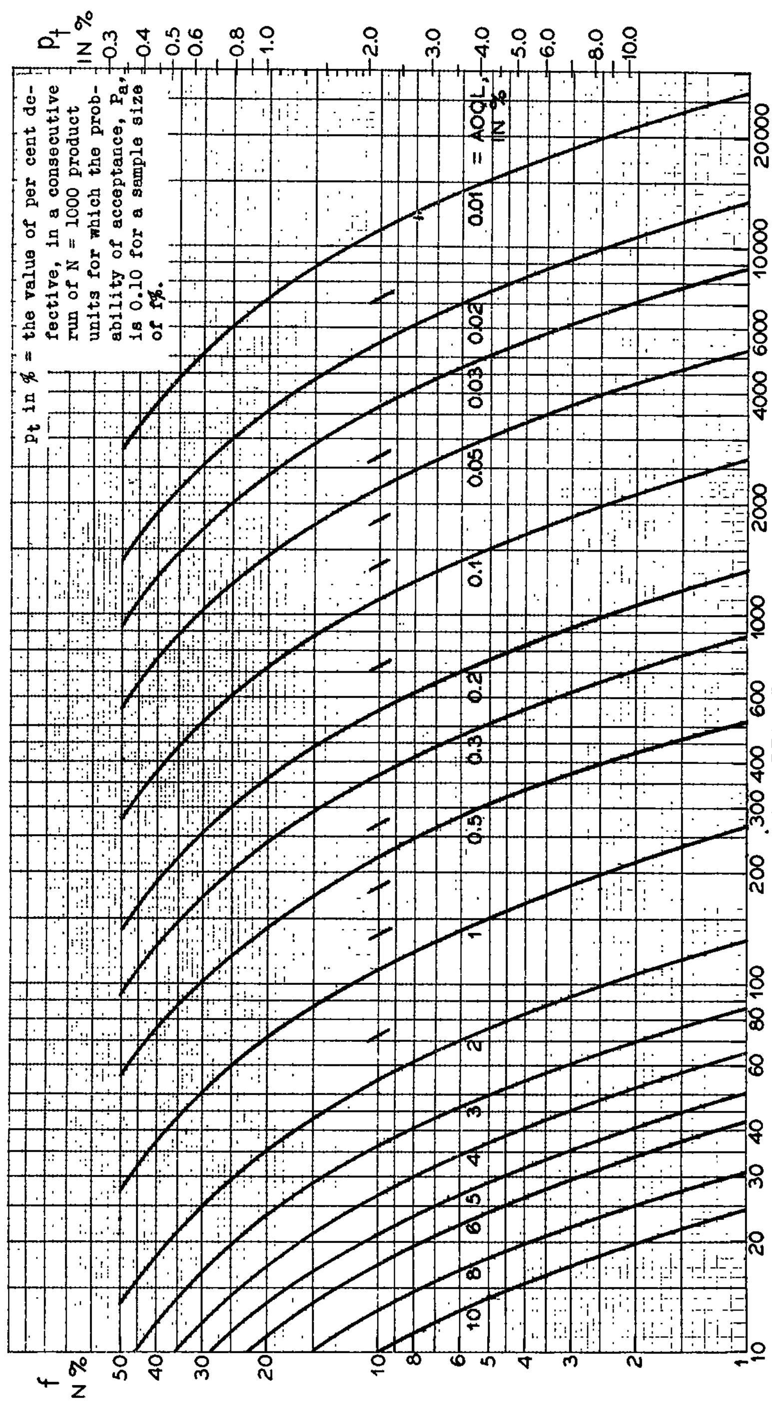

FIG. 3. Curves for determining values of f and i for a given value of AOQL

defective would force a correction of the manufacturing process, if the percentage of total production that would be accepted on a sampling basis falls below a critical value—often, a value of the order of 80% to 90%.

Fig. 5 gives a comparison of the characteristics of several plans having the same AOQL value, 1%. It indiates for example that when the normal level of incoming per cent defective is well below the AOQL, the AOQL value can be assured with less inspection by choosing *f* small and *i* large. But since, for a given AOQL value, the average amount of inspection approaches a minimum as *f* approaches 0, factors other than the minimum amount of inspection have a

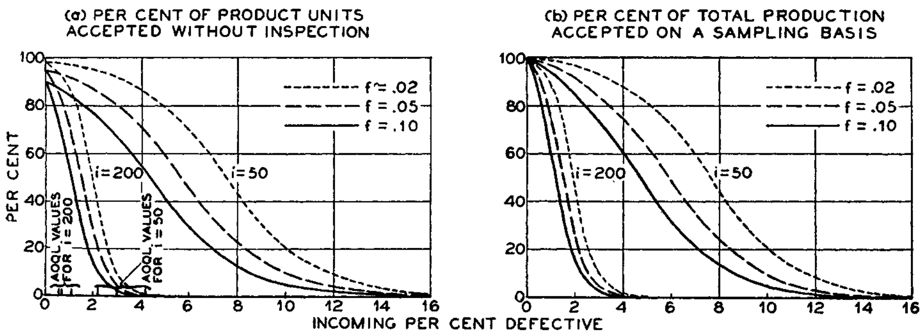

FIG. 4. Curves showing effect of *f* and *i* on operating characteristics of plan

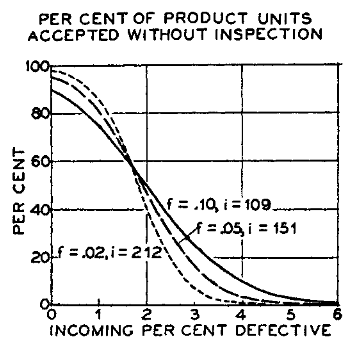

FIG. 5. Characteristics of three plans having the same AOQL of one per cent

more important influence on the choice of the most advantageous combination of *f* and *i* values for a given set of circumstances. For example, when the inspector is located at the end of the production line, it may be desirable to use a value of *i* not greater than some small multiple of the number of product units on the line at any one time. Or again, the value of *f* is often influenced by the normal work loads of the inspector and the operators on the line. Protection against "spotty" quality, such as may arise from temporary irregularities in workmanship or materials, should receive special consideration in connection with the choice of *f*.

11. Protection against spotty quality. The p_t scale at the right of Fig. 3 provides a guide concerning the protection afforded against spotty quality in a continuous run of product. The value of p_t is the per cent defective in a run of 1000 consecutive product units, for which the probability of acceptance by sample is 0.10 for a percentage sample equal to the corresponding f value shown on the chart.

This scale indicates that the protection against spotty quality falls off very rapidly with f and that the protection, considering runs of product of 1000 consecutive units each, becomes quite poor if f is less than 2%.

12. Effect of selecting group samples rather than one unit at a time. The above development assumes selection of individual sample units one at a time from the flow of product and immediate examination of a unit to determine whether or not it is defective. Deviations from this procedure will in general result in giving values of AOQL higher than those shown in Fig. 3.

For example, the actual AOQL may be higher than the theoretical value (a) if the inspector delays looking at the individual units immediately when they are withdrawn from the line, or (b) if he selects a group of units at one time from the production line. The effect of either of these two deviations, both constituting a delay, may be quite large if i is small, or if large group samples are taken.

Although the modification of the theoretical AOQL value resulting from the selection of group samples has not been thoroughly explored, this should not be excessive,

(a) if group samples of $n = 10$ or less are drawn from the line, and

(b) if $i = 50$ or more,

provided there is no delay in examining the group samples drawn from the line.

It should be noted however, that the effect of these delay factors on the AOQL may be compensated for in part if, when a defect is found, the 100% inspection includes some of the units that have already passed the inspection point.

Where appreciable delays are unavoidable, an alternative is to withhold from acceptance a stipulated number of units pending the examination of the sample units that have been selected to represent this quantity of product. Such a procedure provides in effect a lot acceptance plan, the treatment for which is covered in Part III.

13. Administration of inspection operations. The inspection plan is most effective in practice if it is administered in such a way as to provide an incentive to clear up causes of trouble promptly. Such an incentive may be had by imposing a penalty on the operating or manufacturing department when defects are encountered. Normally, no such penalty is imposed if both the sampling inspection and the 100% inspection are performed by the same person or group of persons and the two costs merged; the inspector then merely serves as an agency for screening defects when quality goes bad. It is accordingly recommended that the sampling inspection and the 100% inspection operations be treated as two separate functions.

With this in mind, the inspection work can be performed by two different inspectors, designated inspector C and inspector M. Inspector C may be considered as the *consumer's* representative in that his work is performed as a function independent of the manufacturing group. The term "consumer" is used in the general sense of the recipient of the product after the inspection has been completed. Inspector M is responsible to the Manufacturing Department and the cost of his work is borne by that Department. His work must however be subject to the surveillance and approval of inspector C.

The following method of administering the inspection plan can then be used:

(a) Inspector C inspects the required fraction f. So long as no defects are found, product is considered acceptable and is passed.

(b) When inspector C finds a defect, he
1. continues inspecting the fraction f,
2. places some identification on the succeeding flow of product to indicate nonacceptance (or diverts it from the regular production line if the design of the line permits), such designation to apply until clearance is obtained in accordance with paragraph (c), and
3. calls inspector M to inspect the succeeding flow of product in accordance with paragraph (c).

(c) Inspector M (one or more inspectors as needed) inspects all succeeding units, except those inspected by inspector C in the fraction f, until the required number of units, i, are found clear of defects. Inspector M reports immediately to Inspector C all defects found in the course of his 100% inspection and notifies him when a run of i units has been found clear of defects.

(d) When notified that a run of i units has been found clear of defects, inspector C, if satisfied with the work of inspector M, releases inspector M.

(e) To facilitate speedy correction of causes of trouble, inspector C, on finding a defect, should promptly notify the production foreman or other designated authority and furnish the latter with detailed information regarding the character of the defect found.

It will be noted that the above procedure requires calling inspector M whenever inspector C finds a defect. To avoid taking such action on the occurrence of a single defect, the procedure can be modified so that inspector M is called into the picture only when two defects in succession are observed by inspector C. Where this feature is desired, paragraph (b) above may be modified to read as follows:

(b) When inspector C finds a defect, he
1. proceeds immediately to inspect all succeeding units up to a total of i units, and if no defects are found therein, he again limits his inspection

to the fraction f. If, on the other hand, during the course of inspecting the next i units, inspector C finds a second defect, he immediately discontinues his 100% inspection,

2. places some identification on the succeeding flow of product . . . etc.

While this procedure carries the disadvantage of placing a varying work load on inspector C, it is often preferred since a single defect tends to be regarded as an isolated occurrence whereas two defects in quick succession, (like a first and second offense) are normally accepted as sufficient evidence to justify special action.

14. Inspection for several kinds of defects simultaneously. The procedure given above may be applied directly to an inspection covering two or more kinds of defects, provided that the chosen AOQL value applies to all defects collectively and each unit inspected is always inspected for all of the defects under consideration.

It is sometimes desired, however, when a defect of one kind is observed, to confine the 100% inspection to this one kind of defect alone. This requires a modification of the general procedure and the establishment of a separate AOQL for each kind of defect. A similar modification is required for example where the inspection is to cover several kinds of defects, but where the defects are grouped into two or more classes, according to their seriousness, and the defects in each class treated collectively.

The following paragraphs outline for illustrative purposes a procedure for use where the defects under consideration are to be classified into two groups, Major and Minor, and where all Major defects are to be treated collectively and all Minor defects likewise. By analogy, the procedure to be followed when each kind of defect is to be treated separately will be obvious. In any event, the fraction f is made the same for all classes or all kinds of defects.

Procedure

Several kinds of defects are grouped into two classes with respect to seriousness; designated Major and Minor.

All defects of the same class (Major or Minor) are treated collectively.

Preliminary

(1) Establish an overall AOQL value for Major defects and an overall AOQL value for Minor defects. Select a suitable value for f, applicable to both Major and Minor defects. From Fig. 3 determine a value of i for Major defects, designated i_A, and a value of i for Minor defects, designated i_B.

(2) At the outset, inspect 100% of the units consecutively for both Major and Minor defects until i_{Max} units in succession are found clear of defects ($i_{\text{Max}} = i_A$ or i_B, whichever is the larger).

Routine

(3) When i_{Max} units in succession are found clear of defects, discontinue 100% inspection and inspect only a fraction f of the units *for both Major and Minor defects*, selecting individual sample units one at a time from the flow of product.

(4) If a Major (or Minor) defect is observed during sampling inspection, inspect 100% of the succeeding units *only for defects of the class in question* until i_A (or i_B) units in succession are found clear of defects of this class.
  (4.1) During the 100% inspection referred to in (4) inspect a portion f *for both Major and Minor defects.*
  (4.2) If during the 100% inspection for a particular class of defect (Major or Minor), a defect of the other class is observed on an individual unit of product, start 100% inspection for defects of the new class *only if* the new defect is observed on one of the f units that has been inspected for both Major and Minor defects, and continue such 100% inspection for defects of the new class until i (as determined in (1) for the new class) units in succession are found clear of defects of the new class. Do not take such action, however, if the new defect happens to be observed on one of the non-f units.

(5) When the proper number of successive units are found clear of defects as in paragraph (4) or (4.2), reinstate sampling inspection as in paragraph (3).

From the above it may be appreciated that difficulties of administration are introduced in treating a large number of classes of defects or a large number of individual defects separately. How best to group defects together for collective treatment can generally be determined from the nature of the inspection operations, whether visual or gauging, and the expectancy of defects as determined from the quality history. Items involving visual inspection, can often be treated collectively to advantage.

As is generally true, the layout of an inspection plan depends to a considerable extent on the nature of inspection operations to be performed. Simplicity of administration is always to be desired. From the standpoint of minimizing overall inspection costs, it is often preferable, where several quality characteristics are to be inspected, to break down the inspection work into two or more separate inspection steps, each covering a relatively small number of characteristics.

III. Inspection of a Flow of Individual Lots or Sub-lots

15. Purposes of Inspection. A manufacturer's inspection of his own product serves two purposes[9]:

(a) *Process Control*—To provide a basis for action with regard to the production process with a view to better future product.

(b) *Product Acceptance*—To provide a basis for action with regard to the product already at hand.

The plan outlined in Part II has both of these purposes in mind, but the provision for selecting sample units continuously from the production line places special emphasis on control. It aids, for example, in the prompt detection of defects and location of causes of trouble in the manufacturing process.

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[9] See A. S. A. War Standard, Z1. 3, *Control Chart Method of Controlling Quality During Production*, pp. 5–6, 1942, American Standards Association, New York.

The problem of acceptance of product is often eased, though at some sacrifice to the control aspects of the inspection work, if product is submitted to the inspector in lots or sub-lots and a sample taken from each.

16. Inspection procedure for sub-lots. With minor modifications, the plan and procedure of Part II can be extended to the case where material is offered as a *flow of consecutive sub-lots* of articles. In the inspection of parts, for example, the material may be offered in pan-loads or trays, each containing a collection of parts produced under essentially the same conditions. Or again, the product from a common source for a given short period of time, such as a half-hour, one hour, etc., may often be treated as a sub-lot and offered to the inspector as such for his acceptance. In what follows, however, it is essential that such sub-lots be kept in the *order of their production.*

The theoretical development given in Part II makes use of random-order spacing of defects in a statistically controlled product, with the specific provision that the units inspected be selected in the order of their production. In applying the general plan to the inspection of a *flow of consecutive sub-lots*, we no longer have individual units available in the order of their production. But we can use the same theoretical framework if we consider the random spacing of defects as their spacing in the chain of inspected units arranged in the *order of their inspection*. The probability distribution of the spacing of defects in inspected units will be the same regardless of the manner of selecting the units to be inspected, so long as we hold to the concept of statistical control in our solution.

The "i units in succession to be found clear of defects," discussed in Part II will now be defined as i *consecutively inspected units*. During sampling inspection, a group sample of units will be selected from each sub-lot, and the fraction f will relate to the ratio of the number of units in the sample to the total number of units in the sub-lot. The fraction f will be held constant for all sub-lots. Furthermore, when it is required under the general plan to find i inspected units in succession clear of defects, the 100% inspection must be allowed to extend into immediately succeeding sub-lots if i units in succession are not found clear in the current sub-lot.

17. Procedure B. The procedure is as follows:

(a) At the outset, start inspecting 100% of the units in a sub-lot and continue such inspection until i inspected units in succession are found clear of defects. Extend the 100% inspection, if necessary, into one or more succeeding sub-lots in the order of their production.

(b) When i inspected units in succession are found clear of defects, discontinue 100% inspection and inspect only a fraction f of the units from each of the sub-lots, selecting the sample units in such a way as to fairly represent the sub-lot.

(c) If a sample unit is found defective, start a 100% inspection of the remainder of the sub-lot, and continue the 100% inspection until again i

inspected units in succession are found clear of defects, as in paragraph (a), extending such inspection into succeeding sub-lots, if necessary.

(d) In the event the 100% inspection extends into one or more succeeding sub-lots, if the number of units inspected in the last of such succeeding sub-lots exceeds a fraction f of the number of units in the sub-lot, accept this last sub-lot without further inspection. If on the other hand, the number of units inspected in this last sub-lot is less than the fraction f, inspect additional units from this same sub-lot to make up a sample equal to a fraction f of the number of units in the sub-lot.

(e) Correct or replace with good units all defective units found.

As was the case in Part II, the inspection plan is defined by two constants, f and i, and the protection offered is expressed in terms of AOQL. This sub-lot inspection plan differs from those already published in that the screening action is not confined to a single sub-lot but may extend over a succession of sub-lots, the entire production being regarded as a train of sub-lots that are linked together for purposes of inspection in the order of their production.

IV Remarks

It will have been noted that the plan here outlined should be regarded as a "special purpose" plan applicable under the conditions which have been enumerated—where production is practically continuous, where inspection is to be made during production or immediately thereafter and is to serve not only as a screening *acceptance* agency if necessary, but as an aid to *process control* by disclosing promptly any sub-standard quality conditions in the product. It is believed that the general plan provides a structure, which with possible variations in procedure to serve particular circumstances, may be found useful in designing additional sampling inspection techniques.

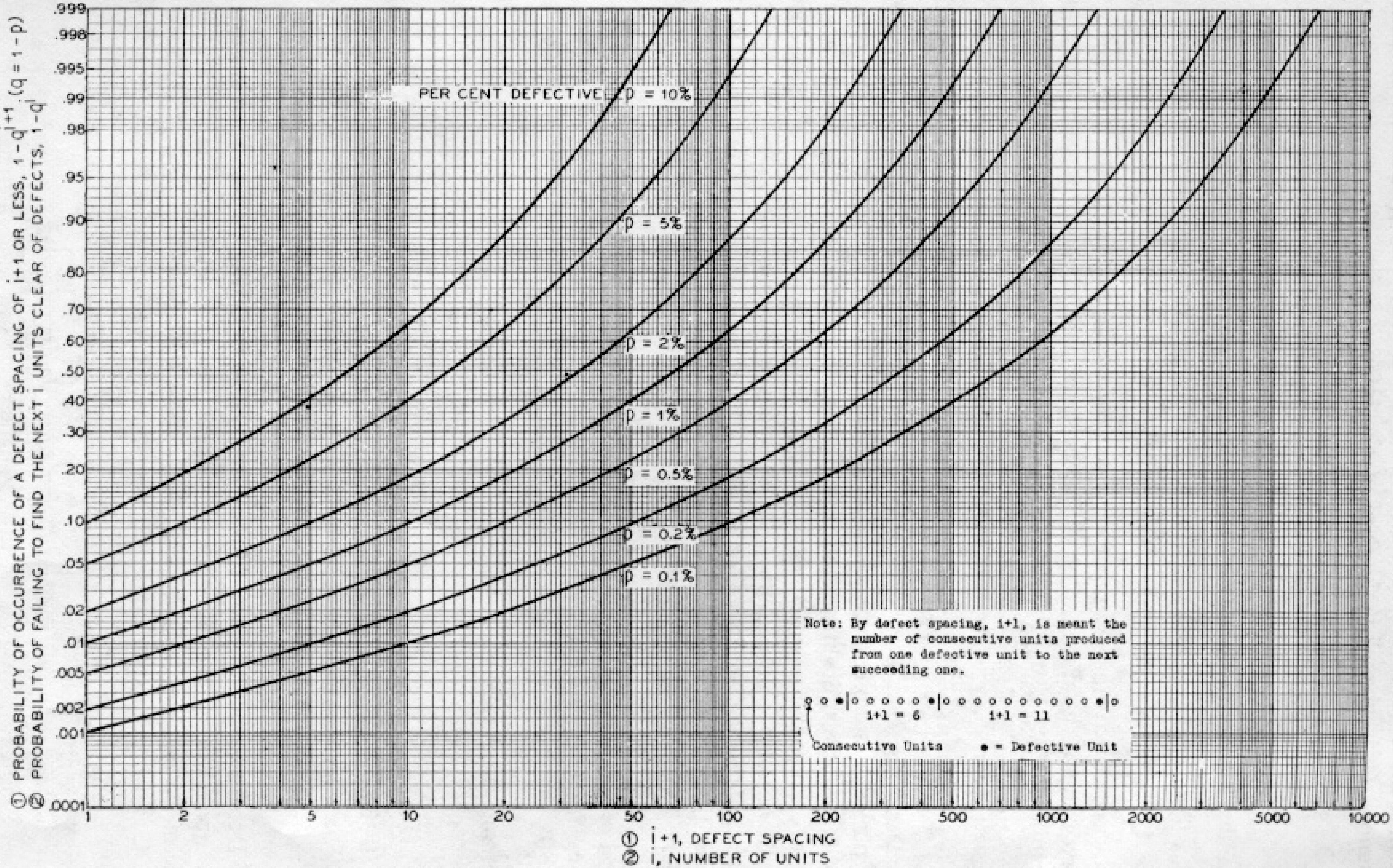

FIG. 2. Curves defining distribution of random order spacing of defects in uniform product

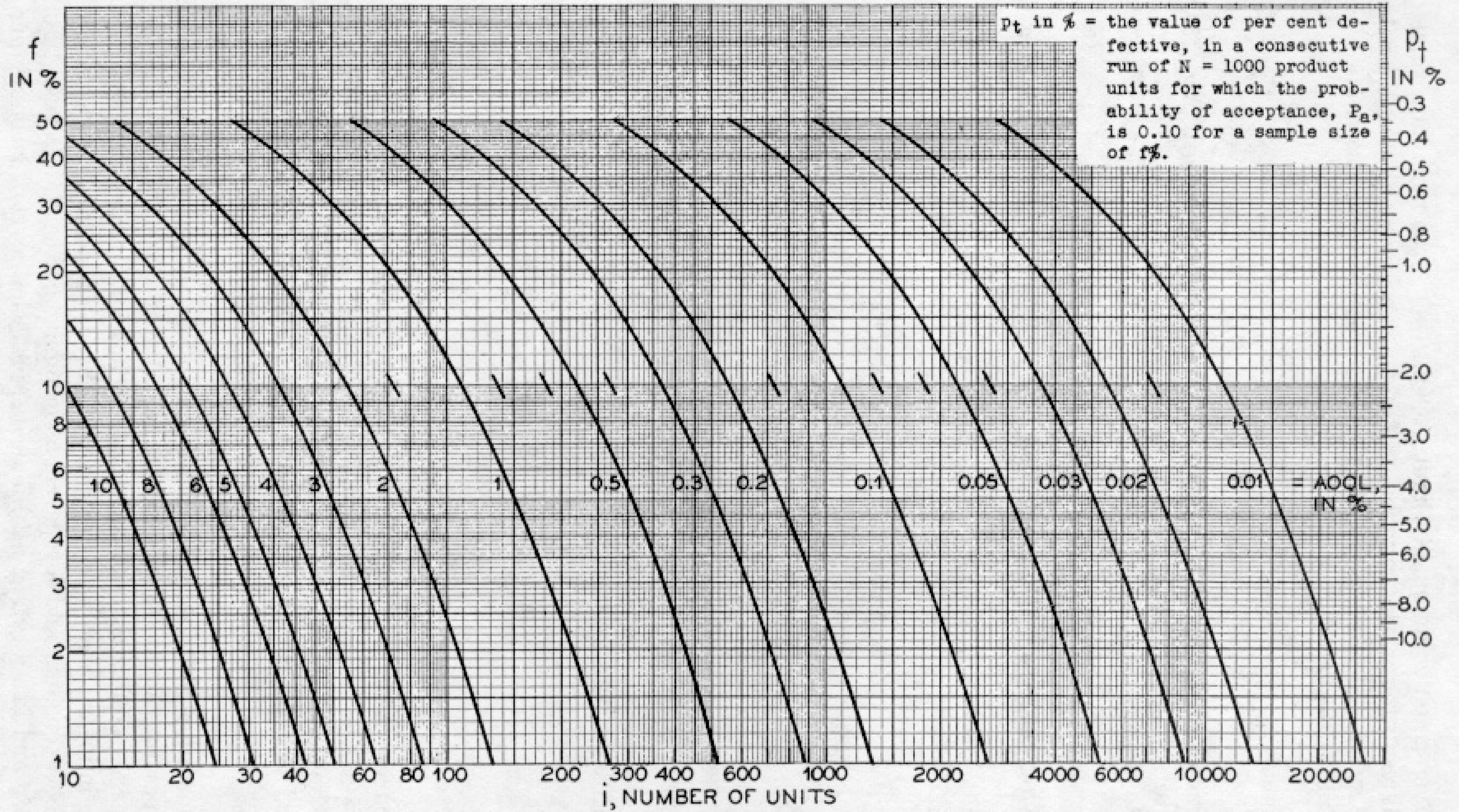

FIG. 3. Curves for determining values of *f* and *i* for a given value of AOQL