# Transactions Letters

# A Scheme for Cancelling Intercarrier Interference using Conjugate Transmission in Multicarrier Communication Systems

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Abstract-To mitigate intercarrier interference (ICI), a twopath algorithm is developed for multicarrier communication systems, including orthogonal frequency division multiplexing (OFDM) systems. The first path employs the regular OFDM algorithm. The second path uses the conjugate transmission of the first path. The combination of both paths forms a conjugate ICI cancellation scheme at the receiver. This conjugate cancellation (CC) scheme provides (1) a high signal to interference power ratio (SIR) in the presence of small frequency offsets (50 dB and 33 dB higher than that of the regular OFDM and linear selfcancellation algorithms [1], [2], respectively, at  $\Delta fT = 0.1\%$  of subcarrier frequency spacing); (2) better bit error rate (BER) performance in both additive white Gaussian noise (AWGN) and fading channels; (3) backward compatibility with the existing OFDM system; (4) no channel equalization is needed for reducing ICI, a simple low cost receiver without increasing system complexity. Although the two-path transmission reduces bandwidth efficiency, the disadvantage can be balanced by increasing signal alphabet sizes.

*Index Terms*—Algorithm, fading channels, intercarrier interference cancellation, OFDM.

#### I. INTRODUCTION

THE OFDM system has high spectral efficiency due to overlapping subcarrier spectra. However, one of the major disadvantages of such a multi-carrier modulated system is the sensitivity of its performance to synchronization error, such as frequency or phase offsets. The frequency offset can result from a Doppler shift due to a mobile environment, as well as from a carrier frequency synchronization error. Such frequency offsets cause a loss of the carriers' *orthogonality*. Hence intercarrier interference (ICI) occurs.

Currently, four different approaches for mitigating ICI have been proposed including: ICI self-cancellation [1], [2], frequency-domain equalization [3], time-domain windowing

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scheme [4], and two-path parallel cancellation scheme [5], [6]. Frequency offset estimation techniques using training sequence such as pilot symbols are proposed in [7]. This study focuses on the ICI cancellation scheme by taking the advantages of the diversity [5]-[6], [8]. One assumption is that the synchronization, including phase, frequency, and timing has been done by using repeated preamble sequence, but the ICI may still exist due to the frequency offset estimation error (constant) or unexpected Doppler velocity (time varying) [9], [10]. Therefore, the frequency offset considered here is smaller than 5% of subcarrier frequency spacing. A twopath algorithm is developed for combating ICI. The first path employs the regular OFDM algorithm. The second path requires a conjugate transmission at the transmitter, and forms a conjugate cancellation (CC) scheme for mitigating ICI of OFDM systems at the receiver. This CC OFDM system works significantly better than a regular OFDM system if the total frequency offsets are less than 5% of the subcarrier frequency spacing in AWGN and frequency selective fading channels.

This paper is organized as follows: The math model of the transmitter and the receiver of a regular OFDM system are described in Section II. Analysis, along with a discussion of the weighting function of the data symbol on the OFDM symbols at the receiver is provided in Section III. Section IV presents the CC scheme and the corresponding sequential and parallel architectures. Additionally, the implementation and its relationship to the space-time coding architecture [11] is presented. The simulation results are discussed in Section V and conclusions are given in Section VI.

#### II. OFDM SYSTEM MODEL

#### A. Regular OFDM Transmitter

A conventional OFDM modulation is employed at the transmitter. The baseband transmitted signal  $x_k$  at the output of the IFFT can be written as

$$x_k = \sum_{n=0}^{N-1} d_n e^{j\frac{2\pi}{N}nk} \quad k = 0, 1, 2, \dots, N-1$$
 (1)

where  $d_n$  is the data symbol, and  $e^{j\frac{2\pi}{N}nk}$ , k = 0, 1, 2, ..., N-1, represents the corresponding orthogonal frequencies of N subcarriers. A group of N different data symbols is mapped onto N subcarriers via the IFFT processor. Note that the IFFT

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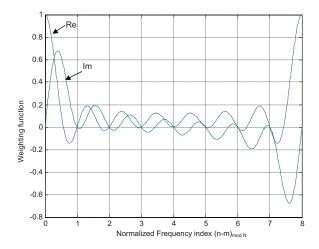


Fig. 1. Continuous weighing function of data symbols.

has  $T_{OFDM}$  seconds to complete its operation. The duration  $T_{OFDM}$  for an OFDM symbol is  $N \cdot T_s$ , where  $T_s$  is the time period of a data symbol. For simplicity, T is used to replace  $T_{OFDM}$  hereafter.

#### B. Regular Receiver Baseband Processing

At the receiver, the OFDM signal is mixed with a local oscillator signal. Assuming it is  $\Delta f$  above the carrier frequency of the received OFDM signal due to frequency estimation error or Doppler velocity, the baseband FFT demodulator output is given by

$$\widehat{d}_m = \frac{1}{N} \sum_{k=0}^{N-1} r_k e^{-j\frac{2\pi}{N}mk} \quad m = 0, 1, 2, \dots, N-1$$
(2)

where  $r_k = x_k e^{j\frac{2\pi}{N}k\Delta fT} + w_k$  represents the received signal at the input to the FFT processor,  $w_k$  is the AWGN, and  $\hat{d}_m$ is the output of the FFT processor. The term  $e^{j\frac{2\pi}{N}k\Delta fT}$ ,  $k = 0, 1, \ldots, N-1$ , represents the corresponding frequency offset of the received signal at the sampling instants, and  $\Delta fT$  is the frequency offset to subcarrier frequency spacing ratio. Note that a complete analytical model is presented in [10]. The model in (2) is a simplified version with assumptions that the receiver is able to compensate for the time varying phase drift that is induced in each block by the carrier frequency offset.

### III. ANALYSIS

Following the approach in [1], [2], we derive expressions for each demodulated subcarrier at the receiver in terms of each transmitted subcarrier and N complex weighting functions. Without loss of generality, the noise  $w_k$  in the received signal is ignored in the following discussion. Substituting (1) into (2) and after some manipulation, it can be shown that

$$\widehat{d}_m = d_m u_0 + \sum_{\substack{n=0\\n \neq m}}^{N-1} d_n u_{n-m} \quad m = 0, 1, 2, \dots, N-1 \quad (3)$$

where

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$$\mu_{n-m} = e^{j\frac{\pi}{N}(N-1)(n-m+\Delta fT)} \\
 \times \frac{1}{N} \left. \frac{\sin\left(\pi\left(n-m+\Delta fT\right)\right)}{\sin\left(\frac{\pi}{N}\left(n-m+\Delta fT\right)\right)} \right|_{(n-m) \mod N}$$
(4)

The complex weighting functions  $u_{0,}u_{1,}\ldots,u_{N-1}$  indicate the contribution of each of the N data symbols  $d_n$  to the FFT output  $\hat{d}_m$ . The first term of (3) is the desirable data  $d_m$  with the weighting function  $u_0$ . Those terms of  $n \neq m$  represent the crosstalk from the undesired data symbols. The weighting function of (4) is a periodic function with a period of N. If the normalized frequency offset  $\Delta fT$  equals zero, then  $\hat{d}_m$ is equal to  $d_n$  at m = n. To illustrate weighting functions graphically with continuous curves over a complete cycle at N = 8, and  $\Delta fT = 0$ , the range of normalized frequency index  $(n - m)_{mod N}$  is set from zero to eight in Fig. 1.

The discrete weighting functions  $u_{n-m}$ , n - m = 0, 1, 2, ..., 7 of 8 symbols are located exactly at n - m = 0, 1, ..., 7 integer-point of the index n - m axis. Given  $\Delta fT = 0$ , all weighting functions are zeroes except that the real part of  $u_0$  equals one. This is because all subcarriers hold the *orthogonality* and have no crosstalk among them at the receiver. However, the curves of the weighting function of Fig. 1 are shifted to the left when the frequency offset  $\Delta fT > 0$  or shifted to the right when the frequency offset  $\Delta fT < 0$ . This implies that there is ICI from undesired data samples to a particular data sample of interest. Such a shift causes a loss of the subcarriers' *orthogonality*, and hence all weights on data symbols are non-zero valued and ICI is self-generated.

#### **IV. CONJUGATE ALGORITHM**

To mitigate the impact of ICI, a conjugate algorithm was developed. The basic idea is to have another algorithm that provides weighting factors with opposite polarities at the zero crossings. This can be achieved by using a second path transmission, assuming that the frequency offset is a constant over the two-path time interval. This operation is illustrated in the next three subsections.

#### A. The Conjugate Algorithm

At the transmitter, the algorithm requires a conjugate operation on the IFFT output as defined in (5):

$$x'_{k} = \left(\sum_{n=0}^{N-1} d_{n} e^{j\frac{2\pi}{N}nk}\right)^{*} = \sum_{n=0}^{N-1} (d_{n})^{*} e^{-j\frac{2\pi}{N}nk}$$
  

$$k = 0, 1, 2, \dots, N-1$$
(5)

where  $d_n$  is the data symbol, and  $e^{j\frac{2\pi}{N}nk}$ , k = 0, 1, ..., N-1, represents the corresponding orthogonal frequencies of Nsubcarriers. Note that in order to demodulate the original signal  $x_k$  and the conjugate signal  $x'_k$  separately,  $x'_k$  needs to be transmitted independently. This can be achieved by using frequency division multiplexing (FDM), or time division multiplexing (TDM), or code division multiplexing (CDM), or other transmission means. At the receiver, the algorithm

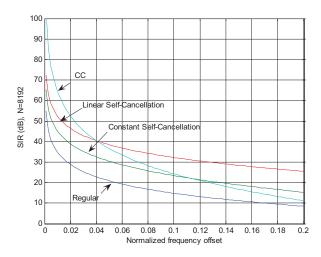


Fig. 2. SIR vs. frequency offsets for four different schemes.

requires a conjugate operation on the received signal first, and then performs the FFT operation as defined in (6):

$$\hat{d}'_m = \frac{1}{N} \sum_{k=0}^{N-1} \left( r'_k \right)^* e^{-j\frac{2\pi}{N}mk} \quad m = 0, 1, 2, \dots, N-1 \quad (6)$$

where  $r'_k = x'_k e^{j\frac{2\pi}{N}k\Delta T} + w'_k$  represents the received signal,  $w'_k$  is the independent AWGN, and  $\hat{d}'_m$  is the output of the FFT processor. The term  $e^{j\frac{2\pi}{N}k\Delta T}$ , k = 0, 1, 2, ..., N - 1, represents the corresponding frequency offset of the received signal at the sampling instants. Without loss of generality, the noise  $w'_k$  is ignored in the following discussion. Substituting (5) into (6) and after some manipulation, it can be shown that

$$\widehat{d}'_{m} = d_{m}\nu_{0} + \sum_{\substack{n=0\\n\neq m}}^{N-1} d_{n}\nu_{n-m} \quad m = 0, 1, 2, \dots, N-1 \quad (7)$$

where the weighting functions for data  $d_n$  at the FFT output is:

$$\nu_{n-m} = e^{j\frac{\pi}{N}(n-1)(n-m-\Delta fT)} \times \frac{1}{N} \frac{\sin\left(\pi\left(n-m-\Delta fT\right)\right)}{\sin\left(\frac{\pi}{N}\left(n-m-\Delta fT\right)\right)} \bigg|_{(n-m) \bmod N}$$
(8)

Equation (8) is similar to (4), but the sign of the frequency offset term,  $\Delta fT$ , is changed from positive to negative. When the frequency offset  $\Delta fT > 0$ , it will result in a shift to the right operation on the weighting function of (8) as opposed to a shift to the left of (4).

#### B. The Conjugate Cancellation Scheme

Assuming that both outputs of a regular OFDM system and a conjugate OFDM system can be combined coherently without interfering with each other at the receiver by using a division multiplexing technique, such as FDM, or TDM, or CDM, the final detected symbol is then chosen as the averaged detected symbols of the regular OFDM receiver and the conjugate algorithm as follows:

$$\widehat{d}_{m}^{"} = \widehat{d}_{m} + \widehat{d}_{m}^{'}$$

$$= \sum_{n=0}^{N-1} d_{n} (u_{n-m} + \nu_{n-m})$$

$$= d_{m} (u_{0} + \nu_{0}) + \sum_{\substack{n=0\\n \neq m}}^{N-1} d_{n} (u_{n-m} + \nu_{n-m})$$

$$m = 0, 1, 2, \dots, N-1$$
(9)

This is called the conjugate cancellation (CC) scheme. From (9), the signal to ICI power ratio (SIR) of the CC algorithm, as a function of frequency offsets is

$$SIR = 10 \log \frac{|u_0 + \nu_0|^2}{\sum_{n=1}^{N-1} |u_n + \nu_n|^2} dB$$
(10)

Note that SIR is independent to m. Hence, the index mis dropped from the complex weighting functions  $u_{n-m}$ and  $\nu_{n-m}$  of (10). The SIR of a regular OFDM system is independent of N. For a small frequency offset, the SIR of the CC algorithm is about the same for different N. It is calculated that the SIR of the CC algorithm is about 50 dB and 30 dB higher than that of the regular algorithm at 0.1% and 1% frequency offset, respectively. Fig. 2 depicts the SIR for four different systems: regular OFDM, self-cancellation schemes with constant and linear components of ICI [1], [2], and this CC scheme at N = 8192. Note that the SIR of the self-cancellation schemes [1], [2] is independent of N, when N > 8. However, this CC algorithm has the highest SIR than others when frequency offsets are small (33 dB and 13 dB higher than that of the linear self-cancellation algorithm, at  $\Delta fT = 0.1\%$  and  $\Delta fT = 1\%$  of subcarrier frequency spacing, respectively).

Alternatively, the received two-path data may be grouped together and form a 2N-element vector as  $\overline{r} = [r_k (r'_k)^*]$ . The receiver employs a 2N-point FFT engine to process  $\overline{r}$  as follows.

$$Y_{l} = \frac{1}{N} \sum_{k=0}^{2N-1} \overline{r} e^{-j\frac{2\pi}{2N}lk}$$
  
=  $\frac{1}{N} \left( \sum_{k=0}^{N-1} r_{k} e^{-j\frac{2\pi}{2N}lk} + \sum_{k=0}^{N-1} (r_{k}')^{*} e^{-j\frac{2\pi}{2N}l(N+k)} \right)$   
 $l = 0, 1, 2, \dots, 2N-1$  (11)

By taking the even FFT output bins, (11) is identical to (9) as follows:

$$Y_{l} = \frac{1}{N} \left( \sum_{k=0}^{N-1} r_{k} e^{-j\frac{2\pi}{2N}lk} + \sum_{k=0}^{N-1} (r'_{k})^{*} e^{-j\frac{2\pi}{2N}lk} \right)$$
  
=  $\hat{d}_{m} + \hat{d}'_{m}$   
=  $\hat{d}_{m}^{*}$   $l = 2m, \quad m = 0, 1, \dots, N-1$  (12)

The odd FFY output bins are ignored. Consequently, the SIR is the same as before.

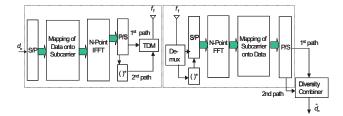


Fig. 3. Postdetection architecture of the CC scheme: transmitter (left) and receiver (right).

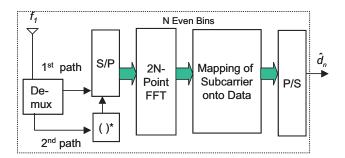


Fig. 4. Predetection receiver architecture of the CC scheme.

#### C. Architectures

Two-path data transmission has been discussed in the previous sections. Naturally, (9) and (12) can be implemented either sequentially or parallel to the required two-path transmission.

1) Sequential Postdetection Architecture: The transmitter and receiver architectures of the CC scheme using a single antenna with TDM are described in Fig. 3. By applying (12), a postdetection time diversity combiner is applied at the receiver. Note that (12) represents the coherent equal gain diversity combining with the assumption that the two-path signals have equal energy symbols. The coherency between these two paths can be achieved by employing preamble sequences or a phaselocked loop. If the two-path signals with unequal energy are identified due to fading channel, the receiver should take advantage of the repeated preamble sequences to estimate the channel parameters, and maximal ratio combining (MRC) should be used to improve its performance. This architecture enjoys simplicity and backward compatibility to the regular OFDM. In fact, the proposed 2<sup>nd</sup> path (conjugate operation) and the TDM circuit can be an optional design to enhance the system performance as needed. Interestedly, this TDM sequential circuit is exactly an OFDM  $[2 \times 1]$  Alamouti scheme [11] where the  $1^{st}$  antenna transmit  $x_k$  and the 2nd transmit 0 and successively the  $1^{st}$  transmit 0 and  $2^{nd}$  transmit  $x_k^*$ . In other words, this CC scheme is a simple and low cost solution without channel estimation. In addition, the Alamouti scheme requires the channel does not change in two successive OFDM symbols. This CC scheme does not have such requirements. In other words, the channel can be the same in two successive OFDM symbols or can be different.

2) Sequential Predetection Architecture: Similarly, a predetection diversity combiner is applied at the receiver as depicted in Fig. 4. The transmitter is the same as that of Fig. 3. At the receiver, a 2N-point FFT engine is employed to process the grouped two-path data. Only the even FFT output bins are used for data symbol detection. The equal gain diversity combining is automatically applied in the FFT processing with

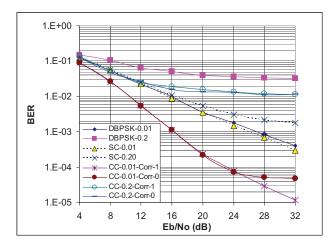


Fig. 5. BER of the regular DBPSK, CC- and SC-DQPSK OFDM systems in mobile fading channels.

the assumption that the two-path signals have equal energy symbols. Clearly, the computation time required for 2N-point FFT is less than that of N-point FFT with two repetitions.

3) Parallel Architecture: The transmitter and receiver architectures of the CC scheme using one antenna with two different frequencies can also be designed, assuming that these two carrier frequencies are far apart and do not interfere with each other. In this case, the architecture is the same as Fig. 4, except the TDM at the transmitter is replaced by a FDM; and the time Demux at the receiver is replaced by a frequency Demux to separate two carrier frequencies and a time Demux for sharing the N-point FFT receiver. Similarly, a CDM can be employed. This parallel architecture also provides signal (frequency or code) diversity and should have the similar performance as the sequential architecture. On the other hands, the CC scheme can be implemented with two-antenna for twopath parallel transmission. In this case, it is the same as the Alamouti's space-time scheme [11] without sacrificing 50% bandwidth efficiency. However, this is not the case we focus on.

#### V. SIMULATION RESULTS

The key of the CC scheme is to reduce the ICI due to the loss of the orthogonality. Both the frequency offset (constant) and the Doppler spread of multipath (time-varying channels) cause the ICI due to the loss of the orthogonality among the subcarriers. Therefore, the CC scheme works in both cases, regardless what the nature of the cause is. The signal processing for the transmitter and receiver, described in Fig. 3, was modeled with the COST 207 typical urban area channel parameters. The frequency domain differential coding is employed in order to avoid channel response estimation. The same frequency selective mobile channel parameters are applied to the two-path CC scheme and other schemes with a block size of N = 8192 in all cases. A quarter of N samples are employed as the cyclic prefix.

#### A. Comparison With Regular OFDM and SC Scheme [1]

Fig. 5 shows the BER comparison between the regular DBPSK, CC-DQPSK scheme and the DQPSK self-

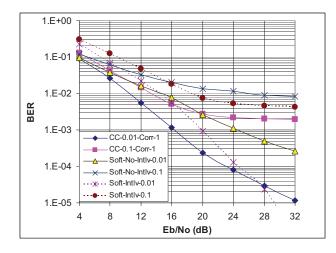


Fig. 6. BER of the CC and coded DQPSK OFDM systems in mobile fading channels.

cancellation (SC) scheme [1]. The maximum Doppler frequency spread to subcarrier frequency spacing ratio  $\Delta f_B T$ is chosen as 0.01 and 0.2. The bandwidth efficiency is the same in all three methods. The transmitted power of SC and each path of the CC scheme is half of that of the regular OFDM. Note that the correlation of two successive channels can be 1 (Corr-1) or 0 (Corr-0) in the CC scheme. The only BER difference between these two extreme conditions is when  $E_b/N_0$  is larger than 24 dB at small  $\Delta f_B T (\Delta f_B T = 0.01)$ . When the signal is strong, the CC scheme of Corr-1 still reduce the ICI caused by Doppler frequency spread, while the CC scheme of Corr-0 treats it as the noise. If  $\Delta f_B T$  is large  $(\Delta f_B T = 0.2)$ , the same BER performance is obtained for both Corr-1 and Corr-0. The CC scheme of both Corr-1 and Corr-0 is superior to the regular OFDM regardless what the value of  $\Delta f_B T$  is. When  $\Delta f_B T$  is small such as 0.01, the BER of the CC scheme of both Corr-1 and Corr-0 is superior to that of SC. The BER of the SC scheme is about the same as the regular DBPSK at small  $\Delta f_B T (\Delta f_B T = 0.01)$ . If  $\Delta f_B T$ is large  $(\Delta f_B T = 0.2)$ , the BER of the SC scheme better than that of CC scheme and regular DBPSK.

#### B. Comparison with Error Correction Coding

The BER performance of both CC-DQPSK Corr-1 scheme and convolution soft coded DQPSK systems with and without inter-leaver is depicted in Fig. 6. The code generator employs octal numbers [117], [155]. The code rate  $\frac{1}{2}$  with constraint length 7 is used in the coded OFDM system which provides the same bandwidth efficiency and same transmission power as that of the CC scheme. If  $\Delta f_B T$  is small ( $\Delta f_B T = 0.01$ ), the coded OFDM with interleaver is better than the CC scheme when  $E_b/N_0$  is greater than 28 dB. However, if  $\Delta f_B T$  is large ( $\Delta f_B T = 0.1$ ), the BER performance of CC scheme is always better than that of the coded OFDM with interleaver. The BER performance of the coded OFDM without interleaver is always worse than that of CC scheme regardless what the value of  $\Delta f_B T$  is. Additionally, the CC scheme can be combined with error correction coding. Such a system is robust to both AWGN and ICI. However, the bandwidth efficiency is further reduced.

## VI. CONCLUSIONS

A low cost receiver with simple architecture is presented with the CC scheme. The use of a frequency domain DQPSK, which avoids channel estimation and equalization for roughly a 3 dB penalty in BER performance is discussed. The key feature of the CC OFDM system is that it provides a much higher SIR over the existing OFDM system. Consequently, the sensitivity of CC OFDM systems to ICI is reduced significantly. Under the condition of the same bandwidth efficiency, the CC scheme performs much better than the regular OFDM systems in both AWGN and mobile channels with either constant frequency offset or multipath Doppler frequency spread. This CC scheme must transmit data twice and the bandwidth efficiency is reduced to half. However, it can be compensated by using larger signal alphabet sizes as depicted in Fig. 5.

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