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BY

H. BIGLARI AND P.H. DIAMOND

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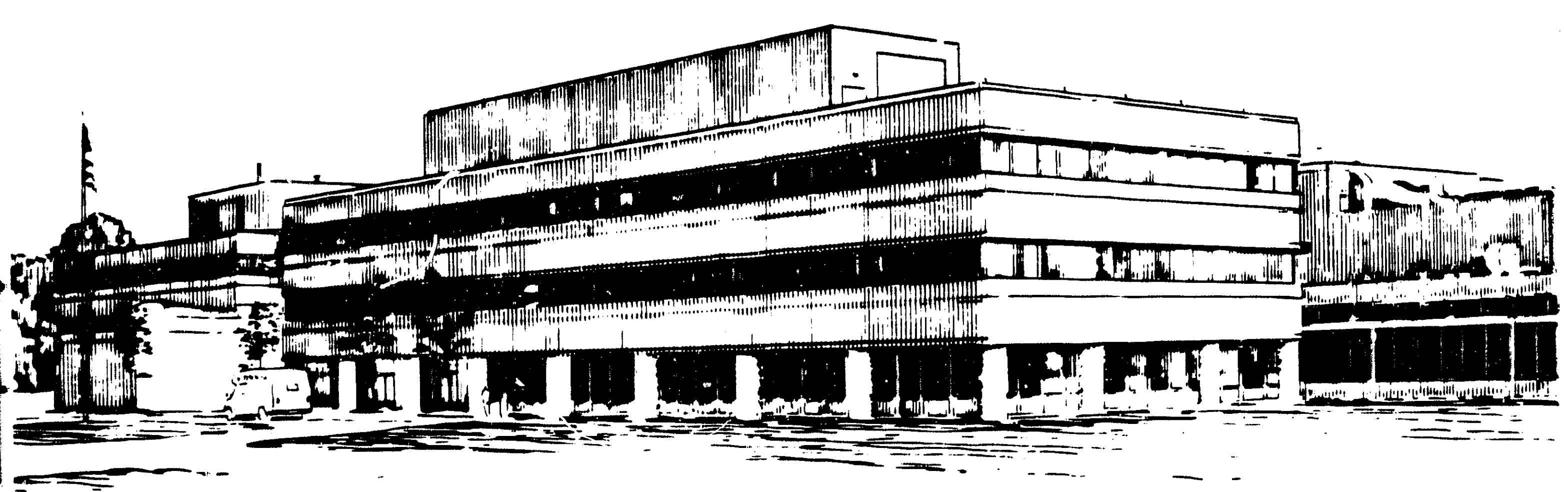

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# A SELF-CONSISTENT THEORY OF COLLECTIVE ALPHA PARTICLE LOSSES INDUCED BY ALFVÉNIC TURBULENCE

**H. Biglari**

*Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543*

**and**

**P. H. Diamond**

*Department of Physics B-019, University of California at San Diego, La Jolla, California 92093*

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The nonlinear dynamics of kinetic Alfvén waves, resonantly excited by energetic ions/alpha particles, is investigated. It is shown that $\alpha$-particles govern both linear instability and nonlinear saturation dynamics, while the background MHD turbulence results only in a nonlinear real frequency shift. The most efficient saturation mechanism is found to be self-induced profile modification. Expressions for the fluctuation amplitudes and the $\alpha$-particle radial flux are self-consistently derived. The work represents the first self-consistent, turbulent treatment of collective $\alpha$-particle losses by Alfvénic fluctuations.

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Arguably, the most critical issue for the next generation of fusion experiments, which will come to grips with the physics of ignited plasmas, will be the confinement of fusion product alpha particles. The importance of the 3.5 MeV $\alpha$-particles produced by the deuterium-tritium reaction stems from the fact that as they thermalize with the bulk plasma particles, they progressively reduce the need for external heating until a point is reached when even in the absence of such heating sources, the plasma continues to "burn" on its own provisions (so-called ignition phase). The promise of thermonuclear self-heating, however, may be nullified if the $\alpha$-particles are lost due to collective processes before they have had time to thermalize. An understanding of the nonlinear interaction between the $\alpha$-particles and the bulk plasma, and the ensuing fluctuation-driven losses is therefore of immediate interest. Previous attempts at nonlinear analyses have been based either on non-self-consistent treatments (i.e., fast ion loss due to orbit stochastization in a *prescribed* turbulent bath),[1] or single-wave trapping.[2] In this work, we present the first *self-consistent, turbulent* treatment of collective energetic particle losses induced by high-frequency, Alfvénic fluctuations.

Our approach is to choose a model that is *a)* tractable enough that fundamental issues can be addressed with a minimum of algebraic complications, and *b)* general enough that qualitative conclusions (e.g., relative importance of the various nonlinearities, nature of saturation dynamics, etc.) with broad applicability can be drawn. It is with this philosophy in mind that we choose as our working paradigm, the nonlinear dynamics of kinetic Alfvén waves (KAW) resonantly excited by energetic ions/$\alpha$-particles. It is well known that KAW's, which are shear Alfvén waves discretized by finite Larmor radius (FLR) effects,[3] can be linearly destabilized via inverse Landau resonances by energetic ions[4–6,1] if the singularity (continuum) associated with $k_\parallel = 0$ [$k_\parallel(r) = (n - m/q)/R$, $q(r)$ is the plasma safety factor, $R$ is the plasma major radius, and $(m, n)$ are the poloidal and toroidal mode numbers, respectively] does not fall inside the plasma.[7] The eigenmodes are radially localized (i.e., $k_\perp \simeq k_r \gg k_\theta$), and form a standing wave between the location of the Alfvén resonance [$r = r_A$, $\omega^2 = \omega_A^2(r_A)$] and the plasma center ($r = 0$). Using the ordering $1 > \omega_{d\alpha}/\omega > k_\perp^2\rho_s^2$ [$\omega_{d\alpha} \simeq i v_{d\alpha} \sin\theta \nabla_r$ is the $\alpha$-particle precessional magnetic drift frequency, $v_{d\alpha} = ({v_\parallel}^2 + {v_\perp}^2/2)/R\Omega_\alpha$, $\rho_s = \tau^{1/2}\rho_i$ is the ion gyroradius at the electron temperature, $\tau = T_e/T_i$ is the electron-ion temperature ratio, and $\Omega_\alpha = e_\alpha B/m_\alpha c$ is the $\alpha$ gyrofrequency], the mode frequency and growth rate can be straightforwardly derived ($T_e = T_i \equiv T_c$ for simplicity): $\omega_r = \pm\omega_A [1 + (7/8)\, k_\perp^2 \rho_s^2]$, and $\gamma_l \simeq \Lambda_\alpha v_A/(|k_\parallel| L_c^2) - \pi^{1/2} k_\perp^2 \rho_s^2 |k_\parallel|\, v_A^2/v_{te} \ll \omega_r$, where $v_A = B/(4\pi m_i n_i)^{1/2}$ is the Alfvén speed, $v_{te}$ is the electron thermal speed, $L_c = qR$ is the connection length,

$$\Lambda_\alpha = \frac{q^2}{4}\,\beta_\alpha \int_0^1 d^3\bar{v}\left(\bar{v}_\parallel^2 + \frac{\bar{v}_\perp^2}{2}\right)^2 \left(\omega_{*pm} - \frac{3\omega_r}{2\bar{E}}\right) \bar{E}^{-3/2} \left[\delta(\omega_r - k_\parallel^{m+1} v_\parallel) + \delta(\omega_r - k_\parallel^{m-1} v_\parallel)\right], \quad (1)$$

$\bar{v} = v/v_m$ is the velocity normalized to the birth speed, $\omega_{*pm} = k_\theta v_m \rho_m / L_{ph}$ is the $\alpha$-particle diamagnetic frequency at birth energy, $L_{p\alpha}^{-1} = -d\ln p_\alpha/dr$ is the $\alpha$-particle pressure scale length, and a slowing-down $\alpha$ distribution function has been adopted $F_{0\alpha} = [p_\alpha(r)/2^{5/2}\pi m_\alpha E_m]\, E^{-3/2}$, $E \leq E_m$ ($E_m = v_m^2/2$ is the birth energy). The first term in the expression for the growth rate is the hot-$\alpha$ destabilization, while the second is stabilization due to electron Landau damping (since these modes are relatively localized, they do not experience continuum damping). A necessary condition for instability is $\omega_{*pm} > \omega_A$, with wavelengths in the range $q^2\beta_\alpha(qR/L_{p\alpha})(v_\alpha/v_A)^2 > k_\theta\rho_i > (T_c/T_\alpha)^{1/2}(L_{ph}/R)(v_A/v_\alpha)$ (where $T_\alpha = m_\alpha E_m$). The radial mode width is set by the balance between FLR and the Alfvén scale length ($L_A^{-1} = d\ln\omega_A/dr$): $\Delta x \simeq (\rho_s L_A)^{1/2}$.

We now turn to nonlinear theory. To begin with, since the modes are dispersive and $\gamma_l \sim \Delta\omega_k \ll \omega_r$ ($\Delta\omega_k$ is the turbulent decorrelation frequency which will be determined as part of the nonlinear calculation), a weak turbulence analysis is called for. We first consider the types of nonlinear interactions that are possible between the energetic $\alpha$'s and the background plasma. As we shall see, there are in general two types of electromagnetic nonlinearities: *i)* the core plasma magnetohydrodynamic (MHD) nonlinearity, and *ii)* the $\alpha$-particle nonlinearity. Each of these nonlinearities, in turn, can couple to the bulk plasma and to the energetic $\alpha$'s. Now although *individually*, the core plasma nonlinearities dominate the hot particle nonlinearities (a simple estimate shows that the ratio of the nonlinear background $\mathbf{J}\times\mathbf{B}$ force to the $\alpha$-particle nonlinearity is of order $\omega_A n_{0c} v_{te}/\omega_{d\alpha} n_{0\alpha} v_m \gg 1$), there can be subtle cancellations which render saturation by MHD turbulence ineffective. Indeed, we find that in the radially local limit, energetic particles govern *both* linear instability *and* nonlinear saturation dynamics, whereas MHD turbulence contributes only a nonlinear real frequency shift. The possibility of saturation by MHD turbulence in a radially nonlocal theory may yet remain viable.

We now proceed to prove these qualitative statements by explicit calculation. Consider first the nonlinear dynamics of the bulk plasma. There are two classes of possible interactions associated with the ambient turbulence, namely nonlinear coupling of the KAW's to other waves in the turbulent spectrum or coupling to ion acoustic turbulence. We explore the former class of interactions first and return to the second later. We adopt a two-field, FLR-modified MHD turbulence model which reproduces the linear theory results. The model is characterized by an Ohm's law and a vorticity equation:

$$\frac{\partial}{\partial t}\left[1+\left(1-i\pi^{1/2}\frac{v_A}{v_{te}}\right)\rho_s^2\nabla_\perp^2\right]\tilde{A}_\parallel + \frac{c}{B}\,\hat{\mathbf{e}}_\parallel\times\nabla\tilde{\phi}\cdot\nabla\tilde{A}_\parallel = -c\nabla_\parallel\tilde{\phi} - \eta c\tilde{j}_\parallel, \qquad (2)$$

$$\frac{c^2}{4\pi v_A^2}\left[\nabla_\perp^2\tilde{\phi} + \frac{c}{B}\,\hat{\mathbf{e}}_\parallel\times\nabla\tilde{\phi}\cdot\nabla\nabla_\perp^2\tilde{\phi} - \nu\nabla_\perp^2\tilde{\phi}\right] = \left(\nabla_\parallel + i\,\frac{\Lambda_\alpha}{L_c^2\nabla_\parallel}\right)\tilde{j}_\parallel + \frac{c}{4\pi B}\,\hat{\mathbf{e}}_\parallel\times\nabla\tilde{A}_\parallel\cdot\nabla\nabla_\perp^2\tilde{A}_\parallel, \qquad (3)$$

where $\tilde{\phi}$, $\tilde{A}_\parallel$, and $\tilde{j}_\parallel = -(c/4\pi)\nabla_\perp^2\tilde{A}_\parallel$ are the perturbed electrostatic potential, parallel vector potential, and parallel current, respectively. The first term in Eq. (2) is the perturbed induction parallel electric field, the second is the modification due to FLR effects, the third is electron Landau damping, the fourth represents the turbulent advection of magnetic flux by the fluid, the fifth is the perturbed electrostatic parallel electric field, while the last represents resistive dissipation. The first two terms in Eq. (3) are associated with the divergence of the ion polarization current and represent the effect of finite ion inertia and FLR, the third term represents the self-advection of fluid vorticity, while the fourth term is associated with viscous dissipation. On the right hand side, the first term is that associated with field-line bending, the second with the hot particle drive, while the last is the nonlinear $\mathbf{J} \times \mathbf{B}$ force. Now, since $\gamma_l \ll \omega_r$ and $k_\perp^2 \rho_s^2 < 1$, we shall treat the bulk nonlinearities *á la* MHD, ignoring the hot particle drive, electron Landau damping, and retaining FLR modifications only when corrections to $\tilde{E}_\parallel \propto \tilde{\phi} - (\omega/k_\parallel c)\tilde{A}_\parallel$ is required. Following standard procedures,[8] we adopt a quasilocal formulation in Fourier space, and follow the evolution of a test KAW (wavenumber $\mathbf{k}$) in the turbulent bath of other KAW's (wavenumber $\mathbf{k}'$). In the direct interaction approximation,[8] the transfer of energy between modes $\mathbf{k}$ and $\mathbf{k}'$ is mediated by a third, beat mode $\mathbf{k}'' = \mathbf{k} + \mathbf{k}'$. In this context, the content of the nonlinear terms in Eqs. (2) and (3) can be physically described as the direct interaction of two slightly asymmetric, counter-propagating KAW's which, in turn, drive a low-frequency beat wave which resonates with the background turbulence. Iteratively substituting the solution of the beat fluctuation equations into the nonlinear terms in Eqs. (4) and (5) yields, after tedious manipulation, the following renormalized set of equations:

$$\frac{\partial}{\partial t}\left[1 - \left(1 - i\pi^{1/2}\frac{v_A}{v_{te}}\right) k_\perp^2 \rho_s^2\right] \tilde{A}_{\parallel k} + (\nu - d_k)\, k_\perp^2 \tilde{A}_{\parallel k} + i\omega_A \tilde{\phi}_k = 0, \tag{4}$$

$$\frac{\partial}{\partial t}\left(1 - \frac{3}{4} k_\perp^2 \rho_s^2\right) \tilde{\phi}_k + (\nu + d_k)\, k_\perp^2 \tilde{\phi}_k - i\omega_A \left(1 + i\frac{\Lambda_\alpha}{k_\parallel^2 L_c^2}\right) \tilde{A}_{\parallel k} = 0, \tag{5}$$

where we have symmetrized the equations with the substitution $\tilde{\phi} \equiv (v_A/c)\,\tilde{\phi}$, assumed $\eta c^2/4\pi = \nu$ for simplicity, and used $\tilde{A}_{\parallel k} = (1 + k_\perp^2 \rho_s^2)(\omega_A/\omega_k)\,\tilde{\phi}_k \simeq (1 + k_\perp^2 \rho_s^2/8)\,\tilde{\phi}_k$. In Eqs. (4) and (5), $d_k = (\pi/16)\sum_{\mathbf{k}'} g_{k,k',k''}\,(k_\perp'^2 - k_\perp^2)\,\rho_s^2 \langle|\tilde{v}_{rk'}|^2\rangle$ is the renormalized nonlinearity, angular brackets denote a spectral average, $\tilde{v}_{rk'} = ck'_\theta \tilde{\phi}_{k'}/B$ is the fluid velocity, and $g_{k,k',k''} \simeq (\pi/2)\,[\delta(\omega''_{k0} + \omega''_A) + \delta(\omega''_{k0} - \omega''_A)]$ is an approximation to the response function of DIA theory.[8] It is clear from the expression for $d_k$ that the spectral transfer of fluctuation energy is nonlocal, and the nonlinearities vanish in the limit of zero Larmor radius, as expected. By checking the limits of $k_\perp \gg k'_\perp$ and $k_\perp \ll k'_\perp$, it may furthermore be verified that mean-squared magnetic flux is scattered to large scales, while fluid energy is transferred to small scales. It is straightforward to show from Eqs. (4) and (5) that the renormalized nonlinearity affects only the real part of the dispersion relation, resulting in a nonlinear frequency shift: $\omega_r^2 + d_k^2 k_\perp^4 = \omega_A^2\,(1 + 7k_\perp^2 \rho_s^2/4)$, without affecting instability dynamics. Physically, the reason is that the pumping of magnetic flux [*negative* turbulent viscosity in Eq. (4)] exactly *cancels* the damping of fluid kinetic energy [*positive* turbulent viscosity in Eq. (5)], thus thwarting the possibility of saturation by MHD turbulence. Note that such subtleties is completely beyond the scope of mixing length theory, which simply replaces the collisional viscosity with a turbulent one. Thus, $\alpha$-particle transport predictions based on mixing-length assumptions[9] should be regarded with serious reservations.

We now recalculate the fluctuation amplitudes assuming that nonlinear $\alpha$-particle interactions rather than those associated with the bulk plasma govern saturation dynamics.

Again, we examine two possibilites: *i)* nonlinear interactions characterized by $\alpha$-Compton scattering, and *ii)* nonlinear saturation through self-induced profile modification.[10] It is found that although both of these processes, in addition to bulk ion Compton scattering lead to saturation, the second is the most efficient. As before, it will be more transparent to perform the calculation in the radially local than nonlocal limit. The $\alpha$-particle dynamics is described by the gyrokinetic equation:

$$(\omega - k_{\|}v_{\|})\,\tilde{h} + i(c/B)\,\hat{\mathbf{e}}_{\|}\cdot\nabla\tilde{L}\times\nabla\tilde{h} = Q_\alpha\omega_{d\alpha}\tilde{\phi}, \tag{6}$$

where $\tilde{h} = \tilde{f}_\alpha - (e_\alpha/m_\alpha)\,\partial_E F_{0\alpha}$ is the nonadiabatic piece of the $\alpha$ distribution function, $Q_\alpha = -(e_\alpha/m_\alpha)\,(\hat{\omega}_{*\alpha}/\omega + \partial_E)\,F_{0\alpha}$ is the free energy source, $\hat{\omega}_{*\alpha} = (\mathbf{k}\times\hat{\mathbf{e}}_{\|}/\Omega_\alpha)\cdot\nabla\ln F_{0\alpha}$, and $\tilde{L} = \tilde{\phi} - (v_{\|}/c)\,\tilde{A}_{\|} \simeq (1 - k_{\|}v_{\|}/\omega)\,\tilde{\phi}$ is the perturbed electromagnetic field. Following the standard weak turbulence expansion,[11] to first (linear) order we get: $\tilde{h}_k^{(1)} \simeq -i\pi\,\delta(\omega - k_{\|}v_{\|})\,Q_\alpha\omega_{d\alpha}\tilde{\phi}_k$. To next order, the beat fluctuation is obtained, and iteratively substituting it into the nonlinear term, we finally obtain the third order (nonlinear) distribution function:

$$\tilde{h}_k^{(3)} \simeq i\frac{e_\alpha}{m_\alpha}\frac{k_r^2 v_{d\alpha}}{\omega}\sum_{\mathbf{k}'}|\hat{\mathbf{e}}_{\|}\cdot\mathbf{k}\times\mathbf{k}'|^2\,\delta(\omega'' - k_{\|}''v_{\|})$$
$$\times\Big[\Big(\frac{1}{k_\theta'\omega'} - \frac{1}{k_\theta\omega}\Big)\frac{k_r^2}{\hat{s}}\,\partial_E + \frac{k_\theta^2 - k_\theta'^2}{\omega^2}\,\hat{v}_{*\alpha}\Big]\,F_{0\alpha}\,\langle|v_{rk'}|^2\rangle\,\tilde{\phi}_k, \tag{7}$$

In deriving Eq. (7), we have used the fact that since the beat mode is at low frequency, $\omega'' \ll \omega \sim \omega'$, while the spectrally averaged beat parallel wavelength is comparable to the test and background parallel wavelengths, i.e., $k_{\|} \sim k_{\|}' \sim k_{\|}''$. Details of the calculation will be presented elsewhere. We may now estimate the saturation amplitudes by balancing $\tilde{h}_k^{(3)}$ against $\tilde{h}_k^{(1)}$. The first set of terms in the square brackets in Eq. (7) dominates the second by a factor $(k_r^2/k_\theta^2)\,(qR/L_{p\alpha}k_\theta\rho_\alpha)\,[T_c/(T_\alpha\beta_c)]^{1/2} \gg 1$, and the balance yields $\langle|v_{rk'}|^2\rangle \sim \omega_{*\alpha}\omega_A\,(\Delta x)^2$, or equivalently, $\tilde{B}_r/B \lesssim [q^2\beta_\alpha\,(L_A\rho_\alpha/L_{p\alpha}^2)\,(v_m/v_A)^3]^{1/2}$ (ignoring electron Landau damping). Physically, saturation is achieved when nonlinear stabilization through velocity space steepening balances linear destabilization associated with the configuration space gradient. The experimental manifestation of nonlinear velocity space steepening is a slowing down of the energetic $\alpha$-particles contributing to instability, thus resulting in a drop in the neutron signal due to the corresponding drop of the fusion cross section.

Bulk ion Compton scattering[12] of $\alpha$-induced KAW's can be calculated in an entirely analogous manner. We present here the final result and relegate details of the calculation elsewhere. The saturation criterion is given by

$$\gamma_l \simeq \frac{\pi}{8}\,(\frac{cT_c}{eB})^2\sum_{\mathbf{k}'}|\hat{\mathbf{e}}_{\|}\cdot\mathbf{k}\times\mathbf{k}'|^2\,\delta(\omega' - \omega)\,\Big(\frac{\omega_{*i}'}{\omega'} - \frac{\omega_{*i}}{\omega}\Big)\,<J_{0k}^2J_{0k'}^2>_\perp\,\Big\langle\Big|\frac{e\phi_{k'}}{T_c}\Big|^2\Big\rangle. \tag{8}$$

where $J_{0k}$ is a Bessel function of argument $k_\perp v_\perp/\Omega_i$, and $<\cdots>_\perp = \int dv_\perp\,(\cdots)\,v_\perp\,\exp(-v_\perp^2/v_{ti}^2)$, and $\omega_{*i} > k_{\|}v_{ti}$ has been assumed. The resulting rms fluctuation levels are $(\tilde{B}_r/B)_{rms} \gtrsim [\beta_\alpha q\,(T_\alpha/T_c)\,(L_{pi}/L_{p\alpha})\,(\rho_s L_A/R^2)]^{1/2}$.

Finally, we calculate the fluctuation amplitudes assuming saturation is achieved through self-induced profile modification. In other words, we ask to what level fluctuations would grow from thermal noise in order to balance the change in resonant particle kinetic energy from the time of instability onset to the time when the instability free energy reservoir is turned off due to profile adjustment. As is obvious from energy conservation, it can be shown that the change in the resonant particle kinetic energy is equal to the wave energy, i.e.,

$$\Delta K \equiv \Delta \int_{v_{min}}^{v_{max}} d^3v \, \frac{m_\alpha v^2}{2} \, F_{0\alpha} = -\Delta \int_{k_{min}}^{k_{max}} d^3k \, \Big[\frac{\partial}{\partial \omega_r}(\omega_r D_r) \, |\tilde{E}_k|^2 + |\tilde{B}_k|^2\Big]/8\pi,$$

where $D_r$ is the real part of the dielectric function. Expanding $F_{0\alpha}$ about the resonance, i.e., $F_{0\alpha} \simeq F_{0\alpha}(0) + \mathbf{v}\cdot\nabla_{\mathbf{v}} F_{0\alpha}$, and using an isotropic slowing-down distribution, the initial slope is given by $\partial_v F_{0\alpha} = -(3/v)\, F_{0\alpha}$, while at saturation, the slope has adjusted to shut off the free energy source: $\partial_v F_{0\alpha} = 2\,(\omega_{*p\alpha}/\omega_A)\,(v/v_m^2)\, F_{0\alpha}$. It is then straightforward to calculate the change in the resonant particle kinetic energy:

$$\Delta K \simeq m_\alpha n_\alpha \left[\frac{7}{8} v_A^2 \, \Delta(k_\perp^2 \rho_s^2)\right] \left(\frac{\omega_{*p\alpha}}{\omega_A} \frac{v_A^2}{v_m^2} - \frac{3}{2}\right)^{1/2}. \tag{9}$$

Equation (9) makes eminent physical sense once it is realized that the term in the square brackets is simply the width of the resonance region in velocity space squared, i.e., $[\Delta(\omega/k_\parallel)]^2$, while the last parenthesis represents the "average" free energy which has been tapped by the instability. The fluctuation amplitudes are then given by

$$\left(\frac{\tilde{B}_r}{B}\right)_{rms} \lesssim \frac{c}{v_A} \frac{v_m^2}{v_A^2} \, q^2 \beta_\alpha \, \frac{qR}{L_{p\alpha}} \left(\frac{\omega_{*p\alpha}}{\omega_A} \frac{v_A^2}{v_m^2} - \frac{3}{2}\right)^{1/2}. \tag{10}$$

Comparing Eq. (10) with the corresponding expressions previously obtained for $\alpha$ and bulk ion Compton scattering, we conclude that self-induced profile modification, where the instability saturates by shutting off the free energy source, is the dominant saturation mechanism. Stated differently, it is more efficient for the instability to saturate by turning off the source of free energy than by coupling to dissipation through nonlinear transfer of fluctuation energy. From the level curves of this quasilinear plateau formation, i.e., $E/\omega_A + x/v_{*\alpha} = const$, it is clear that as the particles slow down in energy through velocity-space steepening, they diffuse *outward* in the radial direction, a topic we turn to next.

Having calculated the saturation amplitudes, we are now in a position to self-consistently calculate the quasilinear radial particle fluxes. The time evolution of the equilibrium $\alpha$-particle distribution function is given by $\partial F_{0\alpha}/\partial t = -\partial \Gamma_{xv}^\alpha/\partial r$, where $\Gamma_{xv}^\alpha = \langle(\tilde{v}_{Er} + v_\parallel \tilde{B}_r/B)\, \tilde{f}_\alpha\rangle$ is the $\alpha$-particle phase space flux. Substituting in for $\tilde{f}_\alpha$, we obtain

$$\Gamma_{xv}^\alpha = \pi \Omega_\alpha v_A^2 \sum_k \delta(\omega - k_\parallel v_\parallel - \omega_{d\alpha})\, k_\theta^{-1} \, \frac{{\omega_{d\alpha}}^2}{\omega^2} \left\langle \left|\frac{\tilde{B}_{rk}}{B}\right|^2 \right\rangle (\hat{\omega}_{*\alpha} + \omega \partial_E)\, F_{0\alpha}, \tag{11}$$

where $\tilde{\psi} = (\omega/k_{\parallel}c)\,\tilde{A}_{\parallel}$. The physics of collective $\alpha$-particle transport can be read off directly from Eq. (11). First, it is the breaking of the longitudinal (i.e., action) invariant [as characterized by the resonance function in Eq. (11)] that allows *circulating* $\alpha$'s to diffuse radially. Indeed, the good confinement of circulating energetic particles in present-day machines must be understood in light of this observation:[13] in the absence of high-frequency (e.g., Alfvénic) fluctuations, the action invariant is conserved, and hence, there can be no net (i.e., secular) radial loss of hot ions. Second, since $\tilde{E}_{\parallel} \simeq 0$ and we are considering low-$\beta$ plasmas, there is negligible ***parallel*** exchange of energy between the MHD wave and $\alpha$-particles. Rather, the wave-particle interaction is characterized by perpendicular energy exchange, i.e., $\tilde{\mathbf{v}}_{d\alpha} \cdot \tilde{\mathbf{E}}_{\perp} \propto \omega_{d\alpha} \tilde{B}_r$. This is the physics content of the reduction factor $(\omega_{d\alpha}/\omega)^2 \ll 1$ appearing in Eq. (11). Thus, orbit stochastization by magnetic perturbations, by itself, is a deficient prescription for $\alpha$-particle transport, and indeed, leads to an overestimate of transport levels. Assuming the correlation between fluctuations at the same poloidal harmonic dominates over those at different harmonics, and integrating over velocity space assuming a slowing-down distribution function, we obtain the $\alpha$-particle diffusion equation: $\partial_t n_{0\alpha} = \partial_r (D_\alpha\, \partial_r n_{0\alpha})$, where

$$D_\alpha \simeq \frac{\pi v_A}{36\sqrt{2}\epsilon^{3/2}} \Big(\frac{\omega_A}{\Omega_\alpha}\Big)^2 \sum_k k_{\parallel}^{-3} \left(\Big\langle \Big|\frac{\partial}{\partial r}\, \frac{\tilde{B}_{r,m-1}}{B}\Big|^2 \Big\rangle + \Big\langle \Big|\frac{\partial}{\partial r}\, \frac{\tilde{B}_{r,m+1}}{B}\Big|^2 \Big\rangle\right) \Big(1 - 2^{5/2} \cdot 3\epsilon\, \frac{\omega_A v_m^3}{\omega_{*pm} v_A^3}\Big) \tag{12}$$

is the $\alpha$-particle diffusion coefficient. Substituting from Eq. (10), and ignoring velocity space stabilization and electron Landau damping, we obtain

$$D_\alpha \sim \frac{\pi}{36\sqrt{2}}\, qRv_A \Big(\frac{q^3 R}{L_{p\alpha}}\, \frac{v_m c}{v_A^2}\Big)^2 \frac{\rho_\alpha^2}{\rho_s L_A}\, \frac{\beta_\alpha}{\epsilon^{3/2}} \Big(\frac{\omega_{*p\alpha}}{\omega_A}\, \frac{v_A^2}{v_m^2} - \frac{3}{2}\Big). \tag{13}$$

In summary, we have systematically investigated four possible routes by which $\alpha$-particle destabilized KAW's could achieve saturation: nonlinear MHD mode-coupling of KAW's, coupling of KAW's to bulk ion acoustic turbulence, nonlinear $\alpha$-Compton scattering, and self-induced profile modification through collective processes. The first of these was shown to give rise only to a nonlinear real frequency shift without affecting instability dynamics, while the last mechanism, by rapidly depleting the free energy reservoir, was shown to be the most efficient means of achieving saturation. The energetic particles diffuse radially out while slowing down in energy, resulting in a reduction of the cross-section for fusion reactions, and a consequent drop in neutron production. We conclude by noting an important implication of the first mechanism we investigated for the so-called toroidal Alfvén gap (TAE) mode.[14] Since the TAE mode and real dispersion c[illegible]cteristics are governed entirely by bulk plasma dynamics, on the basis if the present analysis, we may estimate the level of fluctuations that would be required to nonlinearly push the discrete frequency into the continuum, thereby damping it. The width of the gap is of order $\epsilon\omega_A$, and we estimate $\tilde{B}_r/B \sim \epsilon^{1/2}(\rho_\alpha/R)(v_m/v_A) \sim 5 \times 10^{-3}$. These are rather large fluctuation levels, and so we conclude that the gap is robust to nonlinear modification.

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